

Applied Metrology for Manufacturing Engineering

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Ammar Grous

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Chapter 1

Fundamentals of Error Analysis and their Uncertainties in Dimensional Metrology Applied to Science and Technology

1.1. Introduction to uncertainties in dimensional metrology

In the field of applied science, measurements are not accurate as they are always subject to errors due to various causes, both human and material. Qualifying an error to later quantify an uncertainty proves that the validity of the measurement result is *doubted*. Therefore, evaluating uncertainties of measurements generating errors is quite a complex task. To “buoy” the influencing factors on which the type of measurement depends, first we develop the mathematical principles relevant to this domain [GUI 00, 04, MUL 81, NIS 94, TAY 05].

On reading the *International Vocabulary of Metrology* (VIM) and the *Guide to the Expression of Uncertainty in Measurement* (GUM) [NIS 94, VIM 93] concerning several specific areas of metrology (see ISO 1087-1, 2000, §3.7.2), we note that the definition given for “error” and “uncertainty” is poorly understood and even truncated. For example, in the VIM from 2004 to 2006, there was no fundamental difference in the basic principles of measurements, whether they were carried out in physics or engineering. As the uncertainty in the measurement increases from *classical* or *true value approach* (forevermore unknown) toward uncertainty approach, it leads to the reconsideration of the measurement concepts. We know that both the instruments and the measurements do not provide “this” *true value*. Therefore, it is possible to differentiate two categories of errors. They should be considered differently in terms of *propagation of errors*. However, as no justified rule underlies the combination between systematic and random errors, it results in

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total error, which characterizes the measurement result. The estimated upper limit of the total error is named as uncertainty.

The components of measurement uncertainty are conventionally [VIM 93] grouped into two categories. The first one, *type A*, is estimated using statistical methods, and the second, *type B*, is estimated using other methods. It is *a priori* based on laws. In fact, the person operating is responsible for assessing the sources of errors. Although the manufacturers provide data such as the *class of the device*, the *standard*, and the *resolution*, we should have sound knowledge based on experience. Combination of both categories A and B gives the compound uncertainty $U_c(y)$.

GUM [GUM 93], corrected in 1995, provides a definition for the type B approach of uncertainty. It emphasizes mathematical processing of uncertainty using an explicit measurement model where the measurand is characterized by a unique value. The objective of uncertainty approach in measurement is not to determine the true value but to evaluate the errors. There are several types of measurement errors, such as parallax error, setting zero reference of the device, technique errors, errors in reading the instrument, and even human errors due to various effects such as temperature, dilation, and relative humidity. Therefore, it is difficult to define uncertainty solely based on the standard deviation. We should also consider the parameters given by the manufacturer (Mitutoyo in our study). Moreover, even the most refined measurement cannot reduce the interval to a single value due to the inherently finite amount of information defining the measurand: it is then agreed, in the VIM, that a definitional uncertainty imposes a limitation lower than any measurement uncertainty. The interval is then represented by a *measured value*, which results from the instrumental manipulations.

The VIM third edition of 2008 provides more concise definitions of the terminologies used in metrology. In other fields of engineering, the work is based on reliability indices [GRO 94, GRO 95]. To quantify the probability of assembled structures failure, the Monte Carlo simulation approach plays an important role in metrology. It is one of the reasons we completed this book dedicated to dimensional metrology using a dimensioning approach based on a cross-welded structure.

Similar to the VIM, in the GUM [GUI 00, NIS 94] the objective of measurements is to establish the probability that certain measured values are consistent with the definition of the measurand. The reader will easily notice that the terminology is rather less common in experimental sciences. Nevertheless, *measurement*, *measuring*, *measurand*, *true value* and so on are terms that should not be used inappropriately. The terms given in the VIM third edition, and their formats, are consistent with the frame rules for terminology outlined in international standards ISO 704, ISO 1087-1, and ISO 10241. For further information, the reader can refer to them. The word *mesurage* has been used to describe the act of measurement. The word *mesure* occurs in *various occasions* in the VIM. Other terms include *appareil de mesure*, *unité de mesure*, and *méthode de mesure*.

(respectively, measuring instrument, unit of measurement, and measurement method in English). In general, the usage of French word *mesurage* for *measure* is not permissible.

In addition, the quantity of influence is not subjected to measurement, even when it affects the measurement result (e.g. the temperature of a micrometer). Quantities of influence, or factors or sources of uncertainties, are generally categorized into three types:

- *Human*: handling, maintenance of the test facility, and so on;
- *Technical*: method of testing, properties of test materials, calibration, and so on;
- *Environmental*: test environment, random components, and so on.

