MOORAD CHOUDHRY

AN INTRODUCTION TO VALUE-AT-RISK

FIFTH EDITION

Foreword by Professor Carol Alexander

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AN INTRODUCTION TO VALUE-AT-RISK

Fifth Edition

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Ruth Martin
Managing Director

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Moorad Choudhry

with a contribution from Max Wong

WILEY

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cultured footballers everywhere ...

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FOREWORD

Far too many books have been written on Value-at-Risk (VaR). An Amazon search of texts with 'Value at Risk' in the title produces about 75 distinct results. The sheer quantity of literature on VaR is not surprising, since (for better or worse) this is the metric used by the world's major banks, and other financial firms, to determine both regulatory capital to cover market risks and economic capital to allocate internal resources efficiently. However, many textbooks have achieved relatively modest sales before fading quietly into obscurity.

This little book is one of the few that stand out against the rest, enduring now into its fifth edition, because it clearly fills two significant gaps in the market.

First, it is the perfect self-taught introduction to VaR for the non-specialist. Technical details are kept to the bare minimum as the text focuses on explaining the main concepts, with Moorad's characteristically accessible flair. He has an unusually deep, practical and comprehensive knowledge of risk (a brief scan of his bio shows that his work experience spans banking, trading, structuring and treasury management and that his market knowledge extends far beyond the interest rate and credit-sensitive instruments for which he is so well known) and a real empathy with his readers' needs.

Second, the book covers both market and credit risks, and within market risk it covers the basic building blocks of interest rate sensitive instruments and option portfolios. As such it is an ideal, self-contained textbook for university courses, or indeed for professional training. With seminars covering the worked examples provided, such a course could span approximately 12–18 contact hours of teaching at MBA or undergraduate level.

I am writing this foreword while stranded for the night on the snowbound A23 between Brighton and London. But, as my iPod battery fades, and 101 alternative uses for the energy come to mind, I am at least happy for my friend that I will make the publisher's deadline tomorrow. Indeed, Moorad has been a dear friend for many years. I very much admire his directness, honesty, knowledge, intelligence, kindness, warmth and modesty. I also think he is one of the best writers for not-too-quant-finance audiences of this era. His ideas are always sound and well argued, and he has a natural, fluid writing style which keeps the reader's attention by being both concise and crystal clear – a rare combination.

So, if you are seeking a short, sensible book on VaR that covers all the goal posts – eschewing unnecessary frills – and which is written in an exceptionally clear and readable style, then look no further. Moorad's *Introduction to VaR*, now in its fifth edition, is my strong recommendation.

Professor Carol Alexander
University of Sussex, UK

PREFACE

Here is how I began the Preface in the Fourth Edition of this book:

> *In 1998 I put together an introductory course on value-at-risk for the Securities Institute in London, at the invitation of Zena Doidge. This was a departure for me: my background and work experience up to then had not been in risk management; I had spent the previous 6 years as a bond and money markets trader. However due to personal circumstances I found myself in the position of teaching courses on various bond market subjects. This particular course forced me to actually research the topic though, and not just rely on practical experience and winging it as I might have done when I taught a course on (say) swaps or repo. Thus began an interest in writing, more specifically writing incorporating research and academic rigour rather than just practical experience. Anyway, I delivered the course in June or July that year and it seemed to go down okay. Then Zena suggested that the course companion that I had produced for the course might make a useful textbook on the subject. I had not thought at all about doing this, but as it happened the Institute had its own publishing arm, so there was no need for me to go out and try to get a publishing deal! Thus my first ever book was born, which was published in March 1999.*

And here we are with the Fifth Edition of that book, and as I always keep telling myself, destined to be my last book!

Since that edition came out, we've had a banking crash and ongoing economic recession in the West. And VaR did not emerge from these events unscathed; it didn't have a good crisis. In 2012 JPMorgan reported a trading loss at its London-based investment office that was rumoured to be anything from 10 to 100 times greater than what the bank's VaR model had suggested was its maximum expected loss in a non-crash scenario. Is there any point calculating and reporting VaR if it is going to be this far away from reality?

To be fair to VaR, no mathematical model had a good 2007–08. It was only ever a risk measurement tool, and an estimate at that, and should always have been used under understood assumptions and limitations. Now that we know how inaccurate it can be, we can use its output more judiciously. For example, if one is going to set trading limits and risk limits within a VaR framework, one must ensure that the limit is well below what the model suggests it should be, so that if one is caught short, or the wrong way round, one is less surprised that the lost amount is so large. This book largely ignores the crash, except in the final chapter when we respond to the recent critiques of VaR with some recommendations for a change in approach.

At the end of the day VaR is still widely used in many banks, as well as by bank regulators, which is why we have deemed a revision of this book necessary. As well as a general update, the additional highlights in this Fifth Edition include:

- a comparison of results using different VaR methodologies, using Bloomberg screens to illustrate;
- an introduction to stressed VaR;
- a new set of problems and exercises, with solutions supplied.

The final chapter provides a more than worthy finale to the book. I am pleased to enhance considerably the value of this Fifth Edition with the inclusion of a chapter written by Max Wong, who is in market risk management with RBS in Singapore. Max contributes a technical yet accessible critique of VaR and its performance at the time of the 2007–08 crash. He also makes some recommendations on how VaR might be better employed. In essence, he calls for a different approach to risk management itself, and shows there is still a place for VaR provided it is interpreted in the appropriate way. It's tremendous good fortune for me to have his expertise available here and I am sure readers will value his contribution.

As always we remain true to the spirit of the First Edition: that is, as befits a book directed at newcomers to the market, material is kept simple and accessible throughout. I welcome comments, please feel free to email me direct on *mooradc@hotmail.com*

All the best.

Moorad Choudhry
Surrey, England
4 January 2013

PREFACE TO THE FIRST EDITION

The concept of Value-at-Risk (VaR) has become a mainstay of financial markets risk management since its introduction by JPMorgan in 1994. An increasing number of banks and securities houses, and corporates, now use VaR as their main tool for providing management information on the size of their risk exposure. Initially VaR was used to measure the extent of market risk exposure; this was followed by the application of VaR methodology to the measurement of credit risk exposure.

As this is an introduction to the subject we have attempted to place VaR in context; hence the book begins by defining risk and describing the risk management function and other tools of risk measurement in the financial markets. VaR is best viewed as a tool within an overall risk management framework and hopefully the contents within will communicate this to the reader. An integrated risk management function within a bank or securities house will wish to incorporate VaR as part of its overall risk exposure and control framework. When such a framework is effective it serves an important purpose in providing comfort to a firm's shareholders that the management of trading, market and credit risk is no longer a significant cause for concern. At this point VaR as a risk measurement tool might be said to have come of age, and perhaps have assisted in the realisation of shareholder value.

This book has been written for those with little or no previous understanding of or exposure to the concept of risk management and Value-at-Risk; however, it also describes the subject in sufficient depth to be of use as a reference to a more experienced practitioner. It is primarily aimed at front office, middle office and back office banking and fund management staff who are involved to some extent in risk management. Others including corporate and local authority treasurers may wish to refer to the contents. Undergraduate and postgraduate students and MBA students specialising in financial markets will also find this book useful as a reference text. Comments on the text should be sent to the author care of the Securities Institute Services.

Moorad Choudhry
14 December 1998

ACKNOWLEDGEMENTS

First Edition

Parts of this book were originally written for the introductory course on Value-at-Risk run by the Securities Institute in London. My thanks to Zena Doidge at the Institute, whom it was a pleasure to work with, for giving me the opportunity to teach this course. Thanks to Debra Maddison in the Institute's publishing department, and Simon Chapman and Kalbinder Dhillon for graphics help.

I would also like to thank Richard Thornton at KPMG for lending me his book on VaR.

Fourth Edition

Love, affection and respect to Paul Claxton, Harry Cross, Abukar Ali, Didier Joannas, Khurram Butt, Michael Nicoll, Mo Dualeh, Phil Broadhurst, and all those to whom I want to tell that it's a privilege to call my friends.

Nothing lasts forever. But then again, some things never change ...

Fifth Edition

Thanks to the *Raynes Park Footy Boys*. And to the lads in RBS GBM Treasury, CBD Treasury and Group Treasury that I played footy and volleyball with ...

ABOUT THE AUTHORS

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Moorad is Vice-Chair of the Board of Directors of PRMIA.

Max Wong heads the VaR model validation team at The Royal Bank of Scotland. Prior to this, he worked in roles as market strategist, futures trader and financial analyst at various financial institutions. He was an open outcry pit trader during the Asian crisis in 1997 and a quant risk manager during the credit crisis in 2007. He holds a BSc degree in Physics and an MSc in Financial Engineering.

Max is author of *Bubble Value at Risk: A Countercyclical Risk Management Approach*, Revised Edition, John Wiley & Sons (2013).

Disappointments and setbacks have to be faced in life. There must be no recriminations. I had learnt this lesson when I was dropped from the Marlborough XI on the morning of our match against Rugby at Lord's. There is always something else ahead.

– Lt. Gen. Sir Hugh Stockwell, quoted in Turner, B. (2006), *Suez 1956*, London: Hodder & Stoughton

In my life, why do I give valuable time, to people who don't care if I live or die?
In my life, why do I give valuable time, to people who I'd much rather kick in the eye?

– The Smiths, *Heaven Knows I'm Miserable Now*, Rough Trade Records 1984

Chapter 1

INTRODUCTION TO RISK

The risk management department was one of the fastest growing areas in investment and commercial banks during the 1990s, and again after the crash of 2008. A string of high-profile banking losses and failures, typified by the fall of Barings Bank in 1995, highlighted the importance of risk management to bank managers and shareholders alike. In response to the volatile and complex nature of risks that they were exposed to, banks set up specialist risk management departments, whose functions included both measuring and managing risk. As a value-added function, risk management can assist banks not only in managing risk, but also in understanding the nature of their profit and loss, and so help increase return on capital. It is now accepted that senior directors of banks need to be thoroughly familiar with the concept of risk management. One of the primary tools of the risk manager is *value-at-risk* (*VaR*), which is a quantitative measure of the risk exposure of an institution. For a while VaR was regarded as somewhat inaccessible, and only the preserve of mathematicians and quantitative analysts. Although VaR is indeed based on statistical techniques that may be difficult to grasp for the layman, its basic premise can, and should, be explained in straightforward fashion, in a way that enables non-academics to become comfortable with the concept. The problem with VaR is that while it was only ever a measure, based on some strong assumptions, of approximate market risk exposure (it is unsuited to measuring risk exposure in the banking book), it suffers in the eyes of its critics in having the cachet of science. This makes it arcane and inaccessible, while paradoxically being expected to be much more accurate than it was ever claimed to be. Losses suffered by banks during the crash of 2007–08 were much larger than any of their VaR values, which is where the measure comes in for criticism. But we can leave that aside for now, and concentrate just on introducing the technicalities.

Later in the book we describe and explain the calculation and application of VaR. We begin here with a discussion of risk.

DEFINING RISK

Any transaction or undertaking with an element of uncertainty as to its future outcome carries an element of risk: risk can be thought of as uncertainty. To associate particular assets such as equities, bonds or corporate cash flows with types of risk, we need to define 'risk' itself. It is useful to define risk in terms of a risk *horizon*, the point at which an asset will be realised, or turned into cash. All market participants, including speculators, have an horizon, which may be as short as a half-day. Essentially then, the horizon is the time period relating to the risk being considered.

Once we have established a notion of horizon, a working definition of risk is *the uncertainty of the future total cash value of an investment on the investor's horizon date*. This uncertainty arises from many sources. For participants in the financial markets risk is essentially a measure of the volatility of asset returns, although it has a broader definition as being any type of uncertainty as to future outcomes. The types of risk that a bank or securities house is exposed to as part of its operations in the bond and capital markets are characterised below.

THE ELEMENTS OF RISK: CHARACTERISING RISK

Banks and other financial institutions are exposed to a number of risks during the course of normal operations. The different types of risk are broadly characterised as follows:

- *Market risk* – risk arising from movements in prices in financial markets. Examples include foreign exchange (*FX*) risk, interest rate risk and basis risk. In essence market risk applies to 'tradeable' instruments, ones that are *marked-to-market* in a trading book, as opposed to assets that are held to maturity, and never formally repriced, in a banking book.
- *Credit risk* – something called *issuer risk* refers to risk that a customer will default. Examples include sovereign risk, marginal risk and *force majeure* risk.
- *Liquidity risk* – this refers to two different but related issues: for a Treasury or money markets' person, it is the risk that a bank has insufficient funding to meet commitments as they arise. That is, the risk that funds cannot be raised in the market as and when required. For a securities or derivatives trader, it is the risk that the market for assets becomes too thin to enable fair and efficient trading to take place. This is the risk that assets cannot be sold or bought as and when required. We should differentiate therefore between funding liquidity and trading liquidity whenever using the expression *liquidity*.

Table 1.1 Characterising risk.

	Market	Reinvestment	Credit	Sovereign	FX	Basis	Prepayment	Counterparty
Government bond								
Developed country	■	■	■	■				
Developing country	■	■	■	■	■			
Zero-coupon bond	■		■					
Corporate bond	■	■	■					
Asset-backed bond	■	■	■				■	
Bank deposit	■		■				■	
FRA	■		■					■
Futures contract	■							
Forward contract			■			■		■
Interest rate swap	■							■
Repo	■	■	■					
Equity (listed exchange)	■	■	■					

- *Operational risk* – risk of loss associated with non-financial matters such as fraud, system failure, accidents and ethics. Table 1.1 assigns sources of risk for a range of fixed interest, FX, interest rate derivative and equity products. The classification has assumed a 1-year horizon, but the concepts apply to any time horizon.

Forms of market risk

Market risk reflects the uncertainty as to an asset's price when it is sold. Market risk is the risk arising from movements in financial market prices. Specific market risks will differ according to the type of asset under consideration:

- *Currency risk* – this arises from exposure to movements in FX rates. A version of currency risk is *transaction* risk, where currency fluctuations affect the proceeds from day-to-day transactions.
- *Interest rate risk* – this arises from the impact of fluctuating interest rates and will directly affect any entity borrowing or investing funds. The most common exposure is simply to the level of interest rates but some institutions run positions that are exposed to changes in the shape of the yield curve. The basic risk arises from revaluation of the asset after a change in rates.
- *Equity risk* – this affects anyone holding a portfolio of shares, which will rise and fall with the level of individual share prices and the level of the stock market.
- *Other market risk* – there are residual market risks which fall in this category. Among these are *volatility* risk, which affects option traders, and *basis* risk, which has a wider impact. Basis risk arises whenever one kind of risk exposure is hedged with an instrument that behaves in a similar, but not necessarily identical manner. One example would be a company using 3-month interest rate futures to hedge its commercial paper (*CP*) programme. Although eurocurrency rates, to which futures prices respond, are well correlated with CP rates, they do not invariably move in lock step. If CP rates moved up by 50 basis points but futures prices dropped by only 35 basis points, the 15-bps gap would be the basis risk in this case.

Other risks

- *Liquidity risk* – in banking, this refers to the risk that a bank cannot raise funds to refinance loans as the original borrowing becomes past due. It is sometimes also referred to as *rollover* risk. In other words, it refers to the risk of an inability to continue to raise funds to replace maturing liabilities. There is also another (related) liquidity risk, which refers to *trading liquidity*. This is the risk that an asset on the balance sheet cannot be sold at a previously perceived fair value, or cannot be sold at all, and hence experiences *illiquidity*.
- *Credit risk* – the risk that an *obligor* (the entity that has borrowed funds from you) defaults on the loan repayments.
- *Counterparty risk* – all transactions involve one or both parties in counterparty risk, the potential loss that can arise if one party

were to default on its obligations. Counterparty risk is most relevant in the derivatives market, where every contract is marked-to-market daily and so a positive MTM is taken to the profit & loss (P&L) account. If the counterparty defaults before the contract has expired, there is risk that the actual P&L will not be realized. In the credit derivatives market, a counterparty that has sold protection on the third-party reference name on the credit derivative contract and which subsequently defaults will mean the other side to the trade is no longer protected against the default of that third party.

- *Reinvestment risk* – if an asset makes any payments before the investor's horizon, whether it matures or not, the cash flows will have to be reinvested until the horizon date. Since the reinvestment rate is unknown when the asset is purchased, the final cash flow is uncertain.
- *Sovereign risk* – this is a type of credit risk specific to a government bond. Post 2008, there is material risk of default by an industrialised country. A developing country may default on its obligation (or declare a debt 'moratorium') if debt payments relative to domestic product reach unsustainable levels.
- *Prepayment risk* – this is specific to mortgage-backed and asset-backed bonds. For example, mortgage lenders allow the homeowner to repay outstanding debt before the stated maturity. If interest rates fall prepayment will occur, which forces reinvestment at rates lower than the initial yield.
- *Model risk* – some financial instruments are heavily dependent on complex mathematical models for pricing and hedging. If the model is incorrectly specified, is based on questionable assumptions or does not accurately reflect the true behaviour of the market, banks trading these instruments could suffer extensive losses.

Risk estimation

There are a number of different ways of approaching the estimation of market risk. The key factors determining the approach are the user's response to two questions:

- Can the user accept the assumption of normality – is it reasonable to assume that market movements follow the normal distribution? If so, statistical tools can be employed.

- Does the value of positions change linearly with changes in market prices? If not (as is typical for option positions where market movements are not very small), simulation techniques will be more useful.

Were the answers to both questions to be 'yes' then we could be comfortable using standard measures of risk such as duration and convexity (these concepts are covered later). If the answers are 'no' then we are forced to use scenario analysis combined with simulation techniques. If, as is more likely, the answer to the first question is 'yes' and the second 'no', then a combination of statistical tools and simulation techniques will be required.

For most banks and securities houses the portfolio will almost certainly behave in a non-linear manner because that is the nature of financial markets. Hence, a combination of statistical tools and simulation is likely to be the most effective risk measurement approach. The scenarios used in simulations are often a mixture of observed rate and price changes from selected periods in the past, and judgement calls by the risk manager. The various alternative methods are examined in Chapter 3.

RISK MANAGEMENT

The risk management function grew steadily in size and importance within commercial and investment banks during the 1990s. Risk management departments exist not to eliminate the possibility of all risk, should such action indeed be feasible or desirable; rather, to control the frequency, extent and size of such losses in such a way as to provide the minimum surprise to senior management and shareholders.

Risk exists in all competitive business although the balance between financial risks of the type described above and general and management risk varies with the type of business engaged in. The key objective of the risk management function within a financial institution is to allow for a clear understanding of the risks and exposures the firm is engaged in, such that monetary loss is deemed acceptable by the firm. The acceptability of any loss should be on the basis that such (occasional) loss is to be expected as a result of the firm being engaged in a particular business activity. If the bank's risk management function is effective, there will be no over-reaction to any unexpected losses, which may increase eventual costs to many times the original loss amount.

The risk management function

While there is no one agreed organisation structure for the risk management function, the following may be taken as being reflective of the typical bank set-up:

- an independent, 'middle office' department responsible for drawing up and explicitly stating the bank's approach to risk, and defining trading limits and the areas of the market that the firm can have exposure to;
- the head of the risk function reporting to an independent senior manager, who is a member of the executive board;
- monitoring the separation of duties between front, middle and back office, often in conjunction with an internal audit function;
- reporting to senior management, including firm's overall exposure and adherence of the front office to the firm's overall risk strategy;
- communication of risks and risk strategy to shareholders;
- where leading edge systems are in use, employment of the risk management function to generate competitive advantage in the market as well as control.

The risk management function is more likely to deliver effective results when there are clear lines of responsibility and accountability. It is also imperative that the department interacts closely with other areas of the front and back office.

In addition to the above the following are often accepted as ingredients of a risk management framework in an institution engaged in investment banking and trading activity:

- proactive management involvement in risk issues;
- daily overview of risk exposure profile and profit & loss (*P&L*) reports;
- VaR as a common measure of risk exposure, in addition to other measures including 'jump risk' to allow for market corrections;
- defined escalation procedures to deal with rising levels of trading loss, as well as internal 'stop-loss' limits;
- independent daily monitoring of risk utilisation by middle-office risk management function;

- independent production of daily P&L, and independent review of front-office closing prices on a daily basis;
- independent validation of market pricing, and pricing and VaR models.

These guidelines, adopted universally in the investment banking community, should assist in the development of an influential and effective risk management function for all financial institutions. We say 'should', but of course the experience of JPMorgan, Soc Gen and UBS in the 21st century, shows that the existence of large and seemingly sophisticated risk management infrastructures does not preclude multi-billion dollar trading losses.

Managing risk

The different stakeholders in a bank or financial institution will have slightly different perspectives on risk and its management. If we were to generalise, shareholders will wish for stable earnings as well as the highest possible return on capital. From the point of view of business managers though, the perspective may be slightly different and possibly shorter term. For them, risk management often takes the following route:

- create as diversified a set of business lines as possible, and within each business line diversify portfolios to maximum extent;
- establish procedures to enable some measure of forecasting of market prices;
- hedge the portfolio to minimise losses when market forecasts suggest that losses are to be expected.

The VaR measurement tool falls into the second and third areas of this strategy. It is used to give an idea of risk exposure (generally, to market and credit risk only) so that banks can stay within trading limits, and to feed into the hedge calculation.

QUANTITATIVE MEASUREMENT OF RISK–REWARD

Before introducing the concept of VaR we will consider three standard measures of risk–reward exposure used in the investment community.

Standard deviation

We defined 'risk' above as a prelude to considering its measurement in a VaR context. Investment 'risk' tends to be viewed differently by academics and investors. Academics consider risk within modern portfolio theory to be defined as standard deviation or volatility. To investors risk usually is the probability of loss. Standard deviation is a traditional measure often used by investment professionals. It measures an investment's variability of returns; that is, its volatility in relation to its average return.

While standard deviation has the cachet of science it is a narrow measure and may not provide sufficient information by itself. It is simply a measure of volatility and as a measure of the *probability* of loss is of limited use. However, its usefulness is increased if one pairs it with returns, as in the Sharpe Ratio or the Van Ratio. Moving on from here, the concept of VaR is built on obtaining *probabilities* of loss based on the distribution of returns from a market investment instrument.

Sharpe Ratio

The Sharpe Ratio is a reward–risk ratio. It measures the extent to which the return of an investment (above the *risk-free* return) exceeds its volatility. The higher the ratio, the more reward an investment provides for the risk incurred. The ratio is calculated according to the following equation:

$$\text{Sharpe Ratio} = \frac{R_m - R_f}{V_m} \tag{1.1}$$

where R_m = Rate of return of investment m;
R_f = Risk-free rate of return (e.g., T-Bill);
V_m = Standard deviation of instrument m.

A ratio of 0.5 is considered fair return for risk incurred. For an investor it is more useful as a relative measure, in comparing the ratio of one investment with that of another. For bank trading desks it is a useful measure of the return generated against the risk incurred, for which the return and volatility of individual trading books can be compared with that on the risk-free instrument (or a bank book trading only T-bills).

Van Ratio

The Van Ratio expresses the probability of an investment suffering a loss for a defined period, usually 1 year. For example, a Van Ratio of 20% indicates that there is a 1 in 5 chance of a loss during every four-quarter rolling window. The ratio first uses the following fraction to calculate this probability:

$$\frac{\text{Compound annual return for the measurement period}}{\text{Average four-quarter standard deviation for the measurement period}} \tag{1.2}$$

The probability of a loss is then calculated using standard normal curve probability tables.

The Van Ratio provides an intuitive measure of *absolute* risk, the concept of the probability of a loss. To this end its calculation has assumed a normal distribution of returns. The assumption of normality of returns is important in the concept of VaR as calculated by most of the models and methodologies in use in financial institutions.

Chapter 2

VOLATILITY AND CORRELATION

Value-at-Risk (*VaR*) is essentially a measure of volatility, specifically how volatile a bank's assets are. Assets that exhibit high volatility present higher risk. VaR also takes into account the correlation between different sets of assets in the overall portfolio. If the market price performance of assets is closely positively correlated, this also presents higher risk. So, before we begin the discussion of VaR we need to be familiar with these two concepts. Readers who have an investor's understanding of elementary statistics may skip this chapter and move straight to Chapter 3.

STATISTICAL CONCEPTS

The statistics used in VaR calculations are based on well-established concepts. There are standard formulae for calculating the mean and standard deviation of a set of values. If we assume that X is a random variable with particular values x, we can apply the basic formula to calculate the mean and standard deviation. Remember that the mean is the average of the set of values or observations, while the standard deviation is a measure of the dispersion away from the mean of the range of values. In fact, the standard deviation is the square root of the variance, but the variance, being the sum of squared deviations of each value from the mean divided by the number of observations, has less practical value for us.

Arithmetic mean

We say that the random variable is X, so the mean is $E(X)$. In a time series of observations of historical data, the probability values are the frequencies of the observed values. The mean is:

$$E(X) = \frac{\sum_i x_i}{n} \tag{2.1}$$

where $1/n =$ Assigned probability to a single value among n; and
$n =$ Number of observations.

The standard deviation of the set of values is:

$$\sigma(X) = \frac{1}{n}\sqrt{\sum_i [x_i - E(X)]^2} \tag{2.2}$$

The probability assigned to a set of values is given by the type of distribution and, in fact, from a distribution we can determine the mean and standard deviation depending on the probabilities p_i assigned to each value x_i of the random variable X. The sum of all probabilities must be 100%. From probability values then, the mean is given by:

$$E(X)\frac{\sum_i p_i x_i}{n} \tag{2.3}$$

The variance is the average weighted by the probabilities of the squared deviations from the mean; so, of course, the standard deviation – which we now call the volatility – is the square root of this value. The volatility is given by:

$$\sigma(X) = \sqrt{\sum_i p_i[x_i - E(X)]^2} \tag{2.4}$$

In the example in Table 2.1 we show the calculation of the mean, the variance and the standard deviation as calculated from an Excel spreadsheet. The expectation is the mean of all the observations, while the variance is, as we noted earlier, the sum of squared deviations from the mean. The standard deviation is the square root of the variance.

Table 2.1 Calculation of standard deviation.

Dates	*Observations*	*Deviations from mean*	*Squared deviation*
1	22	4.83	23.36
2	15	−2.17	4.69
3	13	−4.17	17.36
4	14	−3.17	10.03
5	16	−1.17	1.36
6	17	−0.17	0.03
7	16	−1.17	1.36
8	19	1.83	3.36
9	21	3.83	14.69
10	20	2.83	8.03
11	17	−0.17	0.03
12	16	−1.17	1.36
Sum	206	Sum	85.66
Mean	17.17	Variance	7.788
		Standard deviation	2.791

What happens when we have observations that can assume any value within a range, rather than the discrete values we have seen in our example? When there is a probability that a variable can have a value of any measure between a range of specified values, we have a continuous distribution.

Probability distributions

A probability distribution is a model for an actual or empirical distribution. If we are engaged in an experiment in which a coin is tossed a number of times, the number of heads recorded will be a discrete value of 0, 1, 2, 3, 4, or so on, depending on the number of times we toss the coin. The result is called a 'discrete' random variable. Of course, we know that the probability of throwing a head is 50%, because there are only two outcomes in a coin-toss experiment, heads or tails. We may throw the coin three times and get three heads (it is unlikely but by no means exceptional); however, performing the experiment a great number of times should produce something approaching our 50% result. So, an experiment with a large number of trials would produce an empirical distribution which would be close to the theoretical distribution as the number of tosses increases.

This example illustrates a discrete set of outcomes (0, 1, 2, 3); in other words, a discrete probability distribution. It is equally possible to have a continuous probability distribution: for example, the probability that the return on a portfolio lies between 3% and 7% is associated with a continuous probability distribution because the final return value can assume any value between those two parameters.

The normal distribution

A very commonly used theoretical distribution is the normal distribution, which is plotted as a bell-shaped curve and is familiar to most practitioners in business. The theoretical distribution actually looks like many observed distributions such as the height of people, shoe sizes, and so on. The distribution is completely described by the mean and the standard deviation. The normal distribution $N(\mu, \sigma)$ has mean μ and standard deviation σ. The probability function is given by:

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{2.5}$$

The distribution is standardised as $N(0, 1)$ with a mean of 0 and a standard deviation of 1. It is possible to obtain probability values for any part of the distribution by using the standardised curve and converting variables to this standardised distribution; thus, the variable $Z = (X - \mu)/\sigma$ follows the standardised normal distribution $N(0, 1)$ with probability:

$$P(Z = Z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{Z^2}{2\sigma^2}\right] \tag{2.6}$$

The *Central Limit Theorem* (known also as the law of large numbers) is the basis for the importance of the normal distribution in statistical theory, and in real life a large number of distributions tend towards the normal, provided that there are a sufficient number of observations. This explains the importance of the normal distribution in statistics. If we have large numbers of observations – for example, the change in stock prices, or closing prices in government bonds – it makes calculations straightforward if we assume that they are normally distributed.

For both option pricing theory and VaR, it is assumed that the returns from holding an asset are normally distributed. It is often convenient to define the return in logarithmic form as:

$$\ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t = Price today;
P_{t-1} = Previous price.

If this is assumed to be normally distributed, then the underlying price will have a log-normal distribution. The log-normal distribution never goes to a negative value, unlike the normal distribution, and hence is intuitively more suitable for asset prices. The distribution is illustrated as Figure 2.1.

The normal distribution is assumed to apply to the returns associated with stock prices, and indeed all financial time series observations. However, it is not strictly accurate, as it implies extreme negative values that are not observed in practice. For this reason the log-normal distribution is used instead, in which case the logarithm of the returns is used instead of the return values

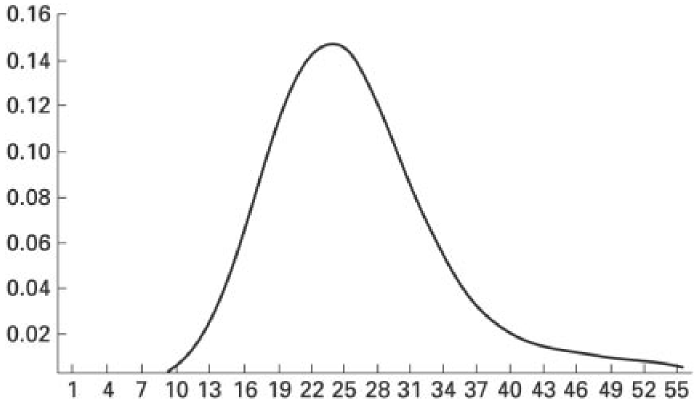

Figure 2.1 The log-normal distribution.

themselves; this also removes the probability of negative stock prices. In the log-normal distribution, the logarithm of the random variable follows a normal distribution. The log-normal distribution is asymmetric, unlike the normal curve, because it does not have negatives at the extreme values.

Confidence intervals

Assume an estimate x of the average of a given statistical population where the true mean of the population is μ. Suppose that we believe that on average $\bar{x}$ is an unbiased estimator of μ. Although this means that on average $\bar{x}$ is accurate, the specific sample that we observe will almost certainly be above or below the true level. Accordingly, if we want to be reasonably confident that our inference is correct, we cannot claim that μ is precisely equal to the observed $\bar{x}$.

Instead, we must construct an interval estimate or confidence interval of the following form:

$$\mu = \bar{x} \pm \text{Sampling error}$$

The crucial question is: How wide must this confidence interval level be? The answer, of course, will depend on how much $\bar{x}$ fluctuates. We first set our requirements for level of confidence; that is, how certain we wish to be statistically. If we wish to be incorrect only 1 day in 20 – that is, we wish to be right 19 days each month

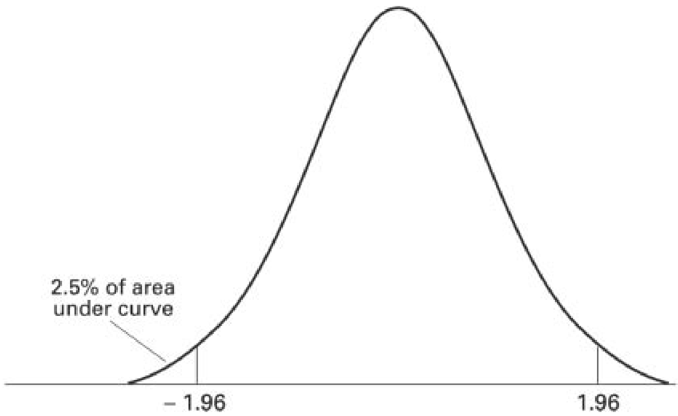

Figure 2.2 Confidence intervals.

(a month is assumed to have 20 working days) – that would equate to a 95% confidence interval that our estimate is accurate. We also assume that our observations are normally distributed. In that case we would expect that the population would be distributed along the lines portrayed in Figure 2.2.

In the normal distribution, 2.5% of the outcomes are expected to fall more than 1.96 standard deviations from the mean. So, that means 95% of the outcomes would be expected to fall within ± 1.96 standard deviations. That is, there is a 95% chance that the random variable will fall between -1.96 standard deviations and $+1.96$ standard deviations. This would be referred to as a 'two-sided' (or 'two-tailed') confidence interval. It gives the probability of a move upwards or downwards by the random variable outside the limits we are expecting.

In the financial markets, we do not however expect negative prices, so that values below 0 are not really our concern. In this scenario, it makes sense to consider a one-sided test if we are concerned with the risk of loss: a move upward into profit is of less concern (certainly to a risk manager anyway!). From the statistical tables associated with the normal distribution we know that 5% of the outcomes are expected to fall more than 1.645 (rounded to 1.65) standard deviations from the mean. This would be referred to as a one-sided confidence interval.

VOLATILITY

In financial market terms, volatility is a measure of how much the price of an asset moves each day (or week or month, and so on). Speaking generally, higher volatility equates to higher profit or loss risk. Bankers must be familiar with volatility, as assets that exhibit higher volatility must be priced such that their returns incorporate a 'risk premium' to compensate the holder for the added risk exposure.

Example 2.1

We demonstrate volatility from first principles here. Table 2.2 shows two portfolios, outwardly quite similar. They have virtually identical means from an observation of portfolio returns over ten observation periods. However, the standard deviation shows a different picture, and we see that Portfolio B exhibits much greater volatility than Portfolio A. Its future performance is much harder to predict with any reasonable confidence. Portfolio B carries higher risk and so would carry higher VaR. We see also from Table 2.2 that the standard deviation is a measure of the dispersion away from the mean of all the observations. To be comfortable that the statistical measures are as accurate as possible, we need the greatest number of observations.

The volatility demonstrated in Table 2.2 is historical volatility; it is based on past performance. Options traders deal in implied volatility, which is the volatility value given by backing out the Black–Scholes options pricing formula from market prices to obtain an implied volatility value for an asset.

Volatility is important for both VaR measurement and in the valuation of options. It is a method of measuring current asset price against the distribution of the asset's future price. Statistically, volatility is defined as the fluctuation in the underlying asset price over a certain period of time. Fluctuation is derived from the change in price between one day's closing price and the next day's closing price. Where the asset price is stable it will exhibit low volatility, and the opposite when price movements are large and/or unstable.

We saw from Table 2.2 that the average values for low- and high-volatility portfolios were similar; however, the distribution of the recordings differ. The low-volatility portfolio showed low variability

Table 2.2

	Excel column B	C	D
	Observations	*Portfolio A*	*Portfolio B*
Excel row 6			
7	1	5.08%	3.50%
8	2	5.00%	5.00%
9	3	5.05%	6.25%
10	4	5.00%	7.10%
11	5	5.05%	3.75%
12	6	5.00%	5.75%
13	7	5.01%	2.50%
14	8	5.20%	4.75%
15	9	5.06%	5.25%
16	10	5.00%	6.75%
17			
18			
19			
20 Mean		5.05%	5.06%
21 Standard deviation		0.000 622 272	0.014 813 282
22			
23 Excel formula		=AVERAGE(C7:C16)	=AVERAGE(D7:D16)
24		=STDEV(C7:C16)	=STDEV(D7:D16)

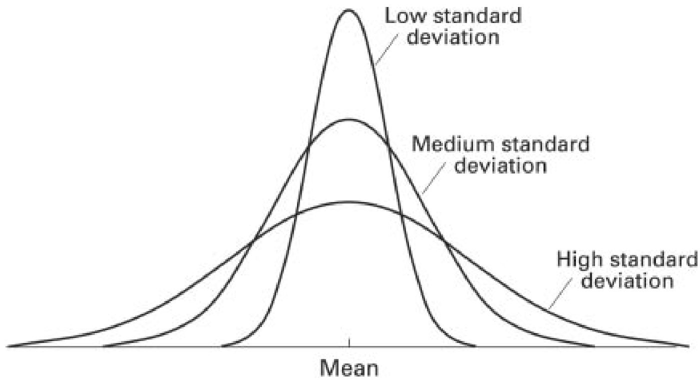

Figure 2.3 Differing standard deviations.

