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ENGLISH FOR STUDENTS OF MATHEMATICS

Introduction

This textbook is English course for students of mathematics. It consists of 12 units based on topics of great interest to everyone studying mathematics. We hope this course will develop the communication skills one needs to succeed in a professional environment and will broaden the knowledge of the history of mathematics and will help to find out connections between mathematics and human progress.

Students are offered a variety of discussion questions as an introduction to each unit. Students will extend their vocabulary by learning useful new words and phrases. Monolingual Glossary at the end of the course will also help to increase math vocabulary.

Students will read authentic articles from the books *The story of mathematics* and *The 17 equations that changed the world* by Ian Stewart one of the most celebrated mathematicians in the world. Students will develop their reading skills. They will also be able to discuss the issues raised in the extracts from the books mentioned above. As a result they will become more accurate in their use of English at level B2.

Contents

Eternal triangles. Trigonometry and Logarithms.

Unit 1**.**

The Logic of Shape

Part 1

Before you read

Discuss these questions.

1. What is the difference between visual and symbolic reasoning in mathematics?
2. What is Euclid's contribution to mathematics?
3. What is most interesting about Euclid's geometry to modern mathematicians?

A Key terms

Match these terms with their definitions.

1. notation	a) not representing any specific value
2. arbitrary	b) having all sides of equal length
3. equilateral	c) any series of signs or symbols used to represent quantities or elements in a specialized system
4. a solid	d) a solid figure having four plane faces
5. tetrahedron	e) a solid figure having eight plane faces
6. octahedron	f) a closed surface in three-dimensional space
7. vertex (pl.vertices)	g) the point of intersection of two sides of a plane figure or angle
8. an obtuse angle	h) (of a triangle) having two sides of equal length
9 .an isosceles triangle	j) (of an angle) lying between 90° and 180°

B Word search

Read the text and find words or expressions in the text that mean the following.

a) forward in time
b) will give proof
c) has been disapproved
d) in spite of that
e) an important role
f) a general idea
g) have within itself
h) careful and thorough
i) began
j) trust
k) a look at
l) a statement taken as a fact without proof
m) had as a result

Introduction

There are two main types of reasoning in mathematics: symbolic and visual. Symbolic reasoning originated in *number notation* and led to the invention of algebra, in which symbols can represent general numbers ('the unknown') rather than specific ones. From the Middle Ages onwards, mathematicians came to rely increasingly heavily on the use of symbols, as a glance at any modern mathematics text will confirm.

As well as symbols, mathematicians use diagrams, opening up various types of visual reasoning. Pictures are less formal than symbols, and their use has sometimes been frowned upon for that reason. There is a widespread feeling that a picture is somehow less rigorous, logically speaking, than a symbolic calculation. Additionally, pictures can contain hidden assumptions –
we cannot draw a ‘general’ triangle; any triangle we draw has a particular size and shape, which may not be representative of any *arbitrary triangle*. Nonetheless, visual intuition is such a powerful feature of the human brain that pictures play a prominent role in mathematics. In fact, they introduce a second major concept into the subject, after number. Namely, *shape*.

(an extract from the book The story of mathematics by Ian Stewart)

C Read the text ‘Euclid’ and fill in the gaps with the following expressions: (a) various supplements and comments on geometry; (b) listing some assumptions and then stating their logical consequences;(c) transforms a three-dimensional scene into a two-dimensional image; (d) not its content, but its logical structure ;(e) breaks up into; (f); a logical consequence of some of the previous steps; (g) merely assert that some theorem is true;

Euclid

The best-known Greek geometer, though probably not the most original mathematician, is Euclid of Alexandria. Euclid was a great synthesizer, and his geometry text, the “Elements “, became an all-time bestseller. Euclid wrote at least ten texts on mathematics, but only five of them survive – all through later copies, and then only in part. The five Euclidean survivors are the “Elements”, the “Division of Figures”, the “Data”, the “Phenomena and the Optics”.

The “Elements” is Euclid’s geometrical masterpiece, and it provides a definite treatment of the geometry of *two dimensions* (*the plane*) and three dimensions (space). The “Division of Figures” and the “Data” contain (1)________________________________. The “Phenomena” is aimed at astronomers, and deals with *spherical geometry*, the geometry of figured drawn on the surface of a sphere. The “Optics” is also geometric, and might be thought of as an early investigation of the geometry of perspective – how the human eye (2)_________________________.

Perhaps the best way to think of Euclid’s work is an examination of the logic of *spatial* relationship. If a shape has certain properties they may logically imply other properties. For example, if a triangle has three sides equal – *an equilateral triangle* – then all three angles must be equal. This type of statement, (3)_____________________, is called a theorem. This particular theorem relates a property of all the sides of a triangle to a property of its angles. A less intuitive and more famous example is Pythagoras’s Theorem.

The “Elements”(4)___________ 13 separate books, which follow each other in logical sequence. They discuss the geometry of the plane, and some aspects of the geometry of space. . The *climax* is the proof that there are precisely five regular solids: the *tetrahedron* (formed from four equilateral triangles), cube (formed from six squares), *octahedron* (formed from eight equilateral triangles), *dodecahedron* (formed from 12 regular pentagons), and *icosahedron* (formed from 20 equilateral triangles). A solid is regular (or Platonic) if it is formed from identical *faces*, arranged in the same way as each *vertex*, with each face a regular polyhedron. The basic shapes permitted in plane geometry are straight lines and circles, often in combination – for instance, a triangle is formed from three straight lines. In spatial geometry we also find planes, cylinders and spheres.

To modern mathematicians, what is most interesting about Euclid's geometry is (5)________________. Unlike his predecessors, Euclid does not (6)______________________. He provides a proof. What is a proof? It is a kind of mathematical story, in which each step is (7)______________________. Every statement that is asserted has to be justified by referring it back to previous statements and showing that it is a logical consequence of them. Euclid realized that this process cannot go back indefinitely: it has to start somewhere, and those initial statements cannot themselves be proved – or else the process of proof actually starts somewhere different.

(an extract from the book The story of mathematics by Ian Stewart)

D Word study

Translate the mathematical terms indicated *in italics* into your native language.

a) of two dimensions (the plane)
b) spherical geometry
c) an equilateral triangle
d) climax
e) spatial
f) tetrahedron
g) octahedron
h) dodecahedron
k) icosahedron
l) an obtuse angle
m) an isosceles triangle
p) postulates
r) common notions
s) faces

E Discussion point

What if you were to give a short presentation on Euclid's most important contribution to mathematics? What would you say?

Part 2

Before you read

Discuss these questions.

1. What is the most influential aspect of Pythagorean cult's philosophy?
2. What is the main empirical support for the Pythagorean concept of a numerical universe?
3. What is meant by mythological symbolism in Pythagorean philosophy? Why did the Pythagoreans believe that the number 10 had deep mystical significance?
4. What is a rational number?

A Key terms

Match these terms with their definitions.

1. multiple	a) any real number that cannot be expressed as the ratio of two integers
2. fraction	b) the product of a given number or polynomial and any other one
3. an irrational number	c) a ratio of two expressions or numbers other than zero
4. a rational number	d) a quotient of two numbers or quantities
5. ratio	e) any real number of the form a/b, where a and b are integers and b is not zero
6. segment	f) one of the products arising from the multiplication of two or more quantities by the same number or quality
7. equimultiple	g) a part of a line or curve between two points

B Word search

Read the text **Pythagoras** and find words or expressions in the text that mean the following.

a) a group of people believing in a particular system of religious worship, principle, etc
b) were expressed by
c) an area of activity
d) an unsuccessful attempt
e) depended on
f) uninterruptedly, eternally
g) fitted perfectly
h) this unpleasant fact
i) on grounds that cause doubts
j) were so angry
k) was dismissed
l) supporters

Pythagoras

Today we almost take it for granted that mathematics provides a key to the underlying laws of nature. The first recorded systematic thinking along those lines comes from the Pythagoreans, a rather mystical cult dating from roughly 600BC to 400BC. Its founder, Pythagoras, is well known mainly because of his celebrated theorem about *right-angled triangles*, but we don't even know whether Pythagoras proved it. We know more about the Pythagoreans' philosophy and beliefs. They understood that mathematics is about *abstract concepts*, not reality. However, they also believed that these abstractions were embodied in 'ideal' concepts, existing in some strange realm of the imagination, so that, for instance, a circle drawn in sand with a stick is *a* flawed attempt to be an ideal circle, perfectly round and infinitely thin.

The most influential aspect of the Pythagorean cult's philosophy is the belief that the universe is founded on numbers. They expressed this belief in mythological symbolism, and supported it with empirical observations. On the mystic side, they considered the number 1 to be the prime source of everything in the universe. The numbers 2 and 3 symbolized the female and male principles. The number 4 symbolized harmony, and also the four elements (earth, air, fire, water) out of which everything is made. The Pythagoreans believed that the number 10 had deep mystical significance, because 10= 1+2+3+4, combining prime unity, the female principle, the male principle and the four elements. Moreover, these numbers formed a triangle, and the whole of Greek geometry hinged upon properties of triangle.

The Pythagoreans recognized the existence of nine heavenly bodies, Sun, Moon, Mercury, Venus, Earth, Mars, Jupiter and Saturn, plus the Central Fire, which differed from the Sun. So important was the number 10 in their view of cosmology that they believed there was a tenth body, Counter-Earth, perpetually hidden from us by the Sun.

As we have seen, the whole numbers 1, 2, 3…, naturally lead to a second type of numbers, fractions, which mathematicians call rational numbers. A rational number is a fraction *a/b* where *a*, *b* are whole numbers (and *b* is non-zero, otherwise the fraction makes no sense). Fractions subdivide whole numbers into arbitrary fine parts, so that in particular the length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. It seems natural to imagine that enough subdivision would hit the number exactly; if so, all lengths would be rational.

If this were true, it would make geometry much simpler, because any two lengths would be whole number *multiples* of a common (perhaps small) length, and so could be obtained by fitting lots of copies of this common length together. This may not sound very important, but it would make the whole theory of lengths, areas and especially similar figures – figures with the same shape but different sizes – much simpler. Everything could be proved using diagrams formed from lots and lots of copies of one basic shape.

Unfortunately, this dream cannot be realized. According to legend, one of the followers of Pythagoras, Hippasus of Metapontum, discovered that this statement was false. Specifically, he proved that the diagonal of a unit square (a square with sides one unit long) is irrational: not an *exact fraction*. It is said (on dubious grounds, but it's a good story) that he made the mistake of announcing this fact when the Pythagoreans were crossing the Mediterranean by boat, and his fellow cult-members were so incensed that they threw him overboard and he drowned. More likely he was just expelled from the cult. Whatever his punishment, it seems that the Pythagoreans were not pleased by his discovery.

The modern interpretation of Hippasus's observation is that 2 is irrational. To the Pythagoreans, this brutal fact was a body-blow to their almost religious belief that the universe was rooted in numbers – by which they meant whole numbers. Fractions – *ratios* of whole numbers fitted neatly enough into this world-view, but numbers that were provably not fractions did not. And so, whether drowned or expelled, poor Hippasus became one of the early victims of the irrationality, so to speak, of religious belief!

(an extract from the book The story of mathematics by Ian Stewart)

C Word search

Choose the correct definition of these words and expressions *in italics* in the context they are used in the text.

a) Today we almost *take it for granted…*

i) accept a fact without question
ii) treat with no attention
iii) admit

b) ... mathematics *provides a key* ...

i) helps to understand
ii) explains
iii) proves

c) …. the fraction *makes no sense*…

i) is exact
ii) is not exact
iii) doesn't have a clear meaning

D Understanding expressions

Make word pairs to form expressions from the text **Pythagoras**.

a) abstract	1 members
b) prime	2 source
c) unit	3 blow
d) cult	4 fraction
e) exact	5 concepts
f) body	6 square

E Understanding expressions

Match the verbs (1-4) with the noun phrase (a-d) to form expressions from the text.

1. provide	this fact
2. form	the mistake
3. make	a triangle
4. announce	a key to

F Use of English

Chose the correct summary of the text **Pythagoras** and explain the difference between them.

A The whole numbers 1, 2, 3, … naturally lead to a second type numbers, fractions, which are called rational numbers. A rational number is a fraction *a/b* where *a, b* are whole numbers and *b* is non-zero. Fractions subdivide whole numbers into arbitrary fine parts, so that in particular the length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. So, all lengths are rational and any two lengths are whole number multiples of a common length, and so can be obtained by fitting lots of copies of this common length together. Everything can be proved using diagrams formed from lots and lots of copies of one basic shape.

B The whole numbers 1, 2, 3,… lead to a second type of numbers, fractions, which mathematicians call rational numbers. A rational number is a fraction *a/b* where *a*, *b* are whole numbers (and *b* is non-zero, otherwise the fraction makes no sense). The length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. It seems natural to imagine that enough subdivision would hit the number exactly; if so, all lengths would be rational. If this were true, it would make geometry much simpler, because any two lengths would be whole number multiples of a common length, and so could be obtained by fitting lots of copies of this common length together.

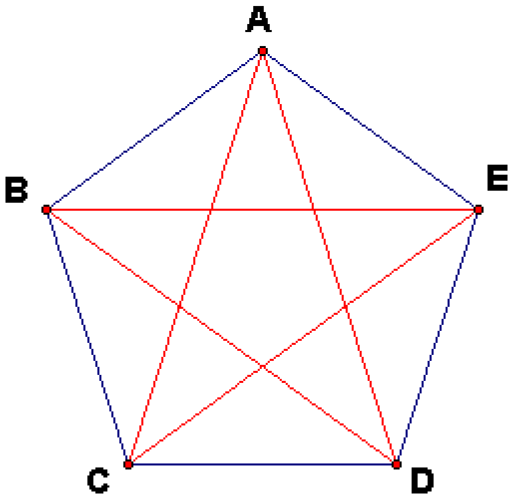
G Discussion point

What if you were to give a short presentation on Pythagoras's most important contribution to mathematics? What would you say? Do you think Pythagoras formulated and proved the theorem named after him?

Part 3

Before you read

Look at the figure. Do you know in what way it is related the golden mean?



A Word search

Read the text **The Golden Mean** and find words or expressions in the text that mean the following.

a) go to
b) not well known
c) geometry that follows accepted rules
d) unclear, wordy jargon
e) meant
f) things of little significance or importance
g) made seem different
h) deep, complete
k) known

l) deal seriously with something difficult
m) write by marking into a surface
n) has the same proportions

The Golden Mean

Book V of the “Elements” heads off in a very different, and rather obscure, direction from Books I-IV. It doesn’t look like conventional geometry. In fact, at first sight it mostly reads like gobbledygook. What, for instance, are we to make of Book V Proposition 1? It reads**: If certain magnitudes are *equimultiples* of other magnitudes, then whatever multiple one of the magnitudes is of one of the others, that multiple also will be of all.** The proof makes it clear what Euclid intended. The 19th-century English mathematician Augustus De Morgan explained the idea in simple language in his geometry textbook: “Ten feet ten inches makes ten times as much as one foot one inch.” What is Euclid up to here? Is it trivialities dressed up as theorems? Mystical nonsense? Not at all. This material may seem obscure, but it leads up to the most profound part of the “Elements”: Eudoxus’s technique for dealing with *irrational ratios.* Nowadays mathematicians prefer to work with numbers because these are more familiar.

Euclid could not avoid facing up to the difficulties of irrational numbers, because the climax to the “Elements” – and, many believe, its main objective – was the proof that there exist precisely five regular solids. Two of the regular solids, the dodecahedron and the icosahedron, involve the regular pentagon: the dodecahedron has pentagonal faces, and the five faces of the icosahedron surrounding any vertex determine a pentagon. Regular pentagons are directly connected with what Euclid called ‘extreme and mean ratio’. On a line AB, construct a point C so that the ratio AB:AC is equal to AC:BC. That is, the whole line bears the same proportion to the larger *segment* as the larger segment does to the smaller. If you draw a pentagon and inscribe a five-pointed star, the edges of the star are related to the edges of the pentagon by this particular ratio. Nowadays we call this ratio the *golden mean.* It is equal to and this number is irrational. Its numerical value is roughly 1.618. The Greeks could prove it was irrational by exploiting the geometry of pentagon. So Euclid and his predecessors were aware that, for a proper understanding of the dodecahedron and icosahedron, they must come to grips with irrationals.

(an extract from the book The story of mathematics by Ian Stewart)

B Word study

Translate the mathematical terms indicated *in italics* into your native language.
a) equimultiples
b) irrational ratios
c) golden mean
d) segment

C Prepositions

Complete the expressions with an appropriate preposition and translate them into your native language
a) to head ___ in a different direction
b) ___ first sight
c) to be up ___

d) dressed ___ as
e) to lead up ___
f) to face up ___
g) to come to grips ___

D Understanding expressions

Complete the following sentences with the expressions from **C.**

a) Euclid and his predecessors must come _________irrationals.
b) This material may seem obscure, but it __________the most profound part of the "Elements" c) What is Euclid_______ here?
d) Is it trivialities _________theorems?
e) In fact, _______it mostly reads like gobbledygook. .
f) Euclid could not avoid __________the difficulties of irrational numbers.
g) Book V of the "Elements" _______and rather obscure direction.

E Discussion point

What makes a single number (Golden ratio) so interesting that ancient Greeks, Renaissance artists etc. would write about it? Discuss this question in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1. Summarize the main points from the texts in your own words.
2. Write a short report on the origin of geometry and its applications.
3. Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "The logic of shape". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants.

Web research task

Unit 2.

Eternal triangles. Trigonometry and Logarithms.

Part 1

Before you read

Discuss these questions.

1. Do you know what the word ‘trigonometry’ means?
2. Do you know that trigonometry originated in astronomy?
3. Can you briefly formulate trigonometric functions?
4. What is Ptolemy’s contribution to trigonometry?

A Key terms

Match these terms with their definitions.

1. chord	a) a section of a curve, graph, or geometric figure
2. arc	b) a straight line connecting two points on a curve or curved surface
3. angle	c) the space between two straight lines that diverge from a common point or between two planes that extend from a common line
4. trigonometry)	d) the branch of mathematics concerned with the properties of trigonometric functions and their application to the determination of the angles and sides of triangles. Used in surveying and navigation
5. sine	e) a trigonometric function that in a right-angled triangle is the ratio of the length of the opposite side to that of the hypotenuse

B Information search

Read the text, and then answer the questions following it according to the information given in the text.

The origins of trigonometry

The basic problem addressed by trigonometry is the calculation of properties of a triangle – lengths of sides, sizes of angles – from other such properties. It is much easier to describe the early history of trigonometry if we first summarize the main features of modern trigonometry, which is mostly a reworking in 18th century notation of topics that go right back to Greeks, if not earlier. This summary provides a framework within which we can describe the ideas of the ancients, without getting tangled up in obscure and eventually obsolete concepts.

Trigonometry seems to have originated in astronomy, where it is relatively easy to measure angles, but difficult to measure the vast distances. The Greek astronomer Aristarchus, in a work of around 260 BC, *On the Sizes and Distances of the Sun and Moon*, deduced that the Sun lies between 18 and 20 times as far from the Earth as the Moon does. (The correct figure is closer to 400, but Eudoxus and Phidias had argued for 10). His reasoning was that when the Moon is half full, the angle between the directions from the observer to the Sun and the Moon is about 87 degrees (in modern units). Using properties of triangles that amount to trigonometric estimates, he deduced (in modern notation) that sin 3degrees lies between 1/18 and 1/20, leading to his estimate of the ratio of the distances to the Sun and the Moon. The method was right, but the observation was inaccurate; the correct angle is 89.8'.

The first trigonometric tables were derived by Hipparchus around 150 BC. Instead of the modern sine function, he used a closely related quantity, which from the geometric point of view was equally natural. Imagine a circle, with two radial lines meeting at an angle *0*. The points where these lines cut the circle can be joined by a straight line, called a chord. They can also be thought of as the Hipparchus drew up a table relating arc and chord length for a range of angles. If the circle has radius l, then the arc length in modern notation is 2sin 0/2. So Hipparchus's calculation is very closely related to a table of sines, even though it was not presented in that way.

(an extract from the book The story of mathematics by Ian Stewart)

1. According to the text, trigonometry…
   a is more complicated now than it was in the past.
   b used sophisticated notation.
   c originated in astronomy.
2. The author implies that the ancient Greeks…
   a measured distances by angles.
   b used accurate observations.
   c managed to measure vast distances
3. According to the text trigonometric functions were…
   a stated in terms of chords.
   b derived by Eudoxus.
   c presented by modern notation.

C Understanding phrases

Complete the text about astronomy with the words and phrases in the box. Two are not used.

appear to lie	goes for	whose vertices lie at	refer to
congruent	can be thought of	poor results	valid

Astronomy

Remarkably, early work in trigonometry was more complicated than most of what is taught in schools today, again because of the needs of astronomy (and, later, navigation). The natural space to work in was not the plane, but the sphere. Heavenly objects (1)___________ as lying on an imaginary sphere, the celestial sphere. Effectively, the sky looks like the inside of a gigantic sphere surrounding the observer, and the heavenly bodies are so distant that they (2)__________ on the sphere.

Astronomical calculations, in consequence, (3)_______ the geometry of a sphere, not that of a plane. The requirements are therefore not plane geometry and trigonometry, but spherical geometry_ and trigonometry. One of the earliest works in this area is Menelaus's Sphaerica of about AD100. A sample theorem, one that has no analogue in Euclidean geometry, is this: if two triangles have the same angles as each other, then they are (4)__________ - they have the same size and shape. (In the Euclidean case, they are similar – same shape but possibly different sizes.) In spherical geometry, the angles of a triangle do not add up to 180', as they do in the plane. For example, a triangle (5)____________ the North Pole and at two points on the equator separated by 90' clearly has all three angles equal to a right angle, so the sum is 270'.Roughly speaking, the bigger the triangle becomes, the bigger its angle-sum becomes. In fact, this sum, minus 180", is proportional to the triangle's total area.

These examples make it clear that spherical geometry has its own characteristic and novel features. The same (6)_______spherical trigonometry, but the basic quantities are still the standard trigonometric functions. Only the formulas change.

(an extract from the book The story of mathematics by Ian Stewart)

D Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Look at Figure 1. What is special about it? In what way is it related to trigonometry?

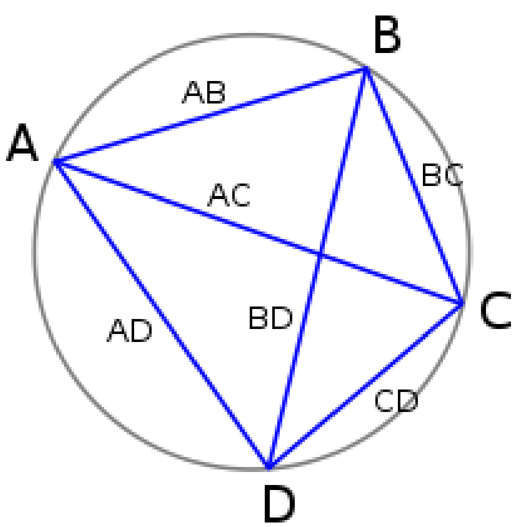


Figure 1.

A Key terms

Match these terms with their definitions.

1. quadrilateral — a) the solid figure bounded by this surface or the space enclosed by it
2. cyclic quadrilateral — b) a polygon having four sides
3. sphere — c) a quadrilateral whose vertices lie on a circle
4. cosine — d) a trigonometric function that in a right-angled is the ratio of the length of the adjacent side to that of the hypotenuse; the sine of the complement
5. epicycle — e) a circle that rolls around the inside or outside of another circle, so generating epicycloid or hypocycloid

Reading tasks

B Understanding phrases

Read the text 'Ptolemy' and fill in the gaps with the following words: requiring , quadrilateral , wander , work out , tour de force, obtaining, deduce, obvious, vertices, responding , complicated , elongated loops , noteworthy ,celestial, chord.

Ptolemy

By far and above the most important trigonometry text of antiquity was the *Mathematical Syntaxis* of Ptolemy of Alexandria, which dates to about AD150. It is better known as the *Almagest*, an Arabic term meaning "the greatest". It included trigonometric tables, again stated in terms of 1)________, together with the methods used to calculate them, and a catalogue of star positions on the 2)________sphere. An essential feature of the computational method was Ptolemy's Theorem which states that if ABCD is a cyclic 3)____________(one whose 4)________ lie on a circle) then

AB x CD +BC x DA= AC x BD

(the sum of the products of opposite pairs of sides is equal to the product of the diagonals).

A modern interpretation of this fact is the remarkable pair of formulas.

$$\sin(\theta+\varphi)=\sin\theta\cos\varphi+\cos\theta\sin\varphi$$

$$\cos(\theta+\varphi)=\cos\theta\cos\varphi-\sin\theta\sin\varphi$$

The main point about these formulas is that if you know the sines and cosines of two angles, then you can easily work the sines and cosines out for the sum of those angles. So, starting with (say) sin 1' and cos 1', you can deduce sin 2' and cos 2' by taking 0=f=1'. Then you can 5) ________ sin 3' and cos3' by taking 0=1',f=2', and so on. You had to know how to start, but after that, all you needed was arithmetic – rather a lot of it, but nothing more 6)__________.

Getting started was easier than it might seem, 7)_________ arithmetic and square roots. Using the 8)_________ fact that 0/2+0/2=0, Ptolemy's Theorem implies that

$$\sin\frac{\theta}{2} = \frac{\sqrt{1-\cos\theta}}{2}$$

Starting from cos90'=0, you can repeatedly halve the angle, 9)_________ sines and cosines of angles as small as you please. Ptolemy used ¼'.) Then you can work back up through all integer multiples of that small angle. In short, starting with a few general trigonometric formulas, suitably applied, and a few simple values for specific angles you can 10) _______ values for pretty much any angle you want. It was an extraordinary 11)__________ , and it put astronomers in business for over a thousand years.

A final 12) __________ feature of the *Almagest* is how it handled the orbits of the planets. Anyone who watches the night sky regularly quickly discovers that the planets 13)________ against the background of fixed stars, and that the paths they follow seem rather complicated, sometimes moving backwards, or travelling in 14) __________ ________.

Eudoxus, 15)___________ to a request from Plato, had found a way to represent these complex motions in terms of revolving spheres mounted on other spheres. This idea was simplified by Apollonius and Hipparcus, to use epicycles – circles whose centers move along other circles, and so on. Ptolemy refined the system of epicycles, so that it provided a very accurate model of the planetary motions.

(an extract from the book The story of mathematics by Ian Stewart)

C Vocabulary tasks. Word search

Find a word or phrase in the text that has a similar meaning.

1. the result of the multiplication of two or more numbers, quantities, etc (para1)
2. (of a polygon) having vertices that lie on a circle (para1)
3. a trigonometric function that in a right-angled triangle is the ratio of the length of the opposite side to that of the hypotenuse (para2)
4. the product of a given number or polynomial and any other one (para4)
5. a circle that rolls around the inside or outside of another circle, so generating an epicycloid or hypocycloid (para6)

D Discussion point

What is the role of Ptolemy's theorem in trigonometry? Discuss this question in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 3

Before you read

Discuss these questions.