In metrology, the measurement is an experimental process aiming to determine the value of a physical quantity, which can be achieved using the *measuring method*. This requires the use of apparatus and measuring instruments, which in many cases prove to be a source of errors. Thus, it makes clear that metrology is mainly based on the concepts of *uncertainty* [MUL 81, PRI 96, TAY 05] and *value*. The uncertainty reflects the way a quantity is measured and the confidence given to a result. The usage of instruments in measurement involves calibrations and manipulations, thereby requiring appropriate procedures and calculations. For these reasons, many systems of calculations and measurements have been introduced. Among the retained systems, we discuss international and Canadian standards. Many countries retain their own standards while using the SI units. This is the case in Canada and the United States. However, the implementation of the SI units is wider in Canada than in the United States. Hence, Canada resorted to the SI units or US standards, rather than Canadian Standards Association (CSA); see the tools of the *American National Standard Drafting Manual* operated by CAD software. In 1960, the General Conference on Weights and Measures, a leading authority on this matter, adopted the SI units. On 16 January 1970, Canada, like the United States and the United Kingdom, decided to convert to SI units.

In Quebec, the system has the comma as the decimal separator, whereas in Canada and the United States the decimal point is still used, refer to CAN3-Z234.1-76 (CSA) of February 1977. For welding standards, Canada has its own codification, reference CSA W47.1-1973. In disciplines dealing with machine production, we associate confidence interval, tolerance, or uncertainty with a nominal dimension. The readers are aware of the quality of the measurement. Obviously metrology standards are generally used by the manufacturer of the apparatus and measuring instruments. We should, therefore, adequately comply with them. In Canada, the National Calibration System, aiming at ensuring the traceability of the reference and measurement instruments to national standards, is based on officially accredited laboratories by the calibration section of the CSA (in French, ANOCR). According to the VIM calibration [VIM 93], it is “the property of the result of a measurement

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or a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties”.

The *verification* consists of confirming by examination and establishing the evidence that specified requirements have been met. Based on the study of the norm by the participants, the following recommendations arise:

- No adjustment shall be made on the meter during the inspection. If an adjustment is made, it must be accompanied by a pre-registration audit/verification result and a record of it after adjustment.
- The verification certificate may contain the measurement results (not compulsory).
- A written record of the verification results must be kept separately in the relevant file of the measuring device.

1.2. Definition of standards

The concept of traceability includes *calibration* and *verification*. Sometimes, there is confusion between the two terms, and they do not cover the same concept. Verification is usually performed in practice. It is agreed that the choice of means of traceability is tricky because of the significant costs incurred. We do not discuss this issue in our context. However, we emphasize, in concordance with the VIM, the definitions of four types of standards, namely, *primary standard*, *reference standard*, *transfer standard*, and *working standard* [ACN 84, FRI 78, MUL 81, TAY 05, VIM 93].

According to ISO, *standard* is defined as the “measure materialized by a meter or measuring system intended to define, realize, conserve or reproduce a unit or many known values of a quantity for the future compared with other measuring instruments.”

Reference standards should never be used as working standards. As a computer is involved in the management of standards, it is clear that the identification of instruments in business is unique to each and rarely corresponds to the serial number of the instrument.

Note that the “service history form” corresponds to an instrument and not a landmark. If the instrument designated by the number $r_1r_2r_3$, which corresponds to the reference R , is changed, the life form follows the number $r_1r_2r_3$ but not the instrument number $r_1r_2r_i$, which will be substituted by the marker R . The system of labeling provided by the quality department indicates the date of next calibration to

allow the programming of calibrations. The identification of non-compliant instruments is carried out by labeling. Programming, if any, also enables us to plan future investments. The traceability documents are in fact the form of non-compliance sheet (internal to laboratory), and the relevant *non-assurance quality* sheet justifies the relationship between the company and the supplier. As we need to be very conscious of the vocabulary used in metrology, definitions from the VIM are proposed as follows:

- *Primary standard* is a “standard that is designated or widely acknowledged as having the highest metrological qualities and whose value is determined without reference to other standards of the same magnitude.”
- *Reference standard* is a “standard, usually having the highest metrological quality available at a given location or in a given organization, based on which measures made therein are derived.”
- *Transfer standard* is a “standard that is routinely used as an intermediary to compare standards between each other.”
- *Working standard* is a “standard that is used routinely to calibrate or verify material measures (materialized), measuring equipment or materials reference.”

The term “device” should also be used wisely. The transfer device should be used when the intermediary is not a standard.

Note that the *working standard* is usually calibrated against the *reference standard*. A working standard is used routinely to ensure that measures are implemented correctly and also known as control standard. Students use this standard during their experiments in metrology laboratories. The chain structures [ACN 84, GUI 04] slightly differ from one laboratory to another depending on the available material resources. The basic structure of a measuring system (Figure 1.1) is generally found in all measurement chains regardless of their complexity and nature, which includes at least three levels. Sensors deliver an electrical signal, which offers huge opportunities. Almost all measuring systems are electronic chains.