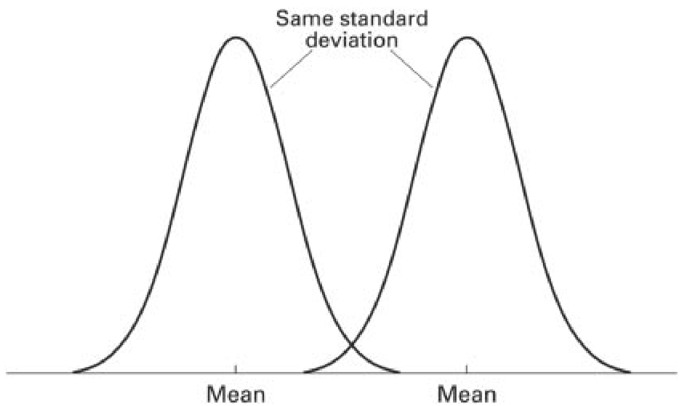

Figure 2.4 Differing means around the same standard deviation.

in the distribution. High-volatility assets show a wider variability around the mean.

Market practitioners wish to obtain a volatility value that approximates around the normal distribution. This is done by recording a sufficiently large volume of data and reducing the price change intervals to as small an amount as possible; this means that the price changes can be described statistically by the normal distribution curve. We saw earlier in the chapter that the normal distribution curve has two numerical properties known as the mean and the standard deviation. The mean is the average reading taken at the centre of the curve, and the standard deviation is a value which

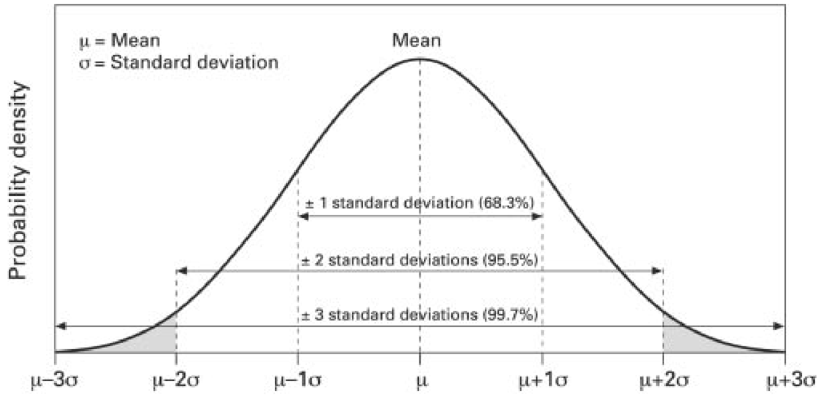

Figure 2.5 Differing standard deviations.

represents the dispersion around the mean. We demonstrate some examples at Figures 2.3 and 2.4.

In Figure 2.5, the standard deviation is shown correlated with dispersion. The curve can be divided into segments which represent specific percentages within each band.

We see from Figure 2.5 that 68.3% of data fall within ± 1 standard deviation, 95.5% of data fall within ± 2 standard deviations and 99.7% fall within ± 3 standard deviations.

The normal distribution curve can also be used to predict future daily share fluctuation over a measured period of time. Future price distribution uses volatility expressed as a 1 standard deviation price change at the end of 1 year.

This can be expressed as a percentage:

$$\text{1 standard deviation price change } (p) = \text{Volatility } (\%) \times \text{Current asset price } (p)$$

Although the value of an option relies upon estimated future volatility, volatility is shown also as historical volatility and implied volatility. *Historical volatility* is the actual price fluctuation in a given time period. The value will depend on the length of the observation period and when the value was observed. This naturally smoothes out day-to-day fluctuations – a moving average of historical volatility can be shown graphically in a similar way as

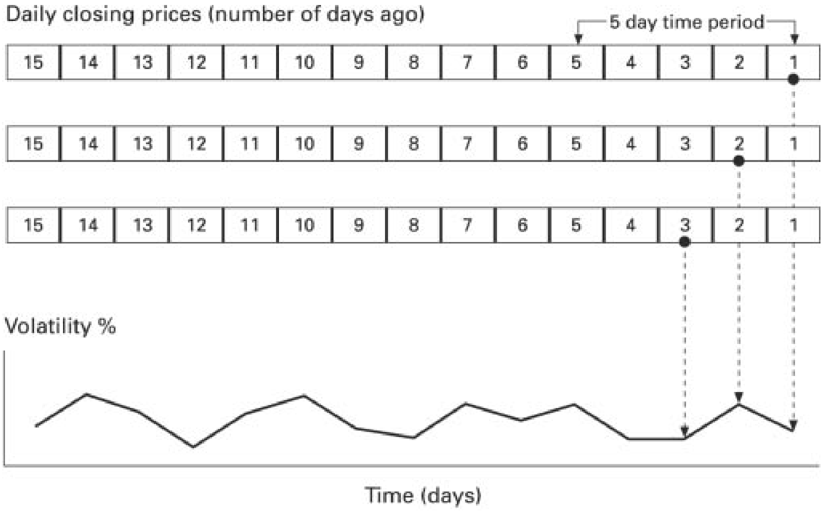

Figure 2.6 Historical volatility chart.

conventional share prices. Figure 2.6 shows a 5-day historical volatility chart reversing through a specified time period.

Although historical volatility can show trends over a greater period of time – for example, 4 years – it can also make distinctly significant and highly variable changes. Therefore, there can be no certainty that a past trend is in any way indicative of a share's future performance.

Implied volatility is a necessary tool to obtain the predicted value of an option which has been obtained from the present value of that option by entering different levels of volatility into an option pricing model, until the current market price is reached. This iterative process effectively reduces the margin of error. In a working situation, most option pricing models allow the calculation of implied volatility by entering the present market price for an option.

Future volatility is the predicted or expected price fluctuation of a period of time until the option has expired. Evidently, this will be affected not only by the calculated implied volatility but also by the expectation of the share's price trend.

Table 2.3 Probabilities extracted from the normal distribution table.

		E	F	G	H	I	J
8	No. of standard deviations	−1.645	−1.000	0.000	1.000	1.650	2.450
9	Probability	5.00%	15.87%	50.00%	84.13%	95.05%	99.29%
10	Excel formula	=NORMSDIST(E8)					

Table 2.4 Normal distribution illustrated for portfolio return.

6	C	D	E	F	G	H	I
7	Observation	1	2	3	4	5	6
8	Mean return	5%	5%	5%	5%	5%	5%
9	Target return	4%	5%	6%	7%	8%	8%
10	Standard deviation of return	1.63%	1.63%	1.63%	1.63%	1.63%	1.63%
11	Number of standard deviations	−0.612 369 871	0	0.612 369 871	1.224 739 743	1.837 109 614	1.837 109 614
12	Probability	27.01%	50.00%	72.99%	88.97%	96.69%	96.69%

Excel formula
Number of standard deviations =(D9-D8)/D10
Probability =NORMSDIST(D11)

THE NORMAL DISTRIBUTION AND VaR

As we will see from Chapter 3 there is more than one way to calculate VaR for an asset portfolio. Many VaR models use the normal curve to calculate the estimation of losses over a specified time period. Normal distribution curve tables, which can be looked up in any number of statistics textbooks or on the Internet, show the probability of an observation moving a specific distance away from the recorded mean. Some specific probabilities are given in Table 2.3.

Table 2.3 shows that 95% of all observations in a normal distribution lie within ± 1.65 standard deviations of the mean. A 95% percentile is often used in VaR calculations. Let us take this further. Consider a gilt portfolio of £20 million with a mean return of 5% per annum. If the standard deviation of the returns is 1.63 what probability is there that returns will fall to 4% within the holding period of 1 year? We require the area of the normal curve at which the 4% value is marked – that is, 27% of the area to the left of the mean. The results are shown at Table 2.4, which also shows the Excel formula.

We should note that, although the markets assume a normal distribution of asset (equity and bond) prices, from observation we know that prices follow a more skewed distribution. In practice, asset prices exhibit what is termed 'leptokurtosis', also known as 'fat tails', which is a normal distribution with fatter tails than the theoretical. In other words, extreme price movements such as stock market corrections occur more frequently than the normal distribution would suggest.

Options traders need to correct for this more than others. The standard option pricing model, the Black–Scholes formula, which we look at in Chapter 5, uses the normal distribution to calculate the delta of the option – the $N(\mathrm{d}1)$ part of the formula – and the probability that the option will be exercised, the $N(\mathrm{d}2)$ part. In practice, the implied volatility of an option is higher if it is deeply in-the-money or out-of-the-money. This is known as the option 'smile', and reflects market understanding that the normal distribution is not a completely accurate description of market price behaviour.

Table 2.5 Correlation.

Cell	*C*	*D*	*E*	*F*	*G*
5	Observation	Government bond 1	Government bond 2	Government bond 3	Government bond 4
6	1	5.35%	11.00%	7.15%	5.20%
7	2	6.00%	9.00%	7.30%	6.00%
8	3	5.50%	9.60%	6.90%	5.80%
9	4	6.00%	13.70%	7.20%	6.30%
10	5	5.90%	12.00%	5.90%	5.90%
11	6	6.50%	10.80%	6.00%	6.05%
12	7	7.15%	10.10%	6.10%	7.00%
13	8	6.80%	12.40%	5.60%	6.80%
14	9	6.75%	14.70%	5.40%	6.70%
15	10	7.00%	13.50%	5.45%	7.20%
16					
17					
18	Mean return	6.30%	11.68%	6.30%	6.30%
19	Volatility	0.006 31	0.018 97	0.007 60	0.006 22
20	Correlation with bond 1		0.357 617 936	−0.758 492 885	0.933 620 205
21					
22	**Excel formula**				
23	Mean return		=AVERAGE(E6:E15)		
24	Volatility		=STDEV(E6:E15)		
25	Correlation with bond 1		=CORREL(E6:E15,D6:D15)		

CORRELATION

The correlation between different assets and classes of assets is an important measure for risk managers because of the role diversification plays in risk reduction. Correlation is a measure of how much the price of one asset moves in relation to the price of another asset. In a portfolio comprised of only two assets, the VaR of this portfolio is reduced if the correlation between the two assets is weak or negative.

The simplest measure of correlation is the correlation coefficient. This is a value between -1 and $+1$, with a perfect positive correlation indicated by 1, while a perfect negative correlation is given by -1. Note that this assumes a linear (straight line) relationship between the two assets. A correlation of 0 suggests that there is no linear relationship.

We illustrate these values at Table 2.5, which is a hypothetical set of observations showing the volatilities of four different government benchmark bonds. Note also the Excel formula so that readers can reproduce their own analyses. We assume these bonds are different sovereign names. Bonds 1, 3 and 4 have very similar average returns, but the relationship between Bond 3 and Bond 1 is negatively closely correlated, whereas Bond 4 is positively closely correlated with Bond 1. Bond 2 has a very low positive correlation with Bond 1, and we conclude that there is very little relationship in the price movement of these two bonds.

What are we to make of these four different sovereign names with regard to portfolio diversification? On first glance, Bonds 1 and 3 would appear to offer perfect diversification because they are strongly negatively correlated. However, calculating a diversified VaR for such a portfolio would underestimate risk exposure in times of market correction – which is, after all, when managers most want to know what their risk is. This is because, even though the bonds are negatively related, they can both be expected to fall in value when the market overall is dropping. Bond 2 is no good for risk mitigation, it is strongly positively correlated. Bond 2 has essentially no relationship with Bond 1; however, it is also the most risky security in the portfolio.

We will apply what we have learned here in Chapter 3.

Chapter 3

VALUE-AT-RISK

The advent of value-at-risk (*VaR*) as an accepted methodology for quantifying market risk and its adoption by bank regulators are part of the development of risk management. The application of VaR has been extended from its initial use in securities houses to commercial banks and corporates, following its introduction in October 1994 when JPMorgan launched RiskMetrics free over the Internet.

In this chapter we look at the different methodologies employed to calculate VaR, and also illustrate its application to simple portfolios. We look first at the variance–covariance method, which is arguably the most popular estimation technique.

WHAT IS VaR?

VaR is an estimate of an amount of exposure cash value. It is based on probabilities, so cannot be relied on with certainty, but reflects rather a level of confidence which is selected by the user in advance. VaR measures the volatility of a company's asset prices, and so the greater the volatility, the higher the probability of loss.

Definition

Essentially VaR is a measure of the volatility of a bank trading book. It is the characteristics of volatility that traders, risk managers and others wish to become acquainted with when assessing a bank's risk exposure. The mathematics behind measuring and estimating volatility is slightly involved, and we do not go into it here. However, by making use of a volatility estimate, a trader or senior manager can gain some idea of the risk exposure of the trading book, using the VaR measure.

VaR is defined as follows:

> **VaR is a measure of market risk. It is the maximum loss which can occur with *X*% confidence over a holding period of *t* days.**

VaR is the expected loss of a portfolio over a specified time period for a set level of probability. So, for example, if a daily VaR is stated as £100,000 to a 95% level of confidence, this means that during the day there is a only a 5% chance that the loss will be *greater* than £100,000. VaR measures the potential loss in market value of a portfolio using estimated volatility and correlations. It is measured within a given confidence interval, typically 95% or 99%. The technique seeks to measure possible losses from a position or portfolio under 'normal' circumstances. The definition of normality is critical to the estimation of VaR and is a statistical concept; its importance varies according to the VaR calculation methodology that is being used.

Broadly speaking, the calculation of a VaR estimate follows four steps:

1. *Determine the time horizon over which one wishes to estimate a potential loss* – this horizon is set by the user. In practice, time horizons of 1 day to 1 year have been used. For instance, bank front-office traders are often interested in calculating the amount they might lose in a 1-day period. Regulators and participants in illiquid markets may want to estimate exposures to market risk over a longer period. In any case a time horizon must be specified by the decision-maker.
2. *Select the degree of certainty required, which is the confidence level that applies to the VaR estimate* – knowing the largest likely loss a bank will suffer 95 times out of 100, or in fact on 1 day out of 20 (i.e., a 95% degree of confidence in this estimate, or confidence interval) may be sufficient. For regulatory requirements a 99% confidence interval may be more appropriate. Senior management and shareholders are often interested in the potential loss arising from catastrophe situations, such as a stock market crash, so for them a 99% confidence level is more appropriate.
3. *Create a probability distribution of likely returns for the instrument or portfolio under consideration* – several methods may be used. The easiest to understand is the distribution of recent historical returns for the asset or portfolio which often looks like the curve associated with the normal distribution. After determining a time horizon and confidence interval for the estimate, and then collating the history of market price changes in a probability distribution, we can apply the laws of statistics to estimate VaR.
4. *Calculate the VaR estimate* – this is done by observing the loss amount associated with that area beneath the normal curve at the critical confidence interval value that is statistically associated with the probability chosen for the VaR estimate in Step 2.

These four steps will in theory allow us to calculate a VaR estimate 'longhand', although in practice mathematical models exist that will do this for us. Bearing these steps in mind, we can arrive at a practical definition of VaR not much removed from our first one:

> **VaR is the largest likely loss from market risk (expressed in currency units) that an asset or portfolio will suffer over a time interval and with a degree of certainty selected by the user.**

We stress, of course, that this would be under 'normal', that is, unstressed conditions. There are a number of methods for calculating VaR, all logically sustainable but nevertheless reliant on some strong assumptions, and estimates prepared using the different methodologies can vary dramatically. At this point it is worthwhile reminding ourselves what VaR is *not*. It is not a unified method for measuring risk, as the different calculation methodologies each produce different VaR values. In addition, as it is a quantitative statistical technique, VaR only captures risks that can be quantified. Therefore, it does not measure (nor does it seek to measure) other risks that a bank or securities house will be exposed to, such as liquidity risk or operational risk. Most importantly, VaR is not 'risk management'. This term refers to the complete range of duties and disciplines that are involved in minimising and managing bank risk exposure. VaR is but one ingredient of risk management, a measurement tool for market risk exposure. So the mean and standard deviation parameters of the statistical distribution are key to the VaR estimate.

METHODOLOGY

Centralised database

To implement VaR, all of a firm's positions data must be gathered into one centralised database. Once this is complete the overall risk has to be calculated by aggregating the risks from individual instruments across the entire portfolio. The potential move in each instrument (i.e., each risk factor) has to be inferred from past daily price movements over a given observation period. For regulatory purposes this period is at least 1 year. Hence, the data on which VaR estimates are based should capture all relevant daily market moves over the previous year. The main assumption underpinning VaR – and which in turn may be seen as its major weakness – is that the distribution of future price and rate changes will follow past variations. Therefore, the potential portfolio loss calculations for

VaR are worked out using distributions from historic price data in the observation period.

Correlation assumptions

VaR requires that the user decide which exposures are allowed to offset each other and by how much. For example, is the Japanese yen correlated to movements in the euro or the Mexican peso? Consider also the price of crude oil to movements in the price of natural gas: if there is a correlation, to what extent is the degree of correlation? VaR requires that the user determine correlations *within* markets as well as *across* markets. The mapping procedures used as part of the VaR process also have embedded correlation assumptions. For example, mapping individual stocks into the S&P 500 or fixed interest securities into the swap curve translate into the assumption that individual financial instruments move as the market overall. This is reasonable for diversified portfolios but may fall down for undiversified or illiquid portfolios.

To calculate the VaR for a single security, we would calculate the standard deviation of its price returns. This can be done using historical data, but also using the *implied volatility* contained in exchange-traded option prices. We would then select a confidence interval and apply this to the standard deviation, which would be our VaR measure. This is considered in more detail later.

There are three main methods for calculating VaR. As with all statistical models, they depend on certain assumptions. They are:

- the correlation method (or variance/covariance method);
- historical simulation;
- Monte Carlo simulation.

Correlation method

This is also known as the variance–covariance, *parametric* or analytic method. This method assumes the returns on risk factors are normally distributed, the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. Using the correlation method, the volatility of each risk factor is extracted from the historical observation period. Historical data on investment returns are therefore required. The potential effect of each component of the portfolio on the overall portfolio value is then worked out from the component's delta (with respect to a particular risk factor) and that risk factor's volatility.

There are different methods of calculating relevant risk factor volatilities and correlations. We consider two alternatives:

(i) Simple *historic volatility* (correlation) – this is the most straightforward method but the effects of a large one-off market move can significantly distort volatilities (correlations) over the required forecasting period. For example, if using 30-day historic volatility, a market shock will stay in the volatility figure for 30 days until it drops out of the sample range and, correspondingly, causes a sharp drop in (historic) volatility 30 days *after* the event. This is because each past observation is equally weighted in the volatility calculation.

(ii) A more sophisticated approach is to weight past observations unequally. This is done to give more weight to recent observations so that large jumps in volatility are not caused by events that occurred some time ago. Two methods for unequal weighting are the generalised autoregressive conditional heteroscedasticity (*GARCH*) models and exponentially weighted moving averages. GARCH models are fine-tuned to each risk factor time series, while exponentially weighted averages can be computed with little more complication than simple historic volatility. Both methods rely on the assumption that future volatilities can be predicted from historic price movements.

Historical simulation method

The historical simulation method for calculating VaR is the simplest and avoids some of the pitfalls of the correlation method. Specifically, the three main assumptions behind correlation (normally distributed returns, constant correlations, constant deltas) are not needed in this case. For historical simulation the model calculates potential losses using actual historical returns in the risk factors and so captures the non-normal distribution of risk factor returns. This means rare events and crashes can be included in the results. As the risk factor returns used for revaluing the portfolio are actual past movements, the correlations in the calculation are also actual past correlations. They capture the dynamic nature of correlations as well as scenarios when the usual correlation relationships break down.

Monte Carlo simulation method

The third method, Monte Carlo simulation, is more flexible than the previous two. As with historical simulation, Monte Carlo simulation allows the risk manager to use actual historical distributions for risk factor returns rather than having to assume normal returns. A large number of randomly generated simulations are run forward in time using volatility and correlation estimates chosen by the risk manager. Each simulation will be different, but in total the simulations will aggregate to the chosen statistical parameters (i.e., historical distributions and volatility and correlation estimates). This method is more realistic than the previous two models and, therefore, is more likely to estimate VaR more accurately. However, its implementation requires powerful computers and there is also a trade-off in that the time to perform calculations is longer.

Validity of the volatility–correlation VaR estimate

The level of confidence in the VaR estimation process is selected by the number of standard deviations of variance applied to the probability distribution. A standard deviation selection of 1.645 provides a 95% confidence level (in a one-tailed test) that the potential estimated price movement will not be more than a given amount based on the correlation of market factors to the position's price sensitivity. This confidence level is advocated by the RiskMetrics version of volatility–correlation VaR.

HOW TO CALCULATE VaR

A conceptual illustration of the normal distribution being applied for VaR is given at Figure 3.1.

A market risk estimate can be calculated by following these steps:

1. Value the current portfolio using today's prices, the components of which are 'market factors'. For example, the market factors that affect the value of a bond denominated in a foreign currency

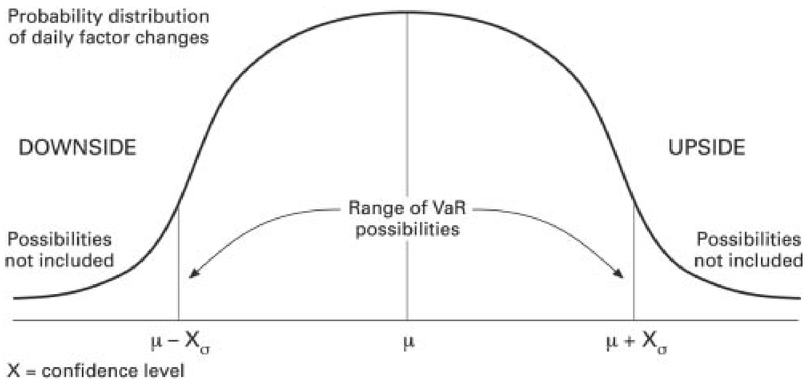

Figure 3.1 VaR and the normal distribution.

are the term structure of that currency's interest rate (either the zero-coupon curve or the par yield curve) and the exchange rate.
2. Revalue the portfolio using alternative prices based on changed market factors and calculate the change in the portfolio value that would result.
3. Revaluing the portfolio using a number of alternative prices gives a distribution of changes in value. Given this, a portfolio VaR can be specified in terms of confidence levels.
4. The risk manager can calculate the maximum the firm can lose over a specified time horizon at a specified probability level.

In implementing VaR the main problem is finding a way to obtain a series of vectors of different market factors. We will see how the various methodologies try to resolve this issue for each of the three methods that can be used to calculate VaR.

Historical method

Values of the market factors for a particular historical period are collected and changes in these values over the time horizon are observed for use in the calculation. For instance, if a 1-day VaR is required using the past 100 trading days, each of the market factors will have a vector of observed changes that will be made up of the 99 changes in value of the market factor. A vector of alternative values is created for each of the market factors by adding the current value of the market factor to each of the values in the vector of observed changes.

The portfolio value is found using the current and alternative values for the market factors. The changes in portfolio value between the current value and the alternative values are then calculated. The final step is to sort the changes in portfolio value from the lowest value to highest value and determine the VaR based on the desired confidence interval. For a 1-day, 95% confidence level VaR using the past 100 trading days, the VaR would be the 95th most adverse change in portfolio value.

Simulation method

The first step is to define the parameters of the distributions for the changes in market factors, including correlations among these factors. Normal and log-normal distributions are usually used to estimate changes in market factors, while historical data are most often used to define correlations among market factors. The distributions are then used in a Monte Carlo simulation to obtain simulated changes in the market factors over the time horizon to be used in the VaR calculation.

A vector of alternative values is created for each of the market factors by adding the current value of the market factor to each of the values in the vector of simulated changes. Once this vector of alternative values of the market factors is obtained, the current and alternative values for the portfolio, the changes in portfolio value and the VaR are calculated exactly as in the historical method.

Variance–covariance, analytic or parametric method

This is similar to the historical method in that historical values of market factors are collected in a database. The next steps are then to:

(i) decompose the instruments in the portfolio into the cash-equivalent positions in more basic instruments;
(ii) specify the exact distributions for the market factors (or 'returns'); and
(iii) calculate the portfolio variance and VaR using standard statistical methods.

We now look at these steps in greater detail.

Decompose financial instruments

The analytic method assumes that financial instruments can be decomposed or 'mapped' into a set of simpler instruments that are exposed to only one market factor. For example, a 2-year UK gilt can be mapped into a set of zero-coupon bonds representing each cash flow. Each of these zero-coupon bonds is exposed to only one market factor – a specific UK zero-coupon interest rate. Similarly, a foreign currency bond can be mapped into a set of zero-coupon bonds and a cash foreign exchange amount subject to movement in the spot foreign exchange (*FX*) rate.

Specify distributions

The analytic method makes assumptions about the distributions of market factors. For example, the most widely used analytic method, JPMorgan's RiskMetrics, assumes that the underlying distributions are normal. With normal distributions all the historical information is summarised in the mean and standard deviation of the returns (market factors), so users do not need to keep all the historical data.

Calculate portfolio variance and VaR

If all the market factors are assumed to be normally distributed, the portfolio, which is the sum of the individual instruments, can also be assumed to be normally distributed. This means that portfolio variance can be calculated using standard statistical methods (similar to modern portfolio theory), given by:

$$\sigma_\rho = \sqrt{\alpha_j^2\sigma_j^2 + \alpha_k^2\sigma_k^2 + 2\alpha_j\alpha_k\rho_{jk}\sigma_j\sigma_k} \tag{3.1}$$

where α_j = Home currency present value of the position in market factor j;
σ_j^2 = Variance of market factor j;
ρ_{jk} = Correlation coefficient between market factors j and k.

The portfolio VaR is then a selected number of portfolio standard deviations; for example, 1.645 standard deviations will isolate 5% of the area of the distribution in the lower tail of the normal curve, providing 95% confidence in the estimate. Consider an example where, using historical data, the portfolio variance for a package of UK gilts is £348.57. The standard deviation of the portfolio would be $\sqrt{348.57}$, which is £18.67. A 95% 1-day VaR would be $1.645 \times$ £18.67, which is £30.71.

Of course, a bank's trading book will contain many hundreds of different assets, and the method employed above, useful for a two-asset portfolio, will become unwieldy. Therefore, matrices are used to calculate the VaR of a portfolio where many correlation coefficients are used. This is considered below.

Matrix calculation of variance–covariance VaR

Consider the following hypothetical portfolio of £10,000,000.00 invested in two assets, as shown in Table 3.1(i). The standard deviation of each asset has been calculated on historical observation of asset returns. Note that *returns* are returns of asset prices, rather than the prices themselves; they are calculated from the actual prices by taking the ratio of closing prices. The returns are then calculated as the logarithm of price relatives. The mean and standard deviation of the returns are then calculated using standard statistical formulae. This would then give the standard deviation of daily price relatives,

Table 3.1(i) Two-asset portfolio VaR.

D	E	F	G	H
		Asset		
8		Bond 1	Bond 2	
9	Standard deviation	11.83%	17.65%	
10	Portfolio weighting	60%	40%	
11	Correlation coefficient			0.647
12	Portfolio value			£10,000,000.00
13	Confidence level			95%
14				
15	Portfolio variance			0.016506998
16	Standard deviation			12.848%
17				
18	95% c.i. standard deviations			1.644853627
19				
20	Value-at-Risk			0.211330072
21	Value-at-Risk £			£2,113,300.72
22				
23				
24				
25				
26				

which is converted to an annual figure by multiplying it by the square root of the number of days in a year, usually taken to be 250.

We wish to calculate the portfolio VaR at the 95% level. The Excel formulae are shown at Table 3.1(ii).

The standard equation is used to calculate the variance of the portfolio, using the individual asset standard deviations and the asset weightings; the VaR of the book is the square root of the variance. Multiplying this figure by the current value of the portfolio gives us the portfolio VaR, which is £2,113,300.72.

Using historical volatility means that we must define the horizon of the time period of observations, as well as the frequency of observations. Typically, a daily measure is used due to the ease of collating information, with the result that we need to use the 'square root of time' rule when moving to another time period. This applies when there are no bounds to returns data. This was illustrated above when we referred to the square root for the number of working days in a year. As an example, if we assume a 2% daily volatility, the 1-year volatility then becomes:

$$\begin{aligned}\sigma_{1\text{ year}} &= \sigma_{1\text{ day}}\sqrt{250}\\ &= 2\% \times 15.811\\ &= 31.622\%\end{aligned}$$

Using this rule we can convert values for market volatility over any period of time.

The RiskMetrics VaR methodology uses matrices to obtain the same results that we have shown here. This is because, once a portfolio starts to contain many assets, the method we described above becomes unwieldy. Matrices allow us to calculate VaR for a portfolio containing many hundreds of assets, which would require assessment of the volatility of each asset and correlations of each asset to all the others in the portfolio. We can demonstrate how the parametric methodology uses variance and correlation matrices to calculate the variance, and hence the standard deviation, of a portfolio. The matrices are shown at Figure 3.2. Note that multiplication of matrices carries with it some unique rules; readers who are unfamiliar with matrices should refer to a standard mathematics text.

As shown at Figure 3.2, using the same two-asset portfolio described, we can set a 2×2 matrix with the individual standard deviations

Table 3.1(ii) Spreadsheet formulae for Table 3.1(i).

D	E	F	G	H
		Asset		
8		Bond 1	Bond 2	
9	Standard deviation	11.83%	17.65%	
10	Portfolio weighting	60%	40%	
11	Correlation coefficient			0.647
12	Portfolio value			£10,000,000.00
13	Confidence level			95%
14				
15	Portfolio variance			=F9^2*F10^2+G9^2*G10^2+2*F9*F10*G9*G10
16	Standard deviation			=H15^0.5
17				
18	95% c.i. standard deviations			=NORMSINV(H13)
19				
20	Value-at-Risk			=H18*H16
21	Value-at-Risk £			=H20*H12
22				
23				

Variance matrix

Bond 1	11.83%	0
Bond 2	0	17.65%

Correlation matrix

Bond 1	Bond 2
1	0.647
0.647	1

VC matrix

0.1183	0.076 54
0.114 196	0.1765

VC matrix

0.1183	0.076 54
0.114 196	0.1765

Variance matrix

11.83%	0
0	17.65%

VCV matrix

0.013 995	0.013 509
0.013 509	0.031 152

Weighting matrix

60%	40%

VCV matrix

0.013 995	0.013 509
0.013 509	0.031 152

WVCV

0.013 801	0.020 566

WVCV

0.013 801	0.020 566

W

60%
40%

WVCVW

0.016 507

Standard deviation	0.128 48

Figure 3.2 Matrix variance–covariance calculation for the two-asset portfolio shown in Table 3.1.

inside; this is labelled the 'variance' matrix. The standard deviations are placed on the horizontal axis of the matrix, and a '0' entered in the other cells. The second matrix is the correlation matrix, and the correlation of the two assets is placed in cells corresponding to the other asset; that is why a '1' is placed in the other cells, as an asset is said to have a correlation of 1 with itself. The two matrices are then multiplied to produce another matrix, labelled 'VC' in Figure 3.2.[1]

The VC matrix is then multiplied by the V matrix to obtain the variance–covariance matrix or VCV matrix. This shows the variance of each asset; for Bond 1 this is 0.013 99, which is expected as that is the square of its standard deviation, which we were given at the start. The matrix also tells us that Bond 1 has a covariance of 0.0135 with Bond 2. We then set up a matrix of the portfolio weighting of the two assets, and this is multiplied by the VCV matrix. This produces a 1×2 matrix, which we need to change to a single number; so, this is multiplied by the W matrix, reset as a 2×1 matrix, which produces

[1] A spreadsheet calculator such as Microsoft Excel has a function for multiplying matrices which may be used for any type of matrix. The function is '=MMULT()' typed in all the cells of the product matrix.

Table 3.2 Asset correlation.

	Asset 1	*Asset 2*
Asset 1	1	0.647
Asset 2	0.647	1

the portfolio variance. This is 0.016 507. The standard deviation is the square root of the variance, and is 0.128 4795 or 12.848%, which is what we obtained before. In our illustration it is important to note the order in which the matrices were multiplied, as this will obviously affect the result. The volatility matrix contains the standard deviations along the diagonal, and '0's are entered in all the other cells. So, if the portfolio we were calculating has 50 assets in it, we would require a 50×50 matrix and enter the standard deviations for each asset along the diagonal line. All the other cells would have a '0' in them. Similarly, for the weighting matrix this is always one row, and all the weights are entered along the row. To take the example just given the result would be a 1×50 weighting matrix.

The correlation matrix in the simple example above is set up as shown in Table 3.2.

The correlation matrix at Table 3.2 shows that Asset 1 has a correlation of 0.647 with Asset 2. All correlation tables always have unity along the diagonal because an asset will have a correlation of 1 with itself. So, a three-asset portfolio of the following correlations

Correlation 1, 2	0.647
Correlation 1, 3	0.455
Correlation 2, 3	0.723

would look like Table 3.3.

The matrix method for calculating the standard deviation is more effective than the first method we described, because it can be used

Table 3.3 Correlation matrix: three-asset portfolio.

	Asset 1	*Asset 2*	*Asset 3*
Asset 1	1	0.647	0.455
Asset 2	0.647	1	0.723
Asset 3	0.455	0.723	1

for a portfolio containing a large number of assets. In fact, this is exactly the methodology used by RiskMetrics, and the computer model used for the calculation will be set up with matrices containing the data for hundreds, if not thousands, of different assets.

The variance–covariance method captures the diversification benefits of a multi-product portfolio because the correlation coefficient matrix is used in the calculation. For instance, if the two bonds in our hypothetical portfolio had a negative correlation the VaR number produced would be lower. It was also the first methodology introduced by JPMorgan in 1994. To apply it, a bank would require data on volatility and correlation for the assets in its portfolio. These data are actually available from the RiskMetrics website (and other sources), so a bank does not necessarily need its own data. It may wish to use its own datasets, however, should it have them, to tailor the application to its own use. The advantages of the variance–covariance methodology are that:

- it is simple to apply and fairly straightforward to explain;
- datasets for its use are immediately available.

The drawbacks of the variance–covariance method are that it assumes stable correlations and measures only linear risk; it also places excessive reliance on the normal distribution, and returns in the market are widely believed to have 'fatter tails' than a true to normal distribution. This phenomenon is known as *leptokurtosis*; that is, the non-normal distribution of outcomes. Another disadvantage is that the process requires mapping. To construct a weighting portfolio for the RiskMetrics tool, cash flows from financial instruments are mapped into precise maturity points, known as *grid points*. We will review this later in the chapter; however, in most cases assets do not fit into neat grid points, and complex instruments cannot be broken down accurately into cash flows. The mapping process makes assumptions that frequently do not hold in practice.

Nevertheless, the variance–covariance method is still popular in the market, and is frequently the first VaR method installed at a bank.

Mapping

The cornerstone of variance–covariance methodologies, such as RiskMetrics, is the requirement for data on volatilities and correlations for assets in the portfolio. The RiskMetrics dataset does not

Table 3.4 RiskMetrics grid points.

1 month	5 years
3 months	7 years
6 months	9 years
1 year	10 years
2 years	15 years
3 years	20 years
4 years	30 years

contain volatilities for every maturity possible, as that would require a value for every period from 1 day to over 10,950 days (30 years) and longer, and correlations between each of these days. This would result in an excessive amount of calculation. Rather, volatilities are available for set maturity periods (these are shown in Table 3.4).

If a bond is maturing in 6 years' time, its redemption cash flow will not match the data in the RiskMetrics dataset, so it must be mapped to two periods, in this case being split to the 5-year and 7-year grid point. This is done in proportions so that the original value of the bond is maintained once it has been mapped. More importantly, when a cash flow is mapped, it must split in a manner that preserves the volatility characteristic of the original cash flow. Therefore, when mapping cash flows, if one cash flow is apportioned to two grid points, the share of the two new cash flows must equal the present value of the original cash flows, and the combined volatility of the two new assets must be equal to that of the original asset. A simple demonstration is given at Example 3.1.

Example 3.1 Cash flow mapping.

A bond trading book holds £1 million nominal of a gilt strip that is due to mature in precisely 6 years' time. To correctly capture the volatility of this position in the bank's RiskMetrics VaR estimate, the cash flow represented by this bond must be mapped to the grid points for 5 years and 7 years, the closest maturity buckets that the RiskMetrics dataset holds volatility and correlation data for. The present value of the strip is calculated using the 6-year zero-coupon rate, which RiskMetrics obtains by interpolating between the 5-year rate and the 7-year rate. The details are shown in Table 3.5.