1. Is trigonometry still applied nowadays?
2. Are older applications of trigonometry obsolete now?

A Reading and Use of English

Read the text and fill in the gaps with the following words

vital, embellishments, spawned, surveying, befits, converting, evolving, obsolete, precision, compilers

Trigonometry __________1 a number of special functions – mathematical rules for calculating one quantity from another. These functions go by names like sine, cosine and tangent. The trigonometric functions turned out to be of __________2 importance for the whole of mathematics, not just for measuring triangles.

Trigonometry is one of the most widely used mathematical techniques, involved in everything from ________3 to navigation to GPS satellite systems in cars. Its use in science and technology is so common that it usually goes unnoticed, as ______4 any universal tool. Historically it was closely associated with logarithms, a clever method for ________5 multiplications (which are hard) into additions (which are easier). The main ideas appeared between about 1400 and 1600, though with a lengthy prehistory and plenty of later __________6, and the notation is still_________**7**.

Many of the older applications of trigonometry are computational techniques, which have mostly become _________8 now that computers are widespread. Hardly anyone now uses logarithms now to do multiplication, for instance. No one uses tables at all, now that computers can rapidly calculate the values of functions to high ______9. But when logarithms were first invented, it was the numerical tables of them that made them useful, especially in areas like astronomy, where long and complicated numerical calculations were necessary. And the _________10 of the tables had to spend years - decades - of their lives doing sums. Humanity owes a great deal these dedicated and dogged pioneers. *(an extract from the book The story of mathematics by Ian Stewart)*

B Discussion point

Discuss questions 1-2 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 4

Before you read

Discuss these questions.

1. Do you know who discovered logarithms?

2. Have you ever heard about Napier rods? If yes, explain how they work.

3. Why was the logarithm so important?

4. Do you think logarithms are important in the age of computers?

A Key terms

Match these terms with their definitions.

1. geometric progression	a) any rational number that can be expressed as the sum or difference of a finite number of units
2. product	b) (also called: exponent, index) a number or variable placed as a superscript to the right of another number or quantity indicating the number of times the number or quantity is to be multiplied by itself
3. integer	c) the result of the multiplication of two or more numbers, quantities etc.
4. power	d) a sequence of numbers, each of which differs from the succeeding one by a constant ratio
5. logarithm	e) the exponent indicating the power to which a fixed number, the base, must be raised to obtain a given number or variable

B Reading tasks

Read the text and fill in the gaps with the following words and expressions:

(a) is obtained from the previous one;(b)which gets rid of those annoying gaps, (c) seemed not to help much; (d) it took him 20 years ;(e) to convert a product into a sum; (f) with the ungainly name; (g) with a common ratio; (h) to perform multiplication; (i) replacing Napier's concept; (j) by successive powers;(k) efficient methods;(l) snowballed; (m) adding the exponents; (n) seized on the idea; (o)credit; (p) beyond the pale;(q) have no influence;(r) Successive powers; (s) ran backwards; (t) took up the task.

Logarithms

Historically, the discovery of logarithms was not direct. It began with John Napier, baron of Murchiston in Scotland. He had a lifelong interest in 1)_____________ for calculation, and invented Napier rods (or Napier bones), a set of marked sticks that could be used 2)_______________ quickly and reliably by stimulating pen-and-paper methods. Around 1594 he started working on a more theoretical method, and his writings tell us that 3)_________________ to perfect and publish it. It seems likely that he started with geometric progressions, sequences of numbers in which each term 4)___________________ by multiplying by a fixed number – such as the powers of 2

1 2 4 8 16 32 …

or powers of 10

1 10 100 1000 10 000 100 000

Here it had long been noticed that 5)______________ was equivalent to multiplying the powers. This was fine if you wanted to multiply two integer powers of 2, say, or two integer powers of 10. But there were big gaps between these numbers, and powers of 2 or 10 6)___________________ when it came to problems like 57.681 x 29.443, say.

While the good Baron was trying to somehow fill in the gaps in geometric progressions, the physician to King James VI of Scotland, James Craig, told Napier about a discovery that was in widespread use in Denmark, 7)____________________ prosthapheiresis. This referred to any process that converted products into sum. The main method in practical use was based on a formula discovered by Vieta:

$$\sin\frac{x+y}{2}\cos\frac{x-y}{2}=\frac{\sin x+\sin y}{2}$$

If you had tables of sines and cosines, you could use this formula 8) _______________________. It was messy, but it was still quicker than multiplying the numbers directly.

Napier 9)__________________, and found a major improvement. He formed a geometric series 10)_________________ very close to 1. That is, in place of the powers of 2 or powers of 10, you should use powers of, say, 1.0000000001. 11)______________ of such a number are very closely spaced, 12)__________________________. For some reason Napier chose a ratio slightly less than 1, namely 0.999999. So his geometric sequence 13) ___________________from a large number to successively smaller ones. In fact, he started with 10,000,000 and then multiplied this 14) _________________ of 0.9999999. If we write Naplogx for Napier's logarithm of x, it has the curious feature that
Naplog10,000,000=0
Naplog 9,999,999= 1 and so on. The Napierian logarithm, Naplog *x*, satisfies the equation

Naplog $(10^7 xy)$= Naplog(x)+Naplog(y)

The next improvement came from Henry Briggs, he suggested 15) _____________ by a simpler one: the (base ten) logarithm, L=$\log_{10}$ x, which satisfies the condition x= 10^L

Now $\log_{10} xy=\log_{10} x+\log_{10} y$

And everything is easy. To find *xy*, add the logarithms of *x* and *y* and then find the antilogarithm of the result.

Briggs 16) __________________ of computing a table Briggsian (base 10, or common) logarithms. He did it by starting from $\log_{10} x=1$ *and* taking successive square roots. In 1617 he published *Logarithmorum Chilias Prima* , the logarithms of the integers from 1 to 1000 to 14 decimal places. His 1624 Arithmetic *Logarithmica* tabulated common logarithms of numbers from 1 to 20,000 and from 90,000 to 100,000, also to 14 places.

The idea 17) ___________. John Speidel worked out logarithms of trigonometric functions (such as log sin *x*) in 1619. The Swiss clockmaker Jobst Burgi published his own work on logarithms in 1620, and may have possessed the basic idea in 1588, well before Napier. But the historical development of mathematics depends on what people publish and ideas that remain private 18)___________ on anyone else. So 19)________, probably rightly, has to go to those people who put their ideas into print, or at least into widely circulated letters. The exception is people who put the ideas of others into print without due credit. This is generally 20) ____________.

(an extract from the book The story of mathematics by Ian Stewart)

C Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 5

Before you read

Discuss the question.

1. Why do we need logarithms now that we have computers?

A Use of English

For questions 1 – 15, read the text about the role of logarithms in the age of computers and think of the word which best fits each gap. Use only one word in each gap .

The logarithm answers a vital question: if you know how (1)____ radioactive material has been released, and of what kind, how (2)_____ will it remain in the environment, where it could be hazardous?

Radioactive elements decay; (3)___ is, they turn into other elements through nuclear processes, emitting nuclear particles as they do so. It is these particles that constitute the radiation. The level of radioactivity falls away over time just as the temperature of a hot body falls (4)___ it cools: exponentially. So, in appropriate units the level of radioactivity N(t) at time t follows the equation

$N(t) = N_0 e^{-kt}$

where N_0 is the initial level and k is a constant depending on the element concerned. More precisely, it depends (5)___ which form, or isotope, of the element we are considering.

A convenient measure of the time radioactivity persists is the half-life, a concept first introduced in 1907. This is the time it (6)____ for an initial level N_0 to drop to half that size. To calculate the half-life, we solve the (7)_______

$$\tfrac{1}{2}N_0 = N_0 e^{-kt}$$

by taking logarithms of both sides. The result is

$t = \dfrac{\log 2}{k} = \dfrac{0.6931}{k}$ and we can work this (8)___ because k is known from experiments.

Unit 3.

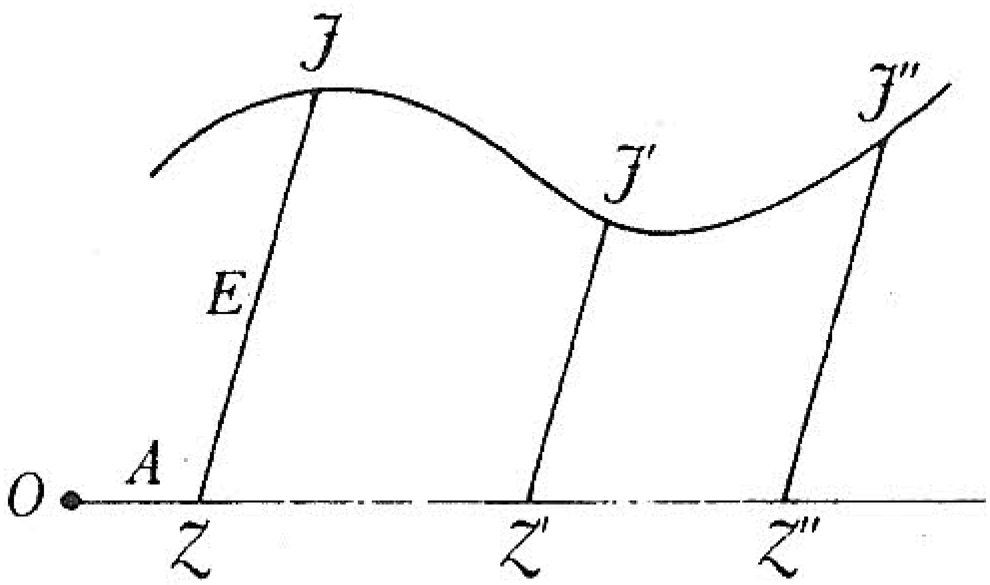
Curves and coordinates

Part 1

Before you read

Discuss these questions.

1. Who discovered a remarkable connection between geometry and algebra and showed that each of these areas can be converted into the other by using coordinates?
2. Do coordinates work in three-dimensional space?
3. What is the most important contribution made by the concept of coordinates?
4. How are coordinates used today? What is the influence of coordinates on everyday life?
5. Look at Figure 1 of the first coordinate system introduced by Fermat. Fermat noticed a general principle: if the conditions imposed on the point can be expressed as a single equation involving two unknowns, the corresponding locus is a curve – or a straight line. He illustrated this principle by a diagram in which two unknown quantities A and E are represented as distances in two distinct directions. Can you explain the difference between Fermat's coordinate system and Cartesian coordinates?



A Key terms

Match these terms with their definitions.

1. locus a) an expression that can be assigned any of a set of values

2. ellipse b) nonperpendicular axis
3. variable c) a set of points whose location satisfies or is determined by one or more specified conditions
4. oblique axis d) a closed conic section shaped like a flattened circle and formed by an inclined plane that does not cut the base of the cone

Reading tasks

B Understanding expressions

Read the text about Fermat Complete the text with the words and phrases in the box. Two are not used.

oblique axes	drawing on	equation	with respect to
turns out	foci	add up to	common
an obsolete term	arises	variables	imposed on
embarked upon		triangles	paved the way
making needless distinctions			

Fermat

The first person to describe coordinates was Pierre Fermat. Fermat is best known for his work in number theory, but he also studied many other areas of mathematics, including probability, geometry and applications to optics. Around 1620, Fermat was trying to understand the geometry of curves, and he started by reconstructing, from what little information was available, a lost book by Apollonius called *On Plane Loci*. Having done this, Fermat (1)_____________ his own investigations, writing them up in 1629 but not publishing them until 50 years later, as *Introduction to Plane and Solid Loci.*

Locus, plural loci, is (2)______________ today, but it was (3)____________even in 1960. It (4)___________ when we seek all points in the plane or space that satisfy particular geometric conditions. For example, we might ask for the locus of all points whose distances from two other fixed points always (5)__________ the same total. This locus (6)____________ to be an ellipse with the two points as its (7)_______ . This property of the ellipse was known to the Greeks.

Fermat noticed a general principle: if the conditions (8)_________ the point can be expressed as a single (9)__________ involving two unknowns, the corresponding locus is a curve – or a straight line, which we consider to be a special kind of curve to avoid (10)______________. He illustrated this principle by a diagram in which the two unknown quantities A and E are represented as distances in two distinct directions.

He then listed some special types of equation connecting A and E, and explained what curves they represent. For instance, if $A^2 = 1 + E^2$ then the locus concerned is a hyperbola.

In modern terms, Fermat introduced (11)________ in the plane (oblique meaning that they do not necessarily cross at right angles). The variables A and E are the two coordinates, which we would call *x* and *y*, of any given point (12)__________ these axes. So Fermat's principle effectively states that any equation in two-coordinate (13)__________ defines a curve, (14)________ the standard curves known to the Greeks.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Understanding expressions

Read the text about Descartes .Complete the text with the words and phrases in the box.

Descartes

above or below the origin , implies , respectively , are not sufficient , to contemplate , we perceive, might be, axes , in its own right, it takes a major effort , origin , are familiar with , came to fruition,

The modern notion of coordinates (1) ______________ in the work of Descartes. In everyday life, we (2)____________ spaces of two and three dimensions, and (3)__________ of imagination for us (4)_____________ other possibilities. Our visual system presents the outside world to each eye as a two-dimensional image – like the picture on TV screen. Slightly different images from each eye are combined by the brain to provide a sense of depth, through which (5)_____________ the surrounding world as having three dimensions.

The key to multidimensional spaces is the idea of a coordinate system, which was introduced by Descartes in the appendix *La Geometrie* to his book *Discours de la Methode*. His idea is that geometry of the plane can be reinterpreted in algebraic terms. His approach is essentially the same as Fermat's. Choose some point in the plane and call it the (6)_______. Draw two (7)_______, lines that pass through the origin and meet at right angles. Label one axis with the symbol *x* and the other with the symbol *y*. Then any point P in the plane is determined by the pair of distances (x,y), which tells us how far the point is from the origin when measured parallel to the x- and y-axes, (8)________.

For example, on a map, *x* (9)_________- the distance east of the origin (with negative numbers representing distances to the west), whereas *y* might be the distance north of the origin (with negative numbers representing distances to the south).

Coordinates work in three-dimensional space too, but now two numbers (10)____________ to locate a point. However, three numbers are. As well as the distances east-west and north-south, we need to know how far the point is (11)____________. Usually we use a positive number for distances above, and a negative one for distances below. Coordinates in space take the form(x, y, z).

This is why the plane is said to be two-dimensional, whereas space is three-dimensional. The *number of dimensions* is given by how many numbers we need to specify a point.

In three-dimensional space, a single equation involving *x, y* and *z* usually defines a surface. For example, $x^2 + y^2 + z^2 = 1$ states that the point (x, y, z) is always a distance 1 unit from the origin, which (12)___________ that it lies on the unit sphere whose centre is the origin.

Notice that the word 'dimension' is not actually defined here (13)___________. We do not find the number of dimensions of a space by finding some things called dimensions and then counting them. Instead, we work out how many numbers are needed to specify where a location in the space is, and that is the number of dimensions.

(an extract from the book The story of mathematics by Ian Stewart)

D Understanding phrases

Read the text and fill in the gaps with the following words:

extended, made, are determined, complicated, to consider, visual aspect, folium of Descartes, quadratic equation, arising, linear equation, reveals

Cartesian coordinates

Cartesian coordinate geometry (1)_______ an algebraic unity behind the conic sections – curves that the Greeks had constructed as sections of a double cone. Algebraically, it turns out that the conic sections are the next simplest curves after straight lines. A straight line corresponds to a (2)________ ax + by + c=0 with constants a, b, c. A conic section corresponds to a (3)___________ $a x^2 + b x y + c y^2 + d x + e y + f = 0$ with constants *a, b, c, d, e, f*. Descartes stated this fact, but did not provide a proof. However, he did study a special case, based on a theorem due to Pappus which characterized conic sections, and he showed that in this case the resulting equation is quadratic.

He went on (4)________ equations of higher degree, defining curves more complex than most of those (5)________ in classical Greek geometry. A typical example is the (6)_________, with equation $x^3 + y^3 - 3axy = 0$ which forms a loop with two ends that tend to infinity.

Perhaps the most important contribution (7)_______ by the concept of coordinates occurred here: Descartes moved away from the Greek view of curves as things that are constructed by specific geometric means, and saw them as the (8)______ of any algebraic formula.

Later scholars invented numerous variations on the Cartesian coordinate system. In a letter of 1643 Fermat took up Descartes' ideas and (9)________them to three dimensions. Here he mentioned surfaces such as ellipsoids and paraboloids, which (10)________by quadratic equations in the three variables *x, y, z.* An influential contribution was the introduction of *polar* coordinates by Jakob Bernoulli in 1691. He used an angle θ and a distance r to determine points in the plane instead of a pair of axes. Now the coordinates are (r, θ).

Again, equations in these variables specify curves. But now, simple equations can specify curves that would become very (11)________ in Cartesian coordinates. For example the equation $r = \theta$ corresponds to a spiral, of the kind known as an Archimedean spiral.

. *(an extract from the book The story of mathematics by Ian Stewart)*

E Vocabulary tasks

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Functions

An important (1) ______ of coordinates in mathematics is a method to represent functions graphically. A *function* is not a number, but a (2)_________ that starts from some number and calculates an associated number. The recipe involved is often stated as a (3)__________, which assigns to each number, x (possibly in some limited range), another number, $f(x)$. For example, the (4)____________ function is defined by the rule $f(x) = \sqrt{x}$, that is, take the square root of the given number. This recipe (5)________ x to be positive. Similarly the square function is defined by $f(x) = x^2$, and this time there is no (6)_________ on x.

We can picture a function (7)_________ by defining the y – coordinate, for a given (8)_______ of x, by $y=f(x)$. This equation states a (9)___________ between the two coordinates, and therefore determines a (10)______. This curve is called the *graph* of the function f.

The (11)______ of the function $f(x) = x^2$ (12)_______ to be a parabola. That of the square root $f(x) = \sqrt{x}$ is half parabola, but lying on its side. More complicated functions (13)______to more complicated curves. The graph of the sine function $y= \sin x$ is a (14)________ wave.

lead wiggly value formula

application

recipe square root geometrically circumference curve

graph turns out prove restriction

relationship

requires

F Complete the sentence

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Coordinate geometry today

Coordinates are one of those simple ideas that has had a marked (1)_________ on everyday life. We use them everywhere, usually without noticing what we are doing. Virtually all computer graphics employ an internal coordinate system, and the geometry that appears on the screen is (2)_______ algebraically. An operation as simple as (3)________ a digital photograph through a few degrees, to get the horizon horizontal, (4)_________ coordinate geometry.

The deeper message of coordinate geometry is about cross-connections in mathematics. Concepts whose physical realizations seem totally different may be different aspects of the same thing. Superficial appearances can be (5)________. Much of the effectiveness of mathematics as a way to understand the universe (6)__________ its ability to adapt ideas, (7)________ them from one area of science to another. Mathematics is the (8)_________ in technology transfer. And it is those cross-connections, revealed to us over the past 4000 years, that make mathematics a single, (9)_______ subject.

influence rotating relies on ultimate

dealt with misleading stems from miscellaneous

invented transferring unified

G Complete the sentence

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Applications of coordinate geometry

Coordinate geometry can be employed on surfaces more complicated than the plane, such as the sphere. The commonest coordinates on sphere are (1)_______and (2)________. So map-making, and the use of maps in navigation, can be viewed as an application of coordinate geometry.

The main navigational problem for a captain was (3)________ the latitude and longitude of his ship. Latitude is relatively easy, because the angle of the Sun above the horizon (4)_______ with latitude and can be tabulated. Since 1730, the standard instrument for finding latitude was the sextant (now made almost (5)_________ by GPS). This was invented by Newton, but he did not publish it. It was independently rediscovered by the English mathematician John Hadley and the American inventor Thomas Godfrey. Previous navigators had used the astrolabe, which goes back to medieval Arabia.

Longitude is (6)_________. The problem was eventually solved by constructing a highly accurate clock, which was set to local time at the start of the voyage. The time of sunrise and sunset, and the movements of the Moon and stars, (7)________ longitude, making it possible to determine longitude by (8)____________ local time with that on the clock. The story of John Harrison's invention of the chronometer, which solved the problem, is famously told in Dava Sobel's *Longitude.*

We continue to use coordinates for maps, but another common use of coordinate geometry occurs in the stock market, where the (9)__________ of some price are recorded as a curve. Here the x-coordinate is time, and the y-coordinate is the price. Enormous quantities of financial and scientific data are recorded in the same way.

longitude obsolete superfluous to determine easier latitude varies depend on comparing fluctuations

trickier

Over to you

1. Summarize the main points from the texts in your own words.
2. Look at Figure 2. Find out what equation corresponds to it.

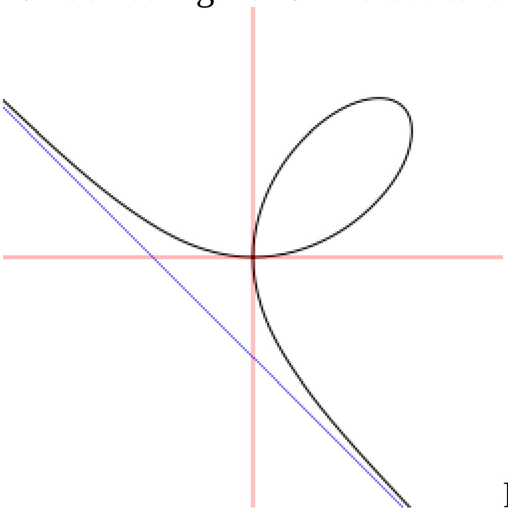
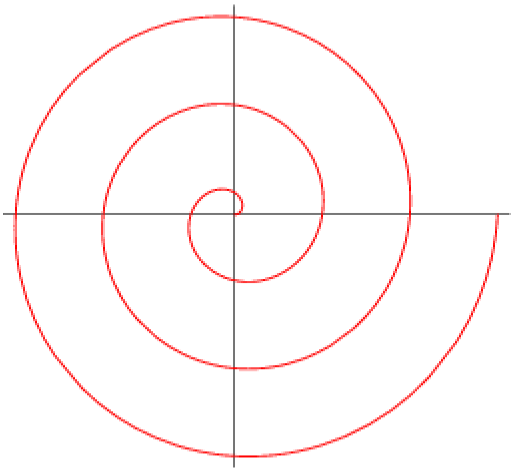


Figure 2.

3. Look at Figure 3. Do you know the equation that defines it?



4. Choose a subject from Unit 3 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template below may help. Try to make you main points as graphic and dramatic as possible.

Presentation template. Work individually or with a partner. Use the template to develop a short presentation with a strong opening, a strong ending and three main stages in between. Make a note of: the main points you want to make; key topic vocabulary you think you may need; expressions that may help you at each stage of the presentation (e.g. I'd like to focus on…, Feel free to interrupt if you have any questions, I'll give a brief overview of…, To sum up,…); signpost language to transition from one stage to the next.(e.g. "To move on", "Turning to the question of…, Getting back to …")

Unit 4.

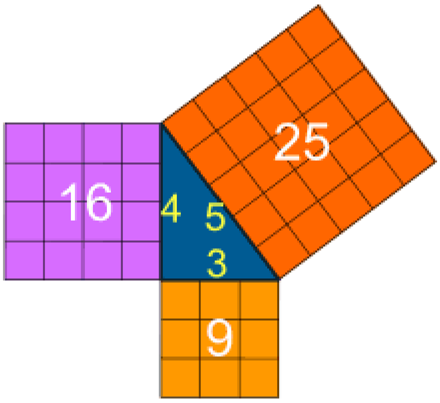
Patterns in Numbers. The origins of number theory.

Part 1

Before you read

Discuss the questions.

1. Look at the 3-4-5 right-angled triangle. What is special about it? Do you remember the name of the ancient mathematician who managed to divide a square into the sum of two squares?



A Key terms

Match these terms with their definitions.

1. prime number — a) a number or quantity to be divided into another number or quantity (the dividend)
2. composite number — b) a number assigned to a quantity and used as a basis of comparison for the measurement of similar quantities
3. magnitude — c) an integer that cannot be factorized into other integers but is only divisible by itself or 1
4. divisor — d) a positive integer that can be factorized into two or more other positive integers
5. factorization — e) one of two or more integers or polynomials whose product is a given integer or polynomial
6. factor — f) the decomposition of an object (for example, a number, a polynomial, or a matrix) into a product of other objects, or factors, which when multiplied together give the original

7. Pythagorean triple g) a set of positive integers a, b and c that fits the rule:

$$a^2 + b^2 = c^2.$$

Reading and Vocabulary tasks

B Word study and Understanding expressions

Read the text given below and complete the sentences using the words and phrases in the box.

seemingly unrelated , the advent, the ideas are thinly disguised as , a very famous conjecture, was given a big boost, heady reaches of the ivory towers, apparently straightforward properties, leaves little scope for, baffling questions , conceals hidden depths.

Introduction

There is something fascinating about numbers. Plain, unadorned whole numbers, 1, 2, 3, 4, 5, … What could possibly be simpler? But that simple exterior (1)_________________, and many of the most (2)_________________ in mathematics are about (3)_________________________ of whole numbers. The area is known as number theory, and it turns out to be difficult precisely because its ingredients are so basic. The very simplicity of whole numbers (4)_____________________ clever techniques.

The earliest serious contributions to number theory – that is, complete with proofs, not just assertions – are found in the works of Euclid, where (5)_________________ geometry. The subject was developed into a distinct area of mathematics by the Greek, Diophantus, some of whose writings survive as later copies. Number theory (6)__________________ in the 1600s by Fermat, and developed by Leonhard Euler, Joseph-Luis Lagrange and Carl Friedrich Gauss into a deep and extensive branch of mathematics which touched upon many other areas, often (7)_________________. By the end of the 20th century these connections had been used to answer some – though not all – of the ancient puzzles, including (8)_______________ made by Fermat around 1650, known as his Last Theorem.

For most of its history, number theory has been about the internal workings of mathematics itself, with few connections to the real world. If ever there was a branch of mathematical thought that lived in the (9)_____________________, it was number theory. But (10)____________ of the digital computer has changed all that. Computers work with electronic representations of whole numbers, and the problems and opportunities raised by computers frequently lead to number theory. After 2500 years as a purely intellectual exercise, number theory has finally made an impact on everyday life.

C Word study and Understanding expressions

Read the text given below and fill in the gaps with the words and phrases in the box.

excites the mathematical curiosity, a unit, successive numbers, to occur somewhat irregularly, the same goes for, can be broken up into, contemplates, provided they can be solved for the primes

,

Primes

Anyone who (1)____________ the multiplication of whole numbers eventually notices a fundamental distinction.

Many numbers (2)______________ smaller pieces, in the sense that the number concerned arises by multiplying those pieces together. For instance, 10 is 2 x 5, and 12 is 3 x 4. Some numbers, however, do not break up in this manner. There is no way to express 11 as the product of two smaller whole numbers; (3)___________________ 2, 3, 5, 7 and many others.

The numbers that can be expressed as the product of two smaller numbers are said to be composite. Those that cannot be so expressed are prime. According to this definition, the number 1 should be prime, but for good reasons it is placed in a special class of its own and called (4)_______. So the first few primes are the numbers

2 3 5 7 11 13 17 19 23 29 31 37 41

As this list suggests, there is no obvious pattern to the primes (except that all but the first are odd). In fact, they seem (5)____________ , and there is no simple way to predict the next number on the list. Even so, there is no question that this number is somehow determined – just test (6)______________ until you find the next prime.