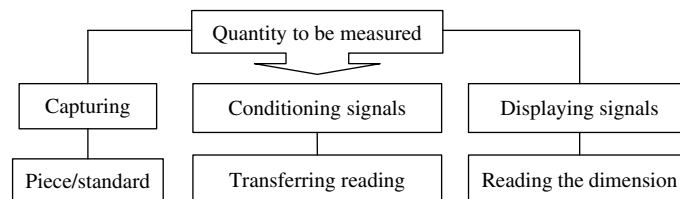


Figure 1.1. Chart illustrating the quantities to be measured “at minimum”

In metrology, inappropriate usage of a term would result in distortion in the measurement and even in the interpretation of its result. Thus, the French term *mesurage* defines the set of operations carried out to determine the value of a quantity. The particular quantity that is to be measured is called the *measurand*. The valuation of a given quantity in comparison with another similar one taken as unity gives rise to the measure X , for example, 3/4 in. The *quantity value* X is a parameter that must be controlled during the development of a product or its transfer.

The physical quantity measurement is carried out by comparing with the earlier set standard. Comparison may be difficult or even impossible due to practical reasons. The measurement may be done directly or indirectly based on the measurable quantity. We know that any measure of a physical quantity is always flawed with errors. Errors are inevitable due to the nature of methods and procedures used in the experiments.

Beware of linguistic “faux amis”: in French, “*étalonnage*” is not “*calibrage*” (not to be confused with “calibration” in English).

First, it should be noted that the calibration of an instrument is not sufficient. The calibration of a micrometer, for example, is only a statement at time τ , under certain conditions, deviation between the indications of the device and a reference standard. The calibration certificate of an instrument [CAT 00] provides the deviation, and uncertainty on this deviation is called *calibration uncertainty*. It is incumbent on the user to take into account the calculation parameters of *measurement uncertainty*, which includes:

- uncertainty about calibration carried out during traceability;
- uncertainty due to the accuracy of the device if uncorrected;
- uncertainty related to drift (fatigue) of the instrument between two calibrations;
- uncertainty linked to the instrument’s characteristics (reading, repeatability, and so on);
- uncertainty linked to the environment, if the conditions are different during calibration.

Based on these, we may conclude that calibration is a *process of comparing* an “unknown” element (a measure obtained) with an equivalent or better standard. A standard measure is considered as a reference. Calibration may include an adjustment to correct the deviation of the obtained value from the standard. This is represented by the standard deviation. In sum, calibration is used for various reasons, for example:

- plan and exchange confidently;
- optimizing resources to be competitive;

– ensuring the compatibility of measurements in different locations at different times but under the same conditions, thus justifying the adequacy of repeatability.

According to the VIM, standards are defined as “a static materialized measure, measuring device or measuring system intended to define, realize, conserve or reproduce a unit or more known values of a quantity (magnitude) in order to forward them (transfer) compared to other measurement instruments.”

In analogy with earlier definitions, the four major standards are defined as follows:

– *International standard* is recognized by international agreement to serve internationally as the basis for fixing the values of all other standards of the quantity concerned.

– *National standard* is recognized by an official national decision to serve in a country as the basis for fixing the values of all other standards of the quantity concerned.

– *Primary standard* is designed as having the highest metrological qualities in a specified domain. So it is a standard with the highest precision order used to calibrate a standard of lower level.

– *Secondary standard* is a designated measurement element, which compares the base value of a primary standard with another test element.

In Canada, the frequency of calibration is done within an interval defined for each calibration according to the norm CAN3-85-Z299.1. The period is different for each calibration (3 months or 2 years). It can be expressed as the calibration cycle required per year. The equipment used in metrology has a proven accuracy as it is provided by the manufacturer. Depending on the environmental conditions, the usage, and the application, the desired accuracy must be established for a specific period of time. Figure 1.2 is a simple illustration for $\tau = (1.5-12)$ and a calibration reliability at 50% = $\lambda = 1/2$:

$$f(\tau) = \lambda \times e^{-\lambda \times \tau} \quad [1.1]$$

Traceability is the property of the measurement result or the value of the standard whereby it can be linked to specified references (usually of national or international standards) through an unbroken chain of comparisons all having stated uncertainties.

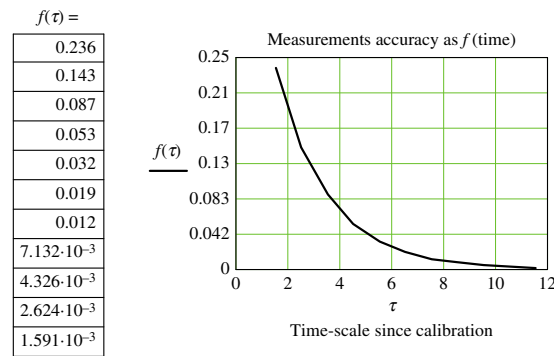


Figure 1.2. Illustration of the measurement accuracy over time

Traceability is supplemented with a succinct document related to metrology to translate the examination of each event of the procedure and its means. Furthermore, the traceability requires ordered and permanent records. This allows the user to know the history of a process or an instrument. Traceability helps know the drift or changes in equipment, thus facilitating the management of a multitude of aspects, such as:

- varied use and appropriate adjustment of equipment in the workplace;
- selection of a piece of equipment among others offered by different suppliers;
- detection of higher or lower precision (based on records).