Table 3.5 Bond position to be mapped to grid points.

Gilt strip nominal (£)	1,000,000
Maturity (years)	6
5-year zero-coupon rate	5.35%
7-year zero-coupon rate	5.50%
5-year volatility	24.50%
7-year volatility	28.95%
Correlation coefficient	0.979
Lower period	5
Upper period	7

Note that the correlation between the two interest rates is very close to 1; this is expected because 5-year interest rates generally move very closely in line with 7-year rates.

We wish to assign the single cash flow to the 5-year and 7-year grid points (also referred to as *vertices*). The present value of the bond, using the 6-year interpolated yield, is £728,347. This is shown in Table 3.6, which also uses an interpolated volatility to calculate the volatility of the 6-year cash flow. However, we wish to calculate a portfolio volatility based on the apportionment of the cash flow to the 5-year and 7-year grid points. To do this, we need a weighting to use to allocate the cash flow between the two vertices. In the hypothetical situation used here, this presents no problem because 6 years falls precisely between 5 years and 7 years. Therefore, the weightings are 0.5 for Year 5 and 0.5 for Year 7. If the cash flow had fallen in less obvious a maturity point, we would have to calculate the weightings using the formula for portfolio variance.

Using these weightings, we calculate the variance for the new 'portfolio', containing the two new cash flows, and then the standard deviation for the portfolio. This gives us a VaR for the strip of £265,853.

Table 3.6 Cash flow mapping and portfolio variance.

Interpolated yield	0.054 25
Interpolated volatility	0.267 25
Present value	728,347.0103
Weighting 5-year grid point	0.5
Weighting 7-year grid point	0.5
Variance of portfolio	0.070 677 824
Standard deviation	0.265 853 012
VaR (£)	265,853

Confidence intervals

Many models estimate VaR at a given confidence interval, under normal market conditions. This assumes that market returns generally follow a random pattern but one that approximates over time to a normal distribution. The level of confidence at which the VaR is calculated will depend on the nature of the trading book's activity and what the VaR number is being used for. The original amendment to the Basel Capital Accord stipulated a 99% confidence interval and a 10-day holding period if the VaR measure is to be used to calculate the regulatory capital requirement. However, certain banks prefer to use other confidence levels and holding periods; the decision on which level to use is a function of asset types in the portfolio, quality of market data available and the accuracy of the model itself, which will have been tested over time by the bank.

For example, a bank may view a 99% confidence interval as providing no useful information, as it implies that there should only be two or three breaches of the VaR measure over the course of 1 year; that would leave no opportunity to test the accuracy of the model until a relatively long period of time had elapsed, in the meantime the bank would be unaware if the model was generating inaccurate numbers. A 95% confidence level implies the VaR level being exceeded around 1 day each month, if a year is assumed to contain 250 days. If a VaR calculation is made using 95% confidence, and a 99% confidence level is required for, say, regulatory purposes, we need to adjust the measure to take account of the change in standard deviations required. For example, a 99% confidence interval corresponds to 2.32 standard deviations, while a 95% level is equivalent to 1.645 standard deviations. Thus, to convert from 95% confidence to 99% confidence, the VaR figure is divided by 1.645 and multiplied by 2.32.

In the same way there may be occasions when a firm will wish to calculate VaR over a different holding period from that recommended by the Basel Committee. The holding period of a portfolio's VaR calculation should represent the period of time required to unwind the portfolio; that is, sell off the assets on the book. A 10-day holding period is recommended but would be unnecessary for a highly liquid portfolio; for example, a market-making book holding government bonds.

To adjust the VaR number to fit it to a new holding period we simply scale it upwards or downwards by the square root of the time period required. For example, a VaR calculation measured for a 10-day holding period will be $\sqrt{10}$ times larger than the corresponding 1-day measure.

COMPARISON BETWEEN METHODS

The three methods produce different VaR estimates and these are more marked with portfolios that contain options. The analytic method usually estimates the market risk of option positions based on delta (or delta and gamma). This results in inaccurate risk estimates for large changes in the price of the underlying; it also ignores the potential effect of changes in the volatility of the underlying. The historic and simulation methods can account for changes in all the market factors that affect an option price, and the revaluation process allows the market risk of options to be more accurately measured for larger changes in market factors.

A comparison of the three methodologies is presented at Table 3.7, summarised from *Risk* in November 1997.

Choosing between methods

The composition of a bank's portfolio is a prime factor in deciding which method to implement. For portfolios with no options the analytic method may be most suitable because it does

Table 3.7 Comparison of VaR methods.

	Historical	*Simulation*	*Analytic*
Ease of implemenation			
Easy to aggregate risk across markets	Yes	Yes	Yes
Data available at no charge	No	No	Yes
Ease of programming (spreadsheet)	Easiest	Hardest	Medium
Distributions for market factors			
Must specific distributions be assumed?	No	Yes	Yes
Are actual volatilities and correlations used?	Yes	Possible	Yes
Handling of individual instruments			
Are pricing models required?	Yes	Yes	No
Is it necessary to map instruments?	No	No	Yes
Accurate handling of options	Yes	Yes	No
Communication with senior management			
Ease of explanation	Easiest	Medium	Hardest
Can sensitivity analyses be done?	No	Yes	Some

Source: Smitson/Minton, *Risk*.

not require pricing models. Publicly available software and data (e.g., RiskMetrics) makes installation simpler.

Historical or simulation methods are more appropriate for portfolios with option positions. The historical method is conceptually simple and the required pricing models are often available as add-ins for spreadsheet packages. The main obstacle to using the simulation method is the complex task of doing Monte Carlo simulations; although the software is available the process is time-consuming.

RiskMetrics

RiskMetrics was launched by JPMorgan in 1994. Its approach is to assume that changes in the prices and yields of financial instruments follow the normal distribution. Given this assumption volatility can be expressed in terms of the standard deviation from the mean. RiskMetrics uses 1.645 (usually rounded to 1.65) standard deviations as its measure of risk, which encompasses 95% of occurrences (in a one-tailed test, or the 'negative' or loss end of the curve).

The assumptions behind RiskMetrics give rise to these two consequences:

- by setting a confidence level of 95%, we say that we are prepared to accept a 5% chance that the market will move beyond our parameters; on 1 day in 20 the market will move more than we predict;
- we accept the risk that in reality some market prices, such as FX rates, move in a non-normal manner; there is considerable evidence that many rates display 'fat tails', implying that the number and size of large movements is higher than forecast by a normal distribution.

The analytical approach of RiskMetrics is a direct application of modern portfolio theory and is summarised by the following equation, which we encountered earlier:

$$\sigma_p^2 = \sum_{i=1}^{n} (\alpha_i \cdot \sigma_i)^2 + 2 \sum_{i=1}^{n} \sum_{i=1}^{n} \alpha_i \alpha_j \rho_{ij} \sigma_i \sigma_j$$

where σ_p^2 = Variance for the entire portfolio;
α_i = Portfolio weighting for asset i;
σ_i^2 = Variance of the logarithmic return of asset i;
ρ_{ij} = Correlation between the logarithmic return of asset i and of asset j;
n = Number of assets in the portfolio.

This equation essentially states that total portfolio risk is a function of two types of factors:

- the volatility of each distinct asset in the portfolio, denoted by σ_i and σ_j; and
- the correlations between assets, denoted by ρ_{ij}.

Box 3.1 Technical addendum.

The calculation of portfolio standard deviations using an equally weighted moving average approach is given by:

$$\sigma_t = \sqrt{\frac{1}{(k-1)} \sum_{s=t-k}^{t-1} (x_s - \mu)^2}$$

where σ_t = Estimated standard deviation of the portfolio at the beginning of time period t;
k = Number of days in the observation period;
x_s = Change in portfolio value on day s;
μ = Mean change in the portfolio value.

The formula for the portfolio standard deviation under an exponentially weighted moving average approach is represented by:

$$\sigma_t = \sqrt{(1-\lambda)\sum_{s=t-k}^{t-1}(x_s-\mu)^2}$$

The parameter λ, referred to as the 'decay factor', determines the rate at which the weights on past observations decay as they become more distant.

Exponentially weighted moving averages emphasise recent observations by using weighted averages of squared deviations. They aim to capture short-term movements in volatility.

Given the assumption of normality of returns we can estimate within a certain level of confidence the range over which the portfolio will fluctuate on a daily, weekly, monthly and annual basis.

For example, for a portfolio with no more than a 5% chance that its market value will decrease by more than £1 million over a 1-day period, the VaR is £1 million.

RiskMetrics defines market risk, first, in terms of absolute market risk: the estimate of total loss expressed in currency terms (i.e., dollars or pounds of risk). Second, it defines risk in terms of a time horizon. Its daily earnings at risk (*DEaR*) number is defined as the expected loss over a 1-day horizon. VaR itself measures the potential loss over a longer horizon, such as 1 month.

RiskMetrics bases its estimates of volatility and correlations on historical data. As we saw in the previous section, this is a methodological choice. There are alternative approaches to estimating future volatility: subjective forecasts can be used; alternatively, implied volatilities from exchange-traded instruments can be used. Of course, many instruments are not exchange-traded, nor is the forecasting power of implied volatility necessarily greater than historical volatility. RiskMetrics uses exponentially weighted moving averages of historical rate movements. Generally, 75 days' worth of data are used, and the weighting means that newer data are given more emphasis than older data. This ensures that the volatility estimates respond quickly to market shocks, but also that they gradually revert back to more normal levels. However, in the immediate aftermath of a significant correction, the volatility figure may be unrealistically high.

To calculate a single position VaR example in simple fashion, consider Example 3.2.

Example 3.2 VaR calculation.

VaR	Amount of position * Volatility of instrument;
Volatility	% of value which may be lost with a certain possibility (e.g., 95%);
Position	A bond trader is long of $40 million US 10-year Treasury benchmark;
Market risk	US 10-year volatility is 0.932%;
VaR =	$40 million * 0.932% = $372,800.

For a two-position VaR example the portfolio now needs to consider correlations and the following expression is applied:

$$VaR = \sqrt{VaR_1^2 + VaR_2^2 + 2\rho VaR_1 VaR_2}$$

where VaR_1 = Value-at-risk for Instrument 1;
VaR_2 = Value-at-risk for Instrument 2;
ρ = Correlation between the price movements of Instrument 1 and 2.

The individual VaRs are calculated as before.

Essentially, RiskMetrics follows the procedure detailed in the previous section for analytic method VaR estimates.

The core of RiskMetrics is:

- a method mapping position, forecasting volatilities and correlations, and risk estimation;
- a daily updated set of estimated volatilities and correlations of rates and prices.

The DEaR and VaR are the maximum estimated loss in market value of a given position that can be expected to be incurred with 95% certainty until the position can be neutralised or reassessed.

Assessment

The key technical assumptions made by RiskMetrics are:

- conditional multivariate normality of returns and assets;
- exponentially weighted moving average forecasts of volatility (as against GARCH or stochastic models);
- variance–covariance method of calculation (as against historical simulation).

The key limitations are:

- limited applicability to options and non-linear positions generally;
- simplicity of its mapping process, assuming cash flows on standardised grid points on the time line;
- like any VaR model, no coverage of liquidity risk, funding risk, credit risk or operational risk.

Comparison with the historical approach

The historical approach is preferred in some firms because of its simplicity. It differs from RiskMetrics in three respects:

- it makes no explicit assumption about the variances of portfolio assets and the correlations between them;
- it makes no assumptions about the shape of the distribution of asset returns, including no assumption of normality;
- it requires no simplification or mapping of cash flows.

To calculate VaR using this approach all that is required is a historical record of the daily profit and loss (*P&L*) of the portfolio under consideration. Hence, a major strength of the historical approach is the minimal analytical capability required. An additional benefit is the lack of cash flow mapping. The simplification process can create substantial risk distortion, particularly if there are options in the portfolio. Under RiskMetrics, options are converted into their delta equivalents.

The main drawback of the historical approach is that since it is based strictly on the past it is not useful for scenario analysis. With RiskMetrics we can alter the assumed variances and correlations to see how the VaR would be affected. This is not possible under the historical approach.

COMPARING VaR CALCULATION FOR DIFFERENT METHODOLOGIES

The different approaches to calculating VaR produce a wide range of results. As we illustrate here, this difference occurs even with the simplest portfolio (in this case, a holding of just one vanilla fixed income instrument). If this variation is so marked for just one asset holding, how much more dispersion must there be for complex portfolios that include exotic instruments? The point here is that one must be aware that care needs to be taken when interpreting and using VaR numbers. The actual loss that might occur in practice can bear no relation to the previous day's VaR calculation. This being the case, it suggests that banks should be ultra conservative when setting VaR limits for credit and market risk, on the grounds that the true risk exposure might be many times what the bank's model is suggesting. Relying excessively on VaR model output is not recommended.

Figure 3.3 shows Bloomberg screen PORT, which needs to be set up by the user to hold the desired securities (in other words, the

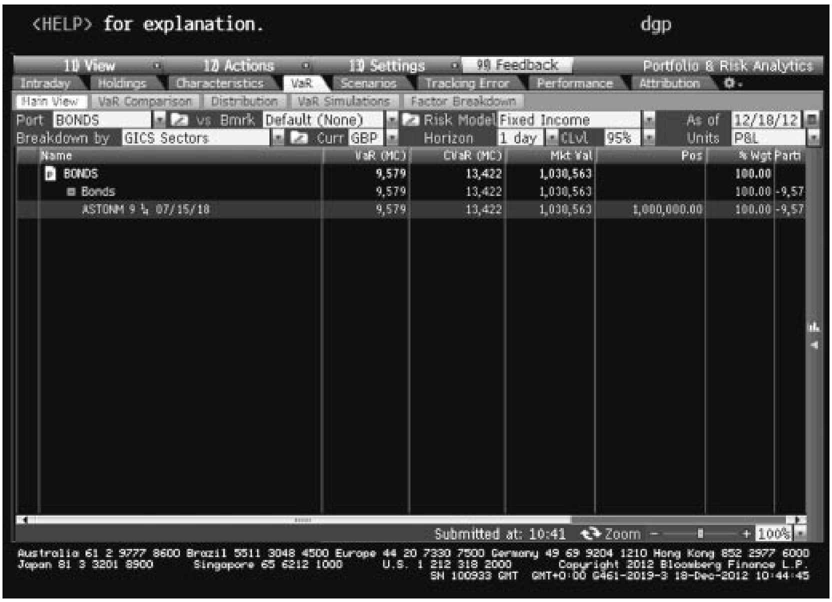

Figure 3.3 PORT screen showing holding of GBP1 million Aston Martin Capital Ltd $9\frac{1}{4}\%$ 2018 sterling corporate bond.

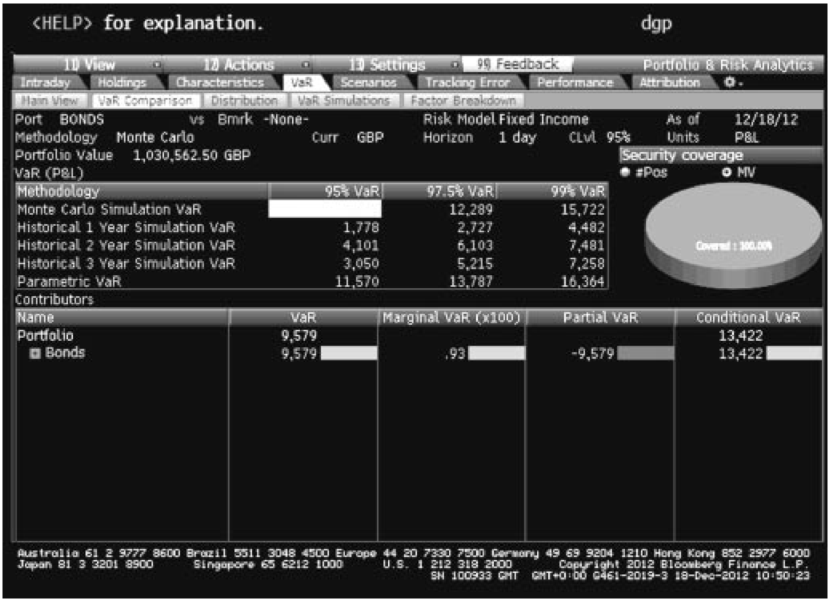

Figure 3.4 PORT screen showing VaR results by methodology, for bond holding at Figure 3.3

portfolio is bespoke to the user's needs). We see that the portfolio consists of one bond, the Aston Martin Capital Ltd $9\frac{1}{4}$% 2018, a £304 million issue from 2011 and quoted at £100.30 as at 18 December 2012. At that price the holding of £1 million had a market value of £1,030,563.

This Bloomberg screen has a VaR functionality built in, which allows you to compare VaR numbers by methodology. We see this at Figure 3.4. From this screen we observe the following values, for the 95% confidence interval:

Monte Carlo VaR	9,579
1-year historical VaR	1,778
2-year historical VaR	4,101
3-year historical VaR	3,050
Parametric VaR	11,570

This variation is so great as to render the results almost unusable. The investor can take the entire range or simply select an average or the worst-case scenario. It is at this point that using VaR itself

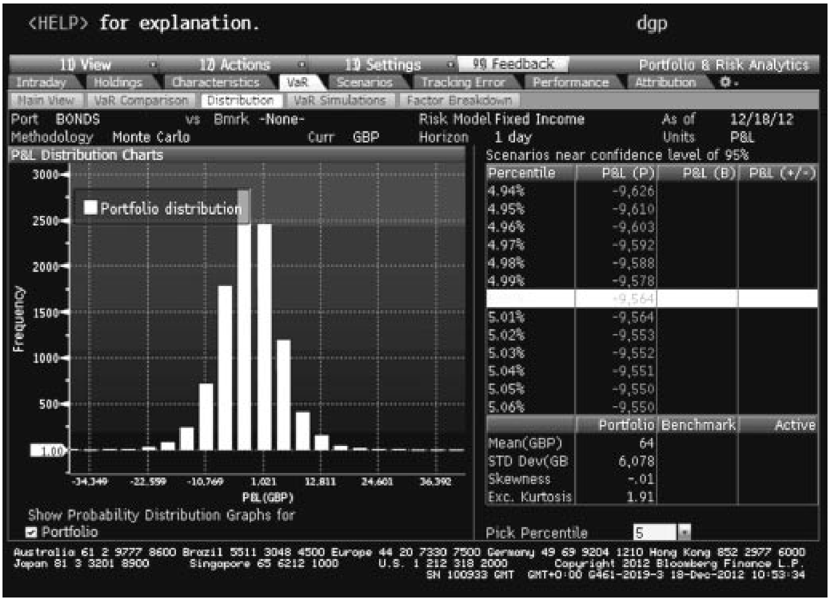

Figure 3.5 PORT screen showing the VaR distribution of values by selected confidence interval

becomes slightly subjective. The other interesting observation is the 2-year historical VaR exceeding the 3-year historical VaR. This forces the question, what time horizon is most appropriate if selecting historical VaR as the preferred methodology? The answer is not clear-cut, and of course there is no one right answer. Banks generally tend to obtain a feel for their preferred approach based on a number of market and operational factors.

Finally, Figure 3.5 is the distribution of VaR results by confidence interval selected. As expected we observe reducing estimates the closer we decrease towards 95% c.i., and this trend continuing the further below 95% c.i. the user falls. It also shows the resulting P&L distribution.

SUMMARY

The different VaR calculations all produce different estimates for VaR from the same data. We illustrated how matrices are used to calculate VaR in the variance–covariance approach, which is the one used by RiskMetrics. The benefits of using matrices are that the user can set up a weighting matrix, containing all the assets in the trading book, a correlation matrix, which contains the correlations of one asset with all the other assets in the portfolio, and a volatility matrix. The user then multiplies the matrices to calculate the standard deviation of the portfolio, which is then used to obtain the portfolio VaR.

The variance–covariance approach is based on the use of matrices to calculate VaR. In theory, using this approach the benefits of diversification are captured because the VaR of the portfolio as a whole tends to be lower than the sum of all the individual VaR numbers. In our illustration we saw that the order in which matrices are multiplied is important, as is their composition. For instance, in the volatility matrix the standard deviations are placed in the diagonal, while the other cells contain '0' values. Thus, in a 10-asset portfolio, we would set a 10×10 matrix and enter the volatility values along the diagonals. The remaining cells in the matrix would contain '0's. The correlation matrix contains '1' values along the diagonals (because the correlation of an asset with itself is 1), and correlation coefficients in the other cells. Once we have calculated the variance–covariance matrix, we use the weighting matrix to calculate the portfolio standard deviation. The weighting matrix is always one row, while the number of columns will be the number of assets in the portfolio.

Possibly the easiest VaR method to use is the variance–covariance one. It is straightforward to implement because the datasets required in its calculation, for liquid currencies, are already available from the RiskMetrics website (*www.riskmetrics.com*). This holds volatility and correlation data for all major currencies and assets. A bank setting up a VaR system for the first time need only construct a weighting matrix for its assets, and then use the volatility and correlations available from the Internet. The drawbacks of the approach are that it assumes constant volatilities, and that asset returns follow a normal distribution. It has been observed that the occurrence of market crashes occurs more frequently than is implied by a normal distribution, causing the measurement to be biased. However, the approach is popular with practitioners because it is easy to implement and explain.

The historical approach is also easy to understand. In this method, risk managers keep an historical record of the daily P&L of the portfolio and then calculate the fifth percentile cut-off for a 95% VaR figure. The historical approach uses actual market data, unlike, say, RiskMetrics whose volatilities and correlations are estimates based on averages over a specified period of time. In extreme situations, such as market corrections or crashes, average values do not hold and so the variance–covariance approach will produce an unrealistic result. The historical method uses actual results to make its calculation, so recent past market events are picked up more accurately. The historical method also does not require any mapping of cash flows to grid points. The mapping approach is straightforward for plain vanilla instruments but makes certain assumptions that are not realistic for more exotic instruments such as options. The weakness of the historical approach is that it does not account for changes in portfolio weightings over time. However, this shortcoming can be overcome using a more complex method known as *historical simulation*. This approach not only uses the current composition of the portfolio, but also historical market data. As it is a simulation it requires a large amount of computer resources. Put simply, if a current portfolio was composed of 80% of Asset A and 20% of Asset B, the user would obtain the asset prices of A and B over a specified period in the past – say, the last 1,000 days – and for each day calculate the value of the portfolio, using 80 : 20 weightings and keeping them constant. The VaR measure is then calculated as before.

Chapter 4

VALUE-AT-RISK FOR FIXED INTEREST INSTRUMENTS

This chapter discusses market risk for fixed income products. The measures described in the first section are regarded as being for 'first-order risk'. Since the advent of value-at-risk (*VaR*), risk managers and traders generally use both types of measure to quantify market exposure. In this chapter we consider calculation of VaR for a fixed interest product. We also note that credit derivatives are now a significant segment of the fixed income markets.

FIXED INCOME PRODUCTS

We consider the basic building blocks of a bond, which we break down as a series of cash flows. Before that, we discuss essential background on bond pricing and duration.

Bond valuation

A vanilla bond pays a fixed rate of interest (coupon) annually or semi-annually, or very rarely quarterly. The *fair price* of such a bond is given by the discounted present value of the total cash flow stream, using a market-determined discount rate.

Yield to maturity (*YTM*) is the most frequently used measure of return from holding a bond. YTM is equivalent to the *internal rate of return* on the bond, the rate that equates the value of discounted cash flows on the bond to its current price. The YTM equation for a bond paying semi-annual coupons is:

$$P = \sum_{t=1}^{2T} \frac{C/2}{\left(1 + \frac{1}{2}r\right)^{t}} + \frac{M}{\left(1 + \frac{1}{2}r\right)^{2T}} \tag{4.1}$$

where
P = Fair price of bond;
C = Coupon;
M = Redemption payment (par);
T = Number of years to maturity;
r = Required rate of return on the bond.

The solution to this equation cannot be found analytically and has to be solved through iteration; that is, by estimating the yield from two trial values for r, then solving by using the formula for linear interpolation.

In practice, one can use the Excel function references '= PRICE' and '= YIELD' to work out a bond price or yield given one or the other.

Table 4.1 Spreadsheet calculation of bond price or yield using an Excel function reference.

B	C	D	E
3	**5% 2016 corporate bond**		
4			
5			
6	06/01/2006	settlement date	
7	15/02/2016	maturity date	
8	5%	coupon	
9	98.95	price	
10	100	par	
11	2	semi-annual coupon	
12	4	30/360 day-count	
13			
14			
15	YIELD	5.134%	0.05132928
16			
17	PRICE	98.95642289	
18			
19	DURATION	7.888769069	
20			

YIELD	=YIELD(C6,C7,C8,C9,C10,C11,C12)
PRICE	=PRICE(C6,C7,C8,E15,C10,C11,C12)
DURATION	=DURATION(C6,C7,C8,D15,C11,C12)

This is demonstrated in Table 4.1 for a plain vanilla semi-annual bullet bond.

While YTM is the most commonly used measure of yield, it has one major disadvantage. The effect of this means that in practice the measure itself will not equal the actual return from holding the bond, even if it is held to maturity. The disadvantage is that implicit in the calculation of YTM is the assumption that each coupon payment as it becomes due is reinvested at rate r. This is clearly unlikely, due to fluctuations in interest rates over time and as the bond approaches maturity.

The bond price equation has illustrated the relationship between a bond's price and discount rate (the yield measure). The percentage increase in price when yields decline is greater than the percentage decrease when yields rise. This is due to the convex relationship, when plotted on a graph, between price and yield.

The sensitivity of a bond to changes in interest rate is measured by *duration* and *modified duration*. Duration is the weighted average maturity of a bond, using the discounted cash flows of the bond as weights.

Duration

The measure of interest rate risk typically used by bond analysts is called 'duration'. Duration is defined as:

> **The weighted average time until the receipt of cash flows from an instrument, where the weights are the present values of cash flows.**

It was developed by Macaulay in 1938, and is sometimes referred to as 'Macaulay's duration'.

Formally, we can write duration as the following expression:

$$D = \frac{\sum_{t=1}^{n} \frac{tC_t}{(1+r)^t}}{P} \tag{4.2}$$

where D = Duration;
P = Price of the bond;
C_t = Cash flow at time t;
r = Yield to maturity.

In the case of a zero-coupon bond there is only one cash flow, the payment at maturity. Therefore, for zero-coupon bonds duration is always equal to the maturity of the bond.

Example 4.1 Duration calculation.

Consider a Eurobond with 8% coupon, maturing in 5 years' time and priced (present-valued) at 100, thus giving a YTM of 8%.

Present value is calculated using the standard formula:

$$\frac{C}{(1+r)n}$$

Cash flow	PV at 8% yield	Time (t)	$PV \times t$
8	7.41	1	7.41
8	6.86	2	3.72
8	6.35	3	19.05
8	5.88	4	23.52
108	73.50	5	367.51
	100.00		**431.21**

$$\text{Duration is } \frac{431.21}{100} \text{ which equals 4.31 years}$$

This also illustrates that for a coupon-bearing bond duration is always less than for the corresponding maturity zero-coupon bond. The 5-year coupon bond in our example has a duration of 4.31 years; the zero-coupon bond would have a duration of 5 years. Duration also varies with coupon, yield and maturity. Figure 4.1 illustrates the price sensitivity profile for a straight bond.

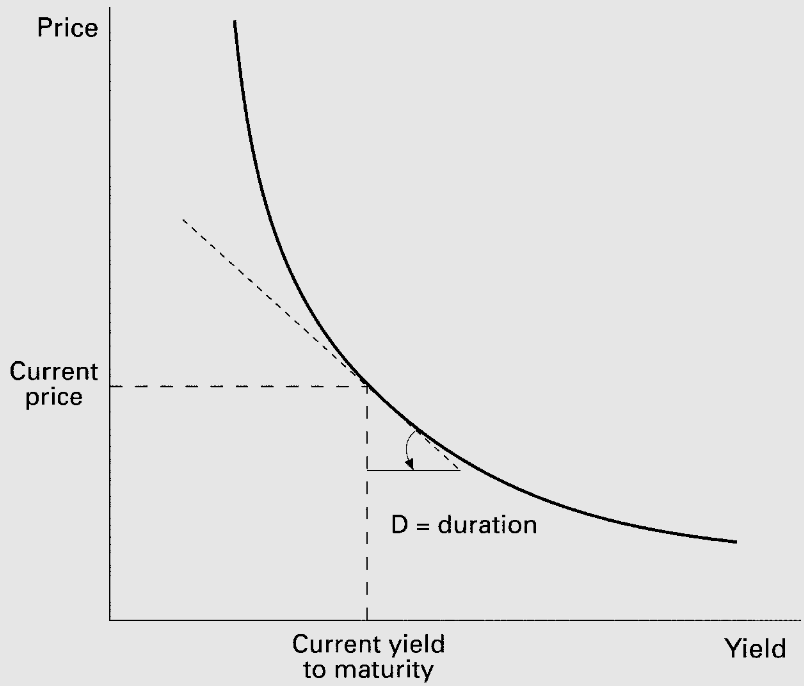

Figure 4.1 Bond price/yield profile.

Modified duration

Duration is important as it measures the interest rate elasticity of the bond price and is therefore a measure of interest rate risk. The lower the duration the less responsive is the bond's value to interest rate moves. *Modified duration* measures the sensitivity of the bond's price to changes in the yield curve. It is related to duration as follows:

$$MD = \frac{D}{1+r} \tag{4.3}$$

where MD = Modified duration;
r = Yield to maturity.

In our example where D is 4.31 years and the yield is 8%, we have:

$$MD = \frac{4.31}{1.08} = 3.99$$

Modified duration measures the proportionate impact on the price of the bond of a change in yield. In our example modified duration is 3.99; if yield rises by 1% the bond price falls by 3.99%.

The duration and modified duration formulae are obtained via a Taylor expansion. This is described in the Appendix (p. 179).

Convexity

Duration is regarded as a first-order measure of interest rate risk: it measures the *slope* of the present value profile (Figure 4.1). *Convexity* is a second-order measure of interest rate risk: it measures the curvature of the present value profile. Convexity describes how a bond's modified duration changes with respect to interest rates. It is approximated by the following expression:

$$\text{Convexity} = 10^8\left(\frac{\Delta P'_d}{\Delta P_d} + \frac{\Delta P''_d}{P_d}\right) \tag{4.4}$$

where $\Delta P'_d$ = Change in bond price if yield increases by 1 basis point (0.01);
$\Delta P''_d$ = Change in bond price if yield decreases by 1 basis point.

It can be shown that convexity decreases with coupon and yield. It becomes a useful measure when considering larger changes in yield. Duration, as a linear approximation to a curved present value profile, is a reasonable estimate for small changes in yield. For large moves this linear approximation will be inaccurate if convexity is high. A bond with higher convexity will outperform one of lower convexity whatever happens to interest rates. The price of such a bond will fall by less for a given rise in yield, and will rise by more if yields fall, than that of a bond of lower convexity.

INTEREST RATE PRODUCTS

For newcomers we introduce a simple forward rate instrument as a prelude to discussing a simple bond portfolio.

Forward rate agreements

A forward rate agreement (*FRA*) is a simple derivative contract that essentially allows banks or corporates to hedge against future interest rate exposure. Say that a corporate expects to borrow funds in 6 months' time, but at the rate prevailing at that time. To assist its budgeting process, it can fix the rate today (or, if it expects interest rates to worsen in 6 months' time, it can deal today) by buying an FRA. If rates indeed have risen in 6 months' time, the extra interest the corporate will pay on the loan will be compensated by the money it receives on the FRA.[1] No money changes hands at the start of an FRA transaction, only the difference in interest between the transacted rate and the rate at 'fixing' (maturity) is paid or received.

We illustrate FRAs with an example. Assume a corporate expects to receive £10 million in 3 months' time as working capital. These funds are not expected to be needed until 3 months after receipt, so they will be placed on deposit until then. The company can simply wait until then and deal, but let us suppose it expects interest rates to have dropped by then, and so wishes to lock in a deposit rate now. The bank it deals with will write a ticket for a forward deposit, and the rate it quotes will be the 3-month rate in 3 months' time; that is, the 3-month forward–forward rate. In FRA terms this would be known as a 'threes–sixes' or '3v6' FRA.

If the 3-month rate is 5.25% and the 6-month rate is 5.75%, the forward rate is the 'breakeven' rate for 3-month money in 3 months'

[1] 'Buying' a FRA is, in effect, 'borrowing' money, so if I buy a FRA at 6% and the rate on settlement is 7% I will have made the 1% difference because, in effect, I have fixed my borrowing at the lower rate.

Table 4.2 FRA rate calculation.

A1	B	C	D	E	F	G	H
2	3-month rate	5.25%					
3	6-month rate	5.75%					
4							
5			Days		Cashflow	Borrow for 3 months	Depo for 6 months
6	Today	09-Jan-06				£9,870,800.68	–£9,870,800.68
7		10-Apr-06	91		£10,000,000.00	£10,000,000.00	
8		10-Jul-06	182		–£10,153,808,71		£10,153,808.71
9							
10							
11					Implied return	1.5381%	
12					Days	91	
13					Forward–forward rate	6.1693%	
14							
15							

3-month 5.25% 6.169% fwd-fwd rate

6-month

5.75%

Table 4.3 Excel spreadsheet formulae for Table 4.1.

A1	B	C	D	E	F	G	H
2	3-month rate	5.25%					
3	6-month rate	5.75%					
4							
5			Days		Cashflow	Borrow for 3 months	Depo for 6 months
6	Today	09-Jan-06				=F7/(1+D7/365*C2)	=-G6
7		=C6+91	91		£10,000,000.00	=G6*(1+C2*D7/365)	
8		=C6+182	182		=-H8		=-H6*(1+D8/365*C3)
9							
10							
11					Implied return	=F8/-F7-1	
12					Days	=D8-D7	
13					Forward–forward rate	=365/G12*G11	
14							
15							

time. The calculation for this is shown in Table 4.2. Think of the calculation in terms of what the bank would do to hedge this exposure: it would borrow £9,870,800.68, which is the present value of £10 million in 3 months' time, and place this on deposit for 6 months. The FRA rate (assuming no bid–offer spreads) is 6.17% and is shown in cell G13 in Table 4.2. The Excel formulae are shown in Table 4.3 (previous page).

Fixed income portfolio

A bond is a series of cash flows (the coupons) and a final coupon and redemption payment. We can view the package as a series of FRAs, in effect. We illustrate bonds by looking at an hypothetical simple bond portfolio comprised of three bonds. This is shown at Table 4.4. Assume the date is 9 January 2006 and the bonds have precise terms to maturity.

Table 4.5 (see p. 69) shows the bond cash flows, their present values based on an assumed zero-coupon term structure and the total market value of the portfolio. Table 4.6 shows the undiversified VaR for this portfolio, which is simply the total of the present values multiplied by their volatility.

Table 4.4 Hypothetical Eurobond portfolio.

	5% Eurobond 2008	*6% Eurobond 2007*	*4.25% Eurobond 2010*
Holding	\$10 million	\$5 million	\$5 million

Table 4.6 Undiversified VaR for portfolio.

Cash flow date	*Cash flows*	*Present values*	*Volatilities*	*Undiversified VaR*
1	6,012,500.00	5,726,190.48	0.315%	18,037.50
2	10,712,500.00	9,670,448.71	0.335%	32,396.00
3	212,500.00	180,967.90	0.374%	676.82
4	212,500.00	169,276.04	0.407%	688.95
5	5,212,500.00	3,895,083.23	0.528%	20,566.04
			Totals	72,365.31

Table 4.5 Bond portfolio valuation.

Term structure	
1y	5.00%
2y	5.25%
3y	5.50%
4y	5.85%
5y	6.00%

	Cash flows			**Discount factors**	**Present values**			**Totals**
Cash flow date	5% 2008	6% 2007	4.25% 2010		5% 2008	6% 2007	4.25% 2010	
1	500,000.00	5,300,000.00	212,500.00	0.9523810	476,190.48	5,047,619.05	202,380.95	5,726,190.48
2	10,500,000.00		212,500.00	0.9027257	9,478,619.51	0.00	191,829.20	9,670,448.71
3			212,500.00	0.8516137			180,967.90	180,967.90
4			212,500.00	0.7965931			169,276.04	169,276.04
5			5,212,500.00	0.7472582			3,895,083.23	3,895,083.23
				Totals	9,954,809.98	5,047,619.05	4,639,537.32	
				Portfolio total				**19,641,966.35**

To calculate the diversified VaR we apply the matrix technique described in the previous chapter, using bond correlations to obtain the portfolio variance and then the portfolio VaR.