Despite, or perhaps because of, their irregular distribution, primes are of vital importance in mathematics. They form the basic building blocks for all numbers, in the sense that larger numbers are created by multiplying smaller ones. Chemistry tells us that any molecule, however complicated, is built from atoms – chemically indivisible particles of matter. Analogously, mathematics tells us that any number, however big it may be, is built from primes – indivisible numbers. So primes are the atoms of number theory.

The feature of primes is useful because many questions in mathematics can be solved for all whole numbers (7)______________________, and primes have special properties that sometimes make the solution of the question easier. The dual aspect of primes – important but ill-behaved – (8)______________________.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Understanding details

Read the text and put the paragraphs into the correct order.

Euclid

(1) If instead we had started from 30=10x3, then we would break down 10 instead, as 10=2x5, leading to 30=2x5x3. The same three primes occur, but multiplied in a different order – which of course does not affect the result. It may seem obvious that however we break a number into primes, we always get the same result except for order, but this turns out to be tricky, to prove. In fact, similar statements in some related systems of numbers turn out to be false, but for ordinary whole numbers the statement is true. Prime factorization is unique. Euclid the fact needed to establish uniqueness in Proposition 30, Book VII of the *Elements:* if a prime divides the product of two numbers, then it must divide at least one of those numbers. Once we know Proposition 30, the uniqueness of prime factorization is a straightforward consequence.

(2) Euclid introduced primes in Book VII of the *Elements*, and he gave proofs of three key properties. In modern terminology, these are:

(i) Every number can be expressed as a product of primes.
(ii) This expression is unique except for the order in which the primes occur.
(iii) There are infinitely many primes.

What Euclid actually stated and proved is slightly different.
(3) Although Euclid's proof employs three primes, the same idea works proves for a longer list. Multiply all primes in the list, add one and then take some prime factor of the result; this always generates a prime that is not on the list. Therefore no finite list of primes can ever be complete.
(4) Proposition 20, Book IX states that: 'Prime numbers are more than any assigned multitude of prime numbers'. In modern terms, this means that the list of primes is infinite. The proof is given in a representative case: suppose that there are only three prime numbers, *a, b* and *c*. Multiply them together and add one, to obtain *abc*+1. This number must be divisible by some prime, but that prime cannot be any of the original three, since they would then divide *abc* exactly, so they cannot also divide *abc*+1, since they would then divide the difference, which is 1. We have therefore found a new prime, contradicting the assumption that *a*, *b* and *c* are all the primes there are.
(5) Proposition 31. Book VII tells us that any composite number is measured by some prime – that is, can be divided exactly by that prime. For example, 30 is composite, and it exactly divisible by several primes, among them 5 in fact 30=6*x*5. By repeating this process of pulling out a prime divisor, or factor, we can break any number down into a product of primes. Thus, starting from 30=6*x*5, we observe that 6 is also composite, with 6=2x3. Now 30=2x3x5, and all three factors are prime. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

Find a word in the text that means the same as the words and phrases below:
1) involving snags or difficulties (adjective, 6 letters)
2) simple; easy (adjective, 15 letters)
3) to be found or be present (verb, 5 letters)
4) to be inconsistent with (verb, 10 letters)
5) one of two or more integers or polynomials whose product is a given integer or polynomial (noun, 6 letters)

F Use of English

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap.

Diophantus

We have mentioned Diophantus of Alexandria (1)______connection with algebraic notation, but his greatest influence (2)____ in number theory. Diophantus studied general questions, (3)_____ than specific numerical ones, although his answers (4)____ specific numbers. For example: 'Find three numbers such that their sum, and the sum (5)_____ any two, is a perfect square.' His answer is 41, 80 and 320. To check : the sum of all three is $441=21^2$. The sums of pairs are $41+80=11^2$, $41+320=19^2$ and $80+320=20^2$. One of the (6)____known equations

solved by Diophantus is a curious offshoot of Pythagoras's Theorem. We can state the theorem algebraically: if a right triangle has sides *a, b, c* with *c* being (7)____longest, then $a^2 + b^2 = c^2$. There are some special right triangles for which the sides are whole numbers. The simplest and the best (8)______ is when *a, b, c* are 3,4,5, respectively; here$3^2 + 4^2 = 9 + 16 = 25 = 5^2$.
Another example, the next simplest, is $5^2 + 12^2 = 13^2$.
In fact, there (9)_____ infinitely many of these *Pythagorean triples*. Diophantus found all possible whole number solutions of (10)_____ we now write as the equation $a^2 + b^2 = c^2$. His recipe is (11)____ take any two whole numbers, and form the difference (12)_______ their squares. These three numbers always form a Pythagorean triple, and all such triangles arise in this manner provided we also allow all three numbers (13)____ be multiplied (14)____ some constant. If the numbers are 1 and 2, for example, we get the famous 3-4-5 triangle. In particular, since there are infinitely many ways to choose the two numbers, (15)_____exist infinitely many Pythagorean triples. .

(an extract from the book The story of mathematics by Ian Stewart)

G Use of English

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap.

Fermat

After Diophantus, number theory stagnated (1)___ over a thousand years, until it was taken (2)___ by Fermat, who made many important discoveries. One of his (3)____ elegant theorems tells us exactly when (4)____ given integer *n* is a sum of two perfect (5)______: $n = a^2 + b^2$. The solution is simplest when n is prime. Fermat observed that (6)_____ are three basic types of prime:

(i) The number 2, the only (7)____ prime.
(ii) Primes that are 1 greater than a multiple of 4, such as 5, 13, 17 and so on – these primes are all (8)____.
(iii) Primes that are 1 (9)____ than a multiple of 4, such as 3, 7, 11 and so on – these primes are also odd.

He proved that a prime is a sum of two squares if it belongs (10)____categories (i) or (ii), and it is not a sum of two squares (11)____ it belongs to category (iii). (12)____ instance, 37 is in category (ii), being 4 x 9+ 1, and $37 = 6^2 + 1^2$, a sum of two squares. In (13)______, 31 = 4 x 8 – 1 is in category (iii), and if you try all possible ways (14)____ write 31 as a sum of two squares, you (15)_____ find that nothing works. (For instance, 31=25 + 6, where 25 is a square**,** but 6 is not.)

The upshot is (16)_____ a number is a sum of two squares if and only (17)____every prime divisor of the form 4k – 1 occurs (18)_____an even power. Using similar methods, Joseph-Louis Lagrange proved in 1770 that every positive integer is a sum of four perfect squares (including one or more 0s if (19)________. Fermat (20)_____previously stated this result, but no proof is recorded.

One of Fermat's most influential discoveries is also one of the simplest. It is (21)______ as Fermat's Last (sometimes called Great Theorem), and it states that if *p* is any prime and *a* is any whole number, then $a^p - a$ is a multiple of *p*. The corresponding property is usually false when *p* is composite, but not always.

Fermat's most celebrated result (22)______ 350 years to prove. He stated it around 1640, and he claimed a proof, all we know of his work is a short note. Fermat owned a copy of Diophantus's *Arithmetica* , which inspired many of his investigations, and he often wrote (23)_____ his own ideas in the margin. At some point he must have (24)______ thinking about Pythagorean equation: add two squares to get a square. He wondered what (25)_____ happen if instead of squares you tried cubes, but found no solutions. The same problem arose for fourth, fifth or higher powers.

In 1670 Fermat's son Samuel published an edition of Bachet's translation of the *Arithmetic*, which included Fermat's marginal notes. One of such note became notorious: the statement that (26)____ $n \geq 3$, the sum of two nth powers is never an nth (27)_____. The marginal note states 'To resolve a cube into the sum (28)______two cubes, a fourth power into two powers or, in general, any power (29)______ than the second onto two of the same kind is impossible; of which fact I have found a remarkable proof. The margin is too small to contain it.' It seems unlikely that his proof, if it existed, was correct. The first, and currently only, proof (30)______ derived by Andrew Wiles in 1994; it uses advanced abstract methods that (31)_____ not exist until the late 20th century.

After Fermat, several major mathematicians worked in number theory, notably Euler and Lagrange. Most of (32)_____ theorems that Fermat (33)____ stated but not proved (34)____ polished (35)____ during this period.

. *(an extract from the book The story of mathematics by Ian Stewart)*

H Information search

1. Give the definition of prime numbers.
2. What is the difference between composite and prime numbers?
3. Is number 1 prime or composite?
4. Give an example of a Pythagorean triple.
5. Formulate Fermat's last Theorem and make comments on its proof.

J Speaking

Imagine you a teacher of mathematics. Explain to your potential students the role of Fermat in Number theory.

Part 2

Before you read

Discuss the questions.

1. Do you know what Gauss's contribution to Number theory is?
2. Do you know what Fermat prime is?
3. Do you think it is possible to construct a 17-sided polygon using a ruler and compass alone?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. quadratic equation	a) a prime number of the form $2^{m} + 1$, where *m* is the *n* th power of 2 (that is, $m = 2^{n}$, where *n*

	is an integer
2. modular arithmetic	b) an odd prime p can be written as $p = x^2 + y^2$ if and only if $p \equiv 1 \pmod 4$
3. the law of quadratic reciprocity	c) a second-order polynomial equation in a single variable x
4. compass	d) the arithmetic of congruences, sometimes known informally as "clock arithmetic"
5. modulus	e) an integer that can be divided exactly into the difference between two other integers
6. congruence	f) an instrument used for drawing circles
7. Fermat prime	g) the relationship between two integers, x and y, such that their difference, with respect to another positive integer called the modulus, n, is a multiple of the modulus

Reading tasks

B Understanding main points

Translate the following expressions into your native language, read the text given below and fill in the gaps with them.

(a)With hindsight (b)takes *m* hours to go full circle; (c)as a sequence of quadratic equations; (d) the law of quadratic reciprocity; (e) holds for; (x) a far-reaching extension;(f) differing in one key respect; (g) the quadratic residues; (h) to capture the spirit of Gauss's idea; (i) using ruler and compass;(j) find significant for their own sake (k) devised general conceptual foundations for number theory, such as modular arithmetic;(l) for regular polygons; (m) set up the foundations; (n) mainly focused on; (o) propelled the theory; (p) who spotted many of the important patterns ;(r) led to the recognition of new kinds of structure in mathematics (s) came to the fore in those areas of application

Gauss

The next big advance in number theory was made by Gauss, who published his masterpiece; the *Disquisitiones Arithmeticae (Investigations in Arithmetic)* in 1801.This book (1) _________________ of numbers to the centre of the mathematical stage. From then on, number theory was a core component of the mathematical mainstream. Gauss (2) _____________ his own, new work, but he also (3)________________ of number theory and systematized the ideas of his predecessors.

The most important of these foundational changes was a very simple but powerful idea: modular arithmetic. Gauss discovered a new type of number system, analogous to integers but (4)__________________________: some particular number, known as the modulus, was identified with the number zero. This curious idea turned out to be fundamental to our understanding of divisibility properties of ordinary integers.

Here is Gauss's idea. Given an integer *m*, say that *a* and *b* are congruent to the modulus *m*, denoted

$$a \equiv b \pmod m$$

if the difference *a-b* is exactly divisible by *m*. Then arithmetic to the modulus *m* works exactly the same as ordinary arithmetic, except that we may replace *m* by 0 anywhere in the calculation. So, any multiple of *m* can be ignored.

The phrase ‘clock arithmetic’ is often used (5)_______________________ . On a clock, the number 12 is effectively the same as 0 because the hours repeat after 12 steps (24 in continental Europe and military activities). Seven hours after 6 o’clock is not 13 o’clock, but 1 o’clock, and in Gauss’s system $13 \equiv 1 (\text{mod } 12)$. So modular arithmetic is like a clock that (6)______________________. Not surprisingly, modular arithmetic crops up whenever mathematicians look at things that change at repetitive cycles.

The *Disquisitiones Arithmeticae* used modular arithmetic as the basis for deeper ideas, and we mention three.

The bulk of the book is (7)____________________ of Fermat’s observations that primes of the form 4k+1 are a sum of two squares, whereas those of the form 4k-1 are not. Gauss restated this result as a characterization of integers that can be written in the form $x^2 + y^2$, with *x* and *y* integers. Then he asked what happens if instead of this formula we use a general *quadratic form*, $ax^2 + bxy + cy^2$. His theorems are too technical to discuss, but he obtained an almost complete understanding of his question.

Another topic is (8)_______________________, which intrigued and perplexed Gauss for many years. The starting point is a simple question: what do perfect squares look like to a given modulus? For instance, suppose that the modulus is 11. Then the possible perfect squares (of the numbers less than 11) are

0 1 4 9 16 25 36 49 64 81 100 which, when reduced (mod 11),
yield 0 1 3 4 5 9 with each non-zero number appearing twice. These numbers are (9)_________________, mod 11.

The key to this question is to look at prime numbers. If *p* and *q* are primes, when is *q* a square (mod *p*)? For example, the list of quadratic residues above shows that q=5 is a square modulo $p \equiv 11$. It is also true that 11 is a square modulo 5 – because $11 \equiv 1$ (mod 5) and $1 \equiv 1^2$. So here both questions have the same answer.

Gauss proved that this law of reciprocity (10)_________ any pair of odd primes, except when both primes are of the form 4k – 1, in which case the two questions always have opposite answers. That is: for any odd primes *p* and *q*,
q is a square (mod *p*) if and only if *p* is a square (mod *q*), unless both *p* and *q* are of the form 4k – 1, in which case
q is a square (mod *p*) if and only if *p* is not a square (mod *q*).

Initially Gauss was unaware that this was not a new observation: Euler had noticed the same pattern. But unlike Euler, Gauss managed to prove that it is always true. The proof was very difficult, and it took Gauss several years to fill one small but crucial gap.

A third topic in the *Disquisitiones* is the discovery that had convinced Gauss to become a mathematician at the age of 19: a geometric construction for the regular 17-gon (a polygon with 17 sides). Euclid provided constructions, (11)_________________________with three, five and 15 sides; he also knew that those numbers could be repeatedly doubled by bisecting angles, yielding regular polygons with four, six, eight and 10 sides, and so on. But Euclid gave no constructions for 7-sided polygons, 9-sided ones, or indeed any other numbers that the ones just listed. For some two thousand years, the mathematical world assumed that Euclid had said the last word, and no other regular polygons were constructible. Gauss proved them wrong.

It is easy to see that the main problem is constructing regular *p*-gons when *p* is prime. Gauss pointed out that such a construction is equivalent to solving the algebraic equation

$$x^{p-1} + x^{p-2} + x^{p-3} + \ldots + x^{2} + x + 1 = 0$$

Now, a ruler-and-compass connection can be viewed, thanks to coordinate geometry, (12)____________________. If a construction of this kind exists, it follows (not entirely trivially) than p -1 must be a power of 2.

The Greek cases p = 3 and 5 satisfy this condition: here p – 1 = 2 and 4, respectively. But they are not the only such primes. For instance 17 – 1 = 16 is a power of 2. This doesn't yet prove that the 17-gon polygon is constructible, but it provides a strong hint, and Gauss managed to find an explicit reduction of his 16th degree equation to a series of quadratics. He stated, but did not prove, that a construction is possible whenever p – 1 is a power of 2 (still requiring p to be prime), and it is impossible for all other primes. The proof was soon completed by others.

These special primes are called Fermat primes, because they were studied by Fermat. He observed that if *p* is a prime and $p - 1 = 2^{2}$, then k must itself be a power of 2. He noted the first few Fermat primes: 2; 3; 5; 17; 257; 65, 537. He conjectured that numbers of the form $2^{2^{m}} + 1$ are always prime, but this was wrong. Euler discovered that when m = 5 there is a factor 641.

It follows that there must also exist ruler-and-compass constructions for the regular 257-gon and 65,537-gon.F.J.Richelot constructed the regular 257-gon in 1832, and his work is correct. J.Hermes spent ten years working on the 65,537-gon, and completed his construction in 1894. Recent studies suggest there are mistakes.

Number theory started to become mathematically interesting with the work of Fermat, (13)________________________________ concealed in the strange and puzzling behaviour of whole numbers. His annoying tendency not to supply proofs was put right by Euler, Lagrange and a few less prominent figures, with the sole exception of his Last Theorem, but number theory seemed to consist of isolated theorems – often deep and difficult, but not very closely connected to each other.

All that changed when Gauss got in on the act and (14)____________________________________. He also related number theory to geometry with his work on regular polygons. From that moment, number theory became a major strand in the tapestry of mathematics.

Gauss's insights (15)____ – new number systems, such as the integers mod *n*, and new operations, such as the composition of quadratic forms. (16)__________, the number theory of the late 18th and early 19th centuries led to the abstract algebra of the late 19th and 20th centuries. Mathematicians were starting to enlarge the range of concepts and structures that were acceptable objects of study. Despite its specialized subject matter, the

Disquisitiones Arithmeticae marks a significant milestone in the development of the modern approach to the whole of mathematics. This is one of the reasons why Gauss is rated so highly by mathematicians.

Until the late 20th century, number theory remained a branch of pure mathematics – interesting in its own right, and because of its numerous applications within mathematics itself, but of little real significance to the outside world. All that changed with the invention of digital communications in the late 20th century. Since communication then depended on numbers, it is hardly a surprise that number theory (17)___________________________________. It often takes a long time for a good mathematical idea to acquire practical importance – sometimes hundreds of years – but eventually most topics that mathematicians (18)______________________ turn out to be valuable in the real world too.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Information search

Look quickly at the texts and answer these questions.

1. What is Fermat prime?
2. Formulate the of quadratic reciprocity
3. What is modular arithmetic? Why is modular arithmetic called “clock arithmetic’?
4. Explain how to construct a 17-sided polygon using a ruler and compass alone?

D Complete the sentences

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap

What number theory does for us

Number theory forms the basis (1)____ many important security codes used (2)____ the Internet commerce. The best known (3)____ code is the RSA (Ronald Rivest, Adi Shamir and Leonard Adleman) cryptosystem, which (4)____ the surprising feature (5)_____ the method for putting messages into code can be (6)_____ public without giving (7)_____ the reverse procedure of decoding the message.

Suppose Alice wants to send a secret message to Bob. Before doing this, they agree (8)____ two large primes *p* and *q* (having at least a hundred digits) and multiply them together to get $M=pq$. They can (9)_____ this number public if they wish. They also compute $K=(p-1)(q-1)$, but keep this secret.

Now Alice represents her message (10)____ a number *x* in the range *0* to *M* (or a series of such numbers (11)____ it’s a long message). To encode the message she chooses some number *a*, which has no factors (12)____ common with *K*, and computes $y=-x^a(mod\ M)$. The number *a* must be known to Bob, and can also be (13)____ public.

To decode messages, Bob has to know a number *b* such that $ab \equiv 1\ mod\ K$. This number (which exists and is unique) is kept secret. To decode *y*, Bob computes $y^b(mod\ M)$.

Why (14)____ this decode? Because $y^b \equiv [\![(x]\!]^a)^b \equiv x^{ab} \equiv x^1 \equiv x(mod\ M)$, using a generalization of Fermat`s Little Theorem due to Euler.

This method is practical because there are efficient tests to find large primes. However, (15)____ are no known methods for finding the prime factors of a large number efficiently. So telling people the product pq does not help them find p and q, and without those, they cannot work out the value of b, needed to decode the message. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Speaking

Explain how to apply Number theory in cryptography.

Over to you

1. Speak about the origin of Number Theory. Comment on the role of Number theory in Internet commerce.
2. Web research tasks. Research applications of number theory and present your findings to the class. The template in Unit 3 may help.

Web search key words: cryptography, internet commerce, RSA code, security codes, decoding messages

3. Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic “Number theory”. As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants.

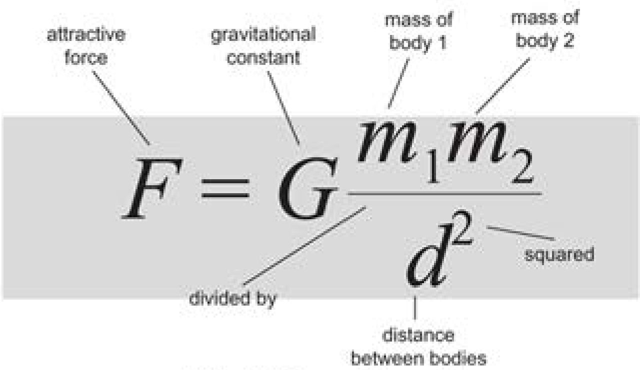
Unit 5.

The system of the world. The invention of calculus.

Part 1

Before you read

1. Do you know who invented Calculus?
2. Can you give a brief history of the Calculus controversy?
3. Do you know whose notation is used in Calculus nowadays?
4. Try to explain the difference between integral calculus and differential calculus?
5. Look at the formula. Can you read it? Do you know what it says? Why is that important?



A Key terms

Match these terms with their definitions.

1. tangent)	a)calculus that concerns accumulation of quantities and the areas under and between curves
2. velocity	b) calculus that deals with the study of the rates at which quantities change
3. integral calculus	c) a geometric line, curve, plane, or curved surface that touches another curve or surface at one point but does not intersect it
4. differential calculus	d) a measure of the rate of motion of a body expressed as the rate of change of its position in a particular direction with time
5. curve	e) the change of a function, f(x), with respect to an infinitesimally small change in the independent variable
6. derivative	f) the graph of a function with one independent variable
7. volume	g) the extent of a two-dimensional surface enclosed within a specified boundary or geometric figure
8. area	h) the rate of change of velocity
9. acceleration	i) the magnitude of the three-dimensional space enclosed within or occupied by an object, geometric solid, etc
10. function	j) a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output

B Understanding main points

Read the text and fill in the gaps with the following words.

plagiarist, terrestrial, heavenly, instantaneous , subsidiary, associated, obtains, assigning, curve, applications, opposite , Geometric , priority, budding, tangent, calculus, reverse, velocity, pure, length, plane, variable, apparently, space, creator, to manifest themselves, insight, acceleration, particles, inversely, proportional, square, to emerge, differential, Algebraic

The system of the world

The most significant single advance in the history of mathematics was calculus, invented independently by Isaac Newton and Gottfried Leibniz. Leibniz published first, but Newton claimed 1) ______ and portrayed Leibniz as a 2) ____.

Even though Leibniz probably deserves priority, Newton turned 3) _____into a central technique of the 4)________ subject of mathematical physics, humanity's most effective known route to the understanding of the natural world. Newton called the theory "The System of the World". This may not have been terribly modest, but it was pretty fair description. Before Newton, human understanding of patterns in nature consisted mainly of the ideas of Galileo about moving bodies. After Newton, mathematical patterns governed almost everything in the physical world: the movement of 5) _______and 6) ________bodies, the flow of air and water, the transmission of heat, light, and the force of gravity.

Newton's unpublished documents known as the *Portsmouth Papers* show that when he was working on the *Principia*, Newton already had the main ideas of calculus.

What is calculus? The methods of Newton and Leibniz are more easily understood if we preview the main ideas. Calculus is the mathematics of 7) _________rates of change – how rapidly is some particular quantity changing at this very instant? For a physical example: a train is moving along a track: how fast is it going right now? Calculus has two main branches. Differential calculus provides methods for calculating rates of change, and it has many geometric 8) ________, in particular finding tangents to curves. Integral calculus does the 9) ______: given the rate of change of some quantity, it specifies the quantity itself. 10) ____________applications of integral calculus include the computation of areas and volumes. Perhaps the most significant discovery is this unexpected connection between two 11)______unrelated classical geometric questions: finding tangents to a 12)______and finding areas.

Calculus is about functions: procedures that take some general number and calculate an associated number. The procedure is usually specified by formula, 13) _______ to a given number *x* (possibly in some specific range) an 14) ______ number *f(x)*.

The first key idea of calculus is differentiation, which 15) _____ the derivative of a function. The derivative is the rate at which *f(x)* is changing, compared to how *x* is changing – the rate of change of *f(x)* with respect to *x*. The other key idea in calculus is that of integration. This is most easily viewed as the 16)_______ process to differentiation.

Inspirations for the invention of calculus came from two directions. Within 17)______ mathematics, differential calculus evolved from methods for finding tangents to curves, and integral calculus evolved from methods for calculating the areas of plane shapes and the volumes of solids. But the main stimulus towards calculus came from physics – the growing realization that nature has patterns. For reasons we still do not really understand, many of the fundamental patterns in nature involve rates of change. So they make sense, and can be discovered, only through calculus.

The invention of calculus was the outcome of a series of earlier investigations of what seem to be unrelated problems, but which possesses a hidden unity. These included calculating the instantaneous 18) ______ of a moving object from the distance it has travelled at any given time, finding the 19) ______ to a curve, finding the 20) ______ of a curve, finding the maximum and minimum values of a 21)________ quantity, finding the area of some shape in the 22) ________ and the volume of some solid in 23)________ Some important ideas and examples were developed by Fermat, Descartes and the more obscure Englishman, Isaac Barrow, but the methods remained special to particular problems. A general method was needed.

The first real breakthrough was made by Leibniz. The other 24) ______ of calculus was Isaac Newton. Newton's main law of motion (there are some 25)______ ones) states that the 26) _____ of a moving body, multiplied by its mass, is equal to the force that acts on the body. Now velocity is the derivative of position, and acceleration is the derivative of velocity. The law of gravity states that all 27) ______ of matter attract each other with a force that is 28)_______ to their masses, and 29) ______ proportional to the 30) _______ of the distance between them.

The most important single idea 31) ______ from the flurry of work on calculus was the existence, and the utility of a novel kind of equation – the 32) _______ equation. 33) _____ equations relate various powers of an unknown number. Differential equations are grander: they relate various derivatives of an unknown function.

Newton's great discovery was that nature's patterns seem 34) ________ not as regularities in certain quantities, but as relations among their derivatives. The laws of nature are written in the language of calculus: what matters are not the values of physical variables, but the rates at which they change. It was a profound 35) _______, and it created a revolution, leading more or less directly to modern science, and changing our planet forever.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word search

Find a word in the text that means the same as the words and phrases below:

a) having or expressing a humble opinion of one's accomplishments or abilities (adjective)

b) quickly, fast (adverb)

c) a calculation involving numbers or quantities (noun)

d) a way of acting or progressing in a course of action (noun)

e) stimulation or arousal of the mind, feelings, etc., to special or unusual activity or creativity (noun pl)
f) to develop or cause to develop gradually (verb in Past Simple)
g) result; consequence (noun)
h) inconspicuous or unimportant (adjective)
i) new, original (adjective)
j) penetrating deeply into subjects or ideas (adj)

D Understanding main points

For questions 1 – 22, read the following text and think of the word which best fits each gap. Use only one word in each gap.

The system of the world.

Even though Leibniz probably deserves priority, Newton turned calculus(1)__a central technique of the budding subject of mathematical physics, humanity's (2)___effective known route (3)___ the understanding (4)___ the natural world. Newton called his theory "The System of the World". This may not have (5) ____ terribly modest, but it (6)___ a pretty fair description. Before Newton, human understanding of patterns in nature consisted mainly (7) ___ the ideas of Galileo about moving bodies, (8) ___ particular the parabolic trajectory of an object (9) ___ as a cannonball, and Kepler's discovery (10)___ Mars follows an ellipse through the heavens. After Newton, mathematical patterns governed almost everything in (11)___physical world: the movement of terrestrial and heavenly bodies, the flow (12)___air and water, the transmission of heat, light and sound, and (13)__ force of gravity.