The term traceability is often inappropriately used by the journalistic world without being accurately defined as in the VIM. Thus, it is poorly understood and sometimes wrong.

Good traceability is required for good analysis defining the periodicity of the traceability, classifying, archiving, and writing a procedure describing the details of the instrument's life form and keep it updated. The *service history form* of an instrument is equivalent to an *individual health book* and should be maintained during the instrument's life cycle. This form is obtained while purchasing the instrument and archived *unlined without ambiguity* even when the instrument becomes out of order.

Again, we recall that calibration is a set of operations that determine, under specified conditions, the relationship between the values indicated by an instrument or a measuring system and the corresponding known values of the measured quantity. It establishes the relationship between the output quantity value and the applied one. Its results are documented in a calibration certificate (report) [CAL 05].

Calibration certificate does not provide information on the satisfaction of the measuring device requirements. It records only the inherent information at that time. The calibration chain proves that calibrating a measuring device requires choosing the options that consider costs and uncertainties. The frequency of calibration is based on appropriate need, such as drift in time expressed through the service history, according to the manufacturer's specifications and regulations.

Certificate of conformity [CAL 05], [CAT 00] certifies a firm that it has made every effort to ensure that the device, specified in the certificate, satisfies the specified requirements. In dimensional metrology, a procedure is described in a simple and accurate concept. Concisely, a procedure is a detailed set of operations performed sequentially in a given method. Each test generally consists of three distinct phases: the configuration of the measurement system (standard and test piece), the measurement, and the assessment of the result. Figure 1.3 shows a general schematic representation of the procedure.

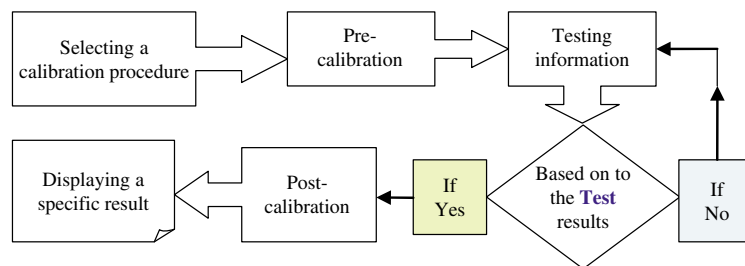


Figure 1.3. Schematic representation of the procedure in dimensional metrology

In metrology laboratories, various vocabularies are used, such as *Accredited or certified? Recorded? Conform to the standard? Meeting the standard? In compliance with the standard(s)?* In Canada, there are four agencies that contribute to the development of national standards, which results in the involvement of the CSA. The number of Canadian professionals involved in the development and implementation of standards is estimated at more than 15,000. In all the cases where the metrology function is necessary, questions relating to the control of the means reveal the basic criteria of metrology.

1.3. Definition of errors and uncertainties in dimensional metrology

Uncertainty is an estimation characterizing the range of values within which the *true value* of the *measured quantity* lies. In fact, uncertainty of measurement comprises many components. Some of them can be estimated based on the statistical distribution of series of measurements often characterized by experimental standard deviation, whereas other components can be estimated based on the experience or

other information. Unfortunately, in some school case studies, *error* and *uncertainty* continue to be confounded. Precise metrology cannot define this flouting of the vocabulary hence VIM exists. For this reason, we try to answer succinctly the following question.

1.3.1. What is the difference between error and uncertainty?

Figure 1.4 shows the error included in the interval between the read value and the true value. However, in metrology, uncertainty [GUI 04, PRI 96] never means error [MUL 81, VIM 93].

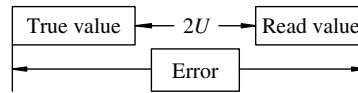


Figure 1.4. Schematic representation of the error over uncertainty

Assuming that T is the permissible tolerance on the measurement, uncertainty can be expressed as:

$$U = \{-T < \text{error} < +T\}$$

Thus, the measurement result is equal to the read value $\pm U$.

Uncertainty factors on the measurement will vary. Figure 1.5 shows the *5M method* boiling down to the uncertainty on the measurand.

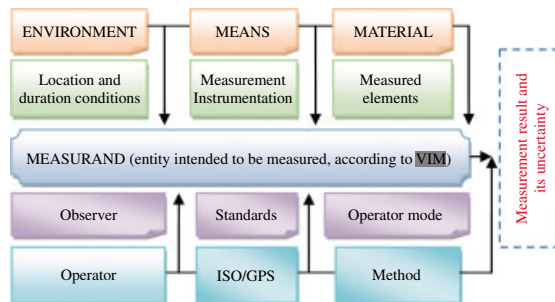


Figure 1.5. Representation of the error with respect to the uncertainty using the 5M methods¹

¹ 5M, with reference to initials of five terms in French: Milieu, Moyen, Matière, Main d'œuvre, and Méthode.