APPLYING VaR FOR A FRA

The VaR calculation for a FRA follows the same principles reviewed in Chapter 3. As we saw earlier the derivation of a FRA rate is based on the principle of what it would cost for a bank that traded one to hedge it; this is known as the 'breakeven' rate. So a bank that has bought 3v6 FRA (remember, this is called a 'threes–sixes FRA') has effectively borrowed funds for 3 months and placed the funds on deposit for 6 months. Therefore, a FRA is best viewed as a combination of an asset and a liability, and that is how it is valued. So, a long position in a 3v6 FRA is valued as the present value of a 3-month cash flow asset and the present value of a 6-month cash flow liability, using the 3-month and 6-month deposit rates. The net present value is taken, of course, because one cash flow is an asset and the other a liability.

Consider a 3v6 FRA that has been dealt at 5.797%, the 3-month forward–forward rate. The value of its constituent (notional) cash flows is shown in Table 4.7. The 3-month and 6-month rates are cash rates in the market, while interest rate volatilities have been obtained from the RiskMetrics website. The details are summarised in Table 4.7.

The undiversified VaR is the sum of individual VaR values, and is £34,537. It has little value in the case of a FRA, however, and would overstate the true VaR, because a FRA is made up of a notional asset and liability, so a fall in the value of one would see a rise in the value of the other. Unless a practitioner was expecting 3-month rates to go in an opposite direction to 6-month rates, there is an element of

Table 4.7 Undiversified VaR for 3v6 FRA.

Cash flow	*Term* (days)	*Cash rate*	*Interest rate volatilities*	*Present value*	*Undiversified VaR*
10,000,000	91	5.38%	0.14%	9,867,765	13,815
10,144,536	182	5.63%	0.21%	9,867,765	20,722

diversification benefit. There is a high correlation between the two rates, so the more logical approach is to calculate a diversified VaR measure.

For an instrument such as a FRA, the fact that the two rates used in calculating the FRA rate are closely positively correlated will mean that the diversification effect will be to reduce the VaR estimate, because the FRA is composed notionally of an asset and a liability. From the values in Table 4.7, therefore, the 6-month VaR is actually a negative value (if the bank had sold the FRA, the 3-month VaR would have the negative value). To calculate the diversified VaR then requires the correlation between the two interest rates, which may be obtained from the RiskMetrics dataset. This is observed to be 0.87. This value is entered into a 2×2 correlation matrix and used to calculate the diversified VaR in the normal way. The procedure is:

- transpose the weighting VaR matrix to turn it into a 2×1 matrix;
- multiply this by the correlation matrix;
- multiply the result by the original 1×2 weighting matrix;
- this gives us the variance; the VaR is the square root of this value.

The result is a diversified VaR of £11,051. The matrix procedure is shown at Figure 4.2.

VaR weighting transposed | Correlation matrix

13,815	−20,722	1	0.87	−4,213.14	−8,702.95
		0.87	1		

VaR weighting

−4,213.14	−8,702.95	13,815	1.22E+08
		−20,722	

VaR	11,051.61

11,051.61

Figure 4.2 Diversified VaR calculation for 3v6 FRA, correlation coefficient 0.87.

VaR FOR AN INTEREST RATE SWAP

An interest rate swap is essentially a series of FRAs. To calculate a variance–covariance VaR for an interest rate swap, we use the process described earlier for a FRA. There are more cash flows that go to make up the undiversified VaR, as the swap is a strip of FRAs. In a plain vanilla interest rate swap, one party pays fixed rate interest on an annual or semi-annual basis, and receives floating rate interest, while the other party pays floating rate interest payments and receives fixed rate interest. Interest payments are calculated on a notional sum, which does not change hands, and only interest payments are exchanged. In practice, it is the net difference between the two payments that is transferred.

The fixed rate on an interest rate swap is the breakeven rate that equates the present value of the fixed rate payments to the present value of the floating rate payments; as the floating rate payments are linked to a reference rate such as the London Interbank Offered Rate (*LIBOR*), we do not know what they will be, but we use the forward rate applicable to each future floating payment date to calculate what it would be if we were to fix it today. The forward rate is calculated from zero-coupon rates today. A 'long' position in a swap is to pay fixed and receive floating, and is conceptually the same as being short in a fixed-coupon bond and being long in a floating-rate bond; in effect, the long is 'borrowing' money, so a rise in the fixed rate will result in a rise in the value of the swap. A 'short' position is receiving fixed and paying floating, so a rise in interest rates results in a fall in the value of the swap. This is conceptually similar to a long position in a fixed rate bond and a short position in a floating rate bond.

Describing an interest rate swap in conceptual terms of fixed and floating rate bonds gives some idea as to how it is treated for VaR purposes. The coupon on a floating rate bond is reset periodically in line with the stated reference rate, usually LIBOR. Therefore, the duration of a floating rate bond is very low, and conceptually the bond may be viewed as being the equivalent of a bank deposit, which receives interest payable at a variable rate. For market risk purposes,[2] the risk exposure of a bank deposit is nil, because its present value is not affected by changes in market interest rates. Similarly, the risk

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[2] We emphasise for *market* risk purposes; the credit risk exposure for a floating rate bond position is a function of the credit quality of the issuer.

Table 4.8 Fixed rate leg of 5-year interest rate swap and undiversified VaR.

Pay date	*Swap rate* (%)	*Principal* (£)	*Coupon* (£)	*Coupon present value* (£)	*Volatility* (%)	*Undiversified VaR*
07-Jun-00	6.73	10,000,000	337,421	327,564	0.05	164
07-Dec-00	6.73	10,000,000	337,421	315,452	0.05	158
07-Jun-01	6.73	10,000,000	335,578	303,251	0.10	303
07-Dec-01	6.73	10,000,000	337,421	294,898	0.11	324
07-Jun-02	6.73	10,000,000	335,578	283,143	0.20	566
09-Dec-02	6.73	10,000,000	341,109	277,783	0.35	972
09-Jun-03	6.73	10,000,000	335,578	264,360	0.33	872
08-Dec-03	6.73	10,000,000	335,578	256,043	0.45	1,152
07-Jun-04	6.73	10,000,000	335,578	248,155	0.57	1,414
07-Dec-04	6.73	10,000,000	337,421	242,161	1.90	4,601
					Total	**10,526**

exposure of a floating rate bond is very low and to all intents and purposes its VaR may be regarded as 0. This leaves only the fixed rate leg of a swap to measure for VaR purposes.

Table 4.8 shows the fixed rate leg of a 5-year interest rate swap of the following terms:

Trade date	3 December 1999
Effective date	7 December 1999
Maturity	7 December 2004
Nominal	GBP10 million
Fixed rate	6.73%
Day-count	Act/365
Semi-annual	

To calculate the undiversified VaR we use the volatility rate for each term interest rate; this may be obtained from the RiskMetrics website, for instance. Below we show the VaR for each payment; the sum of all the payments constitutes the undiversified VaR. We then require the correlation matrix for the interest rates, and this is used to calculate the diversified VaR. The weighting matrix contains the individual term VaR values, which must be transposed before being multiplied by the correlation matrix.

Using the volatilities and correlations supplied by RiskMetrics the diversified VaR is shown to be £10,325. This is very close to the undiversified VaR of £10,526. This is not unexpected because the different interest rates are very closely correlated. The matrices are shown at Figure 4.3 (see opposite).

Using VaR to measure market risk exposure for interest rate products enables a risk manager to capture non-parallel shifts in the yield curve, which is an advantage over the traditional duration measure and interest rate gap measure. Therefore, estimating a book's VaR measure is useful not only for the trader and risk manager, but also for senior management, who by using VaR will have a more accurate idea of the risk market exposure of the bank. The VaR methodology captures pivotal shifts in the yield curve by using the correlations between different maturity interest rates; this reflects the fact that short-term interest rates and long-term interest rates are not perfectly positively correlated.

The weighting matrix W is composed of the individual VaR values for each interest rate period in the swap. This is shown in Figure 4.4.

In order to multiply this by the correlation matrix C it needs to be transposed, and this is shown as Figure 4.3(i), the correlation matrix. These data may be obtained from RiskMetrics direct or downloaded from *http://www.Riskmetrics.com*

164
158
303
324
566
972
872
1,152
1,414
4,601

Figure 4.4 Interest rate swap weighting matrix.

(i) **WV transpose**

164	158	303	324	566	972	872	1152	1414	4601

Correlation matrix

1	0.89	0.91	0.92	0.94	0.87	0.91	0.89	0.95	0.97
0.89	1	0.97	0.96	0.95	0.97	0.89	0.91	0.98	0.93
0.91	0.97	1	0.96	0.95	0.94	0.98	0.91	0.92	0.92
0.92	0.96	0.96	1	0.95	0.94	0.95	0.97	0.98	0.97
0.94	0.95	0.95	0.95	1	0.92	0.93	0.93	0.94	0.96
0.87	0.97	0.94	0.94	0.92	1	0.97	0.98	0.95	0.94
0.91	0.89	0.98	0.95	0.93	0.97	1	0.89	0.91	0.95
0.89	0.91	0.91	0.97	0.93	0.98	0.89	1	0.95	0.97
0.95	0.98	0.92	0.98	0.94	0.95	0.91	0.95	1	0.96
0.97	0.93	0.92	0.97	0.96	0.94	0.95	0.97	0.96	1

(ii) **WVC**

9882.823	9880.203	9806.312	10165.1	9990.291	10022.81	9920.518	10094.76	10082.95	10262.1

The usual procedure is then followed, with the WVC matrix multiplied by the weightings matrix; this gives us the variance, from which we calculate the VaR to be £10,325, as shown below:

WCVW	106614498.1
VaR	10325.42968

Figure 4.3 Example of diversified VaR calculation for a 5-year interest rate swap.

APPLYING VaR FOR A BOND FUTURES CONTRACT

As we noted in Chapter 3, there is more than one way to implement VaR. Most methodologies revolve around estimation of the statistical distribution of asset returns. The main approaches are the variance–covariance methods or *simulation* (or Monte Carlo) methods. Parametric VaR assumes that the distributions of net asset returns are normal. Therefore, the variance–covariance matrix describes the distribution completely. The parametric approach can be summarised by the equation below which we have already encountered; σ_p^2 is the volatility of returns of the portfolio being measured:

$$\sigma_p^2 = \sum (a_i \cdot \sigma_i)^2 + \sum_{i \neq j} \sum_{i \neq j} a_i \cdot a_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j \tag{4.5}$$

The equation shows that portfolio risk, as expressed by its variance, is a function of the variance of the return on each instrument in the portfolio as well as on the correlations between each pair of returns. Unless the returns in the portfolio are perfectly correlated (all $\rho_{ij} = 1$) the variance on the portfolio does not equal the simple sum of the variances of the individual positions.

Calculation illustration

We will now illustrate a VaR calculation for the long gilt bond futures contract as traded on LIFFE.

The position in this illustration is a June 1998 long gilt futures contract purchased on 24 May 1998. Assume the closing price that day was 110-00. The contract represents £50,000 nominal of bonds, hence each £1 change in the futures price results in a £500 change in the value of the position (tick value is £15.625, there are 32 ticks per £1).

VaR is estimated in terms of returns (conceptually similar to prices). The return is calculated as:

$$R_t = \frac{P_t - P_t - 1}{P_{t-1}} \cdot 100$$

where R = Daily return;
P = Price of the instrument.

Table 4.9 Daily price returns.

Date	*Futures price*	*Daily return* (%)
24 May 1998 (today)	110-00	
23 May 1998	109-13	0.524702
22 May 1998	110-05	−0.68085
21 May 1998	110-18	0.541928
20 May 1998	109-25	−0.171086
⋮		
31 May 1997	111-08	0.674157
30 May 1997	111-04	0.112486

The daily returns for the bond futures over the last 12 months are shown in Table 4.9.

To calculate the 1-day VaR of this position we need to estimate the mean of the daily returns and the volatility, as measured by the standard deviation. As our portfolio consists of this one position only we do not need to consider correlations.

Assuming the returns follow the normal distribution, 95% of all returns will fall within 1.96 standard deviations of the mean return (using a double-sided, or two-tailed, estimation). If we require a higher confidence level – say, 98% – this will be covered within 2.33 standard deviations of the mean return.

In our example for the long gilt future, we calculate the following:

$$\text{Mean} = -0.00224\%$$

$$\text{Standard deviation} = 0.605074\%$$

This means that for our desired confidence level of 98% all returns would fall between −1.4098% and 1.4098% (rounded to 2 decimal points giving −1.41% and 1.41%) and only 1% of returns will be lower than −1.41% (see Figure 4.5).

The range is obtained by multiplying the standard deviation by 2.33 (i.e., $0.605\,074 \times 2.33 = 1.4098$), the number of standard deviations required to give us our 98% confidence level.

To convert the negative return of 1.41% to a pounds sterling amount we take the price of the future for the day we are calculating the VaR, which is 110-00.

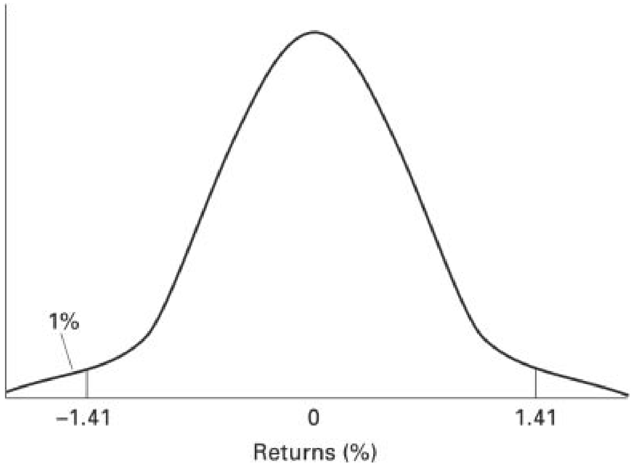


Figure 4.5 1% confidence interval for VaR calculation.

From this we calculate a 1-day VaR at the 98% probability level (two-tail, representing a price fall) to be:

$$\frac{1.41}{100} \times 110 \times £500 = £775.5$$

If the VaR estimate is accurate the daily loss on this position will exceed £775.5 on no more than 1 day out of 100.

The risk manager may feel that the 1-day period is too short and that a 1-week holding period is more appropriate. Assuming that the returns are serially independent – that is, a return on one day does not affect the return on any other day – then a property of the distribution is that the standard deviation increases proportionately with the square root of time. Therefore, if the 1-day standard deviation of returns is 0.605 074%, the standard deviation for 1 week (five business days) is:

$$\sqrt{5} \times 0.605\,074 = 1.3530\%$$

This gives us a 1-week VaR at the 98% probability level of:

$$\sqrt{5} \times £775.5 = £1{,}734.07$$

This means that if we held the position for 1 week we should not expect to lose more than £1,734 more often than 1 week in 100.

The assumptions of normality and serial independence that we make about the distribution of returns that underlines this analytic method – of which RiskMetrics is perhaps the best known – allow us to calculate the VaR using any volatility and correlation for any holding period. We in fact do not require historical returns themselves, which are used in the historical approach.

THE HISTORICAL METHOD

Much empirical research on the statistical properties of asset returns has found deviations from normality. Returns tend to exhibit kurtosis – that is, they are more peaked around the mean and have fatter tails than the normal distribution. Some asset returns tend to be skewed to the left; this indicates a greater incidence of unusually large negative returns, such as crashes, than would be suggested by a normal distribution. For our example we can construct a frequency distribution of the daily returns for the long gilt future between May 1997 and May 1998. The resulting histogram is shown at Figure 4.6, which shows the returns approximating a normal distribution.

A normal curve has been superimposed on the histogram for comparison. The returns do indeed exhibit the typical pattern found in many asset returns, fat tails and left-skewness. It is possible to calculate VaR without assuming normality and this is the approach employed by the historical method. This involves

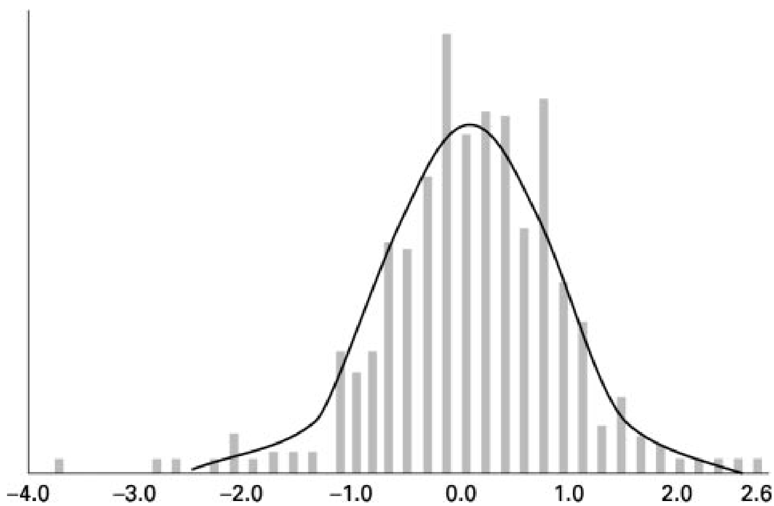


Figure 4.6 The normal approximation of returns.

finding the lowest returns in real historical data. To calculate VaR at the 1% (98%, two-tailed) probability level we need to rank the daily returns and identify the lowest 1% of returns. The first percentile (i.e., the first 1%) return is found to be -1.73%. This gives us a daily VaR of:

$$\frac{1.73}{100} \times 110 \times £500 = £951.50$$

This is almost 23% greater than the VaR estimate using the analytical method.

If we wish to recalculate the VaR for a different holding period, without making the assumption of serial independence, we cannot simply multiply the 1-day VaR by the square root of the time involved. Instead, we have to recalculate all returns for the new holding period (i.e., the weekly returns over a year, say), construct the new frequency distribution and identify the value for the appropriate percentile ranges.

SIMULATION METHODOLOGY

It may not be appropriate to calculate VaR directly by establishing the probability distribution of returns on the instrument itself, as we did for the long gilt future. This occurs because:

- for a large or complex portfolio it is impossible or impractical to maintain historical data on all the instruments involved;
- historical data are unavailable for many instruments, especially customised ones.

In such cases the historic dataset used to calculate VaR will consist of returns not on the instruments themselves, but on their 'risk factors'. There are other instruments or factors that influence their values. For example, for a domestic bond the risk factor is the interest rate; and for equity derivatives the risk factor is the value of the related index, such as the FTSE 100.

In such cases we can improve on the pure historical approach by using 'historic simulation'. Instead of looking at the volatility of actual portfolio returns in the past we can simulate past portfolio returns by using the actual values of the risk factors and the current portfolio composition. We then construct the frequency distribution of the simulated portfolio returns by ranking them into percentiles and determining the VaR at the chosen confidence level.

Volatility over time

By calculating the volatility of daily returns we assume that this volatility is constant throughout the year. Of course, volatility can and does change over time and it may make sense to give more weight to recent observations in forecasting future volatility.

In our example, had we used only the last two months of returns rather that a full year, we would have found the standard deviation to be 0.6513 rather than 0.605 074. This would have resulted in a VaR of £834.63, rather than £775.5, as shown below:

$$\begin{aligned} 0.6513 \times 2.33 &= 1.5175 \\ &= \frac{1.5175}{100} \times 110 \times £500 \\ &= £834.63 \end{aligned}$$

One way to estimate volatility is through the exponential weighting of observations. This emphasises more recent observations at the expense of more distant ones, because the weights attached to past observations decline over time. The volatilities and correlations are updated every day in accordance with the most recent data, as the earliest observation is dropped from the historical series and the newest ones are added.

The formula for the standard deviation (σ) of the daily return (R) and mean return (m) with exponential weights based on a historical period of N days is given as:

$$\sigma = \sqrt{(1-\lambda)\sum_{i=1}^{N} \lambda^i (R_{N-i} - \mu)^2}$$

The parameter λ is known as the decay factor; it determines how fast the weight on past observations decays. The higher the λ, the slower is the rate of decay and the more weight is given to more distant observations. This would have the effect of reducing the VaR in our example.

Application

Using decay factors enables us to incorporate recent trends in volatility. If volatility has recently been trending higher, the portfolio VaR is higher if a decay factor is used at all (higher decay then reduces this number).

Not using a decay factor has the disadvantage of failing to account fully for recent changes in volatility but the advantage of being more stable and less vulnerable to irregularities in a few recent returns.

It is advisable to reduce the decay factor after a significant market correction or crash; otherwise, a risk report will place excessive reliance on recent high volatility, which would be likely to reduce following a downward correction.

The RiskMetrics dataset incorporates a fixed decay factor of 6%, meaning that each day's volatility only counts as 94% the day after. Bloomberg offers a VaR system on its terminal based on RiskMetrics but with the option of setting a decay factor of between 0% and 9%.

BLOOMBERG SCREENS

Bloomberg users can use the screen PVAR to calculate the 1-year VaR for a bond portfolio. The portfolio must be set up first. For illustration we set up an hypothetical portfolio comprised of the following securities:

- £10 million nominal UK Treasury 5% 2014;
- £10 million nominal UK Treasury 5% 2012;
- £10 million cash.

The portfolio is shown at Figure 4.7, which is the portfolio page obtained as part of the menu for the PVAR page. This shows the market value of each of the securities; the market value of the cash is of course unchanged (the 'price' of 1.737 is the USD/GBP exchange rate). The total market value as at 28 December 2005 is £31,373,392. The VaR for each portfolio component is shown alongside.

The actual 1-year VaR is shown at Figure 4.8. This is part of the PVAR screen menu, obtained by running RPT <go>. We see that the VaR for differing probability of loss figures is given. The highlighted 5% value reflects the fact that a 95% c.i. is the most commonly calculated number.

The last exhibit, Figure 4.9, shows the one-currency VaR, in this case in pounds sterling.

1 N200 Client RPT
Screen Printed
PORTFOLIO VALUE-AT-RISK REPORT PAGE 1/ 1

Portfolio: U2251574-2 GILTS
Holder Name: KBC FINANCIAL PRODUCTS UK LTD. Evaluation Date: 12/28/2005
Base Currency: BRITISH POUND Horizon Date: 1/11/2006
# of Issues : 3 of 3 Probability of Loss: 5.000 %

Total Mkt Value (GBP): 31,373,392 Total VaR (GBP): 234,965
VaR from FX rate: Sum of security VaRs 235,555
X-cur risk reduction: Intracurrency risk reduction: 590

0<PAGE> for Component VaR

Security	Price	Mkt Val (GBP)	VaR (GBP)	% VaR
1)UKT 5 09/07/14	106.140	10,768,696	135,995	1.263
2)UKT 5 03/07/12	104.500	10,604,696	99,560	0.939
3)GBP	1.737	10,000,000	0	0.000

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2005 Bloomberg L.P.
0 28-Dec-05 11:47:20

Figure 4.7 Hypothetical gilt portfolio set up on Bloomberg, as of 28 December 2005.

Menu N200 Client RPT
Screen Printed
PORTFOLIO VALUE-AT-RISK REPORT PAGE 1/ 3

Portfolio: U2251574-2 GILTS
Holder Name: KBC FINANCIAL PRODUCTS UK LTD. Evaluation Date: 12/28/2005
Base Currency: BRITISH POUND Horizon Date: 1/11/2006
# of Issues : 3 of 3 Data set: Bloomberg 0% 12/27/2005

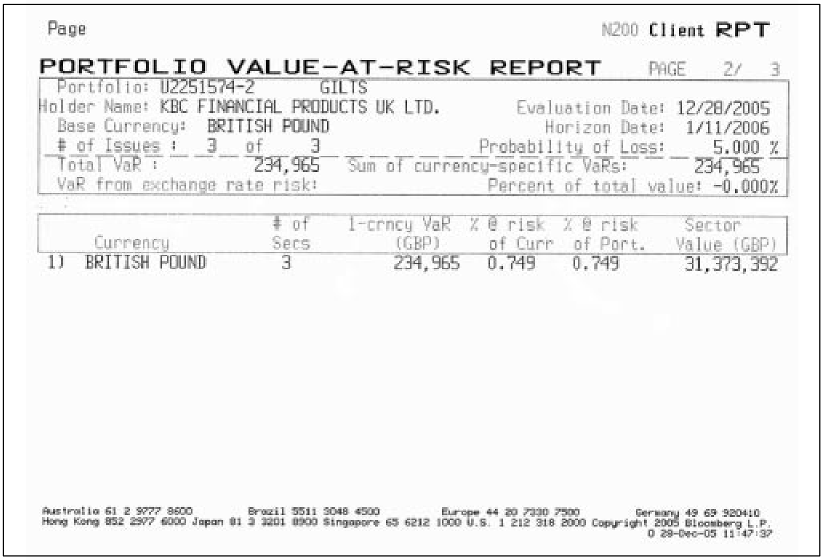
Total Market Value of included securities:
31,373,392 GBP

Probability of Loss	Value-At-Risk	% of total Mkt Val
0.10 %	441,435	1.407
0.20 %	411,141	1.310
0.50 %	367,954	1.173
1.00 %	332,316	1.059
2.00 %	293,375	0.935
5.00 %	**234,965**	**0.749**
10.00 %	183,068	0.584
20.00 %	120,224	0.383

Type 1<Go> to see individual security VaRs.
Type 2<Go> to see securities not included in the analysis.
<Page Fwd> to see breakdowns by single-currency subportfolios.

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2005 Bloomberg L.P.
0 28-Dec-05 11:47:31

Figure 4.8 Portfolio 1-year VaR.



Page N200 Client RPT

PORTFOLIO VALUE-AT-RISK REPORT PAGE 2/ 3

Portfolio: U2251574-2 GILTS
Holder Name: KBC FINANCIAL PRODUCTS UK LTD. Evaluation Date: 12/28/2005
Base Currency: BRITISH POUND Horizon Date: 1/11/2006
# of Issues : 3 of 3 Probability of Loss: 5.000 %
Total VaR : 234,965 Sum of currency-specific VaRs: 234,965
VaR from exchange rate risk: Percent of total value: -0.000%

	Currency	# of Secs	1-crncy VaR (GBP)	% @ risk of Curr	% @ risk of Port.	Sector Value (GBP)
1)	BRITISH POUND	3	234,965	0.749	0.749	31,373,392

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2005 Bloomberg L.P.
0 28-Dec-05 11:47:37

Figure 4.9 Portfolio one-currency VaR.

Chapter

5

OPTIONS: RISK AND VALUE-AT-RISK

It was the increasing use of trading instruments exhibiting non-linear characteristics such as options that was the prime mover behind the development and adoption of value-at-risk type methodologies, as traditional risk measures such as modified duration were deemed to be less and less adequate. This chapter introduces the application of VaR to option instruments. First though, we look at the Black–Scholes option pricing model (*B-S model*).

OPTION VALUATION USING THE BLACK–SCHOLES MODEL

We begin with a brief overview of the valuation of options. For a basic description of options we recommend *Financial Market Analysis* by David Blake, while Robert Kolb's *Futures, Options and Swaps* provides a good description of the B-S model.

Option pricing

The original valuation model, and still commonly used in the market for plain vanilla options, was the Black–Scholes model (*B-S*), which was first presented by its authors in 1973. The basic model is:

$$C = SN(\mathrm{d1}) - e^{-rt}XN(\mathrm{d2}) \tag{5.1}$$

where

$$\mathrm{d1} = \frac{\ln\left(\dfrac{S}{X}\right) + \left(r + \dfrac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$\mathrm{d2} = \mathrm{d1} - \sigma\sqrt{t}$$

and

C is the price of a call option;
S is the current price of the underlying asset;
X is the strike price;
r is the risk-free interest rate;
t is the time to expiry;
$N(\cdot)$ is the cumulative normal distribution function;
σ is the volatility of the underlying asset returns.

The expression $N(\mathrm{d1})$ uses the normal distribution to calculate the *delta* of the option, while N(d2) calculates the probability that the option will be exercised.

The B-S model provides a single formula to enable the fair price of a call option (and through the put–call parity theorem, a put option as well) to be calculated. The formula can be interpreted as measuring the expected present value of an option based on the key assumption that prices follow a log-normal distribution. Note that deriving the B-S model involves some involved mathematics and so will not be shown here!

For the valuation of a plain vanilla European call option, it is quite easy to set up a B-S calculation using Microsoft Excel. Consider the following call option details:

Underlying asset price	£20
Volatility	10.00%
Time to maturity	3 months
Strike price	£18
Interest rate	5.00%

The Excel calculation and cell formulae are given in Table 5.1. The price of the call option, which is in-the-money, is £2.23. The *delta* of the option is given by N(d1) and in the example is 0.99. It is related to the probability that the option will be exercised. The delta value is important with respect to the hedge that the option trader puts on for the option position.

Table 5.1 Call option valuation using the B-S model and Microsoft Excel.

Cell	E	F	H	I	
D7	**Option parameters**				**Cell formulas**
8	Asset price	20	ln(S/X)	0.015361	=LN(F8/F11)
9	Volatility	0.1	Adjusted return	0.01125	=(F12-F9^2/2)*F10
10	Time to expiry	0.25	Time adjusted vol	0.05	=F9*F10^0.5
11	Exercise price	18	d2	2.33221	=(I8+I9)/I10
12	Interest rate	0.05	N(d2)	0.990155	=NORMSDIST(I11)
13					
14					
15			d1	2.38221	=I11+I10
16			N(d1)	0.991395	=NORMSDIST(I15)
17					
18			Discount factor	0.987578	=EXP(-F10*F12)
19					
20			Call price	2.226514	= +F8*I16-F11*I12*I18

Volatility

Of the inputs to the B-S model, the variability of the underlying asset, or its volatility, is the most problematic. The distribution of asset prices is assumed to follow a log-normal distribution, because the logarithm of the prices is normally distributed (we assume log-normal rather than normal distribution to allow for the fact that prices cannot – as could be the case in a normal distribution – have negative values): the range of possible prices starts at 0 and cannot assume a negative value. Returns are defined as the logarithm of the price relatives and are assumed to follow the normal distribution such that:

$$\ln\left(\frac{S_t}{S_0}\right) \sim N(\mu t, \sigma\sqrt{t}) \tag{5.2}$$

where S_0 = Price at time 0;
S_t = Price at time t;
$N(m, s)$ = Random variable with both mean and standard deviation;
μ = Annual rate of return;
σ = Annualised standard deviation of returns;

and the symbol $\sim$ means 'is distributed according to'.

Volatility is defined in the equation above as the annualised standard deviation of returns (prices). Price relatives are calculated from the ratio of successive closing prices. Returns are then calculated according to the following equation as the logarithm of the price relatives:

$$\text{Return} = \ln\left(\frac{S_{t+1}}{S_t}\right) \tag{5.3}$$

where S_t = Market price at time t;
S_{t+1} = Price one period later.

The mean and standard deviation of returns follow standard statistical techniques using the following formula:

$$\mu = \sum_{i=I}^{N} \frac{x_i}{N} \quad \text{and} \quad \sigma = \sqrt{\sum_{i=I}^{N} \frac{(x-\mu)^2}{N-1}} \tag{5.4}$$

where x_i = ith price relative;
N = Total number of observations.

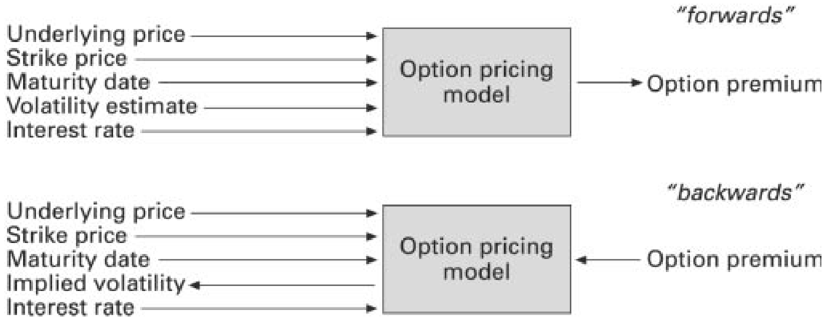


Figure 5.1 Model pricing and implied volatility.

This gives a standard deviation or volatility of daily price returns. To convert this to an annual figure, it is necessary to multiply it by the square root of the number of working days in a year, normally taken to be 250.

Calculations such as these produce a figure for *historic volatility*. What is required is a figure for *future* volatility, since this is relevant for pricing an option expiring in the future. Future volatility cannot be measured directly, by definition. Market-makers get around this by using an option pricing model 'backwards', as shown in Figure 5.1.

An option pricing model calculates the option price from volatility and other parameters. Used in reverse the model can calculate the volatility implied by the option price. Volatility measured in this way is called *implied volatility*. Evaluating implied volatility is straightforward using this method and generally more appropriate than using historic volatility.

THE GREEKS

As we noted in the previous section the price of an option depends on five variables. Of these the strike price is normally (but not always) fixed in advance. This leaves four variables. Option traders follow a number of quantities, each of which measures how the price of an option will change when one of the input parameters changes while the others remain unchanged. The measures are identified by letters from ancient Greek and so are known as the 'Greeks'; the main ones are summarised below.

Delta

The delta of an option is the change in premium (price) for a unit change in the underlying asset price. In the B-S model it is given by $N(\mathrm{d}1)$. It is the main measure of market risk, and its value drives the hedge. The premise behind the B-S model concept is that of the 'risk-neutral' portfolio that mirrors the option. The size of this risk-neutral portfolio is given by the delta. So, for example, from Table 5.1 we see that the delta is 0.991. This means that if the underlying asset goes up in price from £20 to £21, the value of the option will go up by £0.991. Therefore, if an options trader sells 1,000 call options on the underlying security, his delta-neutral portfolio will be long 991 of the security.

We saw how the delta was calculated in Table 5.1.

Gamma

Gamma is like the convexity of a bond, it is a measure of the rate of change of delta. That is, it is the change in *delta* for a unit change in the underlying asset price. Gamma is given by:

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{t}} \tag{5.5}$$

where the inputs are as before. The gamma of an option is always negative for the person who has written (sold) the option.

The existence of gamma means that an options portfolio must be delta-hedged on a regular basis – that is, *dynamically hedged* – because delta is always changing. The higher gamma is, the more often the hedge must be rebalanced, because it indicates a faster changing delta.

Table 5.2 illustrates calculation of gamma, using the same option parameters as Table 5.1, except we change the current underlying price from 20 to 21. This is to demonstrate the gamma value, which was almost identical to the value for $N'(\mathrm{d}1)$ at the price of 20 and may have confused readers. As before, we show the Excel formulae so that students can construct the spreadsheet themselves. Note we set *pi* with a value of 3.1416.

A value of 0.001 352 423 means that if the asset price increases by 1 (i.e., from 21 to 22) then the delta will increase by 0.001 35.

Table 5.2 Option gamma and Excel formula.

Cell	E	F	H	I	J	K
D7	**Option parameters**				**Cell formulas**	
8	Asset price S	21	ln(S/X)	0.15415068	=LN(F8/F11)	
9	Volatility	0.1	Adjusted return	0.01125	=(F12-F9^2/2)*F10	
10	Time to expiry	0.25	Time adjusted vol	0.05	=F9*F10^0.5	
11	Exercise price X	18	d2	3.308013597	=(I8+I9)/I10	
12	Interest rate	0.05	N(d2)	0.999530141	=NORMSDIST(I11)	
13			d1	3.358013597	=I11+I10	
14			N(d1)	0.999607422	=NORMSDIST(I13)	
15						
16			Coefficient	0.398941814	=(2*3.1416)^-0.5	
17			(d1^2/2)	5.638127657	=I13^2/2	
18			Exp-(d1^2/2)	0.003559527	=EXP(-I17)	
19			N′(d1)	0.001420044	=I16*I18	
20						
21			Gamma	0.001352423	=I19/(F8*I10)	

Option gamma, like delta, is at its highest for at-the-money options and will decrease as the option becomes in-the-money or out-of-the-money. Gamma exposure is not captured by some VaR measurement methods, and risk managers must therefore gamma-adjust their calculations. Any calculation for gamma, like that for convexity, is always an approximation and is a dynamic, not static, number.

Vega

Vega is the change in option premium for a unit change in volatility (usually 1%) of the underlying security. It is sometimes called *kappa*. Vega is given by:

$$Vega = S\sqrt{\Delta t}N(\mathrm{d}1) \tag{5.6}$$

The term $N(\mathrm{d}1)$ is the equation for the normal distribution, which is given by:

$$N(x) = \frac{1}{\sqrt{2\Pi}}e^{-x2/2} \tag{5.7}$$

Table 5.3 illustrates the calculation of vega for the same option seen in Table 5.1.

The vega value of 0.234 means that a 1% increase in volatility in the underlying security will produce a 0.234 increase in option premium. Similarly with gamma, vega sensitivity is most acute when the

Table 5.3 Option vega and Excel formula.