Curiously, though, Newton's main publication on the mathematical laws of nature, his *Principia Mathematica*, (14)___ not mention calculus (15)__ all; instead, it relies (16)___ the clever application of geometry in the style of (17)___ancient Greeks. But appearances are deceptive: unpublished documents known (18)___ the *Portsmouth Papers* show that when he (19)____ working on the *Principia*, Newton already had the main ideas of calculus. It (20)___likely that Newton used the methods of calculus (21)___ make many of his discoveries, but chose not to present them that way. His version of calculus (22)___published after his death in the *Method of Fluxions* of 1732.

. *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

For questions 1 – 18, read the text below and choose the most appropriate word from the list (A –Q) for each gap. There are two extra words that you do not need to use.

Calculus

What is calculus? The methods of Newton and Leibniz are more easily understood if we (1) _______ the main ideas. Calculus is the mathematics of instantaneous (2) _____ of change – how rapidly is some quantity (3)_______ at this very instant? For a physical example: a train is moving along a (4) ____: how fast is it going right now? Calculus has two main branches. Differential calculus (5) ______ methods for calculating rates of change, and it has many geometric (6) ______, in particular finding tangents to curves. Integral calculus does the (7) _______: given the rate of change of some quantity, it specifies the quantity (8)____. Geometric applications of integral calculus include the computation of areas and (9)______. Perhaps the most significant discovery is this unexpected connection between two apparently (`10) ________ classical geometric questions: finding tangents to a curve and finding areas. Calculus is about functions: (11) ________ that take some general number and calculate an associated number. The procedure is usually specified by a formula, assigning to a given number x (possibly in some specific range) an associated number $f(x)$. Examples include the square root function $f(x)=\sqrt{x}$ (which requires x to be positive) and the square function $f(x)=x^2$ (where there is no (12)______ on x.

The first key idea of calculus is differentiation, which (13)_______ the derivative of a function. The derivative is the rate at which $f(x)$ is changing, compared to how x is changing – the rate of change of $f(x)$ with (14)______to x. Geometrically, the rate of change is the (15) _____ of the tangent to the graph of f at the value x. It can be approximated by finding the slope of the (16)_____ – line that cuts the graph of f at two nearby points, corresponding to x and $x+h$, respectively, where h is small. The slope of the secant is $\frac{f(x+h)-f(x)}{h}$. Now suppose that h becomes very small. Then the secant approaches the tangent to the graph at x. So in some sense the required slope – the derivative of f at x – the limit of this expression as h becomes (17) _____small.

The main conceptual issue here is to define what we mean by limit. It took more than a century to find a logical definition. The other key idea in calculus is that of integration. This is most easily viewed as the (18) _____ process to differentiation.

. *(an extract from the book The story of mathematics by Ian Stewart)*

A provides	B applications	C track	D opposite	E rates	F volumes
G changing	H procedures	I restriction	J obtains	K unrelated	L preview
M itself	N respect	O approximation	P ellipse	S arbitrary	
T secant	U reverse	Q squares			

F Understanding main points

Read the text The need for calculus and fill in the gaps with these expressions.

a) the most accurate model of the motion of the Sun, Moon and planets was that of Ptolemy

b) did debunk the view of the Earth as the centre of the universe

c) exerted substantial control over its adherents' view of the universe

d) differential calculus evolved from methods for finding tangents to curves, and integral calculus evolved from method for calculating the areas of plane shapes and the volumes of solids

e) leave room for five intervening shapes

f) found some discrepancies between Copernicus's heliocentric theory and some subtle observations

g) AND HUMAN BEINGS WERE THE PINNACLE of creation

h) his spheres spun about gigantic axles, some of which were attracted on other spheres and moved along with them.

(j) consigning it to the waste bin of history
(k)) who discovered mathematical regularities in the movements of a pendulum and in falling bodies

The need for Calculus

Inspiration for the invention of calculus came from two directions. Within pure mathematics, (1) ____________________. But the main stimulus towards calculus came from physics – the growing realization that nature has patterns. For reasons we still do not really understand, many of the fundamental patterns in nature involve rates of change. So, they make sense, and can be discovered, only through calculus.

Prior to Renaissance, (2) ______________. In his system, the Earth was fixed, and everything else – in particular, the Sun – revolved around it on a series (real or imaginary, depending on taste) circles. The circles originated as spheres in the work of the Greek astronomer Hipparchus; (3) ____________. Hipparchus's model was not terribly accurate, compared to observations, but Ptolemy's model fitted the observations very accurately indeed, and for over a thousand years it was seen as the last word on the topic. Even the *Almagest* failed to agree with all planetary movements. In Renaissance Europe, however, the scientific attitude began to take root, and one of the casualties was religious dogma. At that time, the Roman Catholic Church (4) _________. It wasn't just the existence of the universe, and its daily unfolding, were credited to the Christian God. The point was that the nature of the universe was believed to correspond to a very literal reading of the Bible. The Earth was therefore seen as the centre of all things. (5)_________, the reasons for the universe's creation. No scientific observation can ever disprove the existence of some invisible unknown creator. But observations can- and (6) _________. And this caused a huge fuss, and got a lot of innocent people killed, sometimes in hideously cruel ways. The fat hit the fire in 1543, when the Polish scholar Nicholas Copernicus published an astonishing, original and somewhat heretical book: *On the Revolution of the Heavenly Moon, turned around the Sun Spheres*. Like Ptolemy, he used epicycles for accuracy. Unlike Ptolemy, he placed the Sun at the centre, while everything else, including the Earth, but excluding the
. Copernicus's main reason for this radical proposal was pragmatic: it replaced Ptolemy's 77 epicycles by a mere 34. Another advantage of Copernicus's theory was that it treated all the planets in exactly the same manner. The only difference was that the inner planets were closer to the Sun than the Earth was, while the outer planets were further away. Copernicus's theory was complicated and his book was difficult to read. Tycho Brahe (7)__________________, which also disagreed with Ptolemy's theory; he tried to find a better compromise. When Brahe died, his papers were inherited by Kepler who was something of a mystic, in the Pythagorean tradition, and he tended to impose artificial patterns to observational data. (he explained the spacing of the planets in terms of regular solids. In his day, the known planets were six in number. Kepler noticed that six planets (8)__________________, and since there were exactly five regular solids, this

would explain the limit of six planets. There are 120 different ways to rearrange the five solids, which between them give an awful lot of different spacings. It is hardly surprising that one of these was in reasonably close agreement with reality. The later discovery of more planets knocked this particular piece of pattern-seeking firmly on the head, (9)____________ . Along the way, though, Kepler discovered some patterns that we still recognize as genuine, now called *Kepler's Laws of Planetary Motion.* The laws state: (i) Planets move round the Sun in elliptical orbits. (ii) Planets sweep out equal areas in equal times. (iii) The square of the period of revolution of any planet is proportional to the cube of its average distance from the Sun. The most unorthodox feature of Kepler's work is that he discarded the classical circle in favour of the ellipse. Another major figure of the period was Galileo Galilei, (10)__________. That's the physical astronomical background that led up to calculus.

. *(an extract from the book The story of mathematics by Ian Stewart)*

G Information search

1. Give a brief history of the Calculus controversy.
2. Explain the difference between integral calculus and differential calculus.
3. Outline the role of Calculus in science.

H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "The system of the World." As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Choose a subject from Unit 5 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.
2. Web research tasks. Research applications of Calculus and present your findings to the class. Web search key words: integration, differentiation, variable, derivative, slope, plagiarism, velocity etc.

Unit 6

Patterns in nature.

Part 1

Before you read

1. What types of DE do you know?
2. What is ODE? What is PDE?
3. Is the wave equation partial or ordianary?
4. What are the applications of DEs?

A Key terms

Match these terms with their definitions.

1. ordinary differential equation	a) an equation that refers to an unknown function *y* of two or more variables, such as *f(x,y,t)* where *x* and *y* are coordinates in the plane and *t* is time
2. partial differential equation	b) the branch of applied science that concerned with the movement of liquids and gases
3. fluid dynamics	c) an equation that refers to an unknown function *y* of a single variable *x* and relates various derivatives of *y*, such as *dy/dx* and *d2y/dx2*
4. wave equation)	d) to determine the value of (the root of a number)
5. to extract	e) having a constant property
6. homogeneous	f) a second-order linear partial equation for the description of waves

B Understanding main points

Read the text below and fill in the gaps with only one suitable word.

Introduction

The main message in Newton's Principia (1)____ not the specific laws of nature that he discovered and used, but the idea that such laws exist – together with evidence (2)___ the way to model nature's laws mathematically (3)____ with differential equations. While England's mathematicians engaged (4)____ sterile vituperation (5)____ Leibniz's alleged (and totally fictitious) theft of Newton's ideas about calculus, the continental mathematicians were cashing (6)___ on Newton's great insight, making important inroads into celestial mechanics, elasticity, fluid dynamics, heat, light and sound – the core topics of mathematical physics. Many of the equations that they (7)_____ remain in use to this day, despite – or perhaps because of- the many advances in the physical sciences.

C Understanding main points

Read the text and fill in the gaps with the following words and expressions.

celestial, sterile vituperation, concept, velocity, were cashing in on, circumstances, opposite, exceedingly, predicted, derived, arise, applied

The main message in Newton's Principia was not the specific laws of nature that he discovered and used, but the idea that such laws exist – together with evidence that the way to model nature's laws mathematically is with differential equations. While England's mathematicians engaged in 1) ______ over Leibniz's alleged (and totally fictitious) theft of Newton's ideas about calculus, the continental mathematicians 2) __________ Newton's great insight, making important inroads into 3) _______ mechanics, elasticity, fluid dynamics, heat, light and sound – the core topics of mathematical physics. Many of the equations that they 4) _____remain in use to this day, despite – or perhaps because of – the many advances in the physical sciences.

To begin with, mathematicians concentrated on finding explicit formulas for solutions of particular kinds of ordinary differential equation. In a way this was unfortunate, because formulas of this type usually fail to exist, so attention became focused on equations that could be solved by a formula rather than equations that generally described nature.

There are two types of differential equation. An ordinary differential equation (ODE) refers to an unknown function *y* of a single variable *x*, and relates various derivatives of *y,* such as *dy/dx* and *d2y/dx2*. The differential equations described so far have been ordinary ones. Far more difficult, but central to mathematical physics, is the 5)______ of a partial differential equation (PDE). Such an equation refers to an unknown function y of two or more variables, such as *f(x,y,t)* where *x* and *y* are coordinates in the plane and *t* is time. The PDF relates this function to expressions in its partial derivatives with respect to each of the *variables.*

Euler introduced PDEs in 1734 and the big breakthrough came in 1746, when d'Alembert did some work on them. Now we call dAlembert's PDF the wave equation, and interpret its solution as a superposition of symmetrically placed waves, one moving with 6)________ *a* and the other velocity *–a* (that is, travelling in the 7)________ direction).It has become one of the most important equations in mathematical physics, because waves 8)_______in many different 9)_______.

The wave equation is 10) _______important, Waves arise in musical instruments, in the physics of light and sound. Euler found a three dimensional version of the wave equation, which he 11) ______to sound waves. Roughly a century later, James Clerk Maxwell extracted the same mathematical expressions for electromagnetism, and 12) ______the existence of radio waves.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Understanding expressions

For questions 1 - 10, read the rest of the text and then choose from the list (A - J) given below the best phrase to fill each of the spaces. Each correct phrase may only be used once.

A encouraged mathematicians
B assumptions
C …as perfect spheres
D …is valid
E …to derive
F …does not hold for

G…of enormous practical significance
H came up with
I … long-term implications
J… with its revelation of

Another major application of PDEs arose in the theory of gravitational attraction, otherwise known as potential theory. The motivating problem was the gravitational attraction of the Earth, or of any other planet. Newton had modeled planets 1) _______, but their true form is closer to that of an ellipsoid. And whereas the gravitational attraction of a sphere is the same as that of a point particle (for distances outside the sphere), the same 2) _________ ellipsoid.

The fundamental PDE for potential theory is Laplace’s equation. The equation 3)_____outside the body: inside, it must be modified to what is now known as Poisson’s equation.

Successes with sound and gravitation 4) ___________ to turn their attention to other physical phenomena. One of the most significant was heat. Fourier’s first step was 5) _______a PDE for heat flow. With various simplifying 6) _________ - the body must be homogeneous (with the same properties everywhere) and isotropic (no direction within it should behave differently from any other), and so on. He 7) _______what we now call the heat equation, which describes how the temperature at any point in a three-dimensional body varies with time.

No discussion of the PDEs of mathematical physics would be complete without mentioning fluid dynamics. Indeed, this is an area 8)________ , because these equations describe the flow of water past submarines, of air past aircraft, and even the flow of air past Formula 1 racing cars.

Newton’s *Principia* was impressive, 9) ________ deep mathematical laws underlying natural phenomena. But what happened next was even more impressive. Mathematicians tackled the entire panoply of physics – sound, light, heat, fluid flow, gravitation, electricity, magnetism. In every case, they came up with differential equations that described the physics, often very accurately.

The 10) _______have been remarkable. Many of the most important technological advances, such as radio, television and commercial aircraft depend, in numerous ways, on the mathematics of differential equations. The topic is still the subject of intense research activity, with new applications emerging almost daily. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

Find a word in the first paragraph of the text that means the same as the words and phrases below.

a) the act of applying to a particular purpose or use

b) a geometric surface, symmetrical about the three coordinate axes, whose plane sections are ellipses or circles. Standard equation: x2/a2 + y2/b2 + z2/c2 = 1, where ±a, ±b, ±c are the intercepts on the x-, y-, and z- axes

c) a three-dimensional closed surface such that every point on the surface is equidistant from a given point, the centre

d) a complete array

e) to undertake (a task, problem, etc.)

F Use of English

Read the text and fill in the gaps with one suitable word.

Wave equation

We now call d'Alembert's PDE the wave equation, and interpret (1)____ solution as a superposition of symmetrically placed waves, one moving with velocity *a* and the other velocity –*a* (that is, travelling in the opposite direction). It (2)____ become one of the most important equations in mathematical physics, because waves arise in many different circumstances.

D'Alembert's elegant formula is the wave equation. Like Newton's second (3)____, it is a differential equation – it involves (second) derivatives of *u*. (4)____ these are partial derivatives, it is a partial differential equation. The second space derivative represents the net force acting on the string, and the second time (5)______ is the acceleration. The wave equation set a precedent: most of the key equations of classical mathematical physics, and a lot of the modern ones for that matter, are (6)_______ differential equations.

There were two ways (7)___ solve the wave equation: Bernoulli's, which led (8)___ sines and cosines, and d'Alembert's, which led (9)___ waves with any shape whatsoever. (10)___ first it looked (11)___ though d'Alembert's solution must be more general: sines and cosines are functions, but most functions are not sines and cosines. However, the wave equation is linear, so you could combine Bernoulli's solutions (12)____ adding constant multiples of them together.

The wave equation is exceedingly important. Waves arise not only in musical instruments, but in the physics of light and sound. Euler found a three-dimensional version of the wave equation, which he applied (13)____ sound waves. Roughly a century later, James Clerk Maxwell extracted the same mathematical expression from his equations for electromagnetism, and predicted (14)___ existence of radio waves. . *(an extract from the book The story of mathematics by Ian Stewart)*

H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Deferential equations". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Choose a subject from Unit 6 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.
2. Imagine you a teacher of mathematics. Explain to your potential students the role of DEs in mathematical physics.

Unit 7.

Fourier analysis and Fourier transform.

Part 1

Before you read

Discuss these questions.

1. Do you know what mathematicians mean by infinitesimals?
2. What do we mean by derivatives and integrals?
3. What ways of representing a function do you know?
4. Do you know the applications of Fourier series?

Vocabulary tasks

A Key terms

Match these terms with their definitions

1. polynomial	a) a mathematical function such that a small change in the independent variable, or point of the domain, produces only a small change in the value of the function
2. series	b) a value to which a function f(x) approaches as closely desired as the independent variable approaches a specified (x = a) or approaches infinity
3. continuous function	c) any mathematical expression consisting of the sum of a number of terms
4. limit	d) the sum of a finite or infinite sequence of numbers or quantities
5. integral	e) the change of a function, f(x), with respect to an infinitesimally small change in the independent variable
6. derivative	f) the limit of an increasingly large number of increasingly smaller quantities, related to the function that is being integrated (the integrand)
7. infinitesimal	g) of, relating to, or involving a small change in the value of a variable that approaches zero as a limit

Reading tasks

B Understanding main points

Read the text. Fill in the gaps with the words in the box.

underpinning, successors, tangent, circumstances, accurately, abounded, in dispute, precision, possessed, were banned, predecessors, being defined, contemporaries

By 1800 mathematicians and physicists had developed calculus into an indispensable tool for the study of natural world, and the problems that arose from this connection led to a wealth of new concepts and methods – for example, way to solve differential equations – that made calculus one of the richest and hottest research areas in the whole of mathematics. The beauty and power of calculus had become undeniable. However, Bishop Berkeley’s criticisms of its logical basis remained unanswered, and as people began to tackle more sophisticated topics, the whole edifice started to look decidedly wobbly. The early cavalier use of infinite series, without regard to their meaning, produced nonsense as well as insights. The foundations of Fourier analysis were non-existent, and different mathematicians were claiming proofs of contradictory theorems. Words like “infinitesimal” were bandied about without 1) ______; logical paradoxes 2) _______; even the meaning of the word “function” was 3)______. Clearly these unsatisfactory 4)_______could not go on indefinitely. Sorting it all out took a clear head, and a willingness to replace intuition by 5)________, even if there was a cost in comprehensibility. The main players were Bernard Bolzano, Cauchy, Niels Abel, Peter Dirichlet and, above all, Weierstrass. Thanks to their efforts, by 1900 even the most complicated manipulations of series, limits, derivatives and integrals could be carried out safely, 6)________ and without paradoxes. A new subject was created: analysis. Calculus became one core aspect of analysis, but more subtle and more basic concepts, such as continuity and limits, took logical precedence, 7) ________the ideas of calculus. Infinitesimals 8)________, completely.

The mathematicians of the 19th century started separating the different conceptual issues in calculus. One was the meaning of the term, function. Another was the various ways of representing a function – by a formula, a power series, a Fourier series or whatever. A third was what properties the function 9)________. A fourth was which representations guaranteed which properties. A single polynomial, for example, defines a continuous function. A single Fourier series, it seemed, might not. Fourier analysis rapidly became the test case for ideas about the function concept. And it was in a paper on Fourier series, in 1837, that Dirichlet introduced the modern definition of a function. In effect, he agreed with Fourier: a variable y is a function of another variable x if for each value of x (in some particular range) there is specified a unique value of y.

Bohemian priest, philosopher and mathematician Bolzano placed most of the basic concepts of calculus on a sound logical footing; the main exception was that he took the existence of real numbers for granted. He insisted that infinitesimals and infinitely large numbers do not exist. And he gave the first effective definition of a continuous function. Namely, f is continuous if the difference $f(x+a) – f(x)$ can be made as small as we please by choosing a sufficiently small. Cauchy’s concept of continuity amounts to exactly the same thing as Bolzano’s. Bolzano’s ideas made it possible to define the limit of an infinite sequence of numbers. A series that has a finite limit is said to be convergent. A finite sum is defined to be the limit of the sequence of finite sums, obtained by adding more and more terms. If that limit exists, the series is convergent. Derivatives and integrals are just limits of various kinds. They exist – that is, they make mathematical sense – provided those limits converge. Limits are about what certain quantities approach as some other number approaches infinity, or zero. The number does not have to reach infinity or zero.

Weierstrass realized that the same ideas work for complex numbers as well as real numbers. Every complex number z has an absolute value IzI, which by Pythagoras’s Theorem is the distance from 0 to z in the complex plane. If you measure the size of a complex expression using its absolute value, then the real-number concepts of limits, series, and so on, as formulated by Bolzano, can immediately be transferred to complex analysis. Weierstrass's the most surprising theorem proves that there exists a function *f(x)* of a variable *x* which is continuous at every point, but differentiable at no point. The graph is a single, unbroken curve, but it is such a wiggly curve that it has no well-defined 10) _______ anywhere. His 11) _____ would not have believed it; his 12)_______wondered what it was for. His 13) _______ developed it into one of the most exciting new theories of the 20th century, fractals. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Understanding details

Mark the statements as True or False:

1. At first the term 'infinitesimal' was used without being defined.
2. A series that has a finite limit is said to be convergent.
3. Cauchy's concept of continuity is totally different from Bolzano's.
4. Fourier analysis is the test case for ideas about the function concept.
5. Analysis was created after calculus.

Vocabulary tasks

D Word search

Find a word in the text that means the same as the words and phrases below:

a) something that has or makes no sense (noun) (paragr.1)

b) desire, readiness to do sth. (n) (para 1)

c) not immediately obvious or comprehensible (adj.) (para 1)

d) a quality, attribute, or distinctive feature of anything (n) (para 2)

e) a person ordained to act as a mediator between God and man (n) (para 3)

E Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Discuss the question.

Look at Figure 1and make your comments on the square wave and some of its Fourier approximations.

The figure shows how to approximate a square wave as opposed to the sinusoidal waves. The Fourier series requires many more terms to provide the same quality of approximation as we found with the half-wave rectified sinusoid.

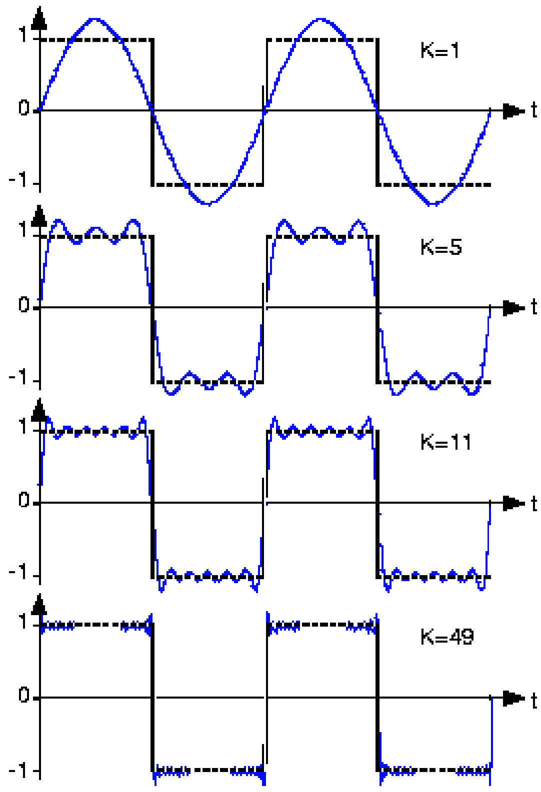


Figure 1

A Understanding details

Read the text, translate the underlined expressions into your native language and think of the word which best fits each space. Use only one word in each space.

Fourier

Before Fourier stuck his oar in, mathematicians (1)____ fairly happy that they knew what a function was. It was some kind of process, *f*, which took a number, *x*, and produced another number, *f(x)*.Which numbers, *x*, make sense depends (2)___ what *f* (3)___. If *f(x)=1/x*, (4)___ instance, then x has to be non-zero. If f(x) = $\sqrt{x}$, and we (5)___ working with real numbers, then *x* must (6)___ positive. But when pressed (7)_____ a definition, mathematicians tend (8)___ be little vague.

The source (9)____their difficulties, we now realize, was that they (10)____grappling with several different features of the function concept – not just what a rule associating a number *x* (11)___another number *f(x)* is, but what properties that rule possesses: continuity, differentiability, capable (12)____being represented by some type of formula and so on.

In particular, they (13)____ uncertain how to handle discontinuous functions, such as

$f(x)= 0$ if $x \le 0$, $f(x)=1$ if $x>0$

This function suddenly jumps from 0 to 1 as x passes through 0. There was a prevailing feeling that the obvious reason (14)______ the jump was the change in the formula: from $f(x)= 0$ to $f(x)= 1$. Alongside that was the feeling that this is the only way that jumps can appear; that any single formula automatically avoided such jumps, so that a small change in x always caused a small change in *f(x)*.

Another source of difficulty was complex functions, where – as we (15)____ seen – natural functions (16)____ the square root are two-valued, and logarithms are infinitely many (17)_____. Clearly the logarithm must be a function – but when there are infinitely many (18)_______, what is the rule for getting *f(z)* from *z*? There seemed to be infinitely many different rules, all equally valid. In order for these conceptual difficulties to be resolved, mathematicians have their noses firmly rubbed in them to experience just how messy the real situation (19)____. And it was Fourier who really got up their noses, with his amazing ideas about writing any function (20____ infinite series of sines and cosines, developed in his study of heat flow.

Fourier's physical intuition told him that his method should be very general indeed. Experimentally, you can imagine holding the temperature of a metal bar (21)___ 0 degrees along half of its length. Physics did not seem to be bothered by discontinuous functions, whose formulas suddenly changed. Physics (22)___ not work with formulas anyway. We use formulas to model physical reality, but that's just technique, it's how we like to think. Of course the temperature will fuzz out a little at the junction of these two regions, but mathematical methods are always approximations (23)___ physical reality. Fourier's method of trigonometric series, applied to a discontinuous function of this kind, seemed to give perfectly sensible results. Steel bars really did smooth out the temperature distribution the way his heat equation, solved using trigonometric series, specified. In *The Analytical Treaty of Heat* he (24) ____ his position plain: "In general, the function *f(x)* represents a succession of values or ordinates each of which is arbitrary. We do not suppose these ordinates to be subject to a common law. They succeed each other in any manner whatever".

Bold words; unfortunately, his evidence in their support did not amount (25)___ a mathematical proof. It was, if anything, even more sloppy than the reasoning employed by people like Euler and Bernoulli. Additionally, if Fourier was right, then his series in effect derived a common law for discontinuous functions. The function above, with values 0 and 1, has a periodic relative, the *square wave*. And the square wave has a single Fourier series, quite a nice one, which works equally well in those regions where the function is 0 and in those regions where the function is 1. So a function that appears to be represented by two different laws can be rewritten in (26) ___ of one law.

Slowly the mathematicians of the 19th century started separating the difference conceptual issues in this difficult area. One was the meaning of the term, function. Another (27) ___ the various ways of representing a function – by a formula, a power series, a Fourier series or whatever. A third was what properties the function possessed. A fourth was which representations guaranteed which properties. A single polynomial, for example, defines a continuous function. A single Fourier series, (28) ___ seemed, might not.

Fourier analysis rapidly became the test case for ideas about the function concept. Here the problems came into sharpest relief and esoteric technical distinctions turned out to be important. And it was in a paper on Fourier series, in 1837, that Dirichlet introduced the modern definition of a function. In effect, he agreed with Fourier: a variable *y* is a function of another variable *x* if for each value of *x* (in some particular (29)______) there is specified a unique value of *y*. He explicitly stated that (30)____ particular law or formula was required: it is enough for *y* to be specified by some well-defined sequence of mathematical operations, applied (31)___ *x*. What at the time must have seemed an extreme example is 'one he made earlier', in 1829: a function *f(x)* taking one value when *x* is rational, and a different value when *x* is irrational. This function is discontinuous (32)____every point. (Nowadays functions like this are viewed as being rather mild; far worse behaviour is possible.)