The VIM vocabulary (XP X07-020) and the GUM [NIS 94] can be used to establish the measurement process as in Table 1.1.

Establishing a measurement process	Define specifications	In line with the expectations of the customer:
		– establish the necessary features of the process
		– establish the desired product characteristics
	Express the measurement needs	What to measure (i.e. the quantity value)?
		Which uncertainties to accept/tolerate?
	Implement resources	Choose methods for equipment
	Master the process of measurement (monitoring)	Check frequently the equipment (maintenance)
	Ensure the conformity of the process	Validate the results in relation to the above

Table 1.1. *Establishing a measurement process*

1.3.2. *Why make a calculation of errors' uncertainty?*

Knowing that the uncertainty serves to choose the necessary means – in the measurement – we deduct that there is a relation between the uncertainty and the tolerance. The greater the accepted tolerance, the greater is the measurement uncertainty. In legal metrology, tolerance is five times greater than uncertainty. The following equation is generally used in industry:

$$4(U) < \text{tolerance} < 10(U)$$

The quantity U gives the capability C_p of measuring equipment T . A case example of uncertainty calculation shows that:

- if the parameter influence is less than $(0.01 \times \text{tolerance})$, then the parameter can be neglected, keeping the track of the calculation that led to neglect it;
- the calculation of uncertainty is a task for experts;
- the calculation of the uncertainty has to be made by a specialist. This calculation could be also relieved in which case we would increase the influence of the factors of influence..

1.3.3. *Reminder of basic errors and uncertainties*

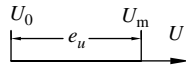
Having discussed the essential pre-conditions for dimensional metrology, we now explain the cases of errors and uncertainties. Uncertainty calculation is based on the types of errors. Therefore, we should differentiate between absolute error and relative error. The latter represents the ratio of measurement error to the *true value*

of the measurand. A relative error is usually expressed as the percentage of the measured quantity. When, for example, a micrometer is used to measure a dimension, we quantify the latter and compare it directly or indirectly to an already existing standard. The quantity thus quantified is a physical observable quantity value because it characterizes a physical condition or a system. In dimensional metrology, the physical quantity characterizes essentially three criteria that are inseparable: the *unity*, the *numerical value*, and its *uncertainty*.

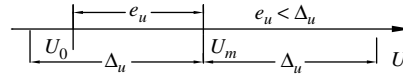
Consider, for example, a measure u_m , that is, the measurement value of a quantity U . Assuming a true value u_0 of this quantity, the error e_u is then defined as follows:

$$e_u = u_m - u_0 \quad [1.2]$$

This may also be converted into the following chart:



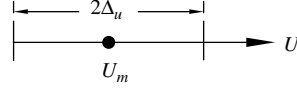
The error e_u can be either a positive or negative number. Obviously, it is not possible to know the error e_u either in measure or in sign. We may only propose to assess – with more or less approximation – an upper limit Δu of the absolute error value; this limit is called *uncertainty* and is schematized:



An upper boundary on the error with an uncertainty domain and a confidence interval can be written as:

$$e_u = \frac{\Delta u}{u_0} e_u \equiv \frac{\Delta u}{u_0} \quad [1.3]$$

According to this equation, Δu is always positive. The accuracy of the measurement is greater when the uncertainty is smaller Δu . This notion may, however, remain vague if we do not specify the datum compared to which the amount Δu is deemed small. This approach consists of comparing Δu with u_0 according to equation [1.2]. This resulting ratio is known as relative uncertainty. It is thus possible to compare two measures of different magnitude orders. The most precise measure is the one for which the relative uncertainty is the smallest. If u_m is known while u_0 is unknown, then the relative uncertainty given in equation [1.2] should be considered.



With u_m being the best estimate of the quantity U , the following description shows how to graphically represent the uncertainty domain associated with the measurement.

$$U = u_m \pm \Delta u \times (100 \times e_m \%) \text{ or } U \in (u_m - \Delta u; u_m + \Delta u) \text{ units} \quad [1.4]$$

1.3.4. Properties of uncertainty propagation

We will not discuss the basic properties often used in physics laboratories. We usually present them to practitioners in laboratories of metrology or applied physics [DIX 51, MUL 81, TAY 05].