Cell	E	F	H	I	J	K
D7	**Option parameters**				**Cell formulas**	
8	Asset price	20	ln(S/X)	0.10536052	=LN(F8/F11)	
9	Volatility	10.00%	Adjusted return	0.01125	=(F12-F9^2/2)*F10	
10	Time to expiry	0.25	Time adjusted vol	0.05	=F9*F10^0.5	
11	Exercise price	18	d2	2.33221031	=(I8+I9)/I10	
12	Interest rate	0.05	N(d2)	0.99015521	=NORMSDIST(I11)	
13			d1	2.38221031	=I11+I10	
14			N(d1)	0.99139548	=NORMSDIST(I13)	
15						
16			Coefficient	0.39894181	=(2*3.1416)^-0.5	
17			(d1^2/2)	2.83746299	=I13^2/2	
18			Exp-(d1^2/2)	0.05857408	=EXP(-I17)	
19			N′(d1)	0.02336765	=I16*I18	
20						
21			Vega	0.2336765	=F8*F10^0.5*I19	

option is at-the-money. Option value increases with volatility, so being long vega is attractive to a trader if he is running a long position.

Other Greeks

The other Greeks include:

Theta The change in the premium for a unit change in the time to expiry (usually 1 day).

Rho The change in premium for a unit change in interest rates (usually 1%).

Lambda The percentage change in premium for a percentage change in the underlying asset price.

RISK MEASUREMENT

As well as using a VaR estimate, option trading desks employ a range of further risk measurement tools. We highlight some of these below.

Spot ladder

This report shows the portfolio value and Greeks for a ladder of values of the underlying asset (it is also known as an asset ladder). It facilitates analysis of a position affected by large market moves. The report measures the sensitivity of the portfolio to changes in the underlying cash market prices.

Maturity ladder

A report showing portfolio Greeks broken down into maturity buckets, measuring the maturity profile from 1 day out to expiry of the longest dated position. It is used to ensure that hedges are adapted to the expiry profile of the portfolio.

Across-time ladder

A report displaying spot ladder values for a range of future dates. It shows how the portfolio value changes as it matures (all else being equal). This enables traders to check whether the hedge is still effective over time.

Jump risk

Derivatives desks often produce reports for trading books showing the effect on portfolio value of a 1-bp move, along each part of the term structure of interest rates. For example, such a report would show that a change of 1 basis point in 3-month rates would result in a change in value of £*x* – this measure is often referred to as price variation per basis point, or sometimes as present value of a basis point (*PVBP*).

Jump risk refers to the effect on value of an upward move of 100 basis points for all interest rates – that is, for all points of the term structure. The jump risk figure is therefore the change in the value of the portfolio for a 1% parallel shift in the yield curve.

Table 5.4 shows an extract from a risk report with the PVBP for selected points along the term structure. The jump risk report will show the effect of a 1% interest rate move across all grid points; the sum of all value changes is the jump risk.

Table 5.4 PVBP per grid point: extract from risk report.

Grid point (days)	*PVBP* (£)
1	1
7	5
91	−1,658
183	928
365	500
730	−1,839
1,643	−944
3,650	1,365
7,300	0
9,125	0

Table 5.5 Jump risk report.

	Limits	*Total VaR*	*Jump risk*
AUD	3,500	1,312	−9,674
CHF	1,750	663	−7,802
DEM	5,000	3,969	−57,246
GBP	7,500	5,695	−74,215
JPY	150,000	49,563	−536,199
USD	4,500	3,339	−33,289
Total	172,250	64,541	−718,425

Table 5.5 shows an extract from the jump risk report for a currency options book of a major investment bank.

APPLYING VaR FOR OPTIONS

For a risk manager a portfolio's market risk may be quantified as a single VaR number; however, management of individual trading books often requires more data to understand risk sensitivities than are provided by VaR alone. This is particularly true for options, where the effect of sensitivity to the Greeks can be significant. The behaviour of options calls for some fine-tuning in applying VaR.

The standard assumption made by most VaR models is that returns (prices) are normally distributed. This allows VaR to be estimated. Returns, in fact, are not distributed normally in practice. Market crashes are more common in the real world than suggested by the normal distribution; it is this factor that calls for the use of stress-testing and scenario analysis. The returns on an option portfolio are also not usually normally distributed. Option instruments have a non-linear pay-out profile. This means that the relationship between a change in the underlying asset price and the resulting change in the option price is not constant. We have already referred to this relationship as the option's delta.

Consider an at-the-money call option on an equity index, which is initially set at a level of 1,020 points. If the index value drops by 10 points to 1,010, the value of the option decreases by roughly 5 points because it has about an equal chance of expiring in-the-money or out-of-the-money. But if the index drops a further 10 points to 1,000, the value of the option may drop by just 3 points, even though the index has moved the same amount as before. This is an example of non-linearity: the option delta is 50% when the index is at 1,020, but has changed to 30% when the index is at 1,010. The relationship between delta and the value of the underlying is the *gamma*. Non-linearity becomes more important as the option becomes more in-the-money and as it approaches expiry.

VaR models allow for delta, which is the linear component of option risk. This allows assumption of the normal distribution, despite inaccuracies. However, for banks running large option books with exposure to significant gamma risk these inaccuracies cannot be ignored. Option prices also are sensitive to the time remaining to expiry (theta) and the underlying volatility (vega). In addition, the Basel CAD II rules will stipulate that banks which write options must measure delta, gamma and vega risk when calculating capital to be held against them.

If the effects of gamma risk are introduced when calculating the potential returns in a portfolio, the distribution of returns becomes asymmetric, or skewed, as shown in Figure 5.2.

Other variables such as mean and variance may also differ from the normal distribution. The statistical measures that describe the normal distribution therefore do not apply. The solution to this is to include the effects of higher order risk sensitivities, such as gamma. The portfolio VaR can then be calculated on the basis of

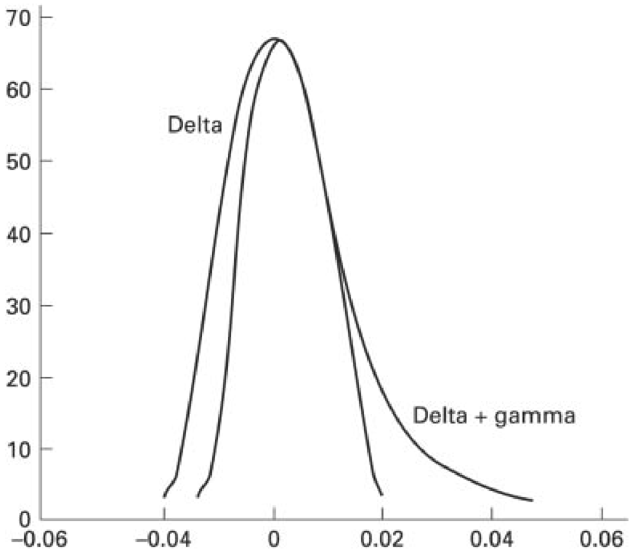


Figure 5.2 The effect of incorporating gamma risk on portfolio distributions.

this skewed distribution. There are three alternatives:

- the skewed distribution can be approximated to a deformed normal distribution;
- the 5th and 95th percentiles can be calculated for the skewed distribution;
- the skewed distribution can be fitted to a more general family of distributions whose statistical measures are known.

However, an analytic approximation to VaR ignores some of the elements which make up the market risk of the portfolio, including the effects of gamma and market crashes. To capture these effects requires other approaches, such as Monte Carlo or historical simulation.

Banks prefer simulation for the reasons mentioned above. The only drawback with simulation, given the large number of trials that are required, is that it is time-consuming and requires considerable computing power. There is, therefore, a trade-off between accuracy and speed of calculation.

Example 5.1 Option VaR.

Call option on £10 million March 1998 long gilt future:

Data on option

Price	= 2.938
Estimated delta (for change in yield)	= −380.2
Estimated gamma (for change in yield)	= 26,440
Estimated vega (for change in volatility of yield)	= 14.95

Risk calculations

(a) Market value of position		= £293,800
(b) The 98% daily VaR is		= £72,500, derived as follows:
	Delta risk	= £75,000
less	Convexity risk	= −£5,000
plus	Volatility risk	= £2,500
equals	Value-at-risk	= £72,500

The gamma effect on the distribution curve is illustrated in Figure 5.3. For a long position the gamma position is always positive, so the distribution is shifted to the right.

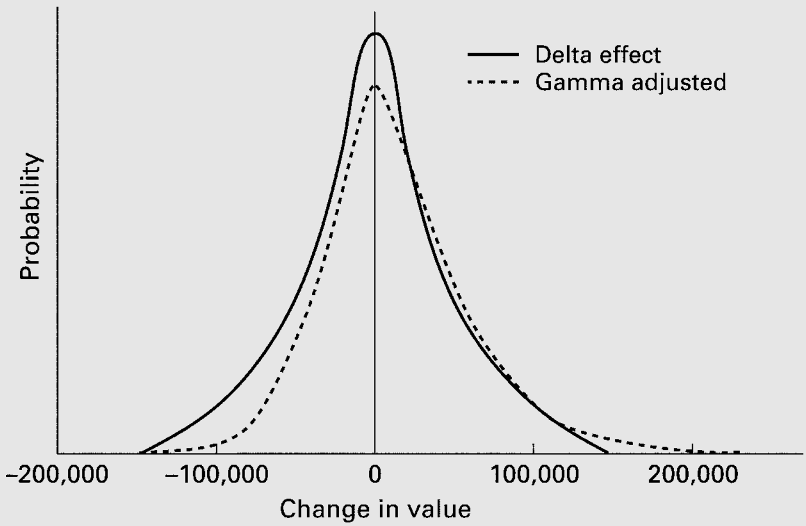


Figure 5.3 Gamma-adjusted VaR calculation.

Chapter 6

MONTE CARLO SIMULATION AND VALUE-AT-RISK

A defining characteristic of options is their non-linear pay-off profile. This makes risk exposure estimation for an options portfolio more problematic compared with a portfolio comprised of linear pay-off profile instruments such as bonds, futures and swaps. For this reason practitioners frequently eschew the variance–covariance methodology in favour of what is called the Monte Carlo simulation approach, because it is believed to provide a more accurate estimation of risk exposure for option instruments. Monte Carlo simulation refers to a process whereby a series of prices for an asset (or assets) is generated by a computer program; these prices are all theoretically possible given certain user-specified parameters. The portfolio is then revalued at each of these possible prices, and this enables the user to calculate a VaR number for the portfolio.

In this chapter we introduce the Monte Carlo simulation method and its use as a value-at-risk (*VaR*) calculation methodology.

INTRODUCTION: MONTE CARLO SIMULATION

We first consider the concept of simulated prices and their application to option valuation.

Option value under Monte Carlo

Table 6.1 reprises our European option contract from Chapter 5. Note an additional parameter, the 'drift'. This reflects what is known as the stochastic nature of asset price movements as defined in the Black–Scholes model (*B-S model*); a stochastic movement is described by volatility and drift. Drift is the measure of how much the price movement moves back around a mean figure.

Assume we have run a simulation program or a random number generator, and from 10 such numbers we have derived 10 possible future asset prices. The series of 10 asset prices is shown in Column H in Table 6.1.On expiry the option is seen to have positive value in eight of the cases; this is given in Column J. Where the option expires out-of-the-money it, of course, carries zero value and would not be exercised. Any positive value is realised on expiry, which is in the future, so we discount the pay-out to obtain a value for the option

Table 6.1 Option valuation following Monte Carlo simulation.

Cell	E	F	G	H	I	J	K
D7	**Option parameters**			**Simulation results**		**Option intrinsic value**	**Option present value**
8	Asset price	20		1	26.3	8.3	8.19690
9	Volatility	10.00%		2	21.2	3.2	3.16025
10	Time to expiry	0.25		3	16.4	0	0.00000
11	Exercise price	18		4	13.8	0	0.00000
12	Interest rate	0.05		5	28.2	10.2	10.07329
13	Drift	6%		6	24.5	6.5	6.41926
14				7	23.7	5.7	5.62919
15	Discount factor	0.9875778		8	18.9	0.9	0.88882
16				9	21.4	3.4	3.35776
17	Average value (option value)	4.52311		10	25.6	7.6	7.50559
18							
19							
20							

Table 6.2 Spreadsheet formulae for Table 6.1.

Cell	E	F	G	H	I	J	K
D7	**Option parameters**			**Simulation results**		**Option intrinsic value**	**Option PV**
8	Asset price	20		1	26.3	=MAX(G8-D11,0)	=H8*D15
9	Volatility	10.00%		2	21.2	=MAX(G9-D11,0)	=H9*D15
10	Time to expiry	0.25		3	16.4	=MAX(G10-D11,0)	=H10*D15
11	Exercise price	18		4	13.8	=MAX(G11-D11,0)	=H11*D15
12	Interest rate	0.05		5	28.2	=MAX(G12-D11,0)	=H12*D15
13	Drift	6%		6	24.5	=MAX(G13-D11,0)	=H13*D15
14				7	23.7	=MAX(G14-D11,0)	=H14*D15
15	Discount factor	=EXP(-F10*F12)		8	18.9	=MAX(G15-D11,0)	=H15*D15
16				9	21.4	=MAX(G16-D11,0)	=H16*D15
17	Average value (option value)	=AVERAGE(K8:K17)		10	25.6	=MAX(G17-D11,0)	=H17*D15
18							
19							
20							

oday. These values are shown in Column K. The discount factor used is the same as that seen in Table 5.1 (p. 87).

The extent of possible values for the option then ranges from £0 to £10. To calculate the fair value we take the average of the range which gives us the theoretical price of the option. The average value is £4.52. Following the logic of the B-S model, the probability that the option will be exercised is 80%, because eight out of the ten simulated values resulted in positive value.

The spreadsheet formulae for Table 6.1 are given at Table 6.2 (opposite).

Monte Carlo distribution

In practice, the parameters input to a Monte Carlo simulation are set to approximate generated values to the normal distribution. This recognises that asset prices revert back to their mean over time; the probability that prices move very far away from the mean is remote. In other words, an asset priced at £20 and with a volatility of 10% is unlikely to be priced at £2 in 1 year's time (and equally unlikely to be priced at £40 in the same period). Note that it is the return on a security that follows the log-normal distribution, and not the prices of the security itself.

We observe this in our simulated prices at Table 6.1. The average of the series is 22, which is actually spot on with the expected value of the underlying in 1 year's time (given by the volatility value). There is a procedure to generate random figures in Excel, which was used in our example. The formulae needed for this are given in Table 6.3.

Table 6.3 Excel formulae for random number generation.

	Trial 1	*Trial 2, etc.*
Random number	=RAND()	
Standard deviations	=NORMSINV(E6)	
Growth	=F13+E7*F9*F10^0.5	
Exponential growth	=EXP(E8)	
Simulated share price	=F8*E9	
Option intrinsic value	=MAX(E10-F11,0)	
Discount factor	=EXP(-E11*E13)	
PV of option	=E11^E12	

Monte Carlo simulation and VaR

We noted that the pay-off profile nature of options means that most VaR calculation methodologies are unsuitable for risk exposure measurement of an option book. The Monte Carlo method is regarded as the most accurate method to use in this case, most especially for exotic options.

To apply the concept to VaR, we simply generate the random number profile and then apply a cut-off at the percentile that suits our purpose. For example, with the example above, rather than just running 10 simulations we can run the program, say, 5,000 or 10,000 times and then cut off the results at the 250th or 500th lowest value – this value would represent a 95% confidence interval VaR number.

Example 6.1 Portfolio volatility using variance–covariance and simulation methods.

A simple two-asset portfolio is composed of the following instruments:

	Gilt strip	*FTSE 100 stock*
Number of units	£100 million	£5 million
Market value	£54.39 million	£54 million
Daily volatility	£0.18 million	£0.24 million

The correlation between the two assets is 20%. Using (3.1) we calculate the portfolio VaR as follows:

$$Vol = \sqrt{s_{bond}^2 + s_{stock}^2 + 2 s_{bond} s_{stock} r_{bond,\, stock}}$$

$$Vol = \sqrt{0.18^2 + 0.24^2 + (2 \times 0.18 \times 0.24 \times 0.2)}$$

$$= 0.327$$

We have ignored the weighting element for each asset because the market values are roughly equal. The calculation gives a portfolio volatility of £0.327 million. For a 95% confidence level VaR measure, which corresponds to 1.645 standard deviations (in a one-tailed test) we multiply the portfolio volatility by 1.645, which gives us a portfolio VaR of £0.538 million.

In a Monte Carlo simulation we also calculate the correlation and volatilities of the portfolio. These values are used as parameters in a random number simulation to throw out changes in the underlying portfolio value. These values are used to reprice the portfolio, and this value will be either a gain or loss on the actual mark-to-market value. This process is repeated for each random number that is generated. In Table 6.4 we show the results for 15 simulations of our two-asset portfolio. From the results we read off the loss level that corresponds to the required confidence interval.

Table 6.4 Monte Carlo simulation results.

Simulation	*Market value*		*Portfolio value*	*Profit/Loss*
	Bond	*Stock*		
1	54.35	54.9	109.25	0.86
2	54.64	54.02	108.66	0.27
3	54.4	53.86	108.26	–0.13
4	54.25	54.15	108.4	0.01
5	54.4	54.17	108.57	0.18
6	54.4	54.03	108.43	0.04
7	54.31	53.84	108.15	–0.24
8	54.3	53.96	108.26	–0.13
9	54.46	54.11	108.57	0.18
10	54.32	53.92	108.24	–0.15
11	54.31	53.97	108.28	–0.11
12	54.47	54.08	108.55	0.16
13	54.38	54.03	108.41	0.02
14	54.71	53.89	108.6	0.21
15	54.29	54.05	108.34	–0.05

As the number of trials is increased, the results from a Monte Carlo simulation approach those of the variance–covariance measure. This is shown in Figure 6.1.

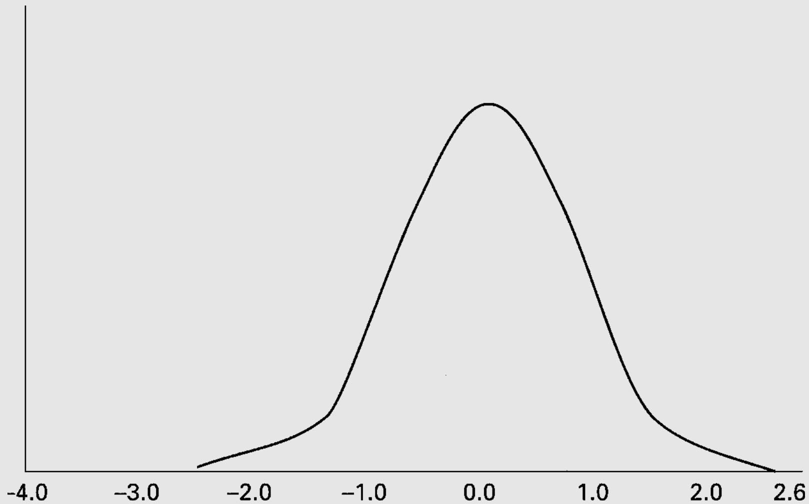


Figure 6.1 Monte Carlo simulation results approaching normal distribution.

Chapter 7

REGULATORY ISSUES AND STRESS-TESTING

We consider briefly, and in overview manner only, some relevant issues in bank regulatory capital. We then discuss the issue of stress-testing of value-at-risk (*VaR*) models.

CAPITAL ADEQUACY

The European Union (*EU*) Capital Adequacy Directive (*CAD*) was introduced in January 1996. It defined the capital requirement calculation that banks and securities houses must comply with based on the relocation of positions between the trading book and the non-trading, or banking, book. The current guise for EU regulation is the Capital Requirements Directive (*CRD*) IV, which is the EU's implementation of Basel III.

Model compliance

Back in the 1990s, as part of complying with CAD I the Bank of England (*BoE*) adopted a procedure to recognise models and the capital requirements which will be generated from these models. Eligible models can cover:

- options risk and interest rate risk in derivatives in the trading book;
- foreign exchange (*FX*) risk in the banking and trading books.

There is a standard approach for banks without recognised models and a more complex approach for those with recognised models, which will normally result in a lower capital requirement for a given quantity of position risk.

Models eligible for recognition fall into two categories:

- pricing models (for complex swaps, vanilla and exotic options);
- risk aggregation models (those which summarise and facilitate management of risk).

The types of risk aggregation models which the BoE can recognise for CAD purposes include the VaR-type models used for the measurement of interest rate risk, position risk and FX risk.

VaR models

Banks wishing to use their own VaR model for calculating capital requirements under CAD must have it recognised by their regulator. No particular type of model is prescribed and banks may use models based on back-testing, variance–covariance matrices, Monte Carlo simulations or simple aggregation of risk numbers.

If using their own models banks must be able to calculate the CAD requirement on a date randomly chosen by the regulatory supervisor, and notify the supervisor of the calculation the following day. The bank will then be required to calculate its capital requirement both according to the standard method and its own VaR model.

The model review process

A model review process includes discussion of the following areas:

- the mathematics of the model and underlying assumptions;
- systems and controls;
- risk management, reporting procedures and limits;
- staffing issues;
- reconciliation and valuation procedures;
- the setting of capital requirements.

Certain conditions and standards need to be met prior to recognition of the model. These standards vary according to the size of the firm, the nature of the business and the type of model being used. The review process takes the form of an on-site visit with follow-up of any outstanding issues to be met before recognition can be granted.

CAD II

CAD II is the key EU directive that allowed financial firms to use internal VaR models to calculate market risk capital requirements in line with Basel standards. This was known as the internal models approach and also underpinned Basel II at larger banks. The directive was implemented in 1999. Firms can base their market risk capital charges for all or part of their business on their internally determined VaR. To use this approach firms must meet certain qualitative and quantitative criteria to the satisfaction of their regulators.

Under both CAD II as adopted by the EU and the 1997 amendment to the Basel Accord – developed by the Bank for International Settlements (*BIS*) – banks can choose whether to use the standard approach to calculating capital requirements for trading books (equities, interest rate instruments, FX and commodities) or to seek supervisory approval to employ their own in-house VaR models as the basis for capital calculation.

Excessive reliance on models raises questions about the necessary safeguards to ensure that the capital requirements generated are adequate. Basel addressed this in a number of ways. One was to lay down simple standards for the construction of models. The standards decreed by the BIS include:

- the model was to express a VaR estimate to a 99% degree of confidence;
- losses must be calculated on the basis of a 10-day holding period;
- historic data must cover at least the last 12 months.

The BIS does not prescribe the *type* of model to be used. An additional requirement from Basel includes a requirement for banks to hold the higher of (i) the VaR number suggested by the model or (ii) three times the 60-day moving average of the VaR numbers generated on the current and past trading books.

Risk factors

Financial firms must be able to identify appropriate risk factors that capture the risks of a portfolio. CAD II required that the following factors be modelled:

- interest rate risk; the yield curve should be divided into a minimum number of six maturity segments;
- FX risk;
- equity risk;
- commodity risk;
- correlations.

Qualitative requirements

CAD II imposed certain qualitative criteria that firms must satisfy if using their internal models for regulatory purposes. These include that:

- the risk model be closely integrated into the daily risk management processes of the firm;
- the firm has an independent risk control unit ('middle office', etc.);
- the firm has sufficient staff skilled in the use of sophisticated models;
- the firm has established procedures for the risk management function;
- the firm frequently conducts a programme of stress-testing.

Specific risk

This is the risk of a price change in an instrument due to factors relating to its issuer or, for a derivative, the issuer of the underlying.

Regulatory capital requirement

A firm's market risk capital charge will be calculated as the higher of a multiple of the previous day's VaR and a multiple of the average of the VaR estimated on each of the preceding 60 business days.

The financial resources requirement element of this can be stated using the following expression:

$$FRR_{VaR} = \text{Max}\left(f_1 \times VaR_t, f_2 \times \frac{1}{60}\sum_{i=0}^{59} VaR_{t-i}\right) + SR$$

where VaR_t = Previous day's VaR number;
VaR_{t-i} = VaR calculated i business days earlier;
f_1, f_2 = Multiplicative factors;
SR = Specific risk add-on.

CAD II requires that f_2 has a minimum value of 3 (f_1 will have a minimum value of 1). Firms are required to carry out back-testing to measure performance and if concerns arise about the accuracy or integrity of the model, the multiplicative factors can be increased by the regulator. FRR_{VaR} will be added under a standard method to obtain the firm's total capital requirement.

Back-testing

This is the process of comparing VaR risk estimates with actual portfolio performance. For each business day firms should compare the 1-day VaR measure calculated by their model and the actual 1-day change in the portfolio value. For each actual loss greater than predicted, an 'exception' (see below) is reported. For multiple exceptions a plus factor of between 0 and 1 is applied to the firm's f_2 multiplicative factor.

The BIS and central bank supervisors carry out back-testing as a check on the accuracy of the models, by comparing actual trading results with model-generated risk measures. This has posed problems because trading results are often affected by changes to portfolios in the period following calculation of the initial VaR estimate. For this reason the BIS has recommended that banks develop their own capability to perform back-testing. Firms that do not meet the Basel back-testing criterion for accuracy suffer additional capital charges. These charges are imposed if, over a 12-month period (250 trading days), a bank's VaR model under-predicts the number of losses exceeding the permitted 1% cut-off point. Such losses are termed 'exceptions'. If a bank's VaR model has generated 0–4 exceptions, it is said to be in the 'green zone'; for 5–9 exceptions the bank is in the 'yellow zone'; and if there are 10 or more exceptions it is in the 'red zone'. The capital requirements for banks whose models are in the yellow zone may be increased by the regulators; if they are in the red zone the requirements will almost certainly be increased.

STRESS-TESTING

Risk measurement models and their associated assumptions are not without limitation. It is important to understand what will happen should some of the model's underlying assumptions break down. 'Stress-testing' is the term used for doing a series of scenario analyses or simulations to investigate the effect of extreme market conditions and/or the effect of violating any of the basic assumptions behind the risk model. If carried out efficiently, stress-testing will provide clearer information on the potential exposures at risk due to significant market corrections; hence, it is recommended practice for financial institutions.

Simulating stress

There is no standard way to do stress-testing. It is a means of experimenting with the limits of a model; it is also a means to measure the residual risk which is not effectively captured by the formal risk model, thus complementing the VaR framework. If a bank uses a confidence interval of 99% when calculating its VaR the losses on its trading portfolio due to market movements should not exceed the VaR number on more than 1 day in 100. For a 95% confidence level the corresponding frequency is 1 day in 20 or roughly 1 trading day each month. The question to ask is 'what are the expected losses on those days?' Also ,what can an institution do to protect itself against these losses? Assuming that returns are normally distributed provides a workable daily approximation for estimating risk, but when market moves are more extreme these assumptions no longer add value.

The 1% of market moves that are not used for VaR calculations contain events such as the October 1987 crash, the bond market collapse of February 1994 and the Mexican peso crisis at the end of 1994. In these cases market moves were much larger than any VaR model could account for; the correlation between markets also increased well above levels normally assumed in models. Figure 7.1 shows how actual distributions differ from the theoretical normal distribution.

An approach used by risk managers is to simulate extreme market moves over a range of different scenarios. One method is to use Monte Carlo simulation. This allows dealers to push the risk factors to greater limits; for example, a 99% confidence interval captures events up to 2.33 standard deviations from the mean asset return level. A risk manager can calculate the effect on the trading portfolio of a 10 standard deviations move. The 1987 crash was a 20 standard deviations move. Similarly, risk managers may want to change the correlation assumptions under which they normally work. For instance, if markets all move down together – something that happened in Asian markets from the end of 1997 and emerging markets generally from July 1998 – losses will be greater than if some markets are offset by other negatively correlated markets.

Only by pushing the bounds of the range of market moves that are covered in the stress-testing process can financial institutions have an improved chance of identifying where losses might occur, and therefore a better chance of managing their risk effectively.

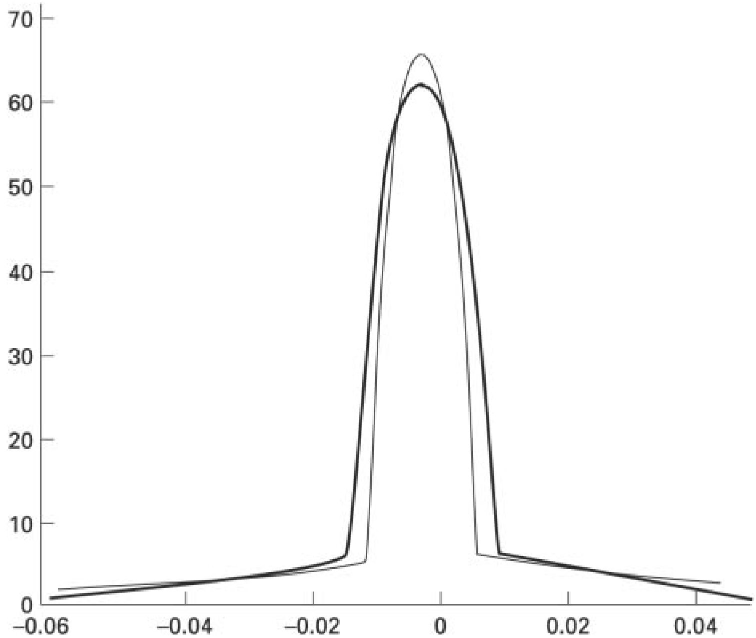


Figure 7.1 Real vs theoretical distributions.

Stress-testing in practice

For effective stress-testing one has to consider non-standard situations. The BIS Policy Group has recommended certain minimum standards in respect of specified market movements; the parameters chosen are considered large moves to overnight-marks, including:

- parallel yield curve shifts of 100 basis points up and down;
- steepening and flattening of the yield curve (2-year to 10-year) by 25 basis points;
- increase and decrease in 3-month yield volatilities by 20%;
- increase and decrease in equity index values by 10%;
- increase and decrease in swap spread by 20 basis points.

These scenarios represent a starting point for a framework for routine stress-testing.

Banks agree that stress-testing must be used to supplement VaR models. The main problem appears to be difficulty in designing appropriate tests. The main issues are:

- difficulty in 'anticipating the unanticipated';
- adopting a systematic approach, with stress-testing carried out by looking at past extremes and analysing the effect on the VaR number under these circumstances;
- selecting 10 scenarios based on past extreme events and generating portfolio VaRs based on reruns of these scenarios.

Issues in stress-testing

Back-testing

It is to be expected that extreme market moves will not be captured in VaR measurements. The calculations will always assume that the probability of certain events – such as the Mexican peso devaluation – are extremely low when analysing historical or expected movements of the currency. Stress tests need to be designed to model for such occurrences. *Back-testing* a firm's qualitative and quantitative risk management approach for actual extreme events often reveals the need to adjust reserves, increase the VaR factor, adopt additional limits and controls and expand risk calculations. With back-testing a firm will take, say, its daily VaR number, which we will assume is computed to a 95% degree of confidence. The estimate will be compared with the actual trading losses suffered by the book over a 20-day period, and if there is a significant discrepancy the firm will need to go back to its model and make adjustments to parameters. Frequent and regular back-testing of the VaR model's output with actual trading losses is an important part of stress-testing.

Procedure

The procedure for stress-testing in banks usually involves:

- creation of hypothetical extreme scenarios; and
- computation of corresponding hypothetical profit and loss (*P&L*) statements.

One method is to imagine *global* scenarios. If one hypothesis is that the euro appreciates sharply against the dollar, the scenario needs to consider any related areas, such as the effect, if any, on the Japanese yen and interest rates generally.

Another method is to generate many *local* scenarios and so consider a few risk factors at a time. For example, given an FX option portfolio a bank might compute the hypothetical P&L for each currency pair under a variety of exchange rate and implied volatility scenarios. There is then the issue of amalgamating the results: one way would be to add the worst case results for each of the sub-portfolios, but this ignores any portfolio effect and cross-hedging. This may result in an overestimate that is of little use in practice.

THE CRASH AND BASEL III

The 2007–08 crash resulted in the emergence of Basel III, the updated international bank regulatory supervision regime. This emphasised stricter capital standards and, for the first time for the BIS, new conservative requirements in liquidity. It also impacts the use of VaR. We cover this impact in Chapter 9.

STRESSED VaR

A key concern of critics of VaR is that the concept of a rolling historical period calculation approach has a pro-cyclical impact on regulatory capital. During periods of low volatility, historical VAR means that banks can lower their capital requirement. But during periods of crisis and high volatility, VaR forces banks to allocate higher capital for the same positions, despite the fact that earlier in the cycle these assets are unlikely to be any 'riskier' than they had been previously.

Basel III has a different approach. The new logic for market risk capital is expected to raise considerably the capital requirements in place now. A large portion of this regulatory capital increase will arise because of *SVaR*, the 'stressed value at risk'. SVaR maintains the standard 99% confidence interval (one-tailed), 10-day holding period test and at least one year's worth of data, but requires it to be calibrated to a known period of volatility (such as during 2008).

This is an improvement on the previous methodology and should help to address the pro-cyclical nature of the approach. However, the Basel Committee did not specify the stress period, and in fact required banks to consider multiple stress periods. For example one could see a case where regional supervisors would choose different stress periods for the same asset classes. That would mean that the same asset held in two different jurisdictions could potentially require different capital requirements.

For more comment on SVaR see Chapter 9.

Chapter 8

CREDIT RISK AND CREDIT VALUE-AT-RISK

Credit risk is a definition of the outcome of banking. However, in increasingly competitive markets, banks and securities houses take on more credit risk. The following are instances:

- credit spreads tightened in the late 1990s and the early part of 2000 to the point where blue chip companies – such as BT or Shell – benefitted from syndicated loan rates for as little as 10–12 basis points over the London Interbank Offered Rate (*LIBOR*); banks are turning to lower rated firms to maintain margin;
- growth in complex financial instruments that are more challenging to manage for credit risk than traditional instruments, such as collateralised debt obligations (*CDOs*);
- investors are finding fewer opportunities in interest rate and currency markets and moving towards yield enhancement through extending and trading credit; for example, in the eurozone participating government bond markets become credit markets and are in some cases very low-rated;
- high yield (junk) and emerging market sectors have been expanding rapidly.

The growth in credit exposures and rise of complex instruments have led to a need for more accurate risk measurement techniques. We discuss the application of the VaR methodology for credit risk exposure.

TYPES OF CREDIT RISK

There are two main types of credit risk:

- credit spread risk;
- credit default risk.

We consider each now.

Credit spread risk

Credit spread is the excess premium required by the market for taking on a certain assumed credit exposure. Credit spread risk is the risk of financial loss resulting from changes in the level of credit spreads used in the marking-to-market of a product. It is exhibited by a portfolio for which the credit spread is traded and marked. Changes in observed credit spreads affect the value of the portfolio.

Credit default risk

This is the risk that an issuer of debt (obligor) is unable to meet its financial obligations. Where an obligor defaults, a firm generally incurs a loss equal to the amount owed by the obligor less any recovery amount which the firm gets back as a result of foreclosure, liquidation or restructuring of the defaulted obligor. All portfolios of risky exposures exhibit credit default risk.

CREDIT RATINGS

The risks associated with holding a fixed interest debt instrument are closely connected with the ability of the issuer to maintain regular coupon payments as well as redeem the debt on maturity. Essentially, *credit risk* is the main risk of holding a bond. Only the highest quality government debt, and a small amount of supra-national and corporate debt, may be considered to be entirely free of credit risk. Therefore, at any time, the yield on a bond reflects investors' views on the ability of the issuer to meet its liabilities as set out in the bond's terms and conditions. A delay in paying a cash liability as it becomes due is known as technical default and is a cause for extreme concern for investors; failure to pay will result in the matter being placed in the hands of a court as investors seek to recover their funds.

Credit ratings

A credit rating is a formal opinion given by a rating agency of the *credit risk* for investors in a particular issue of debt securities. Ratings are given to public issues of debt securities by any type of entity, including governments, banks and corporates. They are also given to short-term debt, such as commercial paper, as well as bonds and medium-term notes.

Purpose of credit ratings

Investors in securities accept the risk that the issuer will default on coupon payments or fail to repay the principal in full on the maturity date. Generally, credit risk is greater for securities with a long maturity, as there is a longer period for the issuer potentially to default. For example, if a company issues 10-year bonds, investors cannot be certain that the company will still exist in 10 years' time. It may have failed and gone into liquidation some time before that. That said, there is also risk attached to short-dated debt securities; indeed, there have been instances of default by issuers of commercial paper, which is a very short-term instrument.