For Dirichlet, the square root was not one two-valued function. It was two one-valued functions. For real *x*, it is natural – but not essential – to take the positive square as one of them, and the negative square root as the other. For complex numbers, there are no obvious natural choices, although a certain amount can be done to make life easier. .

(an extract from the book The story of mathematics by Ian Stewart)

B Understanding details

Read the text and think of the word which best fits each space. Use only one word in each space. The Riemann hypothesis was one of the famous Hilbert problems — number eight of twenty-three. It is also one of the seven Clay Millennium Prize Problems.

Bernhard Riemann's paper, *Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse* (On the number of primes less than a given quantity), (1)____ first published in the *Monatsberichte der Berliner Akademie*, in November 1859. Just six manuscript pages (2)____ length, it introduced radically new ideas (3)___ the study of prime numbers — ideas which led, in 1896, (4)___ independent proofs by Hadamard and de la Vallée Poussin of the prime number theorem. This theorem, first conjectured (5)___ Gauss when he was a young man, states that the number of primes less (6)___ *x* is asymptotic to *x/log(x)*. Very roughly speaking, this means that the probability that a randomly chosen number of magnitude *x* is a prime is *1/log(x)*.

Riemann gave a formula for the number of primes less than *x* in terms the integral of *1/log(x)* and the roots (zeros) of the zeta function, defined by $\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \dots$.

He also formulated a conjecture about the location of these zeros, which fall (7)____ two classes: the "obvious zeros" -2, -4, -6, etc., and those whose real part lies between 0 and 1. Riemann's conjecture was that the real part of the nonobvious zeros is exactly 1/2. That is, they all lie on a specific vertical line in the complex plane.

Riemann checked the first few zeros of the zeta function (8)___ hand. They satisfy his hypothesis. By now over 1.5 billion zeros have been checked by computer. Very strong experimental (9)______. But in mathematics we require a proof. A proof gives certainty, but, just as important, it gives understanding: it helps us understand *why* a result is true.

Why is the Riemann hypothesis interesting? The closer the real part of the zeros lies to 1/2, (10)_____ more regular the distribution of the primes. To draw a statistical analogy, if the prime number theorem tells us something about the average distribution of the primes along the number line, then the Riemann hypothesis tells us something about the deviation (11)_____ the average.

C Use of English

Read the text and think of the word which best fits each space. Use only one word in each space.

Continuous functions

By now it (1)____ dawning on mathematicians that although they often stated definitions of the term ‘function’, they had a habit of assuming extra properties that (2)____ not follow from the definition. For example, they assumed that any sensible formula, (3)____ as a polynomial, automatically defined a continuous function. But they (4)____ never proved this. In fact, they couldn’t prove it, because they (5)_____not defined ‘continuous’. The whole area was awash with vague intuitions, most of (6)_____ were wrong.

The person who made the first serious start (7)___sorting out this mess was a Bohemian priest, philosopher and mathematician. His name was Bernhard Bolzano. He placed most of the basic concepts of calculus (8)___ a sound logical footing; the main exception was that he took the existence of real numbers for (9)_____. He insisted that infinitesimals and infinitely large numbers (10)___ not exist, and so cannot (11)___ used, however evocative they may be. And he gave the first effective definition of a continuous function. Namely, f is continuous if the difference $f(x+a) - f(x)$ can be made as small as we please by choosing a sufficiently small. Previous authors (12)___ tended to say things like ‘if a is infinitesimal then $f(x+a) - f(x)$ is infinitesimal’. But for Bolzano, a was just a number, like any others. His point was that (13)_______ you specify how small you want $f(x+a) - f(x)$ to be, you must then specify a suitable value for a. It wasn’t necessary for the same value to work in every instance.

D Understanding main points

Answer these questions

1. What do mathematicians mean by infinitesimals?
2. Give the definition of a function?
3. Define derivatives and integrals?
4. What is Dirichlet’s contribution to Fourier analysis?
5. Is a series that has a finite limit convergent?

E Discussion point

Discuss questions 1-5 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 3

Before you read

Discuss these questions.

displacement

second partial derivative

second partial derivative

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with respect to time

with respect to space

speed

squared

1. What does this equation say?
2. Why is that important?
3. What did it lead to?
4. Applications of Fourier transform.
5. Does Fourier Transform have a fault?

A Understanding main points

Read the text and discuss these questions again.

B Vocabulary task

Match these words with their definitions:

a reflex	repetitive
a domain	unnecessary things
blip-like	to expose to view
to reveal	to reverberate
to resonate	an analogue sound storage medium
a vinyl record	a field
junk.	an automatic, habitual response

In its most general form Fourier's method represents a signal, determined by a function f, as a combination of waves of all possible frequencies. This is called the Fourier transform of the wave. It replaces the original signal by its spectrum: a list of amplitudes and frequencies for the component sines and cosines, encoding the same information in a different way – engineers talk of transforming from the time domain to the frequency domain. When data are represented in different ways, operations that are difficult or impossible in one representation may become easy in the other. For example, you can start with a telephone conversation, form its Fourier transform, and strip out all parts of the signal whose Fourier components have frequencies too high or too low for the human ear to hear. This makes it possible to send more conversations over the same communication channels, and it's one reason why today's phone bills are, relatively speaking, so small. You can't play this game on the original, untransformed signal, because that doesn't have 'frequency' as an obvious characteristic. You don't know what to strip out.

One application of this technique is to design buildings that will survive earthquakes. The Fourier transform of the vibrations produced by a typical earthquake reveals, among other things, the frequencies at which the energy imparted by the shaking ground is greatest. A building has its own natural modes of vibration, where it will resonate with the earthquake, that is, respond unusually strongly. So the first sensible step towards earthquake-proofing a building is to make sure that the building's preferred frequencies are different from the earthquake's. The earthquake's frequencies can be obtained from observations; those of the building can be calculated using a computer model.

The Fourier transform has become a routine tool in science and engineering; its applications include removing noise from old sound recordings, such as clicks caused by scratches on vinyl records, finding the structure of large biochemical molecules such as DNA using X-ray diffraction, improving radio reception, tidying up photographs taken from the air, sonar systems such as those used by submarines, and preventing unwanted vibrations in cars at the design stage.

Fourier analysis has become a reflex among engineers and scientists, but for some purposes the technique has one major fault: sines and cosines go on forever. Fourier's method runs into problems when it tries to represent a compact signal. It takes huge numbers of sines and cosines to mimic a localised blip. The problem is not getting the basic shape of the blip right, but making everything outside the blip equal to zero. You have to kill off the infinitely long rippling tails of all those sines and cosines, which you do by adding on even more high-frequency sines and cosines in a desperate effort to cancel out the unwanted junk. So the Fourier transform is hopeless for blip-like signals: the transformed version is more complicated, and needs more data to describe it, than the original.

. *(an extract from the book The story of mathematics by Ian Stewart)*

Over to you

1. Choose a subject from Unit 7 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.

2. Prepare a short presentation on The Riemann hypothesis, one of the seven Clay Millennium Prize Problems. The template in Unit 3 may help.

Unit 8.

Symmetry and Group Theory

Part 1

Before you read

Discuss these questions.

1. Discuss these questions.
2. Do you know what branch of algebra emerged from unsuccessful attempts to solve algebraic equations?
3. What is the role of Galois in mathematics?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. permutation	a) an exact correspondence in position or form about a given point, line, or plane
2. group	b) an ordered arrangement of the numbers, terms etc., of a set into specified groups
3. symmetry	c) an equation containing one or more terms in which the variable is raised to the power of two, but no terms in which it is raised to a higher power
4. a quadratic	d) a set that has an associated operation that combines any two members of the set to give another member and that also contains an identity element and an inverse for each element
5. resolvent	e) to modify or simplify the form of (an expression equation), esp. by substitution of one term by another
6. to reduce	f) something that resolves; solvent
7. a quintic	g) of, relating to, or containing roots of numbers or quantities
8. a radical	h) of or relating to the fifth degree
9. translation	j) a transformation in which the direction of one axis is reversed or which changes the sign of one of the variables
10. reflection	k) a transformation in which the origin of a coordinate system is moved to another position so that each axis retains the same direction or, equivalently, a figure or curve is moved so that it retains the same orientation to the axes
11. commutative law	l) (of an operator) giving the same result irrespective of the order
12. a group of prime order	m) a cyclic group whose order is a prime number

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

applying, predecessors, subsequent, insistence, indispensable tool, plausible, upshot, turning – point, intermittently, concept, worth, contribution, commutative

law, profound, dissuade

Around 1850 mathematics underwent one of the most significant changes in its entire history, although this was not apparent at the time. Before 1800, the main objects of mathematical study were relatively concrete: numbers, triangles, spheres. Algebra used formulas to represent manipulations with numbers, but the formulas themselves were viewed as symbolic representations of processes, not as things in their own right. But by 1900 formulas and transformations were viewed as things, not processes, and the objects of algebra were much more abstract and far more general. In fact, pretty much anything went as far as algebra was concerned. Even the basic laws, such as the 1)_________ of multiplication, *had been dispensed* with some important areas.

These changes came about largely because mathematicians discovered group theory, a branch of algebra that emerged from unsuccessful attempts to solve algebraic equations, especially quintic, or fifth degree. But within 50 years of its discovery, group theory had been reorganized as the correct framework for studying the 2) ________ of symmetry. Today, group theory has become an 3)______________ in every area of mathematics and science, and its connections with symmetry are emphasized in most introductory texts. The 4)______________in the evolution of group theory was the work of a young Frenchman, Evariste Galois. There was a long and complicated prehistory. As the centuries went by, with no sign of any success, mathematicians decided to take a closer look at the whole area. The most successful and most systematic work was carried out by Lagrange. He found that partly symmetric functions of the solutions allowed him to reduce a cubic equation to a quadratic. The quadratic introduced a square root, and the reduction process could be sorted out using a cube root. Similarly, any quartic equation could be reduced to a cube, which he called the resolvent cubic. 5)_________the same techniques, you expect to get a resolvent quartic – job done. But, presumably, to his disappointment, he didn’t get a resolvent quartic. He got a resolvent sextic – an equation of the sixth degree. Instead of making things simpler, his method made the quintic more complicated.

As *Lagrange’s ideas started to sink in*, there was growing feeling that perhaps the problem could not be solved. Perhaps the general quintic equation cannot be solved by radicals. Gauss seems to have thought so, privately, but expressed the view that this was not a problem that he thought was 6)_______tackling. Gauss had already initiated some of the necessary algebra to prove the insolubility of the quintic.

The first person to attempt a proof of the impossibility was Paolo Ruffini. In his General Theory of Equations he claimed a proof that “The algebraic solution of general equations of degree greater than four is always impossible”. But Ruffini’s most important 7) ________was the realization that permutations can be combined with each other. Until then, a permutation was a rearrangement of some collection of symbols.

Mathematicians began to doubt that a solution could exist. Unfortunately the main effect of this belief was to 8)________ anyone from working on the problem. An exception was Abel, a young Norwegian with *a precocious talent for* mathematics, who thought that he had solved the quintic while still at school. He eventually discovered a mistake, but remained intrigued by the problem, and kept working on it 9) _______. Abel showed that whenever an equation can be solved by radicals, there must exist a radical tower leading to that solution, involving only the coefficients of the original equation. But the hypothetical tower cannot contain a solution. The quintic is unsolvable because any solution (by radicals) must have self-contradictory properties, and therefore cannot exist.

Galois set himself the task of determining which equations could be solved by radicals, and which could not. Like several of his 10) _________, he realized that the key to the algebraic solution of equations was how the solutions behaved when permuted. The problem was about symmetry. Galois noticed that the permutations that fix some expression in the roots do not form any old collection. They have a simple, characteristic feature. If you take any two permutations that fix the expression, and multiply them together, the result also fixes the permutation. He called such a system of permutations a group. The 11) ________of Galois's ideas are that the quintic cannot be solved by radicals because it has the wrong kind of symmetries. The group of a general quintic equation consists of all permutations of the five solutions. The algebraic structure of this group is *inconsistent* with a solution by radicals.

The concept of a group first emerged in a clear form in the work of Galois. The main architect of this theory was Camille Jordan. He developed his own version of Galois theory. He proved that an equation is soluble if and only if its group is soluble, which means that the simple components all have prime order.
He applied Galois's theory to geometric problems.

The 4000-year-old quest to solve quintic algebraic equations was brought to an abrupt halt when Ruffini, Abel and Galois proved that no solution by radicals is possible. Although this was a negative result, it had a huge influence on the 12)_________development of both mathematics and science. This happened because the method introduced to prove the impossibility tuned out to be central to the mathematical understanding of symmetry, and symmetry turned out to be vital in both mathematics and science. The effects were 13) _______. Group theory led to a more abstract view of algebra, and with it a more abstract view of mathematics. Although many practical scientists initially opposed the move towards abstraction, it eventually became clear that abstract methods are often more powerful than concrete ones, and most opposition has disappeared. Group theory also made it clear that negative results may still be important, and that an 14) _______on proof can sometimes lead to major discoveries. Suppose that mathematicians had simply assumed without proof that quintics cannot be solved, on the 15)________ grounds that no one could find a solution. Then no one would have invented group theory to explain why they cannot be solved. If mathematicians had taken the easy route, and assumed the solution to be impossible, mathematics and science would have been a pale shadow of what they are today. That is why mathematicians insist on proofs.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. inconsistent

a) incompatible b) not enough

2. had been dispensed

a) had been done away (with) or had managed (without

b) had been given out or issued in portions

3. Lagrange's ideas started to sink in

a) Lagrange's ideas started to penetrate the mind

b) Lagrange's ideas started to spread

4. a precocious talent

a) an early talent b) a rare talent

D Understanding main points

Mark these statements T(true) or F(false) according to the information in the text. Find the part of the text that gives the correct information.

1. Quintic equations cannot be solved.
2. Jordan viewed Group theory geometrically.
3. Group theory emerged from unsuccessful attempts to solve algebraic equations.
4. The concept of a group first emerged in a clear form in the work of Galois.
5. Abel discovered a blunder in his work.

E Information search

Look quickly at the text and answer these questions**.**

1. Can quadratic and cubic equations be solved by radicals?
2. Can quintic equations be solved by radicals?
3. What branch of algebra emerged from unsuccessful attempts to solve algebraic equations?
4. What is the role of Galois in mathematics?
5. In whose work did the concept of a group first emerge in a clear form?

F Reading and Use of English

Read the text about Jordan and fill in the gaps with the following expressions: (a) we also encounter screw motions (b) who initiated the mathematical study of crystal symmetries, especially the underlying atomic lattice; (c) discrete translations (by integer multiples of a fixed distance) in other directions (d) developed the entire subject in a systematic and comprehensive way.

The concept of a group first emerged in a clear form in the work of Galois. The main architect of this theory was Camille Jordan, who (1)___________. Jordan exhibited the deep link with geometry in a very explicit manner, by classifying the basic of motion of a rigid body in Euclidean space. More importantly, he made a very good attempt to classify how these motions could be combined into groups. His main motivation was the crystallographic research of August Bravais (a French physicist known for his work in <u>crystallography</u>), (2)________________________.. Technically, Jordan dealt only with closed groups, in which the limit of any sequence of motions in the group is also a motion in the same group. These include all finite groups, for trivial reasons, and also groups like all rotations of a circle about its centre. A typical example of a non-closed group, not considered by Jordan, might be all rotations of a circle about its centre through rational multiples of 360degrees.

The main rigid motions in the plane are translations, rotations, reflections and glide reflections. In three-dimensional space, (3)____________, like the movement of a corkscrew: the object translates along a fixed axis and simultaneously rotates about the same axis.

Jordan began with groups of translations, and listed ten types, all mixtures of continuous translations (by any distance) in some directions and (4)____________________. He also listed the main finite groups of rotations and reflections. Later it became clear that his list was incomplete. For instance, he had missed out some of the subtler crystallographic groups in three-dimensional space. The Jordan-Holder Theorem tells us that the simple groups (a group is simple if it does not break up) relate to general groups in the same way that atoms relate to molecules in chemistry. Simple groups are the atomic constituents of all groups. So, Jordan applied Galois's theory to geometric problems.

G Reading and Use of English

Read the text about applications of Group theory and think of the word which best fits each space. Use only one word in each space.

Group theory is now indispensable throughout mathematics; it turns (1)_____ in theories of pattern formation in many different scientific contexts. One example is the theory of reaction-diffusion equations, introduced (2)___ Alan Turing in 1952 as (3)____possible explanation of symmetric patterns in the markings (4)____ animals. In (5)______ equations, a system of chemicals (6)____ diffuse (7)_____ a region of space, and the chemicals can also react (8)___produce new chemicals. Turing suggested (9)____ some such process might set up a pre-pattern in developing animal embryo, which later on could be turned (10)_____ pigments, revealing the pattern in the adult.

Suppose (11)____simplicity that the region is a plane. Then the equations are symmetric (12)____ all rigid motions. The only solution of the equations that is symmetric (13)_____ all rigid motions is a uniform state, the (14)____anywhere. This would translate into an animal without any specific markings, the same colour all over. However, the (15) ______ state may be unstable; in which case the actual solution observed will be symmetric under some rigid motions but not others. This process is (16)_____ symmetry-breaking. .

(an extract from the book The story of mathematics by Ian Stewart)

H Discussion point

Work in groups. Discuss this statement: "Group theory made it clear that negative results may still be important». Choose a person from your group for a brief summary of your discussion.

Over to you

1. Web research task. Find out as much as you can about Galois and his contribution to Group theory. Web search key words: Galois, group theory, quintics, etc

2. Look at Figure1. Read the text about Rubik's cube. Discuss this question. Is Rubik's cube related to Group theory?

All possible rearrangements of a Rubik's cube can be regarded as an example of a group:

- There is a one-to-one correspondence between the distinct permutations of the cube and the elements of the Rubik's cube group. Each different permutation represents the result of a single element of the group just as the different images of the stop sign represented the result of each element in that group.

- Different moves of the cube could correspond to the same final permutation and would therefore correspond to the same single element in the group. In other words, a single element of the Rubik's cube group can be expressed in different way using different sequence of moves.

To verify that the Rubik's cube group satisfies the three properties of a group consider: - There is an identity element namely "not doing any move" (e.g. "not doing any move" followed by a move *x* equals the move *x*.

Find out as much as you can about this problem and prepare a short review.

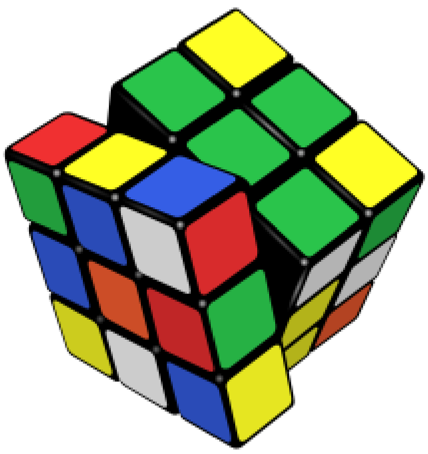
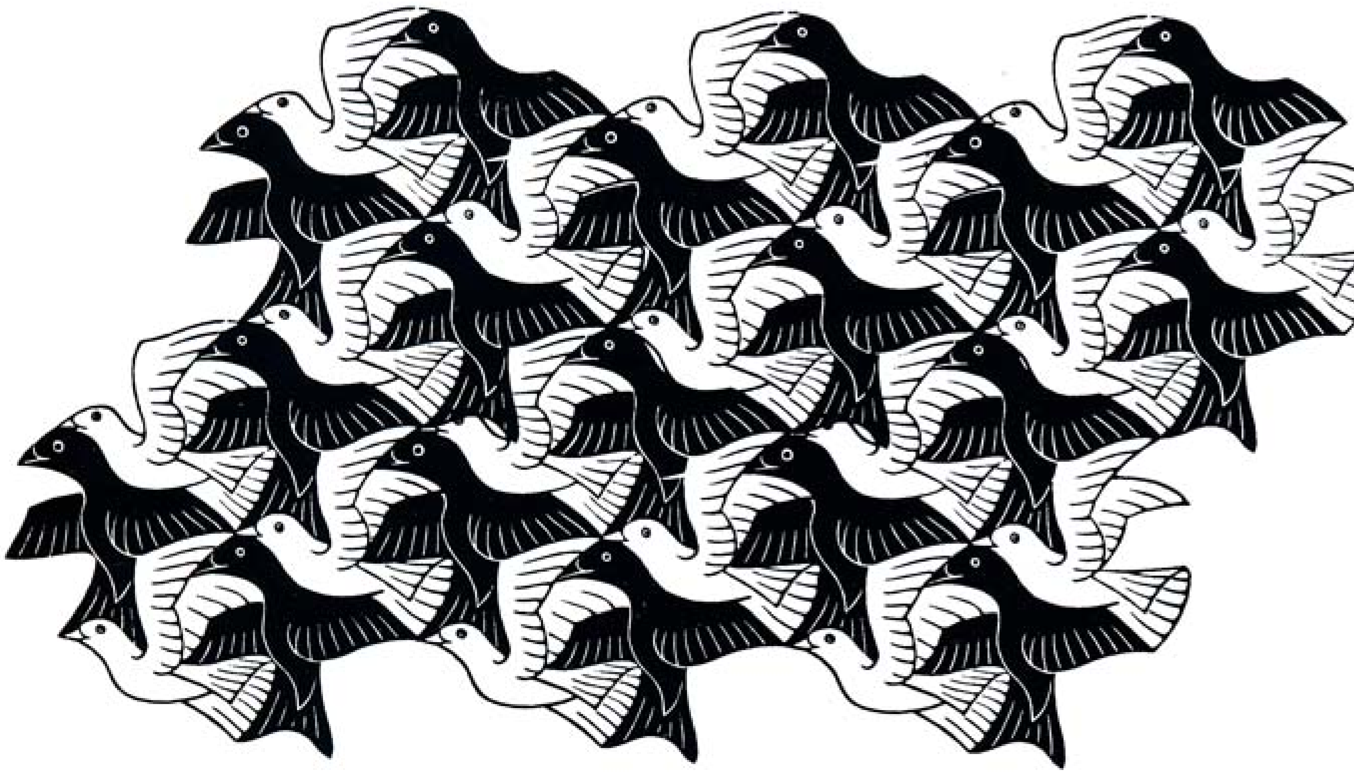


Figure 1.

3. Look at Figure 2. The work, titled *Regular Division of the Plane with Birds* by Escher, uses a tessellation with triangles. As the mathematics of symmetry Group theory is important in art. The important result in Group theory proves that there are only 17 possible wallpaper patterns. Escher exploited these basic patterns in his tessellations, applying what geometers would call reflections, glide reflections, translations, and *r*otations to obtain a greater variety of patterns. He also elaborated these patterns by distorting the basic shapes to render them into animals, birds, and other figures. These distortions had to obey the three, four, or six-fold symmetry of the underlying pattern in order to preserve the tessellation. The effect can be both startling and beautiful. Prepare a short presentation on "Escher and symmetry in his art" using the information given above. Web search key words: Escher, tessellations, symmetry.



Regular Division of the Plane with Birds; wood engraving, 1949

Unit 9.

Topology

Part 1

Before you read

Discuss these questions.

1. What is the Mobius Band?
2. Who is Perelman? What is he famous for?
3. Can you explain the difference between a qualitative theory of shape and more traditional quantitative theory?
4. Can you define the concept of a hole in topology?

A Key terms

Match these terms with their definitions.

1. projective geometry	a) a plural of radius. A straight line joining the centre of a circle or sphere to any point on the circumference or surface
2. a segment	b) a transformation consisting of rotations and translations which leaves a given arrangement unchanged
3. radii	c) the branch of geometry concerned with the properties of solids that are invariant under projection and section
4. a rigid motion	d) the formation of conclusions from incomplete evidence; guess
5. mapping	e) logical sequence, cohesion, or connection
6. a conjecture	f) an entity, quantity, etc., that is unaltered by a particular transformation of coordinates
7. an invariant	g) a topological structure which prevents the object from being continuously shrunk to a point
8. continuity	h) a branch of geometry describing the properties of a figure that are unaffected by continuous distortion, such as stretching or knotting
9. topology	j) a model of the extended complex plane, the complex plane plus a point at infinity
10.a hole in a mathematical object	k) a part of a line or curve between two points
11. Riemann sphere	l) a ring-shaped surface generated by rotating a circle about a coplanar line that does not intersect the circle
12. manifold	m) representing or transforming (a function, figure, set, etc.)
13. torus	n) a topological space having specific properties
14. helix	o) a curve that lies on a cylinder or cone, at a constant

angle to the line segments making up the surface; spiral

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

squashed, stumbled across, removed, conjecture, continuous, rigid, in extremely convoluted ways, flexible, deceptive, Ostensibly

The main ingredients of Euclid's geometry – lines, angles, circles, squares and so on – are all related to measurement. Line segments have lengths, angles are a definite size with 90' differing in important ways from 91' or 89', circles are defined in terms of their radii, squares have sides of a given length. The hidden ingredient that makes all of Euclid's geometry work is length, a metric quantity, one which is unchanged by rigid motions and defines Euclid's equivalent concept to motion, congruence.

When mathematicians first 1) __________ other types of geometry, these too were metric. In non-Euclidean geometry, lengths and angles are defined; they just have different properties from lengths and angles in Euclidean plane. The arrival of projective geometry changed this: projective transformations can change lengths, and they can change angles. Euclidean geometry and the two main kinds of non-Euclidean geometry are 2) _______. Projective geometry is more 3)_________, but even here *subtler* invariants exist, and in Klein's picture what defines a geometry is a group of transformations and the corresponding invariants.

As the 19th century approached its end, mathematicians began to develop an even more flexible kind of geometry; so flexible, in fact, that it is often characterized as rubber-sheet geometry. More properly known as topology, this is the geometry of shapes that can be deformed or *distorted* 4) ______________. Lines can bend, shrink or stretch; circles can be 5)_________ so that they turn into triangles or squares. All that matters here is continuity. The transformations allowed in topology are required to be 6)________ in the sense of analysis; roughly speaking, this means that if two points start sufficiently close together, they end up close together – hence the 'rubber sheet ' image.

There is still a hint of metric thinking here: close together' is a metric concept. But by early 20th century, even this hint had been 7)_______.. and topological transformations *took on a life of their own*. Topology quickly increases its status, until it occupied center stage in mathematics – even though to begin with it seemed very strange, and *virtually content-free*. With transformations that flexible, what could be invariant? The answer, it turned out, was quite a lot. But the type of invariant that began to be uncovered was like nothing ever before considered in geometry. Connectivity – how many pieces does this thing have? Holes – is it all one lump, or are there tunnels through it? Knots – how is it *tangled up*, and can you undo the tangles? To a topologist, a doughnut and a coffee- cup are identical (but a doughnut and a tumbler are not); however both are different from a round ball. An overhand knot is different from a figure-eight knot, but proving this fact required a whole new kind of machinery, and for a long time no one could prove that any knots existed at all. It seems remarkable that anything so diffuse and plain weird could have any importance at all. But all appearances are 8) ________.