1.3.4.1. Addition and subtraction

$$\text{If } U = x + \dots + z - (v + \dots + w), \text{ then } \Delta U = \Delta x + \dots + \Delta z - (\Delta v + \dots + \Delta w) \quad [1.5]$$

1.3.4.2. Multiplication and quotient

$$\text{If } U = \left(\frac{x \times z}{v \times w} \right), \text{ then } \frac{\Delta U}{U} = \frac{\Delta x}{|x|} + \frac{\Delta v}{|v|} + \dots + \left(\frac{\Delta z}{|z|} + \frac{\Delta w}{|w|} \right) \quad [1.6]$$

1.3.4.3. Multiplication by an exact number

$$\text{If } U = (x \times \xi), \text{ then } \Delta U = |\xi| \times \Delta x \quad [1.7]$$

1.3.4.4. Exponentiation

$$\text{If } U = x^\xi, \text{ then } \frac{\Delta U}{U} = |\xi| \times \frac{\Delta x}{x} \quad [1.8]$$

1.3.4.5. Function of one variable

$$\text{If } U = u(x), \text{ then } \Delta U = \Delta x \times \frac{\delta U}{\delta x} \quad [1.9]$$

1.3.4.6. General formula of uncertainty propagation

$$\text{If } U = u(x \dots z), \text{ then } \Delta u = \Delta x \times \left| \frac{du}{dx} \right| + \dots + \Delta z \times \left| \frac{du}{dz} \right| \quad [1.10]$$

1.3.5. Reminder of random basic variables and their functions

We present the basic statistical functions as follows.

1.3.5.1. $E(X)$ or $E(X_{bar})$

The expected value is a parameter of central value that is written as:

$$\bar{x} = E(x) = \int x \cdot f(x) \cdot dx = \frac{\sum_{i=1}^n x_i}{n} = \left(\frac{x_1}{n} + \frac{x_2}{n} + \dots + \frac{x_n}{n} \right) \quad [1.11]$$

where x_i is the i th measured value to the n th value on n measured values; $\bar{x}$ is the arithmetic mean of the measured values.

1.3.5.2. Variance: $V(x)$ or σ^2

The variance is a parameter of dispersion that is often expressed by its standard deviation:

$$V(x) = \int (x - E(x))^2 \times f(x) dx \text{ or } (E(x - E(x)))^2$$

In many cases, we may also consider the following:

$$V(x) = \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)} \quad [1.12]$$

for a population with n measurements ($n < 30$).

1.3.5.3. Covariance: $COV(X_1, X_2)$

The covariance of X_1 and X_2 is a measure of the relationship expressed by:

$$COV(X_1, X_2) = \{E(X_1) - E(X_1)\}, \{(X_2) - E(X_2)\} \quad [1.13]$$

1.3.5.4. Standard deviation $\sigma = \sqrt{V(x)}$

The standard deviation is a dispersion parameter often used in the formulas:

$$\sigma = \sqrt{\int (x - E(x))^2 \times f(x) dx} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad [1.14]$$

1.3.5.5. Probability density function, $f(x)$ or $p(x)$

Probability density function for the normal distribution is given by:

$$p(x) = \frac{1}{\sigma \times \sqrt{2\pi}} \times \text{Exp} \frac{1}{2} \left(\frac{(x_i - \bar{x})}{\sigma} \right)^2 \quad [1.15]$$

1.3.6. Properties of random variables of common functions

Our objective here is not intended for the development of mathematical statistics of basic formulas. We only name them. For further details, the reader may refer to the specialist literature on the subject [MUL 81, TAY 05].

1.3.6.1. Expected value (or mathematical expectation)

$$\begin{cases} E(X_1 + C) = E(X_1) + C \\ E(C \times X_1) = C \times E(X_1) \\ E(X_1 + X_2) = E(X_1) + E(X_2) \\ V(X_1, X_2) = V(X_1) + V(X_2) + 2 \times \text{COV}(X_1, X_2) \\ \text{COV}(X_1, X_2) = E\{(X_1 + X_2) - E(X_1) \times E(X_2)\} \end{cases}$$

1.3.6.2. Variance

$$\begin{cases} V(X_1 + C) = V(X_1) \\ E(C \times X_1) = C^2 \times V(X_1) \\ V(C \times X_1) = E(X_1^2) - [E(X_1)]^2 \\ \text{COV}(X_1, X_2) = R \times \sigma(X_1) \times \sigma(X_2) \end{cases}$$

C is the constant, and R is the correlation coefficient.

1.4. Errors and their impact on the calculation of uncertainties

Measurements are performed to determine the instantaneous value and the evolution of certain quantities [GUI 04, TAY 05], such as information on the status and trends of a given physical phenomenon. In fact, there are two main kinds of errors that are likely to affect measurement: the *bias error* and the *accidental error*.

1.4.1. Accidental or fortuitous errors

Accidental error is caused due to a wrong move, a misuse or malfunction of the apparatus. Usually, they are excluded in determining the measure. They cannot be

quantified without adding them to the error itself. *Random errors* are caused by human and cannot be prevented.