The prospectus or offer document for an issue provides investors with some information about the issuer, so that some credit analysis can be performed on the issuer before the bonds are placed. The information in the offer documents enables investors themselves to perform their own credit analysis by studying this information before deciding whether or not to invest. Credit assessments take up time, however, and also require the specialist skills of credit analysts. Large institutional investors do in fact employ such specialists to carry out credit analysis; however, often it is too costly and time-consuming to assess every issuer in every debt market. Therefore, investors commonly employ two other methods when making a decision on the credit risk of debt securities:

- name recognition;
- formal credit ratings.

Name recognition is when the investor relies on the good name and reputation of the issuer and accepts that the issuer is of such good financial standing, or sufficient financial standing, that a default on interest and principal payments is highly unlikely. An investor may feel this way about, say, Microsoft or BP plc. However, the experience of Barings in 1995 suggested to many investors that it may not be wise to rely on name recognition alone in today's marketplace. The tradition and reputation behind the Barings name allowed the bank to borrow at LIBOR, or occasionally at sub-LIBOR, interest rates in the money markets, which put it on a par with the highest quality clearing banks in terms of credit rating. However, name recognition needs to be augmented by other methods to reduce the risk against unforeseen events, as happened with Barings. Credit ratings are a formal assessment, for a given issue of debt securities, of the likelihood that the interest and principal will be paid in full and on schedule. They are increasingly used to make investment decisions about corporate or lesser developed government debt.

Formal credit ratings

Credit ratings are provided by specialist agencies. The major credit rating agencies are Standard & Poor's, Fitch, and Moody's, based in the United States. There are other agencies both in the US and other countries. On receipt of a formal request, the credit rating agencies will carry out a rating exercise on a specific issue of debt capital. The request for a rating comes from the organisation planning the issue of bonds. Although ratings are provided for the benefit of investors, the issuer must bear the cost. However, it is in the issuer's interest to request a rating as it raises the profile of the bonds, and investors may refuse to buy paper that is not accompanied by a recognised rating. Although the rating exercise involves a credit analysis of the issuer, the rating is applied to a specific debt issue. This means that in theory the credit rating is applied not to an organisation itself, but to the specific debt securities that the organisation has issued or is planning to issue. In practice, it is common for the market to refer to the creditworthiness of organisations themselves in terms of the rating of their debt. A highly rated company, such as Exxon Mobil, is therefore referred to as a 'triple-A rated' company, although it is the company's debt issues that are rated as triple-A.

Ratings changes over time

Ratings transition matrix

We have noted that the rating agencies constantly review the credit quality of firms they have rated. As might be expected, the credit rating of many companies will fluctuate over time as they experience changes in their corporate well-being. As a guide to the change in credit rating that might be expected over a 1-year period, Moody's and S&P publish historical transition matrices, which provide the average rating transition probabilities for each class of rating. An example is shown at Table 8.1, which is Moody's 1-year ratings

Table 8.1 Moody's 1-year rating transition matrix (%).

	Aaa	*Aa*	*A*	*Baa*	*Ba*	*B*	*Caa*	*Default*
Aaa	93.40	5.94	0.64	0.00	0.02	0.00	0.00	0.00
Aaa	1.61	90.55	7.46	0.26	0.09	0.01	0.00	0.02
Aaa	0.07	2.28	92.44	4.63	0.45	0.12	0.01	0.00
Baa	0.05	0.26	5.51	88.48	4.76	0.71	0.08	0.15
Baa	0.02	0.05	0.42	5.16	86.91	5.91	0.24	1.29
Baa	0.00	0.04	0.13	0.54	6.35	84.22	1.91	6.81
Caa	0.00	0.00	0.00	0.62	2.05	4.08	69.20	24.06

Source: Moody's. Reproduced with permission.

transition matrix for 2002. These results are obtained from a sample of a large number of firms over many years. In Table 8.1, the first column shows the initial rating and the first row the final rating. For instance, the probability of an A-rated company being downgraded to Baa in 1 year is 4.63%. The probability of the A-rated company defaulting in this year is 0.00%.

There are some inconsistencies in the ratings transition table and this is explained by Moody's as resulting from scarcity of data for some ratings categories. For instance, an Aa-rated company has a 0.02% probability of being in default at year-end, which is higher than the supposedly lower rated A-rated company. So at all times such results must be treated with care. The clearest conclusion from this table is that the most likely outcome at year-end is that the company rating remains the same. It may be that a 1-year time horizon provides little real value; hence, the rating agencies also publish transition matrices for longer periods, such as 5 and 10 years.

We might expect an increased level of default as we move lower down the credit ratings scale. This is borne out in Figure 8.1, which is a reproduction of data published by Moody's. It shows 1-year default rates by credit rating category, for the period 1985–2000. We see that the average 1-year default rate rises from 0 for the highest rated Aaa to 15.7% for the B3 rating category. As we have just suggested

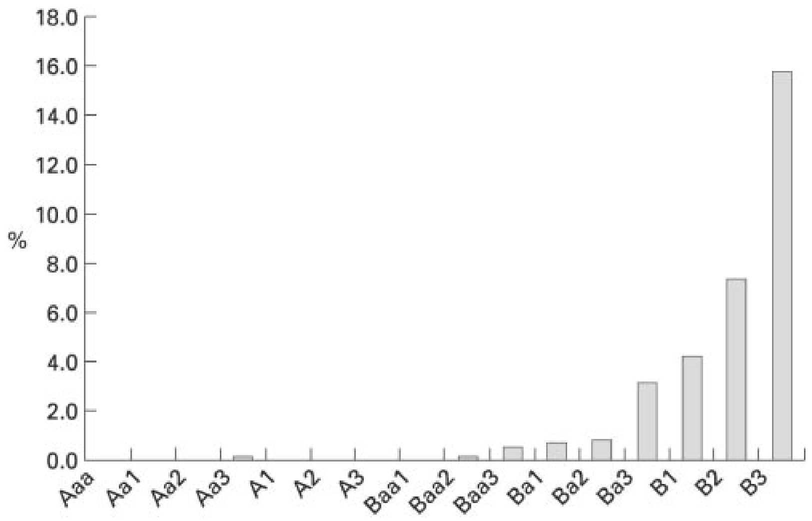


Figure 8.1 One-year default rates 1985–2000.

Source: Moody's. Reproduced with permission.

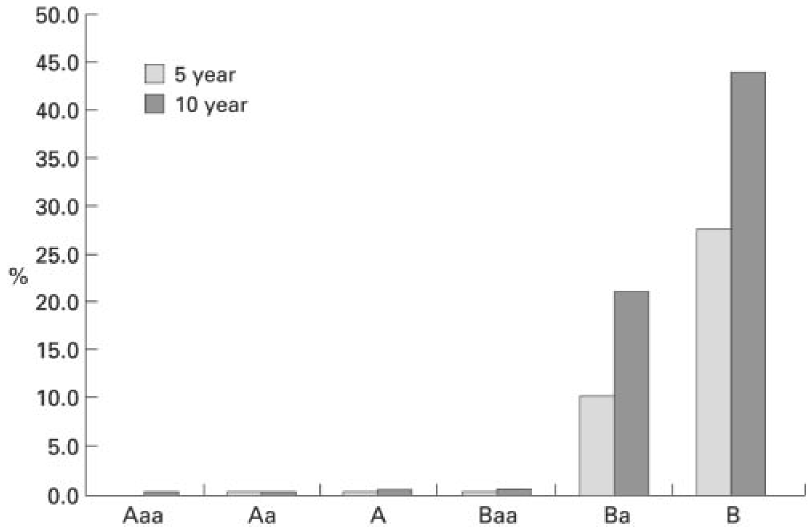


Figure 8.2 Both 5-year and 10-year average cumulative default rates, 1985–2000.

Source: Moody's. Reproduced with permission.

though, some investors attach little value to 1-year results. Figure 8.2 shows average cumulative default rates for 5-year and 10-year time horizons, for the same period covered in Figure 8.1. In fact, this repeats the results shown in Table 8.1, with higher default rates associated with lower credit ratings.

Corporate recovery rates

When a corporate obligor experiences bankruptcy or enters into liquidation or administration, it will default on its loans. However, this does not mean that all the firm's creditors will lose everything. At the end of the administration process, the firm's creditors typically will receive back a portion of their outstanding loans, a *recovery* amount.[1] The percentage of the original loan that is received back is known as the *recovery rate*, which is defined as the percentage of par value that is returned to the creditor.

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[1] This recovery may be received in the form of other assets, such as securities or physical plant, instead of cash.

Table 8.2 Recovery rates according to loan seniority (%).

Seniority	*Mean*	*Standard deviation*
Senior secured bank loans	60.70	26.31
Senior secured	55.83	25.41
Senior unsecured	52.13	25.12
Senior subordinated	39.45	24.79
Subordinated	33.81	21.25
Junior subordinated	18.51	11.26
Preference shares	8.26	10.45

Source: Moody's. Reproduced with permission.

The seniority of a loan strongly influences the level of the recovery rate. Table 8.2 shows recovery rates for varying levels of loan seniority in 2002, as published by Moody's. The standard deviation for each recovery rate reported is high, which illustrates dispersion around the mean and reflects widely varying recovery rates even within the same level of seniority. It is clear that the more senior a loan or a bond is, the higher recovery it will enjoy in the event of default.

CREDIT DERIVATIVES

Credit derivatives have become a major tool for use in managing credit risk exposure. A knowledge of the main credit derivative instruments – credit default swaps (*CDS*) and total return swaps – is essential for risk managers concerned with credit risk. A discussion of the instruments themselves is outside the scope of this book, although we introduce the CDS in Box 8.1.

Box 8.1 The credit default swap

The most common credit derivative is the *credit default swap.* This is a bilateral contract in which a periodic fixed fee or a one-off premium is paid to a *protection seller*, in return for which the seller will make a payment on the occurrence of a specified credit event. The fee is usually quoted as a basis point multiplier of the nominal value. It is usually paid quarterly in arrears, as a

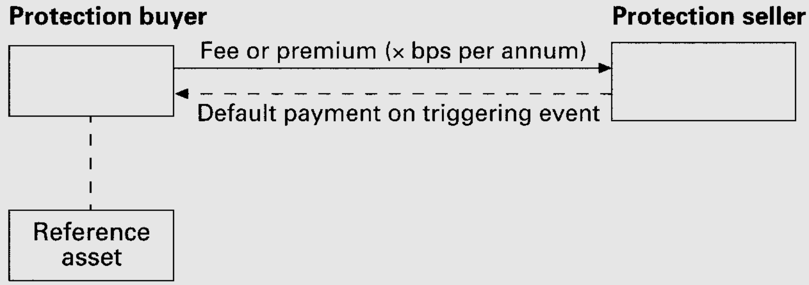


Figure 8.3 Credit default swap.

per-annum fee.[2] The protection seller is buying the credit risk while the protection buyer is selling credit risk. Since no asset is transferred, there is no need for funding the position – so, the CDS is known as an *unfunded* credit derivative.

The CDS can refer to a single asset (known as the reference entity, reference asset or underlying asset) or a basket of assets. The default payment can be paid in whatever way suits the protection buyer or both counterparties. For example, it may be linked to the change in price of the reference asset or another specified asset, it may be fixed at a predetermined recovery rate or it may be in the form of actual delivery of the reference asset at a specified price. Esentially:

$$\text{Pay-out} = 100 - [\text{Recovery value}]$$

Often, the pay-out on a CDS is par minus the market value at the time of default or other credit event.

The basic plain vanilla CDS structure is illustrated at Figure 8.3.

The CDS enables one party to transfer its credit risk exposure to another party. Banks may use default swaps to trade sovereign and corporate credit spreads without trading the actual assets themselves; for example, someone who has gone long a default swap (the protection buyer) will gain if the reference asset obligor suffers a rating downgrade or defaults, and can sell the default swap at a

[2] The counterparty to the protection seller is, of course, the protection buyer. The protection buyer's position can also be defined as a long put option position on the reference asset, as the bond can be put back to the seller in the event.

profit if he can find a buyer counterparty. This is because the cost of protection on the reference asset will have increased as a result of the credit event. The original buyer of the default swap need never have owned a bond issued by the reference asset obligor.

As we stated, the default payment on a CDS will be $(1 - \delta)$ times its notional, where δ is defined as the recovery rate of the reference security. The reason for this pay-out is clear – it allows a risky asset to be transformed into a risk-free asset by purchasing default protection referenced to this credit. For example, if the expected recovery rate for a given reference asset is 30% of its face value, upon default the remaining 70% will be paid by the protection seller. Credit agencies such as Moody's and Standard & Poor's provide recovery rate estimates for corporate bonds with different credit ratings using historical data.

The credit default swap contract has a given maturity, but will terminate early if a credit event occurs. The definition of 'credit event' is crucial to the contract and generally is as defined in standard contract documentation. It can include the default of an issuer or an administration or loan restructuring situation. The maturity of the credit swap does not have to match the maturity of the reference asset and often does not. On occurrence of a credit event, the swap contract is terminated and a settlement payment made by the protection seller or guarantor to the protection buyer. This termination value is calculated at the time of the credit event, and the exact procedure that is followed to calculate the termination value will depend on the settlement terms specified in the contract. This will be either cash settlement or physical settlement, as detailed below.

Measuring risk for a CDS contract

Banks calculate a quantitative measure of the risk exposure of their CDS positions. The approach used follows the same VaR principles used for earlier asset class products; namely, it calculates the sensitivity of a contract to variations in market parameters. The main risk measure regards the sensitivity of the CDS to a change in the primary credit curve, and is known as Spread01 or usually 'Credit01'.

Credit01 is a measure of the change in the mark-to-market value of a CDS contract for a 1-bp parallel shift upwards in the credit-risky curve. The precise definition differs depending on whether one is measuring the risk on a bought or sold protection position. The value of a short credit (buy protection) CDS position increases as credit spreads widen, while the value of a long credit (sell protection) position decreases as credit spreads widen. Generally, the market quotes the Credit01 value of a long credit (sold protection) contract as negative, which matches the sign for a short credit position. Essentially, Credit01 is similar in concept to the present value of a basis point (*PVBP*) or DV01 (Dollar01) interest rate risk measure for a cash bond holding.

The change in the mark-to-market value is given by:

$$\text{Notional} \times \text{Credit01} \times \Delta\text{Spread}$$

with this value being negative or positive depending on whether the holder is buying or selling protection.

There is also an interest rate sensitivity measure for CDS contracts, although this sensitivity is relatively insignificant unless one is experiencing high market volatility. The risk measure of sensitivity to changes in the interest rate yield curve (the LIBOR curve) is known as IR01 or Libor01, and measures the change in value of the contract for a 1-bp upward parallel shift in the LIBOR curve.

MODELLING CREDIT RISK

The main credit risk VaR methodologies take a *portfolio* approach to credit risk analysis. This means that:

- the credit risks to each obligor across the portfolio are restated on an equivalent basis and aggregated in order to be treated consistently, regardless of the underlying asset class;
- correlations of credit quality moves across obligors are taken into account.

This allows portfolio effects – the benefits of diversification and risks of concentration – to be quantified.

The portfolio risk of an exposure is determined by four factors:

- size of the exposure;
- maturity of the exposure;
- probability of default of the obligor;
- systematic or concentration risk of the obligor.

Credit VaR – like market risk VaR – considers (credit) risk in a mark-to-market framework. It arises from changes in value due to credit events; that is, changes in obligor credit quality including defaults, upgrades and downgrades.

Nevertheless, credit risk is different in nature from market risk. Typically, market return distributions are assumed to be relatively symmetrical and approximated by normal distributions. In credit portfolios, value changes will be relatively small upon minor upgrades/downgrades, but can be substantial upon default. This remote probability of large losses produces skewed distributions with heavy downside tails that differ from the more normally distributed returns assumed for market VaR models. This is shown in Figure 8.4.

This difference in risk profiles does not stop City quantitative analysts from assessing risk on a comparable basis. Analytical method market VaR models consider a time horizon and estimate VaR across a distribution of estimated market outcomes. Credit VaR

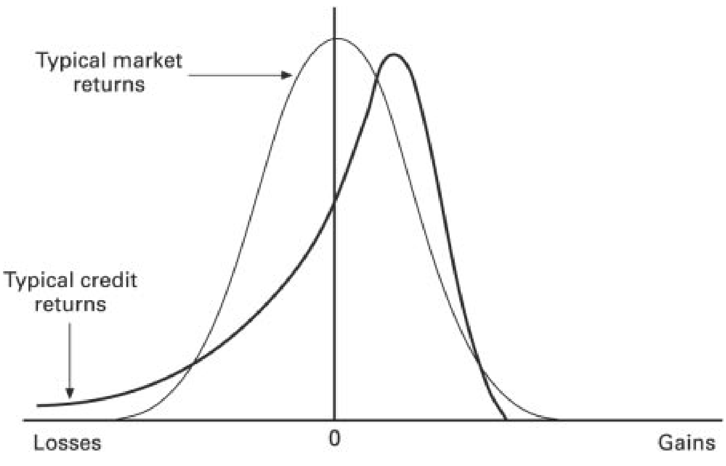


Figure 8.4 Comparison of distribution of market returns and credit returns.

models similarly look to a horizon and construct a distribution of value given different estimated credit outcomes.

When modelling credit risk the two main measures of risk are:

- *distribution of loss* – obtaining distributions of loss that may arise from the current portfolio. This considers the question of what the expected loss is for a given confidence level;
- *identifying extreme or catastrophic outcomes* – this is addressed through the use of scenario analysis and concentration limits.

To simplify modelling, no assumptions are made about the causes of default. Mathematical techniques used in the insurance industry are used to model the event of an obligor default.

Time horizon

The choice of time horizon will not be shorter than the time frame over which risk-mitigating actions can be taken. Most analysts suggest two alternatives:

- a constant time horizon such as 1 year;
- a hold-to-maturity time horizon.

Data inputs

Modelling credit risk requires certain data inputs; generally these are the following:

- credit exposures;
- obligor default rates;
- obligor default rate volatilities;
- recovery rates.

These data requirements present some difficulties. There is a lack of comprehensive default and correlation data, and assumptions need to be made at certain times.

CREDITMETRICS

CreditMetrics was JPMorgan's portfolio model for analysing credit risk and was the first such credit VaR model, providing an estimate of VaR due to credit events caused by upgrades, downgrades and default.

A software package known as CreditManager was made available that allows users to implement the CreditMetrics methodology.

Methodology

There are two main frameworks in use for quantifying credit risk. One approach considers only two states: default and non-default. This model constructs a binomial tree of default *vs* no-default outcomes until maturity (see Figure 8.5).

The other approach, sometimes called the risk-adjusted return on capital (*RAROC*) approach holds that risk is the observed volatility of corporate bond values within each credit rating category, maturity band and industry grouping. The idea is to track a benchmark corporate bond (or index) which has observable pricing. The resulting estimate of volatility of value is then used to proxy the volatility of the exposure (or portfolio) under analysis.

CreditMetrics sits between these two approaches. The model estimates portfolio VaR at the risk horizon due to credit events that include upgrades and downgrades, rather than just defaults. Thus, it adopts a mark-to-market framework. As shown in Figure 8.6 bonds within each credit rating category have volatility of value due to day-to-day credit spread fluctuations. CreditMetrics

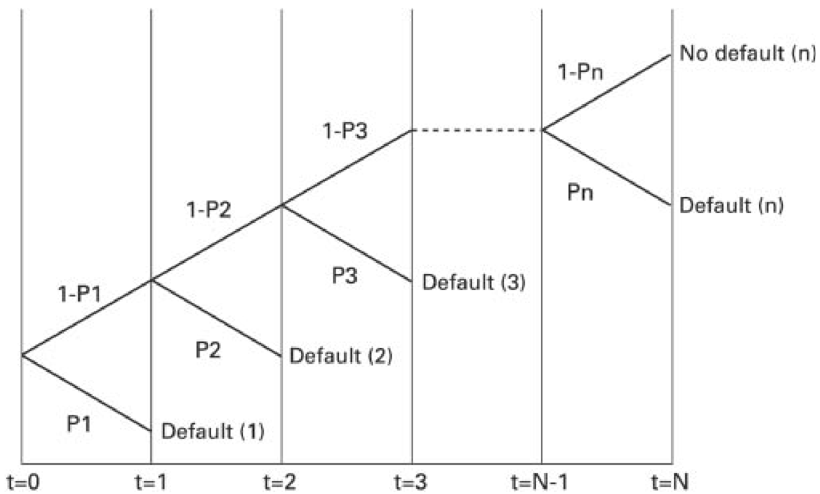


Figure 8.5 A binomial model of credit risk.

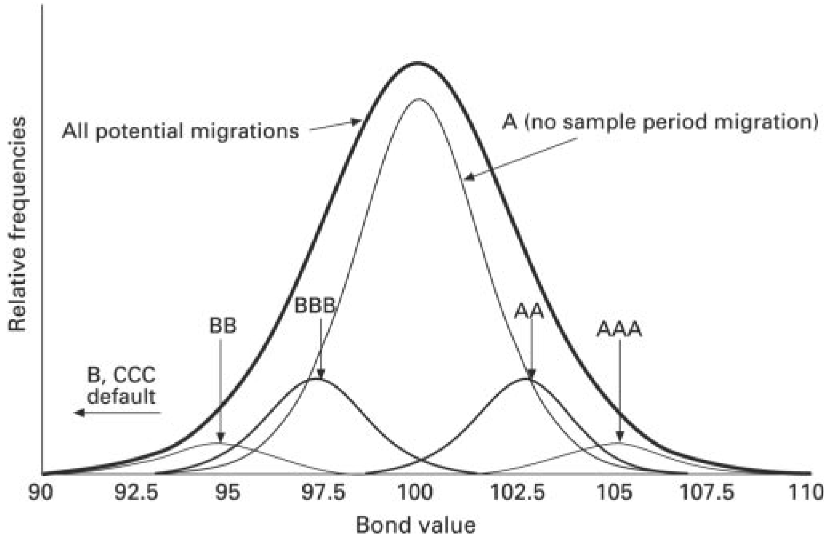


Figure 8.6 Ratings migration distribution.

assumes that all credit migrations have been realised, weighting each by a migration likelihood.

Time horizon

CreditMetrics adopts a 1-year risk horizon mainly because much academic and credit agency data are stated on an annual basis. This is a convenient convention similar to the use of annualised interest rates in the money markets. The risk horizon is adequate as long as it is not shorter than the time required to perform risk-mitigating actions.

The steps involved in CreditMetrics methodology are shown in Figure 8.7, described by RiskMetrics as its analytical 'roadmap'.

The elements in each step are:

- *Exposures*
  - user portfolio;
  - market volatilities;
  - exposure distributions.
- *VaR due to credit events*
  - credit rating;
  - credit spreads;

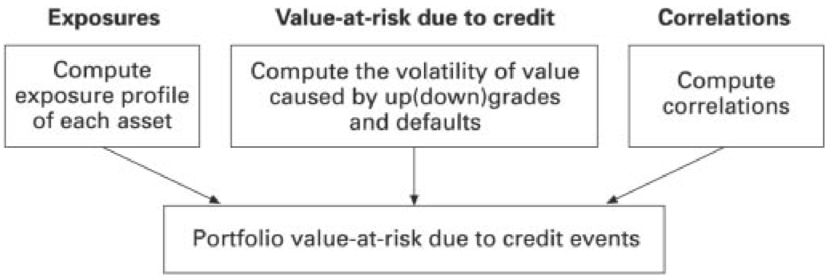


Figure 8.7 Roadmap of the analytics of CreditMetrics.
Source: JPMorgan (1997).

  - rating change likelihood;
  - recovery rate in default;
  - present value bond revaluation;
  - Standard deviation of value due to credit quality changes.
- *Correlations*
  - ratings series;
  - models (e.g., correlations);
  - joint credit rating changes.

Calculating the credit VaR

CreditMetrics' methodology assesses individual and portfolio VaR due to credit in three steps:

Step 1: It establishes the exposure profile of each obligor in a portfolio.

Step 2: It computes the volatility in value of each instrument caused by possible upgrade, downgrade and default.

Step 3: Taking into account correlations between each of these events it combines the volatility of the individual instruments to give an aggregate portfolio risk.

Step 1 Exposure profiles

CreditMetrics incorporates the exposure of instruments such as bonds (fixed or floating rate) as well as other loan commitments and market-driven instruments, such as swaps. Exposure is stated on an equivalent basis for all products. The products covered include:

- receivables (or trade credit);
- bonds and loans;
- loan commitments;
- letters of credit;
- market-driven instruments.

Step 2 Volatility of each exposure from upgrades, downgrades and defaults

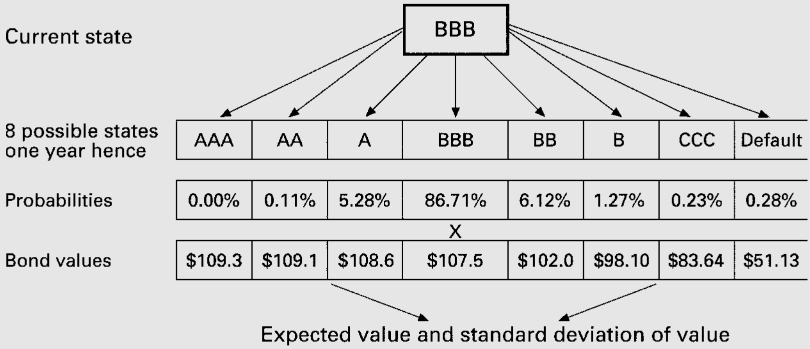
The levels of likelihood are attributed to each possible credit event of upgrade, downgrade and default. The probability that an obligor will change over a given time horizon to another rating is calculated. Each change (migration) results in an estimated change in value (derived from credit spread data and, in default, recovery rates). Each value outcome is weighted by its likelihood to create a distribution of value across each credit state, from which each asset's expected value and the volatility (standard deviation) of value are calculated.

There are three stages to calculating the volatility of value in a credit exposure:

- the senior unsecured credit rating of the issuer determines the chance of either defaulting or migrating to any other possible credit quality state in the risk horizon;
- revaluation at the risk horizon can be by either (i) the seniority of the exposure, which determines its recovery rate in case of default, or (ii) the forward zero-coupon curve (spot curve) for each credit rating category which determines the revaluation upon upgrade or downgrade;
- the probabilities from the two steps above are combined to calculate the volatility of value due to credit quality changes.

Example 8.1 Calculating probabilities.

An example of calculating the probability step is given below. The probabilities of all possible credit events on an instrument's value must be established first. Given these data the volatility of value due to credit quality changes for this one position can be calculated. The process is shown in Figure 8.8.



$$\sigma_T = \sqrt{\sum_{i=1}^{s} p_i(\mu_i^2 + \sigma_i^2) - \mu_T^2} \quad \text{where } \mu_T = \sum_{i=1}^{n} p_i \mu_i$$

Figure 8.8 Constructing the distribution value for a BBB-rated bond.

Source: JPMorgan (1997).

Step 3 Correlations

Individual value distributions for each exposure are combined to give a portfolio result. To calculate the portfolio value from the volatility of individual asset values requires estimates of the correlation between credit quality changes. CreditMetrics itself allows for different approaches to estimating correlations including a simple constant correlation. This is because of the frequent difficulty in obtaining directly observed credit quality correlations from historical data.

CreditManager

CreditManager was the software implementation of CreditMetrics as originally developed by JPMorgan. It is a PC-based application that measures and analyses credit risk in a portfolio context. It measures the VaR exposure due to credit events across a portfolio, and also quantifies concentration risks and the benefits of diversification by incorporating correlations (following the methodology utilised by CreditMetrics). The CreditManager application provides a framework for portfolio credit risk management that can be implemented 'off-the-shelf' by virtually any institution. It uses the following:

- obligor credit quality database – details of obligor credit ratings, transition and default probabilities, industries and countries;
- portfolio exposure database – containing exposure details for the following asset types: loans, bonds, letters of credit, total return swaps, CDS, interest rate and currency swaps and other market instruments;
- frequently updated market data – including yield curves, spreads, transition and default probabilities;
- flexible risk analyses with user-defined parameters – supporting VaR analysis, marginal risk, risk concentrations, event risk and correlation analysis;
- stress-testing scenarios – applying user-defined movements to correlations, spreads, recovery rates, transition and default probabilities;
- customised reports and charts.

CreditManager data sources include Dow Jones, Moody's, Reuters, and Standard & Poor's. By using the software package, risk managers can analyse and manage credit portfolios based on virtually any variable, from the simplest end of the spectrum – single position or obligor – to more complex groupings containing a range of industry and country obligors and credit ratings.

Generally, this quantitative measure is employed as part of an overall risk management framework that retains traditional, qualitative methods.

CreditMetrics can be a useful tool for risk managers seeking to apply VaR methodology to credit risk. The model enables risk managers to apply portfolio theory and VaR methodology to credit risk. It has several applications including prioritising and evaluating investment decisions and, perhaps most important, setting risk-based exposure limits. Ultimately, the model's sponsors claim its use can aid maximising shareholder value based on risk-based capital allocation. This should then result in increased liquidity in credit markets, the use of a marking-to-market approach to credit positions and closer interweaving of regulatory and economic capital.

CreditRisk$^+$

For historical interest, we retain here the description of CSFB's VaR model first introduced in 1996. CreditRisk$^+$ was developed to handle all instruments that give rise to credit exposure including bonds, loans commitments, letters of credit and derivatives.

The modelling process

CreditRisk$^+$ uses a two-stage modelling process as illustrated in Figure 8.9.

CreditRisk$^+$ considers the distribution of the number of default events in a time period, such as 1 year, within a portfolio of obligors having a range of different annual probabilities of default.

The annual probability of default of each obligor can be determined by its credit rating and then mapping between default rates and credit ratings. A default rate can then be assigned to each obligor (an example of what this would look like is shown in Table 8.3). Default rate volatilities can be observed from historic volatilities.

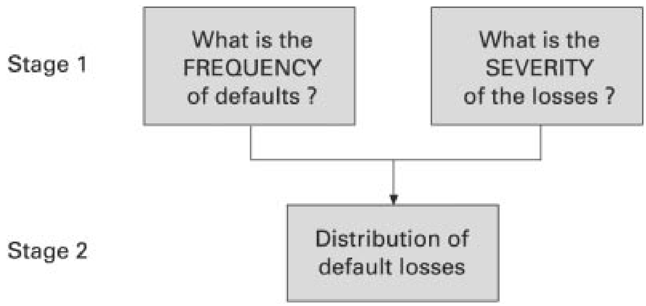


Figure 8.9 CreditRisk$^+$ modelling methodology.

Table 8.3 One-year default rates (%).

Credit rating	*One-year default rate*
Aaa	0.00
Aa	0.03
A	0.01
Baa	0.12
Ba	1.36
B	7.27

Source: CSFB.

Correlation and background factors

Default correlation impacts the variability of default losses from a portfolio of credit exposures. CreditRisk$^+$ incorporates the effects of default correlations by using default rate volatilities and sector analysis.

Unsurprisingly enough, it is not possible to forecast the exact occurrence of any one default or the total number of defaults. Often there are background factors that may cause the incidence of default events to be correlated, even though there is no causal link between them. For example, an economy in recession may give rise to an unusually large number of defaults in one particular month, which would increase the default rates above their average level. CreditRisk$^+$ models the effect of background factors by using default rate volatilities rather than by using default correlations as a direct input. Both distributions give rise to loss distributions with fat tails.

Concentration

As noted above there are background factors that affect the level of default rates. For this reason it is useful to capture the effect of concentration in particular countries or sectors. CreditRisk$^+$ uses sector analysis to allow for concentration. Exposures are broken down into an obligor-specific element independent of other exposures, as well as non-specific elements that are sensitive to particular factors, such as countries or sectors.

Distribution of the number of default events

CreditRisk$^+$ models the underlying default rates by specifying a default rate and a default rate volatility. This aims to take account of the variation in default rates. The effect of using volatility is illustrated in Figure 8.10, which shows the distribution of default rates generated by the model when rate volatility is varied. The distribution becomes skewed to the right when volatility is increased.

This is an important result and demonstrates the increased risk represented by an extreme number of default events. By varying the volatility in this way, CreditRisk$^+$ is attempting to model for real-world shock much in the same way that market risk VaR models aim to allow for the fact that market returns do not follow exact normal distributions, as shown by the incidence of market crashes.

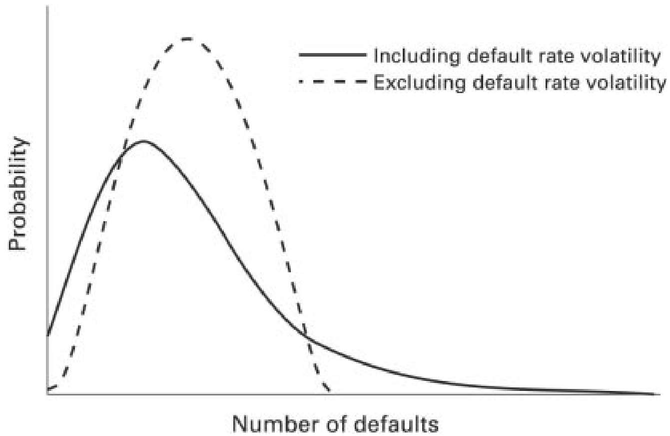


Figure 8.10 CreditRisk$^+$ distribution of default events

Application software

CreditRisk$^+$ is run on Microsoft Excel® as a spreadsheet calculator. The user inputs portfolio statistics into a blank template and the model will calculate his credit exposure. Obligor exposure can be analysed on the basis of all exposures being part of the same sector; alternatively, up to eight different sectors (government, countries, industry and so on) can be analysed. The spreadsheet template allows the user to include up to 4,000 obligors in the static data. An example portfolio of 25 obligors and default rates and default rate volatilities (assigned via a sample of credit ratings) is included with the spreadsheet.

The user's static data for the portfolio will therefore include details of each obligor, the size of the exposure, the sector for that obligor (if not all in a single sector) and default rates. An example of static data is given in Tables 8.4 and 8.5.

An example credit loss distribution calculated by the model is shown in Figure 8.11, which shows the results for a portfolio at the simplest level of assumption; all obligors are assigned to a single sector. The full loss distribution over a 1-year time horizon is calculated together with percentiles of the loss distribution (not shown here), which assess the relative risk for different levels of loss. The model can calculate distributions for a portfolio with obligors grouped across different sectors, as well as the distribution for a portfolio analysed over a 'hold to maturity' time horizon.

Table 8.4 Example default rate data (%).

Credit rating	*Mean default rate*	*Standard deviation*
A+	1.50	0.75
A	1.60	0.80
A−	3.00	1.50
BBB+	5.00	2.50
BBB	7.50	3.75
BBB−	10.00	5.00
BB	15.00	7.50
B	30.00	15.00

Table 8.5 Obligor details.

Company name	*Exposure* (£)	*Rating*	*Mean default rate* (%)	*Default rate standard deviation* (%)	*Sector split* *General economy* (%)
Co. (1)	358,475	B	30.00	15.00	100
Co. (2)	1,089,819	B	3.00	15.00	100
Co. (3)	1,799,710	BBB−	10.00	5.00	100
Co. (4)	1,933,116	BB	15.00	7.50	100
Co. (5)	2,317,327	BB	15.00	6.50	100
Co. (6)	2,410,929	BB	15.00	7.50	100
Co. (7)	2,652,184	B	30.00	15.00	100
Co. (8)	2,957,685	BB	15.00	7.50	100
Co. (9)	3,137,989	BBB+	5.00	2.50	100
Co. (10)	3,204,044	BBB+	5.00	2.50	100

Summary of the CreditRisk^{+} model

- *CreditRisk^{+} captures the main characteristics of credit default events* – credit default events are rare and occur in a random manner with observed default rates varying from year to year. The model's approach attempts to reflect this by making no assumptions about the timing or causes of these events and by incorporating a default rate volatility. It also takes a portfolio approach and uses sector analysis to allow for concentration risk.

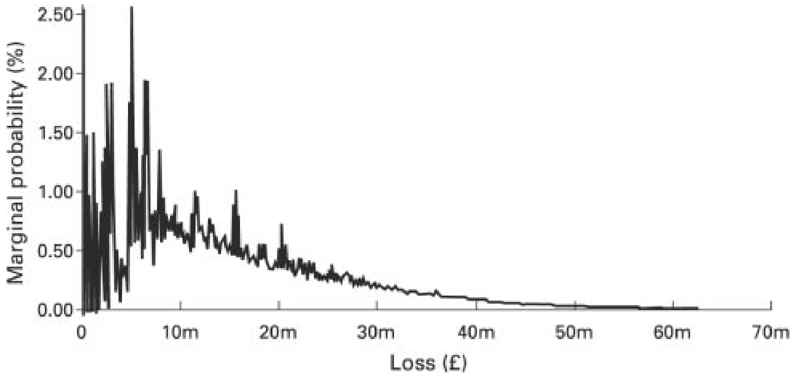


Figure 8.11 Credit loss distribution (obligor portfolio for a single sector).