Continuity is one of the basic aspects of the natural world, and any deep study of continuity leads to topology. And our understanding of the phenomenon of chaos rests on topology. The main practical consumers of topology are quantum field theorists. Another application of topological ideas occurs in molecular biology, where describing and analyzing twists and turns of DNA molecule requires topological concepts. Topology is a rigorous study of qualitative geometric features, as opposed to quantitative one like lengths. A key step was the discovery of connections between complex analysis and the geometry of surfaces, and the innovator was Riemann. The obvious way to think of a complex function *f* is to interpret it as a mapping from one complex plane to another. The basic formula $w=f(z)$ for such a function tells us to take any complex number z, apply *f* to it and deduce another complex number *w* associated with *z* .Geometrically, *z* belongs to the complex plane, and *w* belongs to what is in effect a second, independent copy of the complex plane.

Riemann found it useful to include infinity among the complex numbers, and he found a beautiful geometric way to do this. Place a unit sphere so that it sits on top of the complex plane. Now associate points in the plane with points on the sphere by stereographic projection. This construction is called Riemann sphere. The complex analysis found that, topologically, every Riemann surface is either a sphere, or a torus, with two holes, or a torus with three holes etc. The number of holes, *g*, is known as the genus of the surface, and it is the same *g* that occurs in the generalization of Euler's formula to surfaces.

The natural step after surfaces –is three dimensions. Poincare posed a question, later reinterpreted as the Poincare 9) _______ - if a three-dimensional manifold (without boundary, of finite extent, and so on) has the property that any loop in it can be shrunk to a point, then that manifold must be topologically equivalent to the 3-sphere (a natural three-dimensional analogue of a sphere). In 2002 Grigori Perelman caused a sensation by placing several papers on arXiv, the website for physics and math research. 10)_________these papers were about the Ricci flow, but it became clear that if the work was correct, it would imply the geometrization conjecture, hence that of Poincare.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. subtler
a) less obvious or comprehensible b) thinner
2. distorted
a) twisted b)amplified
3. took on a life of their own
a) acquired a life of their own b) disappeared
4) virtually content-free
a) practically meaningless b) unreal
5. tangled up
a) knotted or coiled together b) ensnared or trapped

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Non-Euclidean geometry is metric.
2. Topology is virtually Projective geometry.
3. The study of continuity leads to Topology.
4. Hole is the concept in Topology.
5. The genus of a surface is in fact a number of holes on it.

E Information search

Look quickly at the text and answer these questions**.**

1. What is Topology?
2. What are the applications of Topology?
3. What are the basic concepts of Topology?
4. Who managed to solve the Poincare conjecture? Give the details.
5. What is Riemann surface?

F Discussion point

Discuss questions 1-5 in small groups of three or four. Choose a person from your group for a brief summary of your discussion

Part 2

A Before you read

Discuss these questions.

1. Describe Euler's contribution to Topology.
2. Can you formulate Euler's Formula for Polyhedra?
3. Do you know that Descartes first noticed the curious numerical pattern of the regular solids, viewed this formula as a minor curiosity and did not publish it?
4. Can you formulate the problem of Konigsberg bridges?

B Key terms

Match these terms with their definitions.

1) face	a) a solid figure having four plane faces
2) vertex (pl vertices)	b) the point opposite the base of a figure
3) edge	c) a solid figure having twelve plane faces
4) tetrahedron	d) a solid figure consisting of four or more plane faces (all polygons), pairs of which meet along an edge, three or more edges meeting at a vertex
5) octahedron	e) a solid figure having eight plane faces
6) dodecahedron	f) a joint made by beveling each of two parts to be joined, usually at a 45° angle, to form a corner, usually a 90° angle
7) a mitred corner	g) a line along which two faces or surfaces of a solid meet
8) polyhedron	h) any one of the plane surfaces of a solid figure

C Reading tasks and Use of English

Read the text and fill in the gaps with the following expressoins:

a)bears no relationship , b) the converse holds; c) a significant hint, d) his manuscript; e) backwards. ;f) the discrepancy; g**)** goes for , h) valid; i) the numerology of regular solids; g**)** stroll**;** k) to take off ; l) had long wondered whether, m) are connected

Read the last paragraph and fill in the gaps with one suitable word

Although topology really began (1)______ around 1900, it made an occasional appearance in earlier mathematics. Two items in the prehistory of topology were introduced by Euler: his formula for polyhedra and his solution of the puzzle of the Konigsberg Bridges. In 1639 Descartes had noticed a curious feature of (2)____________. Consider, for instance, a cube. This has six faces, 12 edges and eight vertices. Add 6 to 8 and you get 14, which is 2 larger than 12. How about a dodecahedron? Now there are 12 faces, 30 edges and 20 vertices. Add 12+20=32, which exceeds 30 by 2. The same (3)________ the tetrahedron, octahedron and icosahedron. In fact, the same relation seemed to work for almost any polyhedron. If a solid has F faces, E edges and V vertices, then F+V=E+2, which we can rewrite as F-E+V=2 . Descartes did not publish his discovery, but he wrote it down and (4)____________ was read by Leibniz in 1675.

Euler was the first to publish this relationship. Is this formula (5)______ for all polyhedra? Not quite. A polyhedron in the form of a picture frame, with square cross-section and mitred corners, has 16 faces, 32 edges and 16 vertices, so that here F+V-E=0. The reason for (6)___________ turns out to be the presence of a hole. In fact, if a polyhedron has *g* holes, then F+V - E=2 – 2*g*. What exactly is a hole? This question is harder than it looks. It is easier to define what ‘no holes’ means. A polyhedron has no holes if it can be continuously deformed, creating curved faces and edges, so that it becomes (the surface of) a sphere. For these surfaces, F+V-E really is always 2. And (7)_________as well: if F+V - E=2 then the polyhedron can be deformed into a sphere.

Euler’s formula is now viewed as (8)____________ of a useful link between combinatorial aspects of polyhedra, such as numbers of faces, and topological aspects. In fact, it turns out to be easier to work (9)__________. To find out how many holes a surface has, work out F+V-E, divide by 2, and change sign: *g*=-(F+V-E)/2.

A curious consequence: we can now define how many holes a polyhedron has, without defining ‘hole’.

At first sight, the problem of Konigsberg Bridges (10)_______________ to the combinatorics of polyhedra. The city of Konigsberg was situated on both banks of the river, in which there were two islands. The islands were linked to the banks, and to each other, by seven bridges. Apparently, the citizens of Konigsberg (11)___________________ it was possible to take a Sunday (12)______ that would cross each bridge exactly once. In 1735 Euler solved the puzzle; rather he proved that there is no solution. He pointed out that what matters is how the islands, the banks and the bridges (13)____________.

The whole problem can (1)___ reduced to a simple diagram of dots (vertices) joined (2)___ lines (edges).(3)___ form this diagram, place one vertex (4)___ each land-mass - north bank, south bank, and the two islands. Join two vertices by an edge (5)_________a bridge exists that links the corresponding land masses. Here we end (6)___ with four vertices A,B,C,D and seven edges, one (7)___ each bridge. Is it possible to find a path – a connected sequence of edges – that includes each edge exactly (8)______? Euler distinguished two types of path: an open tour, which starts and ends (9)___ different vertices, and a closed tour, which starts and ends at the (10)_____ vertices. He proved that for this particular diagram (11)_____ kind of tour exists.

The key (12)___ this puzzle is to consider the valency of each vertex, (13)____is, how many edges meet (14)___ that vertex. First, think of a closed tour. Here, every edge where the tour enters a vertex is matched by another, the next edge, along which the tour leaves that vertex. If a closed tour is possible, then the number of edges at any given vertex must therefore be even. In short, every vertex must have (15)____ valency. But the diagram has three vertices of valency 3 and one of valency 5 – all odd numbers. Therefore no closed tour exists. A similar criterion (16)______ to open tours.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic “Topology”. As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1**.** Web research task. Find out as much as you can about Topology and its applications. Web search key words: hole, knot, vertex, Riemann surface, Klein bottle, Poincare, Perelman.

2. Escher and Topology in his art. The regular solids, known as polyhedra, held a special fascination for Escher. He made them the subject of many of his works and included them as secondary elements in a great many more.

There are only five polyhedra with exactly similar polygonal faces, and they are called the Platonic solids: the tetrahedron, with four triangular faces; the cube, with six square faces; the octahedron, with eight triangular faces; the dodecahedron, with twelve pentagonal faces; and the icosahedron, with twenty triangular faces**.**

In the woodcut *Four Regular Solids* Escher has intersected all but one of the Platonic solids in such a way that their symmetries are aligned, and he has made them translucent so that each is discernable through the others.

Prepare a short presentation on “Escher and Topology in his art”.

Web search key words: Escher, “Four Regular Solids”, wood engraving, “Stars”, woodcut, “Three Intersecting Planes “.

Unit 10

Probability theory, its applications. Statistics

Part 1

Before you read

Discuss these questions.

1. Do you know what expressions are called binominal coefficients?
2. What are the chances of throwing various numbers with two dice?
3. What is the probability of throwing two dice simultaneously that their sum is 5?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. calculations of odds	a) a cube of wood, plastic, etc., each of whose sides has a different number of spots (1 to 6), used in games of chance and in gambling to give random numbers
2. die (pl dice)	b) a triangle consisting of rows of numbers; the apex is 1 and each row starts and ends with 1, other numbers being obtained by adding together the two numbers on either side in the row above
3. binomial coefficient	c) a function representing the relative distribution of frequency of a continuous random variable from which parameters such as its mean and variance can be derived and having the property that its integral from a to b is the probability that the variable lies in this interval
4. Pascal's triangle	d) the function the values of which are probabilities of the distinct outcomes of a discrete random variable
5. normal distribution	e) a continuous distribution of a random variable with its mean, median, and mode equal, the probability density function of which is given by (exp-[(x-μ)2/2σ2/σ√(2π)) where μ is the mean and σ2 the variance
6. measure function	f) any of the numerical factors which multiply the successive terms in a binomial expansion
7. density function	g) calculation of the chances of success in a certain undertaking
8. bell curve	h) a bell-shaped curve
9. probability space	j) a mathematical construct that models a real-world process (or "experiment") consisting of states that occur randomly

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:
emerge, accuracy, clinical trials, likely, assuming, distributed, gambling games, explicit, eliminated, derived

The growth of mathematics in the 20^{th} and early 21^{st} centuries has been explosive. An especially novel branch of mathematics is the theory of probability, which studies the chances associated with random events. It is the mathematics of uncertainty. Earlier ages scratched the surface, with combinatorial calculations of odds in 1) _________ and methods for improving the 2)________of astronomical observations despite observational errors, but only in the early 20^{th} century did probability theory 3) _______ as a subject in its own right.
Nowadays, probability theory is a major area of mathematics, and its applied wing, statistics, has a significant effect on everyday lives. Statistics is one of the main analytic techniques of the medical profession. No drug comes to market, and no treatment is permitted in a hospital, until 4) ________ have ensured that it is sufficiently safe, and that it is effective. Probability theory may also be the most widely misunderstood, and *abused*, area of mathematics. But used properly and intelligently, it is a major contributor to human welfare.

In the Middle Ages, we find discussions of the chances of throwing various numbers with two dice. To see how this works, let's start with one die. 5)__________that the die is *fair*, each of the six numbers should be thrown equally often, in the long run. We expect each number to turn up roughly one time in six; that is, probability 1/6. If this didn't happen, the die would in all likelihood be unfair and *biased*. Back to that medieval question. Suppose we throw two dice simultaneously. What is the probability that their sum is 5? The upshot of numerous arguments, and some experiments, is that the answer is 1/9.
A working definition of the probability of some event is the proportion of occasions on which it will happen. A basic problem here is that of combinations. Given, say, a pack of six cards, how many different subsets of four cards are there? One method is to list these subsets: there are 15 of them. But this method is too *cumbersome* for large numbers of cards, and something more systematic is needed.

Imagine choosing the numbers of the subset, one at a time. We can choose the first in six ways, the second in only five (since one has been 6) ________) etc. The total number of choices in this order is 6 x 5 x 4 x 3 = 360. However, every subset is counted 24 times, there are 24 (4 x 3 x 2) ways to rearrange four objects. So, the correct answer is 360/24, which equals 15. This argument shows that the number of ways to chose *m* objects from a total of *n* objects is $\binom{n}{m} = \frac{n(n-1)\dots(n-m+1)}{1 \times 2 \times 3 \times \dots \times m}$

These expressions are called binominal coefficients, because they also arise in algebra. If we arrange them in a table, so that the nth row contains the binomial coefficients $\binom{n}{0}\ \binom{n}{1}\ \binom{n}{2}\ \binom{n}{n}$ then the result looks like this:

In the sixth row we see the numbers 1,6,15,20,15 6,1. Compare with the formula (x+1)^6 = x^6+6x^5 +15x^4+20x^3+15x^2+6x+1 and we see the numbers arising as the coefficients. This is not a coincidence. The triangle of numbers is called Pascal’s triangle. Binomial coefficients were used *to good effect* in the book on probability written by Jacob Bernoulli. Abraham De Moivre extended Bernoulli’s work on biased coins. When *m* and *n* are large, it is difficult to work out the binomial coefficients exactly, and De Moivre 7) ________ an approximate formula, relating Bernoulli’s binomial coefficients to what we now call the error function or normal distribution.

A major conceptual problem in probability theory was to define probability. Even simpler examples presented logical difficulties. These difficulties caused all sorts of problems, and all sorts of paradoxes. They were finally resolved by a new idea from analysis, the concept of a measure. In many ways the idea of measure was more important that the integral to which it led. In particular, probability is a measure. This property was made 8) _______by Kolmogorov, who laid down axioms for probabilities. More precisely, he defined a probability space. This comprises a set *X*, a collection *B* of subsets of *X* called events and a measure *m* on *B*. The axioms state that *m* is a measure, and that *m(X)+1* (that is, the probability that something happens is always 1.) The collection *B* is also required to have some set-theoretic properties that allow it to support a measure.

The applied arm of probability theory is statistics, which uses probabilities to analyze real world data. It arose from 18th century astronomy, when observational errors had to be taken into account. Empirically and theoretically, such errors are 9) ________ according to the error function or normal distribution, often called the bell curve because of its shape. Here the error is measured horizontally, with zero error in the middle, and the height of the curve represents the probability of an error of given size. Small errors are fairly 10) _______, whereas large ones are very improbable. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. abused

a) incorrect b) misused

2. fair

a) quite good b)) in conformity with standards

3. biased

a) slanting obliquely b) prejudiced

4. cumbersome

a) difficult because of extent or complexity b) awkward

5. to good effect

a) to advantage b) as a result

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Statistics is widely used in medicine.
2. Kolmogorov provided Probability theory with theorems.
3. Pascal’s triangle contains binomial coefficients.
4. Bernoulli defined the concept of a measure.

5. Error function is in fact normal distribution.

E Information search

Look quickly at the text and answer these questions**.**

1. What is Kolmogorov's contribution to Probability theory?
2. Comment on the origin of Probability theory.
3. Outline the role of Fermat in the development of Probability theory.
4. Do you agree that statistics uses probabilities to analyze real world data?
5. Can statistics be used to model social data – births, deaths, divorces, crime and suicide?
6. Can statistical ideas be used as a substitute for experiments?

F Reading and Use of English

Read the text and think of the word which best fits each space. Use only one word in each space.

Probability theory**,** a branch of mathematics concerned (1)____the analysis of random phenomena. The outcome of (2)___ random event cannot (3)___determined before it occurs, but it may be any one (3)____ several possible outcomes. The actual outcome is considered (4)____be determined by chance. The word probability has several meanings (5)____ordinary conversation. Two of these are particularly important (6)____ the development and applications of the mathematical theory of probability. One is (7)___ interpretation of probabilities (8)____relative frequencies, for which simple games involving coins, cards, dice, and roulette wheels (9)______ examples. The distinctive feature of games of chance is that (10)____outcome of (11)____ given trial cannot be predicted (12)____certainty, although the collective results of (13)____large number of trials display some regularity. For example, the statement (14)____ the probability of "heads" in tossing a coin equals one-half, according to the relative frequency interpretation, implies that (15)___ a large number of tosses the relative frequency with which "heads" actually occurs will be approximately one-half, although it contains no implication concerning (16)____ outcome of any given toss. There are many similar examples involving groups of people, molecules of a gas, genes, and so (17)____. Actuarial statements about the life expectancy for persons of a certain age describe the collective experience of a large (18)____ of individuals but do not purport to say what (19)___ happen to any particular person. Similarly, predictions about the chance of a genetic disease occurring in a child of parents having a known genetic makeup are statements about relative frequencies of occurrence in a large number of cases but are not predictions (20) _____ a given individual.

G Reading and Use of English

Read the text and fill in the gaps using the following words in the correct form:

bias	sophistication	arbitrary	oscillation	probable
fledge	define	modification	inspire	supposition
occurrence	double	consistency	coincident	equitably

Probability theory arrived as a fully (1)_________ area of mathematics in 1713 when Jacob Bernoulli published his Ars Conjectandi (‘Art of Conjecturing’). He started with the usual working (2)_________ of the probability of an event: the proportion of occasions on which it will happen, in the long run, nearly all the time. I say ‘working definition’ because this approach to probabilities runs into trouble if you try to make it fundamental. For example, suppose that I have a fair coin and keep tossing it. Most of the time I get a random-looking sequence of heads and tails, and if I keep tossing for long enough I will get heads roughly half the time. However, I seldom get heads exactly half the time: this is impossible on odd-numbered tosses, for example. If I try to (3)_________ the definition by taking (4)____________ from calculus, so that the probability of throwing heads is the limit of the proportion of heads as the number of tosses tends to infinity, I have to prove that this limit exists. But sometimes it doesn’t. For example, suppose that the sequence of heads and tails goes

THHTTTHHHHHHTTTTTTTTTTTT...

with one tail, two heads, three tails, six heads, twelve tails, and so on – the numbers (5)___________ at each stage after the three tails. After three tosses the proportion of heads is 2/3, after six tosses it is 1/3, after twelve tosses it is back to 2/3, after twenty-four it is 1/3,. . . so the proportion (6)_________ to and fro, between 2/3 and 1/3, and therefore has no well-defined limit.

Agreed, such a sequence of tosses is very unlikely, but to define ‘unlikely’ we need to define probability, which is what the limit is (7)__________ to achieve. So the logic is circular. Moreover, even if the limit exists, it might not be the ‘correct’ value of 1/2. An extreme case (8)_________when the coin always lands heads. Now the limit is 1. Again, this is wildly improbable, but. . .

Bernoulli decided to approach the whole issue from the opposite direction. Start by simply defining the probability of heads and tails to be some number p between 0 and 1. Say that the coin is fair if p = 1/2
, and (9)_______ if not. Now Bernoulli proves a basic theorem, the law of large numbers. Introduce a reasonable rule for assigning probabilities to a series of repeated events. The law of large numbers states that in the long run, with the exception of a fraction of trials that becomes (10)___________ small, the proportion of heads does have a limit, and that limit is p. Philosophically, this theorem shows that by assigning probabilities – that is, numbers – in a natural way, the interpretation ‘proportion of occurrences in the long run, ignoring rare exceptions’ is valid. So Bernoulli takes the point of view that the numbers assigned as probabilities provide a (11)________ mathematical model of the process of tossing a coin over and over again.

His proof depends on a numerical pattern that was very familiar to Pascal. It is usually called Pascal’s triangle, even though he wasn’t the first person to notice it. Historians have traced its origins back to the Chandas Shastra, a Sanskit text attributed to Pingala, written some time between 500 BC and 200 BC. The original has not survived, but the work is known through tenth-century Hindu commentaries. Pascal’s triangle looks like this:

where all rows start and end in 1, and each number is the sum of the two immediately above it. We now call these numbers binomial coefficients, because they arise in the algebra of the binomial (two-variable) expression (p + q)^n. Namely,

(p + q)0 = 1
(p + q)1 = p + q
(p + q)2 = p2 + 2pq + q2
(p + q)3 = p3 + 3p2q + 3pq2 + q3
(p + q)4 = p4 + 4p3q + 6p2q2 + 4pq3 + q4

and Pascal's triangle is visible as the coefficients of the separate terms. Bernoulli's key insight is that if we toss a coin n times, with a (12)______________ p of getting heads, then the probability of a specific number of tosses yielding heads is the corresponding term of (p + q)^n, where q = 1 − p. For example, suppose that I toss the coin three times. Then the eight possible results are:

HHH HHT HTH THH
HTT THT TTH
TTT

where I've grouped the sequences according to the number of heads. So out of the eight possible sequences, there are

1 sequence with 3 heads
3 sequences with 2 heads
3 sequences with 1 heads
1 sequence with 0 heads

The link with binomial coefficients is no (13)____________. If you expand the algebraic formula $(H + T)^3$ but don't collect the terms together, you get

HHH + HHT + HTH + THH + HTT + THT + TTH + TTT

Collecting terms according to the number of Hs then gives

H3 + 3H2T + 3HT2+T3

After that, it's a matter of replacing each of H and T by its probability, p or q respectively.

Even in this case, each extreme HHH and TTT occurs only once in eight trials, and more (14)____________ numbers occur in the other six. A more (15)______________calculation using standard properties of binomial coefficients proves Bernoulli's law of large numbers.

. *(an extract from the book 17 equations that changed the world by Ian Stewart)*

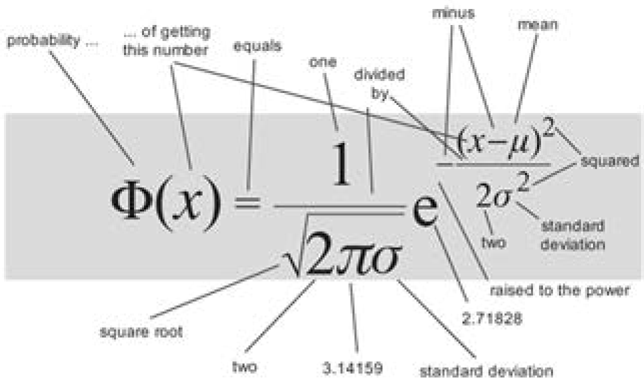
H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Probability and Statistics". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Web research task. Find out as much as you can about Probability theory and its applications. Web search key words: games of chance, Pascal's and Fermat's letters on Probability theory, gambling, binomial coefficients,
2. Web research task. Find out as much as you can about Statistics and its applications. Web search key words: normal distribution, measure, error function, Benford's Law, the mode, the median

3. Look at this formula. What does this formula say? Why is that important? What



did it lead to?
The short answers given below might help you.
The probability of observing a particular data value is greatest near the mean value – the average – and dies away rapidly as the difference from the mean increases. How rapidly depends on a quantity called the standard deviation.
It defines a special family of bell-shaped probability distributions, which are often good models of common real-world observations The concept of the 'average man', tests of the significance of experimental results, such as medical trials, and an unfortunate tendency to default to the bell curve as if nothing else existed..

Unit 11.

Numerical analysis

Part 1

Before you read

Discuss these questions.

1. Do you know the prehistory of modern computers?
2. What do you know about first calculating machines?
3. What are the applications of numerical methods?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. calculator	a) a counting device that consists of a frame holding rods on which a specific number of beads are free to move
2. difference engine	b) a number system having a base of two, numbers being expressed by sequences of the digits 0 and 1: used in computing, as 0 and 1 can be represented electrically as off and on
3. iteration	c) an automatic mechanical calculator designed to tabulate polynomial functions
4. binary notation	d) a device for performing mathematical calculations
5. abacus	e) a computational method in which a succession of approximations, each building on the one preceding, is used to achieve a desired degree of accuracy

B Reading tasks and Use of English

a) Read the first part of the text and fill in the gaps with the following words:

genuinely**,** came up with, successive, obsolete, at high speed, improvements

The rise of the computer

Mathematicians have always dreamed of building machines to reduce the drudgery of routine calculations. Earlier machines were modest, but they saved a lot of time and effort. The first development after abacus was probably Napier's bones, or Napier's rods, a system of marked rods that Napier invented before he (1) _____logarithms. In 1642 Pascal invented the first 2)_____ mechanical calculator, the Arithmetic Machine. It could perform addition and subtraction. In 1671 Leibniz designed a machine for multiplication, and built one in 1694. One of the most ambitious proposals for a calculating machine was made by Charles Babbage. He constructed the difference engine; it was in effect a programmable mechanical computer. A modern reconstruction of the difference engine is in the Science Museum in London – and it works. Augusta Ada Lovelace contributed to Babbage's work by developing some of the first computer program ever written. The first mass-produced calculator, the Arithmometer, was manufactured by Thomas de Colmar in 1820. The next major step was the mechanism of the

Swedish inventor Willgodt. T. Odhner. The motive power was supplied by the operator, who turned a handle to revolve a series of discs on which the digits 0-9 were displayed. With practice, complicated calculations could be carried out 3)___.

The advent of cheap powerful electronic computers in the 1980s made mechanical calculations 4) _____, but their use in business and scientific computing was widespread until that time.

Calculating machines contribute more than just simple arithmetic, because many scientific calculations can be implemented numerically as lengthy series of arithmetical operations. One of the earliest numerical methods, which solves equations to arbitrarily high precision, was invented by Newton, and is appropriately known as Newton's method. It solves an equation f(x)=0 by calculating a series of 5) ____ approximations to a solution, each improving on the previous one but based on it. The method is based on the geometry of the curve y=f(x) near the solution. A second important application of numerical methods is to differential equations. Suppose we wish to solve the differential equation $\frac{dx}{dt}$=f(x) given that x=x_0 at time t=0. The simplest method, due to Euler, is to approximate $\frac{dx}{dt}$ by $\frac{x(t+3)-x(t)}{\varepsilon}$, where ε is very small. Then an approximation to the differential equation takes the form x(t+ε)=x(t)+ εf(x(t)). Starting with x(0) = x_0, we successively deduce the values of f(ε), f(2ε), f(3ε) and, in general, f(nε) for any integer n>). A typical value for ε might be 10^{-6}, say. A million iterations of the formula tells us x(1), another million leads to x(2) and so on. With today's computers a million calculations are trivial, and this becomes an entirely practical method.

However Euler method is too simple-minded to be fully satisfactory, and numerous 6) _____have been devised. The best known, the so called fourth order Runge-Kutta method, is widely used in engineering, science and theoretical mathematics.

b) Read the second part of the text and think of the word which best fits each space. Use only one word in each space.

As well as using computers to help mathematics, we can use mathematics to help computers. In fact, mathematical principles were important in all early designs, 1) ___ as proof of concept or as key aspects of the design. All digital computers in use today work with binary notation, in which numbers 2)___represented as strings of just two digits: 0 and 1. The main advantage of binary is 3)____it corresponds to switching: 0 is off and 1 is 4)___. Or 0 is no voltage and 1 is 5 volts, or whatever standard is employed in circuit design. The symbols 0 and 1 can also 5)____interpreted within mathematical logic, as truth values: 0 means 6) ____and 1 means 7)____. So computers can perform logical calculations as 8) ___as arithmetical ones. Indeed, the logical operations are more basic, and the arithmetical operations 9)____be viewed as sequences 10)___logical operations. Boole's algebraic approach to the mathematics of 0 and 1 provides an effective formalism for the logic of computer calculations. Internet search 11)___ carry out Boolean searches, 12)___is , they search 13)____ items defined by some combination of logical criteria, such as ‘containing the word “cat” but not containing “dog”. .