Aspects such as the *certainty* with which an instrument is handled, the accuracy with which the eye observes the positioning of an indication of the dial caliper read on the scale, and the differential acuity of the observed vision differential are limited. Each experimenter is expected to be aware of accidental errors in measurement, to keep them as low as possible and to estimate or quantify their impact on the measurement result. The measurement result x of a quantity X is not fully defined by a single number. Uncertainties arise from various errors linked to the measurement. A measure should be characterized by, at least, a couple $(x, \delta x)$ and a unit of measurement. Assuming δx the uncertainty on measurement x , we obtain, for example, $1/4 \pm 5\%$ inch or $25.4 \pm 1/10$ mm.

$$\{x - \delta x \leq X \leq x + \delta x\}$$

In fact, the accidental or fortuitous error varies unpredictably both in absolute terms and in signs when making a large number of measurements of the same magnitude in almost identical conditions. For example, we should not disregard an inadvertent error by making a correction to the “gross” value of the measurement result. At the end of a series of measurements, we can only set an upper limit for this error. Hence, a fortuitous error is usually described as accidental error or even random error.

Ultimately, we believe that errors linked to the measured entity and the observation system cannot automatically deduce the true value. The latter remains an *ideal concept* (some would argue that it is a vague linguistic concept) helping model the effect of errors. The metrologist approach consists of finding the *true value* by associating it with the least amount of mistakes, which is the purpose of this chapter. Under the foregoing, we note this statement by:

$$\text{Measurement result} = \text{true value} + \text{error}$$

1.4.2. Systematic errors

Systematic errors are reproducible errors caused by any factors that systematically affect measurement of the variable across the sample; therefore, they could be eliminated by suitable corrections. According to the VIM, the bias is defined as “mean that would result from an infinite number of measurements of

the same measurand carried out under repeatability conditions minus a true value of the measurand.”

$$\text{Systematic error} = \text{error} - \text{random error}$$

As is the case for the true value, the bias and its causes cannot be known completely. As for measuring instruments, we refer to the definition of the bias error written as:

$$\text{Result} = \text{true value} + \text{random error} + \text{systematic error}$$

Systematic errors occur while using the poorly calibrated units such as an *erroneous* scale, an improperly adjusted micrometer, or an inconsistent probing with the sphere of a three-dimensional measuring machine. They also arise due to the negligence of some factors influencing the course of the experiment.

Systematic errors, as long as their cause is known, can be rectified by making the correction to the measurement result. The characteristic of these defects is to act always, in the same direction, on the measurement result, by systematically distorting it with excess or deficiency. These defects introduce *systematic errors* generally known as *defects of accuracy/correctness* (Table 1.2).

Possible origin	Concrete examples
The measuring chain	Displacement from zero and/or faulty calibration, unawareness of the equipment's principles, negligence of significant key elements, and so on
The applied method	Weigh-in a solid, volatile fluid (in chemistry), and so on
The experimenter	Parallax error (confusion between nominal and simulated axes under GPS standards)

Table 1.2. *Some examples of errors and their possible origin*

The following stance should be considered to avoid the errors:

- awareness of the existence of errors and never disregard them;
- make sure to track them, knowing that they always act in a given direction;
- try to reduce their impact through proper use of instruments (zero balancing) or possibly by changing the method of measurement;
- make a correction which includes the detected defect.

1.4.3. Errors due to apparatus

Errors caused by measuring apparatus [CAT 00] are often inherent to mechanical defects or other kinds of defects. The accuracy of a measuring apparatus is defined by the interval of its reading graduations, for example, a simple caliper or a micrometer with mechanical reading. For a 0.02 mm slide caliper, a slide of 1/50 may be affected by one or many of the following reasons:

- unequal intervals of the scale in direct reading;
- inequality in the screw pitch on the micrometer;
- a possible shift of the scale origin, wear of caliper's jaws;
- any defect in the contact surfaces, such as parallelism.

For these reasons, instrument users need to clearly observe the instructions of the manufacturer. Usually, there is a leaflet with the measuring instrument. In terms of faithfulness, in applied metrology of course, here are some examples that would affect the errors, for example, the clearance in the sliding, that is, spouts, screws, joints, and indicators, possible changes in *contact pressure* or limitation of micrometers. Some of the examples of the errors due to apparatus are as follows:

- unequal intervals and shift from the origin;
- distortion of the contacts;
- parallelism defects and the clearance.

1.4.4. Errors due to the operator

Reading errors sometimes result from an erroneous or imprecise assessment. Pitch lines not coinciding the graduation and improper visual position affect the measurement accuracy and therefore the assessment. Imperfections, let us say manipulation flaws, (i.e. misalignment of instrument) shall inevitably lead to errors, thus creating permanent doubt. Though, in dimensional metrology, it is recommended that we doubt. Note that after machining, the existence of burrs and inevitable use of grease or oil would accidentally affect the measurement by the very interposition of foreign materials, such as *graduation on bevel eliminates the risk of misreading, or misalignment of the device*.

1.4.5. Errors due to temperature differences

It is known that moisture conditions (relative humidity) do affect the properties of materials (instruments, appliances, and parts subjected to measurement). The impact is deemed greater when the anisotropic materials are affected more than

the isotropic solid materials. Temperature variations affect both the measuring instruments and the parts intended to be measured, especially on fine-tuning devices. The measuring instruments are calibrated at 20°C. As a consequence, it follows an impact of possible expansions or contractions.