- *CreditRisk$^+$ is capable of handling large exposure portfolios* – the low data requirements and minimum assumptions make the model comparatively easy to implement for firms.

However, the model is limited to two states of the world: default or non-default, as such it is not as flexible as CreditMetrics and, ultimately therefore, not modelling the full exposure that a credit portfolio would be subject to.

APPLICATIONS OF CREDIT VaR

Prioritising risk-reducing actions

One purpose of a risk management system is to direct and prioritise actions. When considering risk-mitigating actions there are various features of risk worth targeting, including obligors having:

- the largest absolute exposure;
- the largest percentage level of risk (volatility);
- the largest absolute amount of risk.

A CreditMetrics' methodology helps to identify these areas and allows the risk manager to prioritise risk-mitigating action.

Exposure limits

Within bank dealing desks, credit risk limits are often based on intuitive, but arbitrary, exposure amounts. This is not a logical

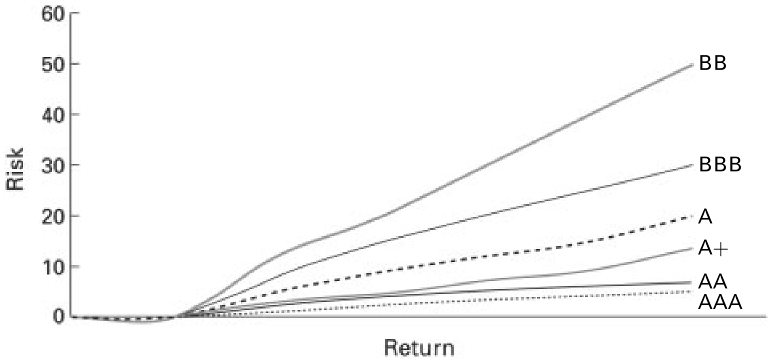


Figure 8.12 Size of total exposure to obligor – risk/return profile.

approach because resulting decisions are not risk-driven. Limits should ideally be set with the help of a quantitative analytical framework.

Risk statistics used as the basis of VaR methodology can be applied to limit setting. Ideally, such a quantitative approach should be used as an aid to business judgement and not as a stand-alone limit setting tool.

A credit committee considering limit setting can use several statistics, such as marginal risk and standard deviation or percentile levels. Figure 8.12 illustrates how marginal risk statistics can be used to make credit limits sensitive to the trade-off between risk and return.

The lines on Figure 8.12 represent risk/return trade-offs for different credit ratings, from AAA to BB. The diagram shows how marginal contribution to portfolio risk increases geometrically with exposure size of an individual obligor, noticeably so for weaker credits. To maintain a constant balance between risk and return, proportionately more return is required with each increment of exposure to an individual obligor.

Standard credit limit setting

In order to equalise a firm's risk appetite between obligors as a means of diversifying its portfolio a credit limit system could aim to have a large number of exposures with equal expected losses. The *expected* *loss* for each obligor can be calculated as:

Default rate × (Exposure amount – Expected recovery)

This means that individual credit limits should be set at levels that are inversely proportional to the default rate corresponding to the obligor rating.

INTEGRATING THE CREDIT RISK AND MARKET RISK FUNCTIONS

It is logical for banks to integrate credit risk and market risk management for the following reasons:

- the need for comparability between returns on market and credit risk;
- the convergence of risk measurement methodologies;
- the transactional interaction between credit and market risk;
- the emergence of hybrid credit and market risk product structures.

The objective is for returns on capital to be comparable for businesses involved in credit and market risk, to aid strategic allocation of capital.

Example 8.2 Integrated risk management.

Assume that at the time of annual planning a bank's lending manager says his department can make £5 million over the year if they can increase their loan book by £300 million, while the trading manager says they can also make £5 million if the position limits are increased by £20 million.

Assuming that due to capital restriction only one option can be chosen, which should it be? The ideal choice is the one giving the higher return on capital, but the bank needs to work out how much capital is required for each alternative. This is a quantitative issue that calls for the application of similar statistical and analytical methods to measure both credit and market risk, if one is to compare like with like.

With regard to the loan issue in the example above the expected return is the mean of the distribution of possible returns. Since the revenue side of a loan – that is, the spread – is known with certainty the area of concern is the expected credit loss rate. This is the mean of the distribution of possible loss rates, estimated from historic data based on losses experienced with similar quality credits.

In the context of market price risk the common denominator measure of risk is volatility (the statistical standard deviation of the distribution of possible future price movements). To apply this to credit risk, the decision maker therefore needs to take into account the standard deviation of the distribution of possible future credit loss rates, thereby comparing like with like.

We have shown that as VaR was being adopted as a market risk measurement tool, the methodologies behind it were steadily applied to the next step along the risk continuum, that of credit risk. Recent market events, such as bank trading losses in emerging markets and the meltdown of the Long Term Capital Management hedge fund in summer 1998, have illustrated the interplay between credit risk and market risk. The ability to measure market and credit risk in an integrated model would allow for a more complete picture of the underlying risk exposure. (We would add that adequate senior management understanding and awareness of a third type of risk – liquidity risk – would almost complete the risk measurement picture.)

Market risk VaR measures can adopt one of the different methodologies available; in all of them there is a requirement for estimation of the distribution of portfolio returns at the end of a holding period. This distribution can be assumed to be normal, which allows for analytical solutions to be developed. The distribution may also be estimated using historical returns. Finally, a Monte Carlo simulation can be used to create a distribution based on the assumption of certain stochastic processes for the underlying variables. The choice of methodology is often dependent on the characteristics of the underlying portfolio plus other factors. For example, risk managers may wish to consider the degree of *leptokurtosis* in the underlying asset returns distribution, the availability of historical data or the need to specify a more sophisticated stochastic process for the underlying assets. The general consensus is that Monte Carlo simulation, while the most IT-intensive methodology, is the most flexible in terms of specifying an integrated market and credit model.

Earlier sub-sections in this section have shown that credit risk measurement models generally fall into two categories. The first category includes models that specify an underlying process of default. In these models firms are assumed to move from one credit rating to another with specified probabilities. Default is one of the potential states that a firm could move to. The CreditMetrics model is of this type. The second type of model requires specification of a stochastic process for firm value. Here default occurs when the value of the firm reaches an externally specified barrier. In both models, when the firm reaches default the credit exposure is impacted by the recovery rate. Again market consensus would seem to indicate that the second type of methodology, the firm value model, most easily allows for development of an integrated model that is linked not only through correlation but also the impact of *common* stochastic variables.

Chapter 9

A REVIEW OF VALUE-AT-RISK[1]

[1] This chapter was written by Max Wong, Head of VaR Model Testing, Markets & International Banking, Royal Bank of Scotland.

Under the Basel risk regulation of banks, Value-at-Risk is the *de facto* risk model for the computation of regulatory capital. The model has been severely criticized post the 2008 credit crisis because many assumptions behind the model broke down during this stressful period; it was found that VaR understated risks at the exact time that it was most needed. Put simply, VaR as a risk measure is just 'too little, too late' under fire, some call it a 'peace-time tool'. This chapter addresses the weaknesses of VaR and discusses some of the new initiatives that came about as a result of crisis response under the banner of Basel III. We also track the evolution of VaR in recent years and look at some interesting work on the research frontier.

VaR models appear in various guises in banks. In the banking book,[2] we have the internal ratings based (*IRB*) model and the interest rate risk in the banking book (*IRRBB*) model. In the trading book, we have VaR, the stressed VaR, the incremental risk charge (*IRC*) model, the specific risk model and the comprehensive risk model (*CRM*). In operational risk, we have the OpVaR model and more recently, in counterparty risk, we have the CVA VaR model. They all have something in common – they are all actuarial based or 'VaR-like' in nature. This involves sampling an empirical distribution or specifying an assumed distribution and then estimating a quantile from the distribution.

So why do banks use VaR models? This may seem obvious, but having a clear understanding of the 'why' will help us to appreciate that some key weaknesses of VaR mentioned in this chapter are simply unacceptable given its intended use.

First, VaR is a regulatory required model for the purpose of calculating capital. Since this so-called 'minimum' capital is set aside to buffer against crisis events that would threaten the survival of the bank, it seems prudent to set the confidence level to be extremely high, typically at 99% or higher. Indeed, we are interested in the correctness of the tail segment of the loss distribution. This would imply, for example, that care should be exercised when

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[2] In essence, the trading book refers to the investment banking area within a bank which takes positions with a trading intent and records them using mark-to-market accounting. In contrast, the banking book deals with loans and deposits and has a longer risk horizon; positions are recorded using accrual accounting.

estimating the correlation between risk variables – it is well known that the correlation at the tails (stress period) can be very different from the correlation within the body of the distribution (benign period). Many researchers also believe that there are (at least) two distributions in action, one during the regular regime and one during a crisis regime, and the reality we observe is a dynamic mixture of the two. The sampled distribution is the unconditional one in which any regime shifts are not differentiated, and the whole recent sample set is used for VaR. If this is true, it means that current industry best practice is at best a simplified toy representation of a much richer underlying phenomenon.

The second purpose of VaR is for internal, day-to-day risk management control and attribution analysis. As such, a lower confidence level is acceptable and desirable since it will afford a more precise estimation of VaR. A weighting scheme may be used to make the VaR respond more quickly to day-to-day market changes. Again, care must be exercised to estimate the dependence structure and ensure coherence because these will ultimately determine how risks are broken down to individual risk drivers.

We limit the scope of this discussion to historical simulation VaR (or *hsVaR*), which was introduced in Chapter 3, to keep the flow of thought consistent and also because this approach is by far the most common.

VAR IN CRISIS

Let us look at the performance of VaR prior to and post the credit crisis in 2008. Figure 9.1 shows the market VaR for the Dow Jones index and Figure 9.2 is the credit spread VaR for the JPMorgan 5-year CDS spread. Here we use a 99% VaR calculated using a 250-day rolling sample of daily asset returns (X). Strictly speaking one should calculate VaR using $g(X)$, the pricing function of the derivative instrument, but for simple linear products, $g(X) = X$ is a valid simplification, which we shall assume in this chapter.

Both positive and negative VaRs are charted; the vertical bars are daily returns which exceeded VaR. Thus the figures are in fact backtest charts where the bars are incidences of backtest breaches.

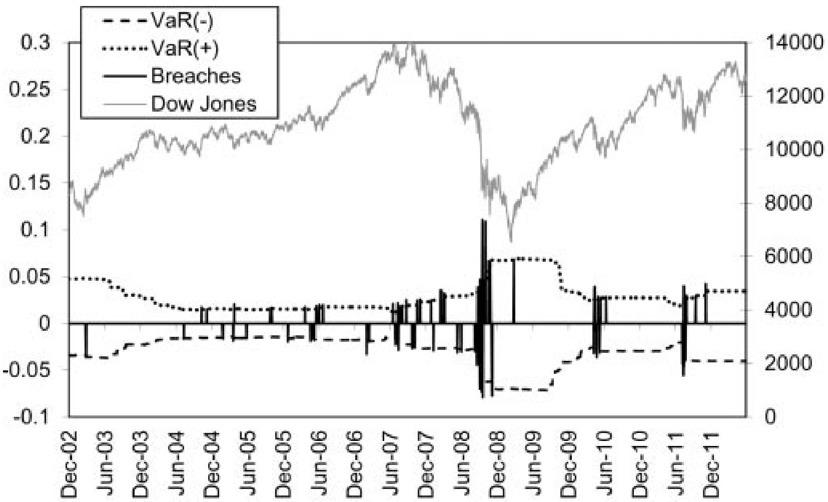

Figure 9.1 VaR for Dow Jones index.

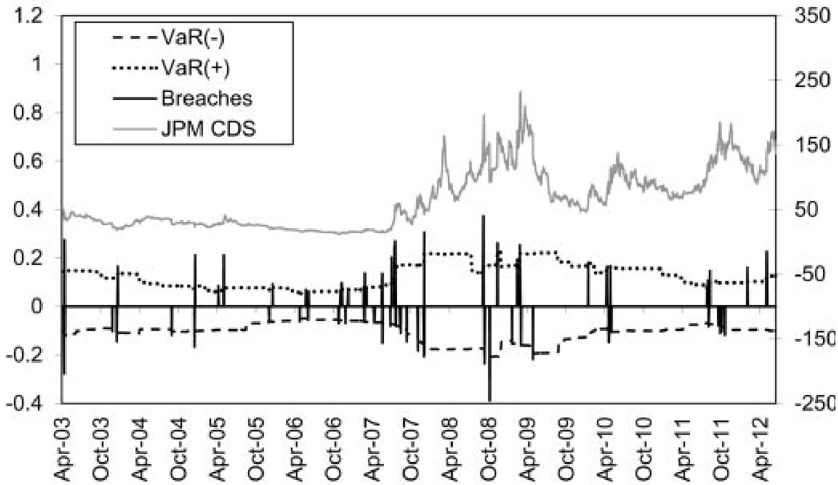

Figure 9.2 VaR for JPMorgan 5-year CDS spreads.

WEAKNESSES REVEALED

Let's look at some stylized facts observed in the markets by using the Dow Jones index and JPMorgan CDS spread as examples.

Market risk

In Figure 9.1, before the 2008 crisis the VaR was relatively mild and was in fact declining as the Dow made new highs. At the height of the crisis (end of 2008) a few observations are noteworthy.

The first observation is that the VaR increased *suddenly* and almost doubled when the Dow Jones collapsed during November 2008. As a risk measure, VaR is perpetually late in reaction and is thus useless for crisis detection. It is also harmful as a tool for determining capital because the burden of capital is increased just as banks enter a crisis. Consequently, they are forced to cut back on their holdings of assets. This exacerbates the fall in asset prices and creates systemic risk. During the four years leading up to the crisis, VaR declined gradually as the market rallied. The accommodative capital requirement encourages balance sheet expansion and accumulation of assets. In summary, VaR as a capital tool will amplify the business cycle, a risk which we now identify as *procyclicality*.

The second observation is that the size of the losses (VaR breaches) seen during the crisis is beyond expectation. To get an intuitive feel, it helps to express the size of the move in 'number of years before such an event is expected to occur, assuming a normal distribution'.[3] This can be calculated using the Excel function:

$$\frac{1}{250T} = \text{Probability}(x) = \text{NORMSDIST}(x, 0, \sigma/\sqrt{250}, \text{TRUE}) \quad (9.1)$$

where the observed event (a log return equal to x) is calculated to occur once every T years (assuming 250 business days per year). We assume an annualized volatility of $\sigma = 25\%$ typical of equity indices. The result in Table 9.1 is clearly ludicrous, as we have already witnessed four such events in 2008 alone!

[3] Although, after the crisis, most people are keenly aware that financial markets are seldom normally distributed, it is a convenient scale for comparison, just as a light year is a convenient unit of measure when looking at galactic distances.

Table 9.1 Large single-day losses for the Dow Jones index during the credit crisis (2008)

Event date	*Daily log return*	*Mean number of years between occurrences*
15-Oct-08	−8.2%	37,326
01-Dec-08	−8.0%	19,952
09-Oct-08	−7.6%	5,482
29-Sep-08	−7.23%	1,684

Nassim Taleb, author of *The Black Swan* (2007) and a critic of VaR described such crisis events as *extremistan*, a term which refers to the fact that at the extreme tails of the distribution, events are statistically non-reproducible (atypical). If this is true as suggested by the self-contradiction of Table 9.1, frequentist statistics (which hinge on reproducibility) will fail under extreme situations and it will be difficult to estimate VaR with any degree of precision and accuracy.

To make matters worse, the use of a rolling observation period in hsVaR means that VaR (as a quantile) can never exceed the worst loss number in the sampled data.[4] There is no compelling reason why this should be the case. Since most crises are preceded by a period of calm often lasting years, when a crisis hits, hsVaR will always be surprised by the loss magnitude. In other words, the tail of the realized distribution is often found (from bad crisis experiences) to be fatter than what the model suggests.

To address the problems of fat tails, some researchers propose the use of extreme value theory (*EVT*) to model a theoretical distribution for VaR. Once this is modeled, one can extrapolate out into the tails to obtain extremely high quantiles (such as 99.97%). However, given the scarcity of extreme observations, the few sample data points can fit different EVT distributions and hence calibrate to different parameters. An EVT distribution with slightly different parameters can extrapolate to a vastly different VaR loss. So you may have a case where a VaR is \$10 million with a ±\$6 million error range – the VaR becomes too inexact for any meaningful use. EVT VaR is often a

[4] A notable exception is the method of volatility updating VaR by Hull and White (1998), but banks mostly use equally weighted or exponentially weighted hsVaR.

static number that does not respond in a timely way to day-to-day market movements. EVT is also not well-developed for large portfolios, because the correlation structure is poorly estimated. The data hardly exist – if it is difficult enough to observe extreme events for a single variable, it will be near impossible to observe extreme events occurring *concurrently* for multiple variables.

The third observation is the *clustering* of VaR breaches around the crisis period (end of 2008). Mathematically speaking, clustering comes from stochastic (changing) volatility. We do expect large P&Ls to occur during a crisis, but we are really talking about *breaches*. For a VaR model to be accurate in its prediction, breaches must be independent (i.e., evenly spread out over time, not clustered together). To see why, suppose that whenever a VaR violation occurs (for 2.5% quantile), it always occurs in pairs, so that whenever one violation occurs it is immediately followed by another of the same loss. Let's say VaR is at $10 million. When one violation has just occurred with a $10.5 million loss, the probability of the second violation is now 100% and no longer 2.5%. Hence, the reported VaR, now at $10.5 million, no longer represents a 2.5% probability of loss exceedence – it is understating the true risk. To be correct, the VaR result should be farther out in the tail (say $30 million) so as to reduce its exceedence probability back to 2.5%. This example shows that in the presence of clusters, VaR is inaccurate because its probability interpretation is distorted.

The only way to overcome this is to use decaying weights (or more generally conditional models such as GARCH) which are more responsive to market changes. These fast-moving models are more adept at keeping in step with the changing volatility of the market – increasing the VaR number more rapidly when multiple breaches occur (and decreasing during quiet periods) such that VaR remains an unbiased measure of the real quantile. Currently, most banks are still using unconditional (equally weighted) models for reasons of simplicity.

Credit risk

Credit risk had been growing towards the autumn of 2007, when the US sub-prime mortgages market began to show signs of weakness and by 2008 the credit crisis was in full swing. As shown in Figure 9.2, spread VaR increased as expected, but this measures only the *volatility* of spreads, a continuous kind of movement. Credit products are also exposed to other forms of risk which are absent from Figure 9.2.

A key missing component is *default risk*. Issuers of credit-sensitive products are companies that can undergo default or bankruptcy. When this happens, credit products may quite abruptly lose their entire value, less some recovery portion. This type of 'jump' risk is not captured in spread VaR. Another type of discontinuous risk is *migration risk*, which occurs when the issuer is downgraded by rating agencies. Spread *levels* may suddenly jump or fall by a quantum and this may not necessarily be reflected in spread VaR. By dealing exclusively with changes (*returns*), spread VaR is missing critical information on default risk which is hidden in the spread *levels*.

As an example, compare Figure 9.3 with Figure 9.4; spread fluctuations and issuer default are quite distinct risks – both risks should be considered in any realistic model.

Another risk pertinent to the (relatively) less liquid credit trading market is *liquidity risk*. The consequence is that the spread level quoted in the markets is untradeable during market stress, causing additional losses for traders looking to exit holdings. Finally, a bank is also exposed to the credit risk of its counterparties, what is now termed *counterparty risk*. It is possible that counterparties on the

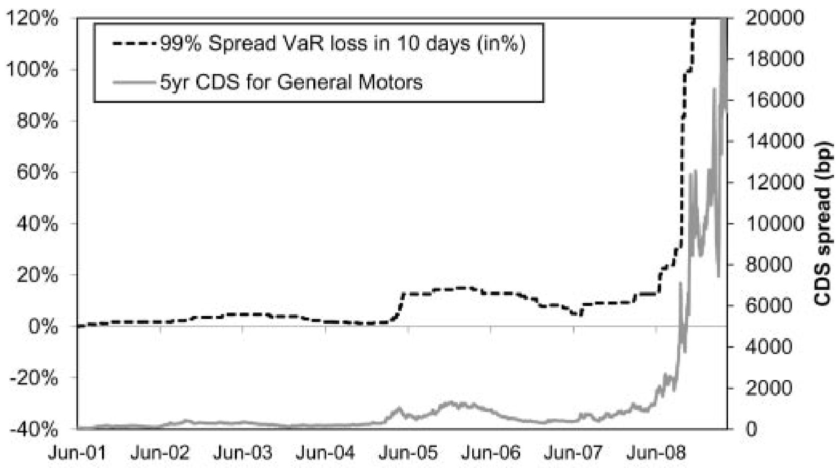

Figure 9.3 VaR due to movement in credit spreads for General Motors CDS. Note the spread VaR went above 100% of notional, an impossible loss.

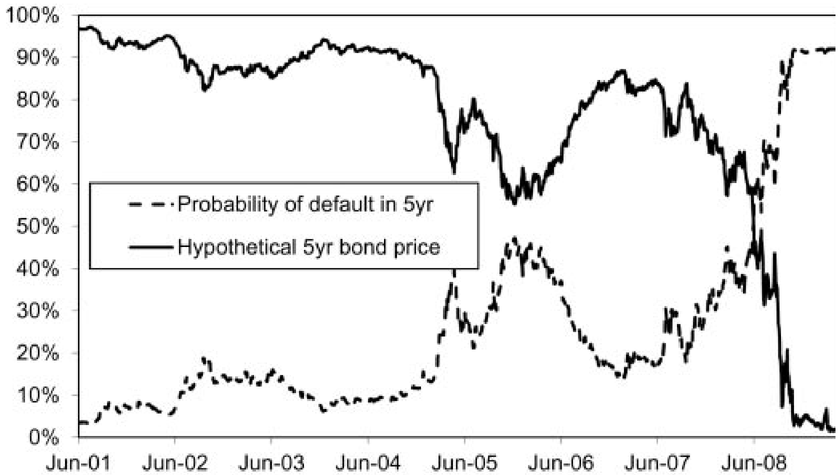

Figure 9.4 Default risk – at any point in time, the bond price has a chance of losing all (from default over a 5-year horizon) with probability given by the dotted line. This probability is derived from the CDS spread level.

losing end of a trade cannot fund their losses, especially during a crisis when the banking system is hoarding liquidity.

Back in 2008, internal models for Basel capital considered only spread VaR for credit trading, which did not capture these four additional risk types. Banks were undercapitalized to weather the storm that followed. Under Basel III, the industry has developed new models to handle these other risks.

Portfolio effects

There are additional complications when a bank deals with a large portfolio of products. There are two subtle problems that are practically insoluble and can result in extremely imprecise and possibly biased VaR numbers ('subtle' in the sense that it is an open secret which banks have to live with, as they continue to report VaR to regulators).

First, *non-linear* products such as options will amplify the 'measurement problem' of extremistan. In practice, VaR is always estimated using the P&L calculated from $g(X)$ and never on X, where $g(\,.\,)$ is the pricing function of the product and X is the risk factor (or a vector of risk factors). For realistic portfolios, $g(\,.\,)$ is often non-linear and this can cause the distribution to be extremely sparse at the tails and skewed. The sparse tail compounds the estimation difficulties brought about by extremistan. The skewed marginal distributions use a correlation matrix to arrive at the portfolio VaR (joint distribution) but this is questionable in theory. Correlation matrices are used in parametric VaR and Monte Carlo VaR. For the correlation to be perfectly correct, the joint distribution has to be elliptical, but elliptical distributions[5] are symmetric – never skewed.

For example, in Figure 9.5 the top panel shows the joint distribution of two random variables, which is obviously elliptical. The correlation can be derived by drawing the best straight line through the 'cloud'. In contrast, in the bottom panel we introduce optionality to one of the variables, causing the joint distribution to be skewed. In this case, the best straight line can still be drawn but it will be *biased*, and so will the correlation.

Second, the hsVaR method can easily produce a portfolio VaR number for a large portfolio with many risk factors. Unfortunately, this number will be very imprecise because of the *curse of dimensionality*. This problem occurs because the length of the sample (about 250 days) is much smaller than the dimension of the system (about 50,000 risk factors). To understand this, consider the following: it takes more than two points to define a one-dimensional object (a line), it takes more than three points to define a two-dimensional object (a plane as in Figure 9.5) and it takes more than four points to define a three-dimensional object. Hence, the number of sampled points required to represent (in a rich way) an N-dimensional object is indeed extremely large as N grows. So a bank with 50,000 risk factors will need data of much more than 50,000 days. Since we only have one year's data, the dependence structure and thus the hsVaR will be poorly measured. Most risk managers are quite content with having a VaR number to report, unaware that it is in fact a very 'blurred image' of true risk.

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[5] The elliptical distribution is a general family of multivariate distributions whose projection onto a two-dimensional plane produces an elliptical contour (or a hyper-ellipse in higher dimensions), hence the name. The normal distribution is a special case of an elliptical distribution.

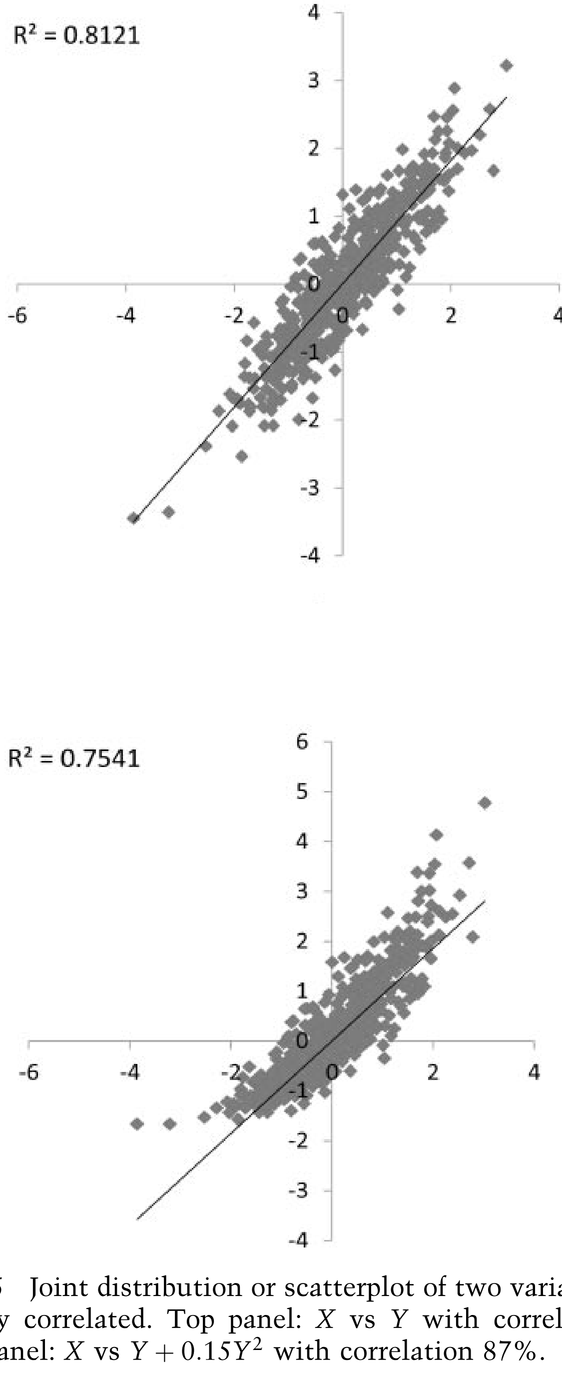

Figure 9.5 Joint distribution or scatterplot of two variables which are highly correlated. Top panel: X vs Y with correlation 90%. Bottom panel: X vs $Y + 0.15Y^2$ with correlation 87%.

In short, the presence of non-linearity and large portfolios can be an insidious couple that makes the measurement of VaR a tricky task.

NEW REGULATION AND DEVELOPMENT

After the 2008 crisis, the BIS initiated a programme of regulatory reform covering various aspects of supervision including risk models that are used for Pillar I capital. The new rules eventually formed the Basel III framework, formally released in December 2010. It outlined various new risk models and the mandatory principles such models must satisfy. It is then left to the industry to develop and implement them within a given time frame.

The new principles are designed to address the weaknesses found during the credit crisis. Here we will only discuss the key models for the trading book.[6] For a full description of the Basel III proposal, see the BIS report *Basel III: A global regulatory framework for more resilient banks and banking systems.*

Procyclicality: stressed VaR (SVaR)

Traditionally, the Basel market risk charge is based on a 99% VaR of a 10-day holding horizon estimated using a rolling observation period of at least 12 months. SVaR is similar in every aspect except that the chosen period is relatively static and must represent a period of extreme stress such as that during the 2008 crisis.

For the purpose of regulatory capital, the VaR and SVaR are simply added, without any diversification offset. The intention is that the (more than) doubling of capital will act as a buffer against the procyclical drawback of VaR. Double counting is criticized by the industry as it is expensive in capital, and from a (purely) modeling perspective the market cannot be in *both* a normal and a crisis state simultaneously. A more realistic approach would be to build a countercyclical buffer into the conventional VaR model itself (see

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[6] There are other Basel III initiatives related to the banking book – such as the countercyclical capital buffer and liquidity ratios – but these are outside the scope of this chapter. We will also not address the *comprehensive risk charge* designed to model the complete risks of credit correlation products in the trading book, as we do not foresee a significant revival of this asset class in the future given its negative role in the credit crisis.

buVaR in the next section). This idea was first mooted by the *Turner Review* (2009), a report by the UK regulator, which recommended an *overt countercyclical* capital buffer such that the required capital would increase during a boom to restrain an overheated market and to build a safety buffer that could be utilized during a future downturn.

Default and migration risks: incremental risk charge (IRC)

The *incremental risk charge* (*IRC*) was introduced by Basel 2.5 (a precursor to Basel III) in recognition that the default risk of illiquid credit products is not well-reflected in VaR. The IRC should capture default risks, rating migration risks and default correlation risks at a horizon of one year at the 99.9% confidence level (in contrast to credit spread VaR which captures continuous spread movement over a 10-day horizon at the 99% confidence level). Basel recommended high-level principles for the IRC without prescribing specific models. For example, the IRC should have a soundness standard comparable with the IRB approach already used in the banking book to avoid possible 'regulatory arbitrage' between the trading and banking books. Since the IRB model is based on a one-factor Gaussian copula model, banks naturally lean towards a similar construction. Another requirement is for the modeling of cross correlation between default, migration and other risk factors.

A novel feature is the introduction of multiple time horizons as a way to address liquidity risk. First, the IRC must assume a *constant level of risk* over a one-year risk horizon. The reason: banks can seldom respond efficiently in the short-term when in distress, so a one-year period seems appropriate for crisis response. Also, consistent with a 'going concern' view of a bank, the bank must continue to trade to support its income-producing activities despite losses – this suggests the concept of rebalancing of positions to maintain a constant level of risk.

Second, the IRC must differentiate between credit products of different liquidity by assigning them a differing *liquidity horizon*. In theory, this horizon represents the shortest amount of time taken to sell off/hedge a position in a stressed market without adversely affecting prices. In practice, the liquidity horizon for a product is determined subjectively. The liquidity horizon must be at least 3 months by Basel rule. A natural way to implement the IRC is via a multi-step simulation approach (in time steps of 3 months) where products are grouped into 3-, 6-, 9- and 12-month horizons.

To model default, it is convenient to use a Merton-style approach whereby default occurs when the asset (for issuer *i*) falls below some threshold.[7] For the purpose of this simulation, the asset return is modelled using a one-factor Gaussian copula model such as:

$$Z_j = \sqrt{\rho_i} Z_{\mathrm{Sys}} + \sqrt{1 - \rho_i}\varepsilon_j \tag{9.2}$$

This assumes that all asset-*i* are driven by a common systematic factor Z_{sys} and their own idiosyncratic risk process ε_i. More realistically (9.2) can be extended to include country and sector systematic drivers. Z_{sys} and ε_i both follow a standard normal distribution, $N(0,1)$.[8] The weight ρ_i can be interpreted as the correlation of issuer-*i* with the systematic factor. This implicitly defines asset correlations between each pair of issuers *i* and *j*. Empirical data analysis indicates that asset correlations between sectors are typically low; on average about 15%. Thus, it is convenient for banks to use a common correlation number for all issuers (so $\rho_i = \rho$) and conservatively set a higher number. Indeed, this is in line with regulation, which calls for stressed calibration of model parameters.

The asset return is simulated; if it falls below thresholds designated for migrations (or default), we calculate the corresponding P&L. The relationship between credit spreads *vs* ratings can be found empirically – typically, spread increases exponentially with lower rating – this profile can be charted. Given a simulated rating migration between two states, we can read from the profile the corresponding spread change and compute the P&L of the product. With many simulations, a distribution of P&L can be derived at the liquidity horizon. Recall there are four liquidity horizons – at 3, 6, 9 and 12 months – and the risk horizon is at 12 months. So a product with a 3-month liquidity horizon will need to be rebalanced three times (at 3 months, 6 months, and 9 months) to reach the risk

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[7] The threshold can be backed out from rating migration matrices. Issuer-*i* is usually categorized by a rating, sector, country combination. Migration matrices for these categories are usually available from rating agencies.

[8] While this appears questionable given the non-normality seen during crises, we are really saying that the *driver* of the default process is normally distributed, not that the asset return is normally distributed; in fact, the calculated loss distribution is highly fat tailed.

horizon – with rating reinstated to its original level at each rebalancing step (even if migrations have occurred in the previous simulation steps). This satisfies the constant risk requirement.

The systematic risk drivers for each 3-month step – hereby denoted Z_{1-3}, Z_{-3-6}, Z_{6-9}, Z_{9-12} – are independently distributed as $N(0,1)$. For example, a product of the 6-month horizon – the P&L at 3 months – is simulated using Z_{1-3}; the P&L at 6 months is simulated using a 6-month systematic risk factor made from a function of Z_{1-3} and Z_{-3-6} such that:

$$Z_{1-6} = \theta Z_{1-3} + \sqrt{1-\theta^2} Z_{3-6} \tag{9.3}$$

where θ is the autocorrelation (or momentum) of systematic migration. Then rebalancing to the original rating is done at 6 months, before commencing the simulation step to 9 months using Z_{6-9} and so on. Briefly speaking, (9.2) allows the capture of cross default correlation and the independent factors $\{Z_{1-3}, Z_{-3-6}, Z_{6-9}, Z_{9-12}\}$ allow for rebalancing to sustain a constant level of risk. In between rebalancing, (9.3) allows for some persistence in the migration process. For more details on IRC modelling, please refer to Wilkens et al. (2012).

Liquidity risks: differing liquidity horizons

In the trading book context, liquidity risk refers to the risk that bid–ask spreads may widen such that a trader looking to exit his positions will incur an extra loss measured from the mid price (fair value) of the market. In a crisis situation the bid–ask can widen tremendously and often disappear altogether. Traditionally, banks control this kind of risk by simply keeping a bid–offer reserve. In academia on the other hand, researchers commonly model the bid–ask loss as an additional fluctuation in the returns used for VaR calculation, leading to an increase in the variance of the distribution. This so-called liquidity-adjusted VaR (*L-VaR*) disregards the fact that it is a sunken cost which will be a *loss* regardless of whether the return is positive or negative – it means the distribution should in fact be shifted to the left (instead of having a larger variance) in the presence of liquidity risk.

The BIS has taken a different approach by requiring that liquidity risk be modelled using liquidity horizons of differing length. The basic idea is that since VaR numbers usually increase with the estimation horizon, by requiring less liquid products to be mapped to a longer risk horizon, VaR will be adjusted for liquidity risk. This construction also merges naturally with the multi-step approach of the IRC model. The IRC is essentially a testing ground for this novel approach.