(an extract from the book The story of mathematics by Ian Stewart)

C Word study

Match the words and phrases 1-9 to their definitions a-j

1) drudgery	a) to move or cause to move around a centre or axis; rotate
2) to revolve	b) at a random manner
3) to implement	c) hard, menial, and monotonous work
4) arbitrarily	d) to carry out; put into action; perform
5) precision	e) rigour, accuracy
6) trivial	f) out of use or practice; not current
7) entirely	g) of little importance; petty or frivolous
8) obsolete	h) solely or exclusively; only
9) successively	j) in a serial or successive manner, one following another

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text.

1. The first mechanical calculator was made by Charles Babbage.
2. Ada Lovelace was the first programmer.
3. There are more than two ways of solving equations numerically.
4. Computer can not perform logical operations.
5. The first computers appeared in the 20th century.

E Information search

Look quickly at the text and discuss these questions**.**

1. Describe Newton’s method for solving an equation numerically.
2. Outline the role of Napier, Pascal, Leibniz in the development of the computer.
3. Describe the role of Ada Laplace in the rise of the computer.
4. Denote binary notation.

F Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Discuss these questions.

1. Have you ever heard about P= NP? problem, the solution of which will win a million-dollar prize?
2. Do you now what P and NP stand for?
3. Can you define the concept of algorithm?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. algorithm	a) an integer or a polynomial whose product is a given integer or polynomial
2. factor	b) the amount of time that grows exponentially
3. exponential running time	c) a recursive procedure whereby an infinite

	sequence of terms can be generated
4. linear running time	d) the amount of time that grows at a rate proportional to the size of the input
5. Euclidean algorithm	e) a rough and practical approach, based on experience, rather than scientific or precise one based on theory
6. a rule of thumb	f) an algorithm for finding the greatest common divisor of two numbers
7. prime factor	g) a factor that cannot itself be factored
8. remainder	h) the amount left over when one quantity cannot be exactly divided by another

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

split second, compute, Roughly speaking, plausible, depend on , unsolved, replace, In contrast

ALGORITHMS

Mathematics has aided computer science, but in return computer science has motivated some fascinating new mathematics. The notion of algorithm – a systematic procedure for solving a problem – is one. An especially interesting question is: how does the running of an algorithm 1) ____ the size of the input data? For example, Euclid's algorithm for finding the highest common factor of two whole numbers *m* and *n,* with $m<n$,$m=n$, is as follows: - Divide *n* by *m* to get remainder *r*. If $r=0$ then the highest common factor is m: STOP. - If $r>0$ then 2) _____ *n* by *m* and *m* by *r* and go back to the start. It can be shown that *n* has *d* decimal digits (a measure of the size of the input data to the algorithm) then the algorithm stops after at most *5d* steps. That means, for instance, that if we are given two 1000-digit numbers, we can 3) _____ their highest common factor in at most 5000 steps – which takes a 4) _____on a modern computer.

The Euclidean algorithm has linear running time: the length of the computation is proportional to the size (in digits) of the input data. More generally, an algorithm has polynomial running time, or is of class P, if its running time is proportional to some fixed power (such as the square or cube) of the size of the input data. 5) ____, all known algorithms for finding the prime factors of a number have exponential running time – some fixed constant raised to the power of the size of the input data. This is what makes the RSA cryptosystem (conjecturally) secure.

6) ____, algorithms with polynomial running time lead to practical computation on today's computers, whereas algorithms with exponential running time do not – so the corresponding computations cannot be performed in practice, even for quite small sizes of initial data. This distinction is *a rule of thumb*: a polynomial algorithm might involve such a large power that it is impractical, and some algorithms with running time worse than polynomial still turn out to be useful.

The main theoretical difficulty now arises. Given a specific algorithm, it is fairly easy to work out how its running time depends on the size of the input data and to determine whether it is class P or not. However, it is *extraordinary difficult* to decide whether a more efficient algorithm might exist to solve the same problem more rapidly. So, although we know that many problems can be solved by algorithm in class P, we have no idea whether any sensible problem is not-P.

Sensible here has a technical meaning. Some problems must be not-P, simply because outputting the answer requires non-P running time. For example list all possible ways to arrange *n* symbols in order. *To rule out* such obviously non-P problems, another concept is needed: the class NP of non-deterministic polynomial algorithms. An algorithm is NP if any guess at an answer can be checked in a time proportional to some fixed power of the size of input data. For example, given a guess at a prime factor of a large number, it can quickly be checked by a single division sum.

A problem in class P is automatically NP. Many important problems, for which P algorithms are not known, are also known to be NP. And now we come to the deepest and most difficult 7) ______ problem in this area, the solution of which will win a million-dollar price from Clay Math Institute. Are P and NP the same? The most 8) ______answer is no, because P=NP means that a lot of apparently very hard computations are actually easy – there exists some *short cut* which we've not yet thought of.

The P=NP? problem is made more difficult by a fascinating phenomenon, called NP-completeness. Many NP problems are such that if they are actually class P, then every NP problem is class P as well. Such a problem is said to be NP-complete. If any particular NP-complete problem can be proved to be P, then P=NP. On the other hand, if any particular NP-complete problem can be proved to be not-P, then P is not the same as NP. .

(an extract from the book The story of mathematics by Ian Stewart)

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. a rule of thumb.
a) empirical b) significant
2. extraordinary difficult
a) extremely difficult b) fairly difficult
3. To rule out
a) to dismiss from consideration b) to skip
4. short cut
a) a means of saving time b) short way

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Euclid's algorithm has a polynomial running time.
2. The RSA cryptosystem is secure.
3. Polynomial running time is the amount of time that grows at a rate proportional to its size.
4. P is equal to NP.
5. The P=NP problem is worth a million-dollar.

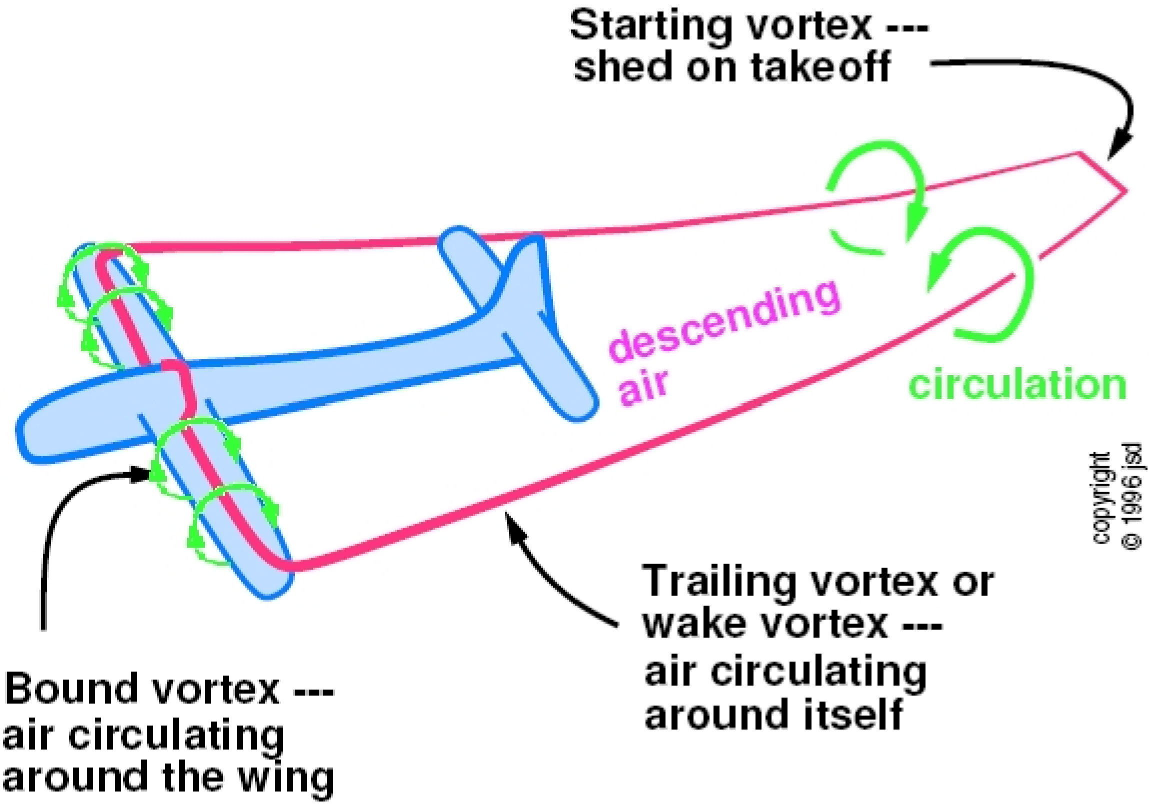
E Discussion point

Look quickly at the text and discuss these items**.**

1. Denote the notion of an algorithm.
2. Describe the difference between algorithms with polynomial running time and exponential running time.
3. Give a brief account of the modern use of numerical analysis

F Reading and Use of English

Read the text about the numerical computation of flow of air past an aicraft and think of the word which best fits each space. Use only one word in each space.



Numerical analysis plays a central role in (1)___ design of modern aircraft. Not so (2)___ ago, engineers worked (3)___out how air would flow (4)___ the wings and fuselage of an aircraft using wind-tunnels. They (5)____ place a model of the aircraft in the tunnel, force air past (6)___ with a system of fans and observe the flow patterns. Equations such as (7)____ of Navier and Strokes provided various theoretical insights, but it was impossible to solve them (8)___ real aircraft because of their complicated shape.

Today's computers are so powerful, and numerical methods for solving PDEs (9) ___ computers have become so effective, (10)___ in many cases the-wind tunnel approach has (11___ discarded in (12)____ of a numerical wind-tunnel – (13)___ is, a computer model of the aircraft. The Navier-Strokes equations are (14)___ accurate that they can safely be used in this way. The advantage of the computer approach is (15)___ any desired feature of the flow can be analysed and visualized.

F Discussion point

Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean's office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. *The P=NP problem.*
Course 2. *The RSA cryptosystem.* Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1**.** Web research task. Find out as much as you can about Numerical analysis and its applications. Web search key words: P vs NP, unsolved problems, Clay Institute, RSA cryptosystem, binary notation

Unit 12.

Chaos theory

Part 1

Before you read

Discuss these questions.

1. Do you know what is meant by the Lorenz attractor?
2. Can you define the concept of butterfly effect?
3. Do you think Chaos theory has a wide range of applications nowadays?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. non-linear dynamics	a) the hallmark of chaos (individual trajectories are unstable but the dynamics of the whole is stable) The coexistence of chaos and hence the instability of individual trajectories and structural stability, a global property is absolutely remarkable
2. horseshoe system	b) a phenomenon in which a small perturbation in the initial condition of a system results in large changes in later condition
3. the butterfly effect	c) a result about discrete dynamical systems. One of the implications of the theorem is that if a discrete dynamical system on the real line has a periodic point of period 3, then it must have periodic points of every other period.
4. the Lorenz attractor	d) a strange attractor in the form of a two-lobed figure formed by a trajectory which spirals around the two lobes, passing randomly between them
5. Sharkovskii's Theorem	e) study of systems governed by equations in which a small change in one variable can induce a large systematic change

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

to yield a system , become amplified , inevitable consequences, tiny disturbances , integer steps, fundamental discoveries, differential equations

Chaos

One topic, which achieved public prominence in the 1970s, is chaos theory, the media's name for nonlinear dynamics. This topic evolved naturally from traditional models using calculus. Another is complex system, which employs less *orthodox ways of thinking*, and is stimulating new mathematics as well as new science.

Before the 1960s, the word chaos had only one meaning: formless disorder. But since that time, 1)________ in science and mathematics have endowed it with a second, more *subtle*, meaning – one that combines aspects of disorder with aspects of form. Newton's Mathematical Principles of Natural Philosophy had reduced the system of the world to 2)__________ , and these are deterministic. That is, once the initial state of the system is known, its future is determined uniquely for all time.

The growth of scientific determinism was also accompanied by a vague but *deep-seated belief* in the conservation of complexity. This belief causes us to look at a complex object or system, and wonder where the complexity comes from. Where, for example, did the complexity of life come from, given that it must have originated on a lifeless planet? It seldom occurs to us that complexity might appear *of its own accord*, but that is what the latest mathematical techniques indicate.

In the early 1960s the American mathematician Stephan Smale developed Poincare's discovery of complex motion in the restricted three-body problem, simplifying the geometry 3)________ known as 'Smale's horseshoe'. He proved that the horseshoe system, although deterministic, has some random features. Other examples of such phenomena were developed by the American and Russian schools of dynamics, with especially notable contributions by Oleksandr Sharkovskii and Vladimir Arnold, and a general theory began to emerge. The term 'chaos' was introduced by James Yorke and Tien-Yien Li in 1975, in a brief paper that simplified one of the results of the Russian school: Sharkovskii's Theorem of 1964, which described a curious pattern in the periodic solutions of a discreet dynamical system – one in which time runs in 4)_______ instead of being continuous.

Meanwhile, chaotic systems were appearing sporadically in the applied literature – largely unappreciated by the whole scientific community. The best known of these was introduced by the meteorologist Edward Lorenz in 1963. He discovered that if the same equations are solved using initial values of the variables that *differ only very slightly from each other*, then the differences 5)_______ until the new solution differs completely from the original one. His description of this phenomenon in subsequent lectures led to the currently popular term, butterfly effect, in which flapping of a butterfly's wings leads, a month later, to a hurricane on the far side of the globe. In the real world, weather is influenced not only by one butterfly but by the statistical features of trillions of butterflies and other 6)__________.

Using topological methods, Smale, Arnold and other coworkers proved that the *bizarre solutions* observed by Poincarre were the 7)___________ of strange attractors in the equations. A strange attractor is a complex motion that the system *inevitably* homes in on. It can be visualized as a shape in the state-space formed by the variables that describe the system. The Lorenz attractor, which describes Lorenz's equation in this matter, looks like a mask, but each apparent surface has infinitely many layers.

The structure of attractors explains 8)________of chaotic systems: they can be predicted in the short term (unlike, say, rolls of a die) but not in the long term. Why cannot several short-term predictions be strung together to create a long-term prediction? Because the accuracy with which we can describe a chaotic system deteriorates over time, at an even-growing rate, so there is a prediction horizon beyond which we cannot penetrate. Nevertheless, the system remains on the same strange attractor – but its path over the attractor changes significantly.

David Ruelle and Floris Takens quickly found a potential application of strange attractors in physics: *the baffling problem* of turbulent flow in a fluid. They suggested that turbulence is a physical instance of a strange attractor. Initially this theory was received with some skepticism, but we now know that it was correct in spirit, even if some of the details were rather questionable. Other successful applications followed, and the word chaos was enlisted as a convenient name for all such behavior. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Match the words and phrases 1-4 to their definitions a-d

1) vague a) to make or become worse or lower in quality, value, character, etc

2) endow b) to provide (with qualities, characteristics, etc.)

3) deteriorate c) to give forth or supply (a product, result, etc.), esp. by cultivation, labour, etc.; produce or bear

4) yield d) not clearly perceptible or discernible; indistinct

D Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. orthodox ways of thinking

a) accepted ways of thinking b) Christian ways of thinking

2. subtle

a) requiring mental acuteness or ingenuity b) delicate or faint

3. a deep-seated belief

a) situated far below the surface b) firmly established

4. of its own accord

a) of its own free will b) in accord with

5. differ only very slightly from each other

a) differ substantially b) differ in a small degree

6. bizarre solutions

a) amusing solutions b) unusual and solutions

7. the baffling problem

a) the difficult problem b) the obsolete problem

8. inevitably

a) eventually b) invariably

E Understanding main points

Mark these statements T (true) or F (false) according to the information in the text.

1. Poincare formulated Chaos theory.

2. According to Sharkovskii's Theorem time always runs continuously.

3. Chaos theory can be defined as nonlinear dynamics.

4. Chaotic systems can be predicted like rolls of dice.
5. The horseshoe system has some haphazard features.

F Information search

Look quickly at the text and discuss these questions.

1. What is meant by the Lorenz attractor?
2. Define the concept of butterfly effect?
3. Does Chaos theory have a wide range of applications nowadays?
4. Briefly formulate Sharkovskii's Theorem.
5. When was the term"chaos" introduced?

G Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Chaos theory". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Part 2

Before you read

1. Can you define the concept of 'fractal'?
2. Have you ever heard about Sierpinski triangle? Can you define it?
3. Do you know where the examples of fractals can be found?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. Sierpinski triangle	a) a continuously bending line that has no straight parts
2. fractal	b) also known as the Sierpinski gasket, is a self similar structure that occurs at different levels of iterations, or magnifications
3. curve	c) a figure or surface generated by successive subdivisions of a simpler polygon or polyhedron, according to some iterative process
4. equilateral	d) having all sides of equal length
5. velocity	e) a place where securities are regularly traded
6. stock market	f) speed of motion, action, or operation; rapidity; swiftness
7. rectangle	g) a parallelogram having four right angles

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

boost; six-pointed; bears; intricate; rectangles; irregular; ground; assumption; wrong; denounced; finite; change; fractals; predecessors;

Theoretical monsters

Between 1870 and 1930 a diverse collection of maverick mathematicians invented a series of bizarre shapes the sole purpose of which was to show up the limitations of classical analysis. During the early development of calculus, mathematicians had assumed that any continuously varying quantity must possess a well-defined rate of 1)______ almost everywhere. For example, an object that is moving through space continuously has a well-defined speed, except at relatively few instants when its speed changes abruptly. However, in 1872 Weierstrass showed that this long-standing 2)_________ is wrong. An object can move in a continuous fashion, but in such an 3)_______ manner that – in effect – its speed is changing abruptly at every moment of time. This means that it doesn't actually have a sensible speed at all.

Other contributions to this strange zoo of anomalies included a curve that fills an entire region of space (one was found by Peano in 1890, another by Hilbert on 1891) a curve that crosses itself at every point (discovered by Waclaw Sierpinski in 1915) and a curve of infinite length that encloses a 4)______ area. This last example of geometric weirdness, invented by Helge von Koch in 1906, is the snowflake curve, and its construction goes like this. Begin with an equilateral triangle, and add triangular promontories to the middle of each side to create a 5)_______ star. Then add smaller promontories to the middle of the star's twelve sides, and keep going forever. Because of its sixfold symmetry, the end result looks like an 6)_______ snowflake. Real snowflakes grow by other rules, but that's a different story.

The mainstream of mathematics promptly (7)________ these oddities as 'pathological' and a 'gallery of monsters', but as the years went by several embarrassing fiascos emphasized the need for care, and the mavericks' viewpoint gained (8)_______. The logic behind analysis is so subtle that leaping to plausible conclusions is dangerous: monsters warn us about what can go (9)_____. So, by the turn of the century, mathematicians had become comfortable with the newfangled goods in the mavericks' curiosity shop – they kept the theory straight without having any serious impact on applications. Indeed by 1900 Hilbert could refer to the whole area as a paradise without causing ructions.

In the 1960s, against all expectations, the gallery of theoretical monsters was given an unexpected (10)______ in the direction of applied science. Benoit Mandelbrot realized that these monstrous curves are clues to a far-reaching theory of irregularities of nature. He renamed them (11)_______. Until then, science had been happy to stick to traditional geometric forms like (12)_______ and spheres, but Mandelbrot insisted that this approach was too far restrictive. The natural world is littered with complex and irregular structures – coastlines, mountains, clouds, trees, glaciers, river systems, ocean waves, craters, cauliflowers – about which traditional geometry remains mute. A new geometry of nature is needed.

Today, scientists have absorbed fractals into their normal ways of thinking, just as their (13)___________ did at the end of the 19th century with those maverick mathematical monstrosities. The second half of Lewis Fry Richardson's 1926 paper 'Atmospheric diffusion shown on a distance-neighbour graph' (14)_______ the title ‘Does the wind have a velocity?’ This is now seen as an entirely reasonable question. Atmospheric flow is turbulent, turbulence is fractal and fractals can behave like Weierstrass’s monstrous function – moving continuously but having no well defined speed. Mandlebrot found examples of fractals in many areas both in and outside science – the shape of a tree, the branching pattern of a river, the movements of the stock market.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Match the words and phrases 1-7 to their definitions a-g

1) abruptly a) any of various projecting structures
2) promontories b) a person of independent or unorthodox views
3) maverick c) newly come into existence or fashion, esp. excessively modern
4) newfangled d) suddenly; unexpectedly
5) ructions e) extensive in influence, effective
6) far-reaching f) (plural) a violent and unpleasant row; trouble
7) monstrosity g) an outrageous or ugly person or thing; monster

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text

1. Any quantity possesses continuous speed.
2. It is possible for a finite area to have an infinite perimeter.
3. A Hilbert curve is a continuous space-filling curve.
4. The discovery of a continuous space-filling curve don’t have any applications.
5. Fractals can be found everywhere.

E Discussion point

Work in pairs. Discuss statements 1-5.

Part 3

Before you read

1. Do you know what is meant by cellular automaton?
2. Do you know any applications of cellular automaton?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. cellular automaton
2. blueprint
3. cutting edge
4. cell
5. grid

a) an original prototype that influences subsequent design

b) a collection of cells arranged in a grid, such that each cell changes state as a function of time according to a defined set of rules that includes the states of neighboring cells.

c) the leading position in any field; forefront

d) a network of horizontal and vertical lines superimposed over a map, building plan, etc., for locating points

e) a small unit of volume in a mathematical coordinate system

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words and phrases: replicate; went ignored; deceptive; blueprint; blindly obeys; came to prominence; falls for

Cellular automaton

In one type of new mathematical model, known as a cellular automaton, such things as trees, birds and squirrels are incarnated as tiny coloured squares. They compete with their neighbours in a mathematical computer game. The simplicity is 1)__________– these games lie at the cutting edge of modern science.

Cellular automata 2)______________ in the 1950s, when John von Neumann was trying to understand life's ability to copy itself. Stanislaw Ulam suggested using a system introduced by the computer pioneer Konrad Zuse in the 1940s. Imagine a universe composed of a large grid of squares, called cells, like a giant chessboard. At any moment, a given square can exist in some state. This chessboard universe is equipped with its own laws of nature, describing how each cell's state must change as time clicks on to the next instant. It is useful to represent that state by colours. Then the rules would be statements like: 'If a cell is red and has two blue cells next to it, it must turn yellow.' Any such system is called a cellular automaton – cellular because of the grid, automaton because it 3)___________ whatever rules are listed.

To model the most fundamental feature of living creatures, von Neumann created a configuration of cells that could 4)__________ – make copies of itself. It had 200,000 cells and employed 29 different colours to carry around a coded description of itself. This description could be copied blindly, and used as a 5)_________ for building further configurations of the same kind. Von Neumann did not publish his work until 1966, by which time Crick and Watson had discovered the structure of DNA and it had become clear how life really does perform its replication trick. Cellular automata 6)__________ for another 30 years.

By the 1980s, however, there was a growing interest in systems composed of large numbers of simple parts, which interact to produce a complicated whole. Traditionally, the best way to model a system mathematically is to include as much detail as possible: the closer the model is to the real thing, the better. But this high-detail approach 7)_______ very complex systems. Suppose, for instance, that you want to understand the growth of a population of rabbits. You don't need to model the length of the rabbits' fur, how long their ears are or how their immune systems work. You need only a few basic facts about each rabbit; how old it is, what sex it is, whether it is pregnant. Then you can focus your computer resources on what really matters.

For this kind of systems, cellular automata are very effective. The make it possible to ignore unnecessary detail about the individual components, and instead to focus on how these components interrelate. .

(an extract from the book The story of mathematics by Ian Stewart)

C Understanding main points

Mark these statements T (true) or F (false) according to the information in the text

1. Cellular automaton is a dynamical system which is discrete in space and time.
2. Cellular automaton operates on a uniform, regular lattice.
3. Cellular automaton was described by John von Neumann.
4. In cellular automaton all cells are necessarily identical.
5. Cellular automaton works for complex systems.

D Use of English

Read the text below and think of the word which best fits each space. Use only one word in each space.

Some scientists, especially those with backgrounds in computing, think that it's time we abandoned traditional equations altogether, especially continuum ones like ordinary and partial differential equations. The future is discrete, it comes (1)____ whole numbers, and the equations should give (2)____ to algorithms – recipes (3)_____ calculating things. (4)_______ of solving the equations, we should simulate the world digitally (5)____ running the algorithms. Indeed, the world (6)_____ may be digital. Stephen Wolfram (7)_____ a case for this view in his controversial book A New Kind of Science, which advocates a type of complex system (8)_____ a cellular automaton. This is an array of cells, typically small squares, (9)_____ existing in a variety of distinct states. The cells interact (10)_____ their neighbours according (11)____ fixed rules. They look a bit (13)____ an eighties computer game, with coloured blocks chasing each (12)_____ over the screen.

Wolfram puts (14)_______ several reasons why cellular automata should be superior (15)_____ traditional mathematical equations. In particular, some of them can carry (16)_____ any calculation that could be performed (17)____a computer, the simplest being the famous Rule 110 automaton. This can find successive digits of π, solve the three-body equations numerically, implement the Black–Scholes formula for a call option – whatever. Traditional methods for solving equations are more limited. We (18)_____ not find this argument terribly convincing, because it is also true that any cellular automaton can be simulated by a traditional dynamical system. What counts is not (19)_____ one mathematical system can simulate another, but which is most effective for solving problems or providing insights. It's quicker to sum a traditional series for π by hand (20)_____ it is to calculate the same number of digits using the Rule 110 automaton.

However, it is still entirely credible that we might soon find new laws of nature based on discrete, digital structures and systems. The future may consist of algorithms, not equations. But until that day dawns, if ever, our greatest insights into nature's laws take the form of equations, and we should learn to understand them and appreciate them. Equations have a track record. They really (21)_____ changed the world – and they will change it again.

. *(an extract from the book 17 equations that changed the world by Ian Stewart)*

E Use of English

Read the text below. Use the word given in capitals at the end of some of the lines to form a word that fits in the space in the same line. There is an example at the beginning:

Chaotic dynamics is (0) sensitive to initial conditions. This phenomenon **SENSE**
is often called the butterfly effect: a butterfly flaps its wings, and a
month later the weather is completely different from what it would
otherwise have been. The phrase is generally credited to Lorenz.
He didn't introduce it, but something similar featured in the title
of one of his lectures. However, someone else invented the title
for him, and the lecture wasn't about the (1)______ 1963 article, but **FAME**
a lesser-known one from the same year. Whatever the phenomenon
is called, it has an important practical (2)_____________. **CONSEQUENTLY**
Althoug chaotic dynamics is in principle (3)____________, **DETERMINE**
in practice it becomes (4)_____________very quickly, because **PREDICT**
any (5) _________ in the exact initial state grows exponentially fast. **CERTAIN**
There is a prediction horizon beyond which the future cannot be
foreseen. For weather, a familiar system whose standard computer
models are known to be chaotic, this horizon is a few days ahead.
For the Solar System, it is tens of millions of years ahead. For
simple laboratory toys, such as a double pendulum
(a pendulum hung from the bottom of another one) it is
a few seconds ahead. The long-held (6) __________ that **ASSUME**
'deterministic'and (7)'_________' are the same is wrong. **PREDICT**
It would be (8)_____ if the present state of a system could be **VALIDITY**
measured with perfect (9)_______, but that's not possible. **ACCURATE**
The short-term (10)___________ of chaos can be used to **PREDICT**
distinguish it from pure (11)________. Many different techniques **RANDOM**
have been devised to make this (12)________, and to work out **DISTINCT**
the underlying dynamics if the system is behaving deterministically
but chaotically. . *(an extract from the book The story of mathematics by Ian Stewart)*

F Discussion point

Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean's office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. *The Lornz attractor.*
Course 2. *Cellular automaton.* Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1**.** Web research task. Find out as much as you can about Chaos theory. Web search key words: chaos and the horseshoe, Smale, the butterfly effect, the Lorenz attractor, the Sierpinski gasket, fractals.