Counterparty risks: CVA VaR

Credit valuation adjustment (CVA) is the fair price of counterparty risk for a derivatives transaction; it is computed as a chargeable fee adjustment when pricing derivative deals for trades that are done without posting collateral. During the credit crisis, it was found that two-thirds of losses from counterparty risk came from CVA pricing losses and only one-third were from actual defaults. Hence, Basel III has called for a counterparty risk capital charge computed using CVA VaR. The CVA for an internal model is given by:

$$\mathrm{CVA} = \mathrm{LGD}\sum_{i=1}^{T}\mathrm{Max}\left\{0, \exp\left(-\frac{s_{i-1}\,.\,t_{i-1}}{\mathrm{LGD}}\right) - \exp\left(-\frac{s_i\,.\,t_i}{\mathrm{LGD}}\right)\right\} \times \left\{\frac{EE_{i-1}\,.\,D_{i-1} + EE_i\,.\,D_i}{2}\right\} \quad (9.4)$$

Let's explain the formula in simple language. It is calculated on a stand-alone basis for each counterparty, with *no* offsets allowed across different counterparties or against other VaR measures.[9] Time is bucketed from today (t_0) to the longest contractual maturity (t_T) along the exposure netting sets with the counterparty. S_i is the CDS spread at tenor t_i with the counterparty as obligor. LGD is the loss given default (or one minus the recovery rate) estimated for the counterparty. The term $\exp(-st/\mathrm{LGD})$ is the survival probability at time t and usually decays with time, thus the marginal survival rate (difference between adjacent survival probabilities in the equation) is usually positive. However, should the CDS spread curve become inverted the marginal survival rate may become negative and reduce the CVA. To be conservative, this is floored at zero by the Max(.) function.

EE_i refers to the *expected exposure* at time t_i and D_i is its discount factor. Figure 9.6 illustrates the meaning of EE. In banks, exposures are simulated using a Monte Carlo engine which reprices deals done

[9] Exception: it can be offset against a CDS (where that counterparty is the obligor) held for counterparty risk hedging.

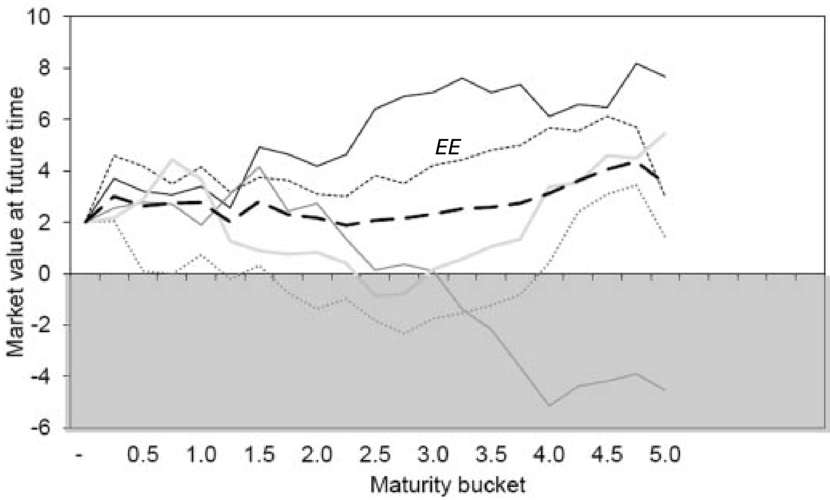


Figure 9.6 The *EE* (dotted line) is the average of simulated market values at a future time bucket, after negative values (in the shaded area) have been floored at zero.

with a counterparty with netting allowed between deals. This generates many future paths of *net* market values, but only positive values (gains) will give rise to counterparty risk (exposures). Hence, negative values are floored at zero. Expected exposure (*EE*) is the average (across many paths) of exposures at a given time bucket. This is discounted to today's money as per the second parenthesis { }. The product of the two { } is summed across all time buckets up to T.

There are two stages of simulation – the first one to generate the *EE* across maturities, and then simulating different shocks to the spread S which gives the distribution of CVA. The quantile of this distribution is the CVA VaR.

The Basel III CVA VaR formula is a simplification of what is still a developing subject. Certain well-known effects are absent from the model; for example, wrong-way risk, bilateral CVA charge and default risk of the counterparty. Wrong-way risk occurs in cases where the *EE* itself is positively correlated to changes in S. In the simple model, the two are assumed to be independent. Bilateral CVA is a *net* fee that also includes the CVA fee that the counterparty charges the bank in a transaction. The CVA distribution is due to changes in spread of the counterparty (i.e., the volatility of S) and excludes the default risk of the counterparty. Basel rules will likely evolve in the coming years to account for such risks as the subject matures.

Fat tail risk: over-buffering

Risk management in banks is a team effort – there are often hundreds of staff managing different aspects of the risk architecture and in different geographic locations. A simple VaR model provides a common language in the chain of command and avoids potential modelling errors. Thus the regulator does not actively encourage more sophisticated modelling of fat tails, such as using EVT. Instead, to protect against tail risks, the current preference is for a larger capital requirement achieved by having multiple risk models (described above). This piecemeal approach leads to a tremendous amount of double counting of capital, as diversification among different models is not accounted for. This may be well intended but it does lead to bad (incoherent) modelling.

A corporate bond, for example, will incur capital from market risk VaR, spread VaR, stressed VaR and incremental risk charge – all summed without offsets. If the bond is trading at a very low price (at distressed levels), it is not impossible for the total capital charge to be larger than the bond price itself,[10] which is illogical.

New framework for trading book

In May 2012, the BIS published the consultative document, *Fundamental Review of the Trading Book*, which describes the supervisor's preferred direction for the development of market risk models and the regulatory framework. Some key proposals pertaining to *models* are:

1. The replacement of VaR with *expected shortfall* (*ES*) defined as:

$$\mathrm{ES} = E(-X \mid X \leq -\mathrm{VaR}) \tag{9.5}$$

where X is the P&L. The expectation is computed by simply taking an average of all the values of X lower than VaR in the observation

[10] Looking at Figure 9.3 as an example, spread VaR on its own has already exceeded 100% because this measures fluctuation (change) only. Thus the spread VaR model is unsuitable when the spread level is extremely high – it does not recognise that a bond can never lose more than its full notional.

period. This means that gap risk[11] which is missed by VaR can be integrated into ES. ES is more sensitive to tail data and more likely to be interpreted correctly by management.[12] However, ES does not really solve the fundamental problems mentioned in the second section ("Weaknesses revealed"). Another advantage is that ES is a *coherent measure* – one of the important properties of coherence is sub-additivity:

$$\mathrm{ES}(X_A + X_B) \leq \mathrm{ES}(X_A) + \mathrm{ES}(X_B) \tag{9.6}$$

Clearly, adding risk factors together in a portfolio should not increase total risk. Strangely, VaR does not satisfy this intuitive criterion all of the time.

2. The BIS emphasized the importance of using *stressed calibration* for risk models for the purpose of capital. This recognises that risk models which use recent sampled data, even if they truthfully represent the current market state, are inadequate when a crisis occurs; after all, a crisis is an unexpected sudden event by definition. Hence, the IRC model, for example, has parameters such as correlation calibrated to reflect stressful periods. This forces an element of conservativeness into risk modelling – in spirit, Basel III modelling is more like an engineering project than a scientific enquiry.

3. *Comprehensive* incorporation of the risk of market illiquidity. First, the BIS reaffirmed its preference for dealing with liquidity risk via the concept of differing liquidity horizons as per the IRC model (credit risk modelling). Second, it suggests that this concept also be applied in the market risk VaR model as a next development. Third, banks should have a capital add-on for (bespoke) instruments which may experience a jump in liquidity premia in times of stress, where this liquidity risk is not adequately covered by the first two points. The BIS consultative paper read in totality seems to suggest that Basel may be gradually directing the industry to develop a comprehensive risk model using a multi-step liquidity horizon as a backbone. This would eliminate the much criticized capital double

[11] This refers to sudden jump losses which are beyond VaR, typically caused by idiosyncratic news or the triggering of stop losses.

[12] VaR was blamed during the 2008 crisis for giving management a false sense of security. This has to do with the common misinterpretation that it is an *expected* loss whereas in reality it is more accurately described as a *minimum* loss at a stated probability. Also, its statistical imprecision is often overlooked and not reported.

counting and model various forms of trading risk under a coherent (diversifiable) framework.[13] However, the BIS commented that the CVA risk charge will be implemented on a 'stand-alone' basis under Basel III, and excluded from an integrated model of trading risk, as the topic of CVA models is still not well established.

BEYOND THE CURRENT PARADIGM

The paradigm of viewing risk as *volatility* is deeply ingrained in current risk management practice and the academia. Is this as logical as it appears to be? Ask a person who has never studied Markowitz portfolio theory, what is his risk once he has bought an investment in stocks? His answer will inevitably be that he is afraid prices may fall by $Y\%$ (and certainly not that his volatility will increase by $X\%$). After all, if the market crashes, it can only crash down, never up! (This fear is even reflected in the option prices of different strikes in the so-called 'volatility smile'.)

Thus, the most natural (cognitive) way to perceive risk is in terms of *directional* risk, not dispersion. Let's look at a more sophisticated example – a trader has bought a stock and short sold another as part of a so-called basis trading strategy; what is his risk? If he uses his cognitive sense, his answer will (rightly) be an adverse move of the basis in the wrong direction (never an $X\%$ increase in basis volatility).

Arguably, the only instance where it is natural to think in terms of volatility is when dealing with an option, and even then, specifically a delta-hedged option. So why do risk managers seldom consider directional risk in their risk measure? A subtle reason is risk management has always been framed in the language of *frequentist* statistics and the variance paradigm has dominated the risk curriculum. Under this school of thought, variables (risk factors) have to be filtered or transformed to be *independent and identically distributed* (*i.i.d.*) so that they are fit for purpose for statistical inferences; the Markowitz mean–variance framework is an elegant classic example.

[13] Naturally, banks will pour resources into such projects, as the realized regulatory capital relief can be large enough to influence their performance. Hopefully, this economic incentive will drive the development of more coherent and sensible models.

Put simply, if they are not i.i.d., models become intractable, meaning that we cannot make precise statements about modeled conclusions, since the results are biased. Unfortunately, the strict requirement for i.i.d. variables means that risk management as an industry deals only with the *returns* of risk factors ($X_n = \ln(P_n/P_{n-1})$) and never the price *levels*[14] (P_n). By ignoring the latter, we are oblivious to information on directional risks, such as overbought/oversold conditions, the potential explosiveness of prices surrounding key levels, and the prevailing cycle of the market.

To make our point clear, look at Figure 9.1. VaR is calculated using returns; hence, both the positive and negative VaR for short and long Dow Jones index positions, respectively, have about the same magnitude – the risk measure is *effectively* symmetric. But shouldn't investors who were long the Dow Jones in 2008 be exposed to greater 'crash' risk than shorts? After all, in an uptrend, the crash can only happen downwards (never up). This is intuitive for anyone who has observed the markets. Conversely, in a downtrend, a sharp bounce can only happen upwards. The VaR model is clearly missing the obvious directional risk, capturing just volatility.

The astute reader will argue that in hsVaR, the return distribution is *not* strictly symmetric – it could be skewed. This may be true in theory but in practice the skew (the third moment of a distribution) is unreliable and is often erratic. This is because it is calculated as a summation of many X^3 terms. Hence, any occasionally large noise in sample X gets magnified and can distort this number, and X can be quite noisy when the market price tests support and breaks resistance levels. Thus, skew is an unreliable measure of directional risk and this gives rise to the near symmetry seen in Figure 9.1.

The idea of integrating a 'directional element' into VaR is at the heart of the countercyclical capital buffer, for without the knowledge of price levels, how could the model 'know' if it is in a boom cycle or a downturn?

A new strand of research involving *quantile regression models* (*QRMs*) uses external variables Y in order to improve the forecastability of VaR and to search for countercyclical potential. In such

[14] Market prices are seldom i.i.d. due to the presence of trends in the series. For most financial time series, taking the first difference (or return) will make the return series i.i.d. Unfortunately, this process washes out useful information on trends and cycles.

models, the VaR of quantile-q of variable X is made a linear function of lagged X and external variables Y. One possibility is:

$$\mathrm{VaR}_t(q, X) = \alpha_q + \beta_q Y_{t-1} + \gamma_q X_{t-1} \tag{9.7}$$

The subscript t refers to time and the estimated parameters are dependent on the chosen quantile q. Y could be macroeconomic or microeconomic variables or risk indicators such as the VIX index, option implied skew, bond–swap spreads, etc. Some notable applications: Adrian and Brunnermeier (2009) suggested *CoVaR* (contagion VaR) where the VaR of Bank A could be a function of the VaR of Bank B. Engle and Manganelli (2004) proposed *CaViaR* (conditional autoregressive value-at-risk) where VaR could be a function of lagged VaR in various forms.

Yet another innovative approach treats VaR as coming from a mixture of two regimes: an extreme state where VaR is calculated using EVT and a regular state where the usual VaR applies:

$$\mathrm{VaR} = \omega(Y)\mathrm{VaR}_{\mathrm{Extreme}} + (1 - \omega(Y))\mathrm{VaR}_{\mathrm{Regular}} \tag{9.8}$$

The weight $\omega(Y)$ ranges between $[0, 1]$ and is driven by proprietary risk indicators Y. An example of this idea is given by Satchkov (2010) in his *instability VaR*. The problem with risk indicators is that no one has ever found one that is countercyclical (i.e., leads the market *consistently*). Most risk indicators are at best contemporaneous with the market, otherwise traders would be able to use them to profit from the market. The pressure from such trading would eliminate the advantage quicky. Thus, the lack of a leading indicator is indeed a sign of market efficiency.

The author's own approach which attempts to incorporate countercyclicality into the model is *bubble VaR* (buVaR) [Wong (2011)]. The intuition behind the model is simple – a crash (or bounce) can only happen in the countertrend direction and often abruptly. The likelihood of a crash is highest when a financial bubble is largest. When it occurs, a crash can only hurt longs, a bounce can only hurt shorts. Can these features be modeled?

Briefly speaking, the measure of a financial bubble is modeled in the time series B_t which is a function of price levels P. If the bubble is on the upside ($B_t > 0$) then we inflate the return distribution on the negative side, which would penalize longs. If the bubble is on the downside ($B_t < 0$), we inflate the positive returns, which would penalize shorts. At time t, for the risk factor of price P_n, its daily

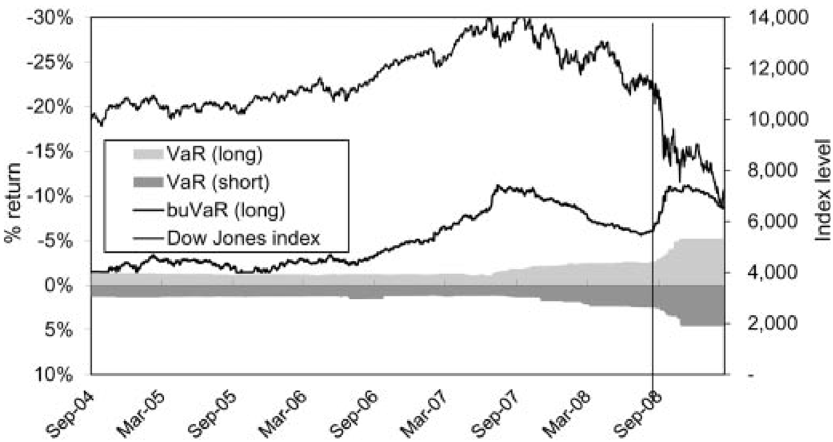


Figure 9.7 buVaR *vs* hsVaR for the Dow Jones index at the 2.5% quantile.

return variable $X_n = \ln(P_n/P_{n-1})$ undergoes a transformation:

$$X_n \rightarrow \begin{cases} \Delta_t X_n & \text{if sign}(X_n) \neq \text{sign}(B_t) \\ X_n & \text{if sign}(X_n) = \text{sign}(B_t) \end{cases} \tag{9.9}$$

where Δ_t (≥ 1) the *inflator* is a constant which is a function of B_t, and n is the scenario number in the hsVaR approach. For a one-year period, $n = 1, \ldots, 250$. For a description of the functional form of B_t, see the paper by Wong (2011). The quantile is estimated from the transformed X sample to arrive at buVaR.

Figure 9.7 illustrates the buVaR for the Dow Jones index. Note that buVaR is asymmetrical – it penalizes the long position more than the short during the unsustainable rally prior to the crisis. The second observation is that buVaR builds up a capital buffer *ahead* of the crash, in contrast to normal hsVaR which rises only *after* the crash.

Such a countercyclical property is ideal for the purpose of capital; it creates a safety buffer in anticipation of a crisis and has all the intuitive features mentioned. The drawback is that the bubble B_t is modeled using prices P (a non-i.i.d. series); the resulting model is not mathematically tractable – hence buVaR will find it difficult to pass strict statistical backtests. In effect buVaR sacrifices testability for countercyclicality, in the same way that using a stressed calibration would make a VaR model untestable at regular times.

In conclusion, the 2008 crisis was an expensive stress test for the VaR models of Basel II. The VaR concept failed during the crisis. The ensuing regulatory reform led to a growth in modeling complexity driven by Basel III principles, which resembles an engineering project (with many safety buffers). Even though ES may soon replace VaR, 'VaR-like' or actuarial models will likely be the mainstay in the Basel framework of risk management. The current expensive piecemeal approach will hopefully evolve into a more coherent model in the future.

EXERCISES

SIMPLE EXERCISES

1. VaR – analytic method.

Calculate the

(a) 1-day 95% one-tail VaR, and
(b) the 2-week VaR

for the following fixed income position:

Single bond position: 100 lots in LIFFE long gilt future.

Contract size: £100,000 nominal (as of the September 1998 contract; therefore, £1 change in price equals £1,000 change in position value).

Closing price: 105.75.

1-year price returns variance: 0.161 604.

Standard deviation:

Range over normal:

2. Portfolio VaR.

A simple formula to calculate a one-position 1-day VaR (DVaR) is:

$$\begin{aligned} \text{DVaR} = [&\text{No. of standard deviations for c.i.} \\ &\times \text{Daily volatility} \\ &\times \text{Position value}] \end{aligned}$$

For a 99% one-tailed test the number of standard deviations is 2.33.

Position in £100 million 10-year gilt. The daily volatility is 0.71% (equivalent to 11.23% annual volatility, which is the daily figure multiplied by $\sqrt{250}$). Calculate the

(a) 1-day VaR for this position in isolation, and
(b) 2-week VaR for the same position.

3. RiskMetrics example – two-position VaR.

Calculate the portfolio VaR using the RiskMetrics™ formulae for the following portfolio:

Position: A trader is long £10 million 10-year gilts and short £20 million 5-year gilts.

Market/Risk:	10-year volatility:	0.999%
	5-year volatility:	0.632%
	Correlation:	0.47%

Volatility here is defined as 'the % of value which may be lost with a certain probability' – for RiskMetrics™ this is 95%. Calculating on this basis, therefore, already takes into account the number of standard deviations used in the VaR equation (unlike in the previous exercise).

Calculate individual position VaRs, the undiversified risk (assuming perfect correlation) and then the portfolio VaR.

What is the diversification benefit to the trader?

Answers

1. Standard deviation: $\sqrt{0.161\,604} = 0.402$.

   Range: $1.645 \times 0.402 = 0.661\,29$.

   (a) 1-day VaR: $0.66/100 \times 105.75 \times £1{,}000 = £697.95$ for one lot – for 100 lots the 1-day VaR is therefore £69,795.

   (b) 2-week VaR: $\sqrt{10} \times £34,897.5 = £220{,}711.2$.

2. (a)
$$\begin{aligned}\text{DVaR} &= 2.33 \times 0.71\% \times £100 \text{ million}\\ &= £1.6543 \text{ million}\end{aligned}$$

(b) $$\text{2-week VaR} = \sqrt{10} \times £1.6543$$
$$= £5.231 \text{ million.}$$

3. $$\text{VaR} = \text{Amount of position} \cdot \text{Volatility}$$

$$\text{VaR}_{\text{port}} = \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2 + 2\rho \text{VaR}_1 \text{VaR}_2}$$

$$\text{10-year bond risk} = £10 \text{ million} * 0.999\%$$
$$= £99{,}900$$

$$\text{5-year bond risk} = £20 \text{ million} * 0.632\%$$
$$= £126{,}400.$$

The undiversified risk is simply the sum of the individual risks, which here is £226,300 (this assumes perfect correlation).

Portfolio risk

$$= \sqrt{(99{,}900)^2 + (126{,}400)^2 + 2(0.47) \times (99{,}900) \times (126{,}400)}$$
$$= £194{,}490$$

The diversified risk is calculated using the formula above, and the difference between the two is the diversification benefit to the trader due to the correlation. Here it is £226,300 minus £194,490, which is £31,810.

BANK RISK EXPOSURE AND VALUE-AT-RISK

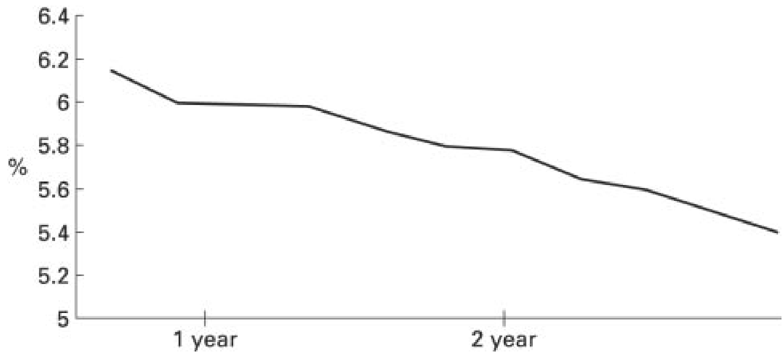
Further questions

1. A risk manager updates a daily volatility forecast using the RiskMetrics method of weighting observations, which uses a decay factor of 0.97. The volatility forecast for the previous day was 1%, and the actual market returns were 2%. What is the risk manager's new forecast? (*Hint*: Use the RiskMetrics decay formula.)

2. A bond portfolio has a one-day VaR measure of £1 million. The market has been observed to be following an autocorrelation trend of 0.10. Calculate the two-day VaR using $\sigma_{2\text{-day}} = \sqrt{\sigma^2_{1\text{-day}} + \sigma^2_{1\text{-day}} + 2\sigma^2_{1\text{-day}}\rho}$.

3. What is the one-year probability of a triple-B rated bond going into default?

4. The cumulative probability of a B-rated counterparty defaulting over the next 12 months is approximately 6.00%. From your observation of the graph below, what is the expected probability of the counterparty defaulting in the first month?



5. To fulfil regulatory requirements, a risk manager converts a one-day holding period VaR measure to a 10-day holding period. How would he do this?

6. What requirements are stipulated by the Basel Committee for banks wishing to calculate VaR for their trading books?

7. What methodology is JPMorgan's RiskMetrics based on?

8. A bank calculates its overnight VaR measure to be £12.85 million, given a 95% confidence interval. What is the appropriate interpretation of this measure?

9. Estimate the approximate VaR of a $23 million long position in a 10-year Brady bond if the 10-year volatility level is 5.78%.

10. A commodity trader has an option position in wheat with a delta of 5,000 bushels and a gamma of −200 per dollar move in price. Using the delta–gamma approximation calculate the VaR on the trader's position, assuming that the volatility level for wheat is equivalent to $3 per bushel.

11. A bond portfolio has a one-day VaR measure, at 95% confidence, of $1 million. How would you convert this measure to meet Basel Committee VaR measure requirements? What would the equivalent VaR measure be?

12. ABC Bank plc calculates its VaR with more observations and a higher confidence level (99%, as opposed to 95%) than XYZ Bank plc. Which bank is likely to have a smaller measurement error due to sampling variation?

13. From the following observations of returns, what is the correlation between A and B?

    A: 20, 18, 16, 14, 12, 10
    B: 10, 12, 18, 14, 16, 20

14. The VaR of one instrument is 1,000, while the VaR of another instrument is 800. The combined VaR of both instruments is 1,200. What is the correlation between the instruments?

Solutions

1. The daily volatility forecast can be given by the recursive formula:

   $$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 = 0.97 \times 0.01^2 + 0.03 \times 0.02^2 = 0.000109$$

   so

   $$\sigma_t = 1.044\%.$$

2. A simple application of the formula gives:

   $$\begin{aligned}\sigma_{2\text{-day}} &= \sqrt{\sigma_{1\text{-day}}^2 + \sigma_{1\text{-day}}^2 + 2\sigma_{1\text{-day}}^2\rho} = \sqrt{1+1+2\times 0.1} \\ &= £1.4832 \text{ million.}\end{aligned}$$

3. According to the transition probability matrix provided by Moody's, the one-year probability of a triple-B rated bond going into default should be about 1%.

4. According to the graph, the probability of the counterparty defaulting in the first month should be about 6.15%.

5. To adjust the VaR number to fit it to a new holding period we simply scale it upwards or downwards by the square root of the time period required. In this case the risk manager should multiply the VaR calculation measured for a one-day holding period with the square root of 10 in order to get the equivalent 10-day holding period.

6. The *Market Risk Amendment to the Basel Capital Accord* stipulates a 99% confidence interval and a 10-day holding period if the VaR measure is to be used to calculate the regulatory capital requirement. However, certain banks prefer to use other confidence levels and holding periods; the decision on which level to use is a function of asset types in the portfolio, the quality of market data available and the accuracy of the model itself, which will have been tested over time by the bank. In addition, the Basel Committee capital adequacy rule update stipulates that when calculating volatility, historical observations and associated decay factors should be for a minimum period of six months.

7. The matrix method for calculating the standard deviation is more effective than the first method we described, because it can be used for a portfolio containing a large number of assets. In fact, this is exactly the methodology used by RiskMetrics, and the computer model used for the calculation will be set up with matrices containing the data for hundreds, if not thousands, of different assets. The variance–covariance method captures the diversification benefits of a multi-product portfolio because the correlation coefficient matrix is used in the calculation.

8. This means that there is a 5% chance that the next day loss will be greater than £12.85 million.

9. The 95% VaR will be $1.645 \times 5.78\% \times 10$ million $=$ £950,810. This figure is the maximum loss the portfolio may sustain over 10 years with 95% probability.

10. The delta–gamma VaR is: $5{,}000 \times 3 + \frac{1}{2} \times (-200) \times 3^2 = \$14{,}100$.

11. The Basel Committee requires a 10-day horizon. In order to meet this requirement the bank has to multiply the VaR measure by the square root of 10. Generally, if we would like to change a value at risk measure of 1-day horizon into an N-day horizon, we simply multiply the value at risk measure by the square root of N days.

12. An increase in the number of observations used to estimate the inputs in the VaR calculation has an impact on reducing measurement error. However, the higher confidence level used means that the point we seek is farther in the tail of the distribution and thus harder to estimate precisely.

13. We put the data in an Excel file and use the function CORREL (array 1; array 2). Thus, we get a correlation of −0.82857. This means that the returns are negatively correlated; that is, we expect the returns of asset B to decrease when the returns of asset A are increasing

14. Using the portfolio variance equation and solving for the correlation components we get:

$$\mathrm{VaR} = \sqrt{\mathrm{VaR}_1^2 + \mathrm{VaR}_2^2 + 2 \times \mathrm{Corr}_{1,2} \times \mathrm{VaR}_1 \times \mathrm{VaR}_2}$$

$$\Rightarrow 1{,}200 = \sqrt{1{,}000^2 + 800^2 + 2 \times \mathrm{Corr}_{1,2} \times 1{,}000 \times 800}$$

$$\Rightarrow 1{,}440{,}000 = 1{,}000^2 + 800^2 + 2 \times \mathrm{Corr}_{1,2} \times 1{,}000 \times 800$$

$$\Rightarrow \mathrm{Corr}_{1,2} = \frac{200{,}000}{2 \times 1{,}000 \times 800} = 0.125.$$

APPENDIX: TAYLOR'S EXPANSION

Taylor's Expansion is an important technique in mathematics and was used by Frederick Macaulay when he developed the duration concept for bonds. It was also used by Black and Scholes when they formulated their option pricing model. Latterly, it has been used in developing the delta–gamma approximation for value-at-risk (VaR) measurement.

Taylor's Expansion is given by:

$$\Delta G = \frac{dG}{dx}\Delta x + \frac{1}{2}\frac{d^2G}{dx^2}\Delta x^2 + \frac{1}{6}\frac{d^3G}{dx^3}\Delta x^3 + \cdots$$

This is, in effect, a calculus differentiation. The first term is the first derivative and can be viewed as the duration of a bond, derived from the price formula. The second term is the second derivative, which is what convexity is in relation to the bond price formula. The third term has little practical impact because it is materially insignificant, so in financial markets it is not considered.

Applying Taylor's Expansion enables us to derive the duration and convexity formulae from the bond price equation. The price of a bond is the present value of all its cash flows, discounted at the appropriate internal rate of return (which becomes the yield to maturity). It is given by:

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n} \qquad \text{(A1.1)}$$

assuming complete years to maturity paying annual coupons, and with no accrued interest at the calculation date. If we take the first derivative of this expression we obtain:

$$\frac{dP}{dr}=\frac{(-1)C}{(1+r)^2}+\frac{(-2)C}{(1+r)^3}+\cdots+\frac{(-n)C}{(1+r)^{n+1}}+\frac{(-n)M}{(1+r)^{n+1}} \quad \text{(A1.2)}$$

If we re-arrange (A1.2) we will obtain the expression (A1.3), which is our equation to calculate the approximate change in price for a small change in yield:

$$\frac{dP}{dr}=-\frac{1}{(1+r)}\left[\frac{1C}{(1+r)}+\frac{2C}{(1+r)^2}+\cdots+\frac{nC}{(1+r)^n}+\frac{nM}{(1+r)^n}\right] \quad \text{(A1.3)}$$

The expression above gives us the approximate measure of the change in price for a small change in yield. If we divide both sides of (A1.3) by P we obtain the expression for the approximate percentage price change, given at (A1.4):

$$\frac{dP}{dr}\frac{1}{P}=-\frac{1}{(1+r)}\left[\frac{1C}{(1+r)}+\frac{2C}{(1+r)^2}+\cdots+\frac{nC}{(1+r)^n}+\frac{nM}{(1+r)^n}\right]\frac{1}{P} \quad \text{(A1.4)}$$

If we divide the bracketed expression in (A1.4) by the current price of the bond P we obtain the definition of Macaulay Duration:

$$D=\frac{\dfrac{1C}{(1+r)}+\dfrac{2C}{(1+r)^2}+\cdots+\dfrac{nC}{(1+r)^n}+\dfrac{nM}{(1+r)^n}}{P} \quad \text{(A1.5)}$$

Equation (A1.5) is frequently re-written as:

$$D=\frac{\displaystyle\sum_{n=1}^{N}\frac{nC_n}{(1+r)^n}}{P} \quad \text{(A1.6)}$$

where C represents the bond cash flow at time n, or as:

$$D=\frac{C}{P}\sum_{n=1}^{N}\frac{n}{(1+r)^n}+\frac{M}{P}\frac{N}{(1+r)^N} \quad \text{(A1.7)}$$

where n = Time in years to the nth cash flow;
N = Time to maturity in years.

This is obviously measuring the same thing, but the expression has been re-arranged in a slightly different way.

The markets commonly use a measure of bond price sensitivity to interest rates[1] known as *modified duration*. If we substitute the expression for Macaulay Duration (A1.5) into equation (A1.6) for the approximate percentage change in price we obtain (A1.8):

$$\frac{dP}{dr}\frac{1}{P} = -\frac{1}{(1+r)}D \tag{A1.8}$$

This is the definition of modified duration, given as (A1.9):

$$MD = \frac{D}{(1+r)} \tag{A1.9}$$

So, modified duration is clearly related to duration; in fact, we can use it to indicate that, for small changes in yield, a given change in yield results in an inverse change in bond price. We can illustrate this by substituting (A1.9) into (A1.8), giving us (A1.10):

$$\frac{dP}{dr}\frac{1}{P} = -MD \tag{A1.10}$$

Taking the second derivative of the duration expression and again dividing by the bond price P gives us the formula for convexity. Note that the second term in the Taylor Expansion contains the coefficient $\frac{1}{2}$. Hence, we multiply the convexity by $\frac{1}{2}$ to obtain the convexity adjustment.

With respect to options, the Taylor Expansion is applied the same way; the first term is the equivalent of delta while the second term is the equivalent of gamma. That is, delta and gamma are the first and second derivatives of the Black–Scholes pricing formula.

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[1] Referred to as *interest rate sensitivity* or interest rate *risk*.

ABBREVIATIONS

B-S model	Black–Scholes option pricing model
BIS	Bank for International Settlements
BoE	Bank of England
bp, bps	Basis point(s)
buVaR	bubble VaR
c.i.	confidence interval
CAD	Capital Adequacy Directive
CaViaR	Conditional autoregressive Value-at-Risk
CDO	Collateralised Debt Obligation
CDS	Credit Default Swap
CoVaR	Contagion VaR
CP	Commercial Paper
CRD	Capital Requirements Directive
CRM	Comprehensive Risk Model
CSFB	Credit Suisse First Boston
CVA	Credit Valuation Adjustment
DEaR	Daily Earnings at Risk
EE	Expected Exposure
EMU	European Monetary Union
ES	Expected Shortfall
EU	European Union
EVT	Extreme Value Theory
FRA	Forward Rate Agreement
FRR	Financial Resources Requirement
FSA	Financial Services Authority
FTSE	Financial Times and Stock Exchange
FX	Foreign eXchange
GARCH	Generalised AutoRegressive Conditional Heteroscedasticity (model)
hsVaR	historical simulation VaR
i.i.d.	independent and identically distributed

IRB	Internal Ratings Based
IRC	Incremental Risk Charge
IRRBB	Interest Rate Risk in the Banking Book
L-VaR	Liquidity-adjusted VaR
LGD	Loss Given Default
LIBOR	London Interbank Offered Rate
LIFFE	London International Financial Futures and Options Exchange
OpVaR	Operational risk VaR
OTC	Over The Counter
P&L	Profit & Loss
PV	Present Value
PVBP	Present Value of a Basis Point (Price Variation per Basis Point)
QRM	Quantile Regression Model
RAROC	Risk Adjusted Return On Capital
SFA	Securities and Futures Authority
SVaR	Stressed VaR
VaR	Value at Risk
VIX	The Volatility IndeX futures contract (Chicago Board Options Exchange)
YTM	Yield To Maturity

SELECTED BIBLIOGRAPHY

Adrian, T. and Brunnermeier, M. K. (2009). *CoVaR* (working paper). Princeton University/Federal Reserve Bank.

Beder, T. S. (1995). Analysis VaR. *Financial Analysts Journal*, Sept./Oct.

Bessis, J. (2002). *Risk Management in Banking* (2nd edn). John Wiley & Sons.

Blake, D. (2000). *Financial Market Analysis* (2nd edn). FT Prentice Hall.

BoE (1996). *Working Paper No. 79.* Bank of England.

BoE (1998). *Quarterly Bulletin*. Bank of England.

Butler, C. (1999). *Mastering Value-at-Risk*. FT Pitman.

Choudhry, M. (2004). *Fixed Income Markets: Instruments, Applications, Mathematics*. John Wiley & Sons.

CSFB (1997). *CreditRisk*$^{+}$. Credit Suisse First Boston.

Engle, R. F. and Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, **4**(2/3), 187–212.

Financial Analysts Journal, September/October 1995.

Hull, J. (2012). *Risk Management and Financial Institution*s (3rd edn). John Wiley & Sons..

Hull, J. and White, A. (1998). Incorporating volatility updating into the historical simulation method for VaR. *Journal of Risk*, **1**, 5–19.

Joon, K. and Ramaswamy, K. (1993) Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management*, **117**, Autumn.

Jorion, P. (2006). *Value at Risk: The New Benchmark for Managing Financial Risk* (3rd edn). McGraw-Hill.

JPMorgan (1997). *Introduction to CreditMetrics.*

Kolb, R. (2000). *Futures, Options and Swaps* (3rd edn), Blackwell.

Macaulay, F. (1938 [2000]). *Interest Rates, Bond Yields and Stock Prices in the United States since 1856*. RISK Classics Library.

MIT Study (1996). *The New VaR Methodology*. MIT Press.

Satchkov, D. (2010). When swans are grey: VaR as an early warning signal. *Journal of Risk Management in Financial Institutions*, **3**(4), 366–379.

SFA (1998). *Board Notice 458*, February.

Smitson, P. and Minton, D. (1997). *Risk*, February.

University of Reading ISMA Centre (1996). *Risk Management in International Securities Markets: Are Today's Standards Appropriate?*

Wilkens, S., Brunac, J. B. and Chorniy, V. (2012). *IRC and CRM: Modelling Framework for the 'Basel 2.5' Risk Measures* (working paper). Social Science Research Network.

Wisniewski, M. (1994). *Quantitative Methods for Decision Makers*. Pitman.

Wong, M. (2011). Market BuVaR: A countercyclical risk metric. *Journal of Risk Management in Financial Institutions*, **4**(4), 1–14.

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The Global Money Markets (with Frank J. Fabozzi and Steven V. Mann), John Wiley & Sons, 2002.

Bond Market Securities, FT Prentice Hall, 2001.

The Bond and Money Markets: Strategy, Trading, Analysis, Butterworth-Heinemann, 2001.