2. Prepare a short presentation on the Lorenz attractor.

3. Prepare a short presentation on the applications of nonlinear dynamics.

Glossary

Absolute value: The magnitude of a number. It is the number with the sign (+ or -) removed and is modulus.

Abstract number: A number with no associated units.

Acute angle: An angle with degree measure less than 90.

Addition: The process of finding the sum of two numbers, which are called addend and the augend symbolised using two vertical straight lines (|5|). Also called (sometimes both are called the addend).

Algorithm: Any mathematical procedure or instructions involving a set of steps to solve a problem.

Arctan: The inverse of the trigonometric function **tangent** shown as arctan(x) or $\tan^{-1}(x)$. It is useful in vector conversions and calculations **Arithmetic mean**: $M = (x_1 + x_2 + ... x_n) / n$ (n = sample size).

Arithmetic sequence: A sequence of numbers in which each term (subsequent to the first) is generated by adding a fixed constant to its predecessor.

Associative property: A binary operation (*) is defined associative if, for a*(b*c) = (a*b)*c. For example, the operations addition and multiplication of natural numbers are associative, but subtraction and division are not.

Asymptote: A straight line that a curve approaches but never meets or crosses. The curve is said to meet the asymptote at infinity. In the equation y = 1/x, y becomes infinitely small as x increases but never reaches zero.

Axiom: Any assumption on which a mathematical theory is based.

Average: The sum of several quantities divided by the number of quantities (also called mean).

Avogadro's number: The number of molecules in one mole is called Avogadro's number (approximately 6.022×10^{23} particles/mole).

Binary operation: An operation that is performed on just two elements of a set at a time.

Binomial distribution: A probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions.

Butterfly effect: In a system when a small change results in an unpredictable and disproportionate disturbance, the effect causing this is called a butterfly effect.

Calculus: Branch of mathematics concerned with rates of change, gradients of curves, maximum and minimum values of functions, and the calculation of lengths, areas and volumes. It involves determining areas (integration) and tangents (differentiation), which are mutually inverse. Also called **real analysis**..

Cartesian coordinates: Cartesian coordinates (x,y) specify the position of a point in a plane relative to the horizontal x and the vertical y axes. The x and y axes form the basis of two-dimensional Cartesian coordinate system.

Chaos: Apparent randomness whose origins are entirely deterministic. A state of disorder and irregularity whose evolution in time, though governed by simple exact laws, is highly sensitive to starting conditions: a small variation in these conditions will produce wildly different results, so that long-term behaviour of chaotic systems cannot be predicted. This sensitivity to initial conditions is also known as the **butterfly effect** (when a butterfly flaps its wings in Mexico, the result may be a hurricane in Florida a month later).

Chord: A straight line joining two points on a curve or a circle. See also **secant line**.

Circle: A circle is defined as the set of points at a given distance (or radius) from its centre. If the coordinates of the centre of a circle on a plane is (a,b) and the radius is r, then $(x-a)^2 + (y-b)^2 = r^2$. The equation that characterises a circle has the same coefficients for x^2 and y^2. The area of a circle is $A = \pi r^2$ and circumference is $C = 2\pi r$. A circle with centre (a,b) and radius r has parametric equations: $x = a + r.\cos\pi$ and $y = b + r.\sin\pi$ $(0 \leq \pi \leq 2\pi)$. A ‘tangent’ is a line, which touches a circle at one point (called the point of tangency) only. A ‘normal’ is a line, which goes through the centre of a circle and through the point of tangency (the normal is always perpendicular to the tangent). A straight line can be considered a circle; a circle with infinite radius and centre at infinity. **Circumference**: A line or boundary that forms the perimeter of a circle.

Closure property: If the result of doing an operation on any two elements of a set is always an element of the set, then the set is closed under the operation. For example, the operations addition and multiplication of natural numbers (the set) are closed, but subtraction and division are not.

Coefficient: A number or letter before a variable in an algebraic expression that is used as a multiplier.

Common denominator: A denominator that is common to all the fractions within an equation. The smallest number that is a common multiple of the denominators of two or more fractions is the **lowest (or least) common denominator** (LCM).

Common factor: A whole number that divides exactly into two or more given numbers. The largest common factor for two or more numbers is their **highest common factor** (HCF).

Common logarithm: Logarithm with a base of 10 shown as $\log_{10}$ [$\log_{10}10^x = x$].

Common ratio: In a geometric sequence, any term divided by the previous one gives the same common ratio.

Commutative property: A binary operation (*) defined on a set has the commutative property if for every two elements, a and b, a*b = b*a. For example, the operations addition and multiplication of natural numbers are commutative, but subtraction and division are not.

Complementary angles: Two angles whose sum is 90^o. See also **supplementary angles**.

Complex numbers: A combination of real and imaginary numbers of the form $a + bi$ where a and b are real numbers and i is the square root of -1 (see **imaginary number**). While real numbers can be represented as points on a line, complex numbers can only be located on a plane.

Composite number: Any integer which is not a prime number, i.e., evenly divisible by numbers other than 1 and itself.

Congruent: Alike in all relevant respects.

Constant: A quality of a measurement that never changes in magnitude.

Coordinate: A set of numbers that locates the position of a point usually represented by *(x,y)* values.

Cosine law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: $a^2 = b^2 + c^2 - 2bc \cos A$ etc. The law of cosines is useful to determine the unknown data of a triangle if two sides and an angle are known.

Counting number: An element of the set C = {1,2,3,...}.

Cube root: The factor of a number that, when it is cubed (i.e., x^3) gives that number.

Curve: A line that is continuously bent.

Decimal: A fraction having a power of ten as denominator, such as 0.34 = 34/100 (10^2) or 0.344 = 344/1000 (10^3). In the continent, a comma is used as the decimal point (between the unit figure and the numerator).

Degree of an angle: A unit of angle equal to one ninetieth of a right angle. Each degree (^{0}) may be further subdivided into 60 parts, called minutes *(60')*, and in turn each minute may be subdivided into another 60 parts, called *seconds* (60''). Different types of angles are called acute ($<90^0$)< right (90^0) < obtuse (90^0-180^0) < reflex (180^0-360^0). See also **radian** (the SI unit of angle).

Denominator: The bottom number in a fraction.

Derivative: The derivative at a point on a curve is the gradient of the tangent to the curve at the given point. More technically, a function ($f'(x_0)$) of a function $y = f(x)$, representing the rate of change of *y* and the gradient of the graph at the point where $x = x_0$, usually shown as *dy/dx*. The notation *dy*/*dx* suggests the ratio of two numbers *dy* and *dx* (denoting infinitesimal changes in *y* and *x*), but it is a single number, the limit of a ratio (k/h) as they both approach zero. Differentiation is the process of calculating derivatives. The derivatives of all commonly occurring functions are known.

Differential Equations: Equations containing one or more derivatives (rate of change). As such these equations represent the relationships between the rates of change of continuously varying quantities. The solution contains constant terms (constant of integration) that are not present in the original differential equation. Two general types of differential equations are ordinary differential equations (ODE) and partial differential equations (PDE). When the function involved in the equation depends upon only a single variable, the differential equation is an ODE. If the function depends on several independent variables (so that its derivatives are partial derivatives) then the differential equation is a PDE. **Diameter**: A straight line that passes from side to side thorough the centre of a circle.

Differential calculus: Differentiation is concerned with rates of change and calculating the gradient at any point from the equation of the curve, $y = f(x)$.

Differential equation: Equations involving total or partial differentiation coefficients and the rate of change; the difference between some quantity now and its value an instant into the future.

Digit: In the decimal system, the numbers 0 through 9.

Dimension: Either the length and/or width of a flat surface (two-dimensional); or the length, width, and/or height of a solid (three-dimensional).

Distributive property: A binary operation (*) is distributive over another binary operation (^) if, a*(b^c) = (a*b)^(a*c). For example, the operation of multiplication is distributive over the operations of addition and subtraction in the set of natural numbers.

Division: The operation of ascertaining how many times one number, the *divisor*, is contained in another, the *dividend*. The result is the *quotient*, and any number left over is called the *remainder*. The dividend and divisor are also called the *numerator* and *denominator*, respectively.

Dynamics: The branch of mathematics, which studies the way in which force produces motion.

e: Symbol for the base of natural logarithms (2.7182818285...), defined as the limiting value of $(1 + 1/m)^m$.

Equilibrium: The state of balance between opposing forces or effects.

Even number: A natural number that is divisible by two.

Exponent (**power**, **index**): A number denoted by a small numeral placed above and to the right of a numerical quantity, which indicates the number of times that quantity is multiplied by itself. In the case of X^n, it is said that X is raised to the power of n. When a and b are non-zero real numbers and p and q are integers, the following rules of power apply:

$a^p \times a^q = a^{p+q}$; $(a^p)^q = a^{pq}$; $(a^{1/n})^m = a^{m/n}$; $a^{1/2} \times b^{1/2} = (ab)^{1/2}$.

Exponential function: A function in the form of $f(x) = a^x$ where x is a real number, and a is positive and not 1. One exponential function is $f(x) = e^x$.

Extrapolation: Estimating the value of a function or a quantity outside a known range of values. See also **interpolation**.

Factorial: The product of a series of consecutive positive integers from 1 to a given number (n). It is expressed with the symbol (!). For example, 5! = 5x4x3x2x1 = 120. As a rule (n!+n) is evenly divisible by n.

Factor: When two or more natural numbers are multiplied, each of the numbers is a factor of the product. A factor is then a number by which another number is exactly divided (a divisor).

Factorisation: Writing a number as the product of its factors which are prime numbers.

Fermat's little theorem: If p is a prime number and b is any whole number, then b^p-b is a multiple of p (2^3 - 2 = 6 and is divisible by 3).

Fermat prime: Any prime number in the form of 2^{2n} + 1 (see also **Mersenne prime**).

Fibonacci sequence: Sequence of integers, where each is the sum of the two preceding it. 1,1,2,3,5,8,13,21,... The number of petals of flowers forms a Fibonacci series.

Fractals: Geometrical entities characterised by basic patterns that are repeated at ever decreasing sizes. They are relevant to any system involving self-similarity repeated on diminished scales (such as a fern's structure) as in the study of chaos.

Fraction (quotient): A portion of a whole amount. The term usually applies only to ratios of integers (like 2/3, 5/7). Fractions less than one are called common, proper or vulgar fractions; and those greater than 1 are called improper fraction.

Function (*f*): The mathematical operation that transforms a piece of data into a different one. For example, f(x) = x^2 is a function transforming any number to its square.

Geometric mean: G = $(x_1.x_2...x_n)^{1/n}$ where n is the sample size. This can also be expressed as antilog ((1/n) S log x)..

Geometric sequence: A sequence of numbers in which each term subsequent to the first is generated by multiplying its predecessor by a fixed constant (the **common ratio**).

Goldbach conjecture: Every even number greater than 4 is the sum of two odd primes (32 = 13 + 19). Every odd number greater than 7 can be expressed as the sum of three odd prime numbers (11 = 3 + 3 + 5).

Gradient: The slope of a line. The gradient of two points on a line is calculated as rise (vertical increase) divided by run (horizontal increase), therefore, the gradient of a line is equal to the tangent of the angle it makes with the positive x-axis (y/x).

Harmonic mean: Of a set of numbers (y_1 to y_i), the harmonic mean is the reciprocal of the arithmetic mean of the reciprocal of the numbers [H = N /∑(1/y)].

Hierarchy of operations: In an equation with multiple operators, operations proceed in the following order: (brackets), exponentiation, division/multiplication, subtraction/summation and from left to right.

Highest common factor (HCF): The greatest natural number, which is a factor of two or more given numbers.

Hypotenuse: The longest side of a right triangle, which lies opposite the vertex of the right angle.

i: The square root of -1 (an **imaginary number**).

Identity element: The element of a set which when combined with any element of the same set leaves the other element unchanged (like zero in addition and subtraction, and 1 in multiplication or division).

Imaginary number: The product of a real number x and i, where $i^2 + 1 = 0$. A complex number in which the real part is zero. In general, imaginary numbers are the square roots of negative numbers.

Improper fraction: A fraction whose numerator is the same as or larger than the denominator; i.e., a fraction equal to or greater than 1.

Infinite: Having no end or limits. Larger than any quantified concept. For many purposes it may be considered as the reciprocal of zero and shown as an 8 lying on its side (∞).

Infinitesimal: A vanishingly small part of a quantity. It equals almost zero.

Integer: Any whole number: positive and negative whole numbers and zero.

Integral calculus: This is the inverse process to differentiation; i.e., a function which has a given derived function. For example, x^2 has derivative 2x, so 2x has x^2 as an integral. A classic application of integral is to calculate areas.

Integration: The process of finding a function given its derived function.

Intersection: The intersection of two sets is the set of elements that are in both sets.

Intercept: A part of a line/plane cut off by another line/plane.

Interpolation: Estimating the value of a function or a quantity from known values on either side of it.

Inverse function: A function which 'does the reverse' of a given function. For example, functions with the prefix arc are inverse trigonometric functions; e.g. arcsin x for the inverse of sin(x).

Irrational number: A real number that cannot be expressed as the ratio of two integers, and therefore that cannot be written as a decimal that either terminates or repeats. The square root of 2 is an example because if it is expressed as a ratio, it never gives 2 when multiplied by itself. The numbers π = 3.141592645..., and ***e*** = 2.7182818... are also irrational numbers. **Iteration**: Repeatedly performing the same sequence of steps. Simply, solving an algebraic equation with an arbitrary value for the unknown and using the result to solve it again, and again.

Least squares method: A method of fitting a straight line or curve based one minimisation of the sum of squared differences (residuals) between the predicted and the observed points. Given the data points (x_i, y_i), it is possible to fit a straight line using a formula, which gives the y=a+bx. The gradient of the straight line b is given by $[\sum (x_i - m_x)(y_i - m_y)] / [(\sum (x - m_x))^2]$, where m_x and m_y are the means for x_i and y_i. The intercept a is obtained by $m_y - bm_x$.

Linear: A model or function where the input and output are proportional.

Linear expression: A polynomial expression with the degree of polynomial being 1, i.e., that does not include any terms as the power of a variable. It will be something like, $f(x)=2x^1+3$, but not x^2+2x+4 (the latter is a **quadratic expression**). Linear equations are closely related to a straight line.

Literal numbers: Letters representing numbers (as in algebraic equations).

Logarithm: The logarithm of a number N to a given base b is the power to which the base must be raised to produce the number N. Written as $\log_b N$. Naturally, $\log_b b^x = x$. In any base, the following **rules** apply: log (ab) = log a + log b; log (a/b) = log a - log b; log (1/a) = -log a; log a^b = b log a; log 1 = 0 and log 0 is undefined. S

Logistic model (map, sequence)

Lowest common multiple (LCM): The smallest non-zero natural number that is a common multiple of two or more natural numbers (compare with the **highest common factor**).

Matrix: A matrix (plural: matrices) is a rectangular table of data. **Mechanics**: Study of the forces acting on bodies, whether moving (dynamics) or stationary (statics).

Mean: 1. The expected value of a random variable 2. The arithmetic mean is the average of a set of numbers, or the sum of the values divided by the number of values

Median: The middle number or average of the two middle numbers in an ordered sequence of numbers

Mersenne prime: A Mersenne number, M_p, has the form 2^p-1, where p is a prime. If M_p itself a prime, then it is called a Mersenne prime. There are 32 such primes known (i.e., not all primes yield a Mersenne prime). **Mixed number**: A number that contains both a whole number and a fraction.

Mode: The most frequent value.

Modulus: The absolute value of a number regardless of its sign, shown as | *x* | or mod *x*. For a vector ***u***, the modulus | ***u*** | is used to indicate its magnitude calculated using **Pythagoras' theorem**: | ***u*** | = $(a^2 + b^2)^{1/2}$.

Multiplication: The process of finding the *product* of two quantities that are called the *multiplicand* and the *multiplier*.

Natural logarithm: Logarithm with a base of *e*, usually abbreviated *ln* (*ln* $e^x = x$).

Natural number: Any element of the set N = {0,1,2,3,...} (positive integers). The inclusion of zero is a matter of definition.

Normal distribution:

Numerator: The top number in a fraction.

Obtuse angle: An angle with a degree measure between 90 and 180

Odd number: A natural number that is not divisible by 2.

Odds: The odds of a success is defined to be the ratio of the probability of a success to the probability of a failure (p/(1-p)).

Ordinate: The vertical coordinate on a plane.

Origin: The point on a graph that represents the point where the *x* and *y* axes meet: (*x*,*y*) = (0,0).

Parallel: Lines or planes that are equidistant from each other and do not intersect.

Perfect number: A number which is equal to the sum of its proper divisors. 6, 28, and 496 are the three of seven known perfect numbers. [6 is a perfect number because its proper divisors (1,2, and 3) total 6.]

Permutation: A permutation of a sequence of objects is just a rearrangement of them.

Perpendicular: At right angles to a line or plane.

Pi(π): The ratio of the circumference of a circle to its diameter. The value of π is 3.1415926, correct to seven decimal places. The sum of the three angles of a triangle is π radians.

Poisson distribution: The probability distribution of the number of occurrences of random (usually rare and independent) events in an interval or time or space.

Polar equation: A system which describes a point in the plane not by its Cartesian coordinates (x,y) but by its polar coordinates: angular direction (□) and distance r from the origin (r, □).

Polygon: A geometric figure that is bound by many straight lines such as triangle, square, pentagon, hexagon, heptagon, octagon etc.

Polynomial: An algebraic expression of the form $a_0x^n + a_1x^{n-1} + ... + a_n$, where a_0, a_1, ..., a_n are members of a field (or ring), and n is the degree of the polynomial.

Prime factors: Prime factors of a number are a list of prime numbers the product of which is the number concerned. When n=1, for example, $f(x)=2x^1+3$, this is a linear expression. If n=2, it is quadratic (for example, $x^2 + 2x + 4$); if n=3, it is cubic, if n=4, it is quartic and if n=5, it is quintic.

Prime number: A natural number other than 1, evenly divisible only by 1 and itself. The numbers 2,3,5,7,11,13,17,19,... Apart from 2, all primes are odd numbers and odd primes fall into two groups: those that are one less than a multiple of four (3,7,11,19) and those one more than a multiple of four (5,13,17). Every natural number greater than 1 may be resolved into a product of prime numbers; eg $8316 = 2^2 \times 3^3 \times 7 \times 11$.

Probability density function: A function representing the relative distribution of frequency of a continuous random variable from which parameters such as its mean and variance can be derived and having the property that its integral from a to b is the probability that the variable lies in this interval. Its graph is the limiting case of a histogram as the amount of data increases and the class intervals decrease in size.

Product: The result of a multiplication problem.

Proper divisor: Any number divides another without leaving a remainder.

Proper fraction: A fraction in which the numerator is smaller than the denominator; i.e., a fraction smaller than 1.

Proportion: A type of ratio in which the numerator is included in the denominator. It is the ratio of a part to the whole ($0.0 \leq \mathbf{p} \leq 1.0$) that may be expressed as a decimal fraction (0.2), vulgar fraction (1/5) or percentage (20%).

Pythagoras' Theorem: For any right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides.

Quadratic equation: An algebraic equation of the second degree (having one or more variables raised to the second power). The general quadratic equation is $ax^2 + bx + c = 0$, in which a, b, and c are constants (or parameters) and 'a' is not equal to 0.

Quotient (fraction): An algebraic expression in which the numerator is divided by the denominator. Turning a fraction upside down gives the fraction's **reciprocal**.

Radian (rad): The SI unit for measuring an angle formally defined as 'the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle' (the angle of an entire circle is 2π radians; radians equal 180^0 (sum of the three angles of a triangle); this is the basis of circumference of a circle formula $2\pi r$). Sum of angles of a triangle equals π radians.

Radius: The distance between the centre of a circle and any point on the circle's circumference.

Random variable: A quantity that may take any of a range of values, either continuous or discrete, which cannot be predicted with certainty but only described probabilistically.

Rate: The relationship between two measurements of different units such as change in distance with respect to time (miles per hour).

Ratio: The relationship between two numbers or measurements, usually with the same units like the ratio of the width of an object to its length. The ratio a:b is equivalent to the quotient a/b.

Rational number: A number that can be expressed as the ratio of two integers, e.g. 6/7. The set of rational numbers is denotes as '**Q**' for quotient.

Real number: **Rational** (fractions) and **irrational** (numbers with non-recurring decimal representation) numbers. The set of real numbers is denoted as '**R**' for real. In computing, any number with a fractional (or decimal) part. Basically, real numbers are all numbers except imaginary numbers (such as the square root of -1).

Reciprocal: The multiplicative inverse of a number (i.e., 1/x). It can be shown with a negative index (x^{-1}).

Reflex angle: An angle with a degree measure between 180 and 360.

Repeating decimal: A decimal that can be written using a horizontal bar to show the repeating digits.

Right angle: An angle with a degree measure 90. An angle which is not an right angle is called oblique angle.

Root: If, when a number is raised to the power of n gives the answer a, then this number is the n^{th} root of a ($a^{1/n}$).

Rounding: To give a close approximation of a number by dropping the least significant numbers. For example 15.88 can be rounded up to 15.9 (or 16) and 15.12 can be rounded down to 15.1 (or 15).

Scalar: A real number and also a quantity that has magnitude but no direction, such as mass and density.

Scientific notation (exponential notation, standard form): One way of writing very small or very large numbers. In this notation, numbers are shown as $(0<N<10)$ x 10^q. An equivalent form is N.Eq. For example; 365,000 is 3.65×10^5 or 3.65E5.

Secant line: A line that intersects a curve. The **intercept** is a **chord** of the curve.

Sequence: An ordered set of numbers derived according to a rule, each member being determined either directly or from the preceding terms.

Sigma (S, s): Represents **s**ummation (□, □).

Significant figure (s.f.): The specific degree of accuracy denoted by the number of digits used. For example 434.64 has five s.f. but at 3 s.f. accuracy it would be shown as '435 (to 3 s.f.)'. From the left, the first nonzero digit in a number is the first significant figure, after the first significant number, all digits, including zeros, count as significant numbers (Both 0.3 and 0.0003 have 1 s.f.; both 0.0303 and 0.303000 have 3 s.f.). If a number has to be reduced to a lower s.f., the usual rounding rules apply (2045.678 becomes 2046 to 4 s.f. and 2045.7 to 5 s.f.). The final zero even in a whole number is not a s.f. as it only shows the order of magnitude of the number (2343.2 is shown as 2340 to 3 s.f.).

Sine law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: sin A / a = sin B / b = sin C / c. The law of sines is useful for computing the lengths of the unknown sides in a triangle if two angles and one side are known.

Skew lines: Two lines in three-dimensional space, which do not lie in the same plane (and do not intersect).

Skewness: A measure of the symmetry of a distribution around its mean, esp. the statistic $B_1 = m_3/(m_2)$^2/3 where m_2 and m_3 are respectively the second and third moments of the distribution around the mean. In a normal distribution,

$B_1 = 0$.

Standard deviation: A measure of dispersion obtained by extracting the square root of the mean of the squared deviations of the observed values from their mean in a frequency distribution.

Stationary point: Point at which the derivative of a function is zero. Includes maximum and minimum turning points, but not all stationary points are turning points.

Straight line: A straight line is characterised by an equation (y = a + bx), where a is the intercept and b is the gradient/slope. One of the methods for fitting a straight line is the **least squares method**.

Subtend: To lie opposite and mark out the limits of an angle.

Subtraction: The inverse operation of addition. In the notation a - b = c, the terms a, b, and c are called the *minuend*, *subtrahend* and *difference*, respectively.

Supplementary angles: Two angles whose sum is 180^o. See also **complementary angles**.

Tangent: The tangent of an angle in a right-angled triangle is the ratio of the lengths of the side opposite to the side adjacent [tan(x) = sin(x) / cos(x)]. A tangent line is a line, which touches a given curve at a single point. The slope of a tangent line can be approximated by a secant line.

Tangent law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: (a+b) / (a-b) = {tan[1/2(A+B)]} / {tan[1/2(A-B)]}.

Transcendental number: A real number that does not satisfy any algebraic equation with integral coefficients, such as $x^3 - 5x + 11 = 0$. All transcendental numbers are irrational and most irrational numbers (non-repeating, non-terminating decimals) are transcendental. Transcendental functions (such as exponential, sine and cosine functions) can burst into chaos under certain circumstances.

Triangle: A three-sided figure that can take several shapes. The three inside angles add up to 180^o. Triangles are divided into three basic types: obtuse, right and acute; they are also named by the characteristics of their sides: equilateral, isosceles, and scalene. The area of a triangle is 1/2 x perpendicular height x base.

Trigonometry: The branch of mathematics that is concerned with the trigonometric functions. Trigonometric identities are the results that hold true for all angles. Sin, Cos and Tan are trigonometric ratios; Cosec, Sec and Cot are reciprocal of trigonometric ratios; Arcsin ($\sin^{-1}$), Arccos ($\cos^{-1}$) and Arctan ($\tan^{-1}$) are inverse of trigonometric functions.

Union: The union of two sets is the set of elements that are in either of the two sets (compare with intersection).

Unit: A standard measurement.

Variable: An amount whose value can change.

Variance: A measure of dispersion obtained by taking the mean of the squared deviations of the observed values from their mean in a frequency distribution.

Vector: A quantity characterised by a magnitude and a direction represented by (1) column form: two numbers (components) in a 2x1 matrix; (2) geometric form: by arrows in the (x,y)-plane; or (3) component form: the Cartesian unit vectors $\boldsymbol{i}$ (x-axis unit vector) and $\boldsymbol{j}$ (y-axis unit vector). The magnitude of a vector $|\, \boldsymbol{u} \,|$ is the length of the corresponding arrow and the direction is the angle (θ) the vector makes with the positive x-axis. When a_1 and a_2 are the components of the vector $\boldsymbol{a}$ (magnitude $|\, \boldsymbol{a} \,| = (a_1^2 + a_2^2)^{1/2}$), it equals to $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j}$ in component form, which equals to $\boldsymbol{a} = |\, \boldsymbol{a} \,| \cos(\theta)\boldsymbol{i} + |\, \boldsymbol{a} \,| \sin(\theta)\boldsymbol{j}$. The angle ($\theta$) can be found as arctan (a_2 / a_1). **Cosine rule** and **sine rule** are used for conversion of vectors from one form to another.

Vertex: The point where lines intersect.

Whole number: Zero or any positive number with no fractional parts.

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