The expansion is expressed as:

$$\Delta L = L \times \lambda \times (\tau_1 - \tau) \quad [1.16]$$

where L is the initial length (nominal dimensions) of a piece, expressed in millimeters or inches; λ is the expansion coefficient (or linear expansion) of materials; $\Delta\tau = (\tau_1 - \tau)$ is the change of temperature in degree Celsius.

If the coefficient of expansion λ is negative (–), we understand that it is a contraction. If it is positive (+), we deduce that there is expansion. In the exercises section, we present an instructive example in this regard.

1.4.5.1. *Vocabulary of the quantity intended to be measured (or measurand)*

The quantity that is to be measured is known as the *measurand*. The comparison system and the standard constitute, in turn, the *measuring system*. It would be unrealistic to consider that any measure really reflects what is translated by an instrument, whatsoever its accuracy. Thus, any measurement process is flawed and hence admits a certain degree of uncertainty. The origins of these multiple *errors* are sometimes difficult to identify. However, metrologists agree to classify them into three categories, namely, errors due to the measurand itself, the measurement system, and the (technical) approach to measurement or observation.

According to section 2.6 of the VIM, these three sources of errors are described. First, we should bear in mind the outline of Figure 1.6 (ASME, American Society of Mechanical Engineers Standards; ISO 1101: GPS*, Geometrical Product Specifications; AIAG, Automotive Industry Action Group).

1.4.5.2. *Measurand*

It is imperative to properly define the measurand because a wrong definition may inevitably distort the interpretation. Some say it is a language problem, whereas others say it is a communication problem. It is both in our view. We should be wary as much as possible to avoid, or even deflect, this source of error. For example, in a dimensional metrology laboratory, sometimes students are taught to measure the length of a gauge block using the width of their thumb (in.). Often we wonder about what is left unsaid by such users of block gauges. We have rarely told them the temperature at which the result should be returned. Our questioning is not simply due to the fact that the observing system has an *accuracy* and *faithfulness* of the order of micrometer (or μ inch). Does this remain sufficient? No.

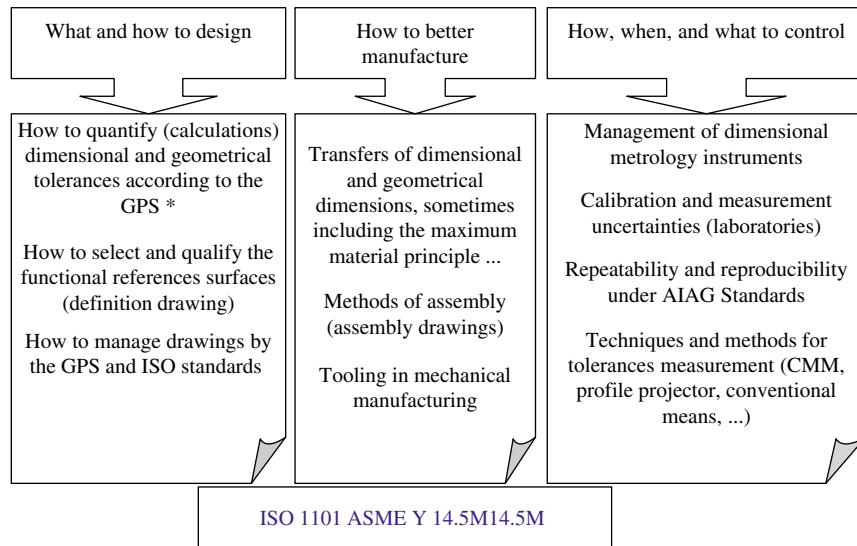


Figure 1.6. Illustration of three sources of errors according to ISO 1101

If, for example, we seek accurate measurement based on such gauge blocks, physical interests should be specified, such as the position of the gauge block relative to the direction of the acceleration of gravity, cleaning of the gauge blocks with appropriate preservatives, considering humidity conditions. Chapter 3 dedicated to standards details it with examples.

If students carry out this verification in a metrology laboratory, certainly this would be a good guarantor of the good habits to be taught and a sound way to achieve commendable results. We know that in physics, when the gauge block is placed vertically, the distance is shorter than that placed horizontally on a plane. It compresses due to its own weight. This simple recommendation seems derision for unwarned users, and it even happened in many laboratories. We have seen in the Continuum Mechanics course, if the gauge block is based on support, its length would depend on the position of that support. There are many cases similar to the preceding. However, a warned metrologist would be interested in endorsing the good conduct during the assessment of the quantity to be measured.

1.4.5.3. Measurement system and measuring technique and/or observation

In practice, a measurement system is never perfect. Any system is subject to environmental factors such as pressure and temperature. This fact is realized when the same measurement is repeated several times. The resulting dispersions prove this laboratory common fact. Sometimes, the same standards that were

used for calibration are inaccurate. School laboratories rarely carry out a periodic check up.

The *primary standard* is an imperfect realization of the definition of the unit that it is supposed to represent. The unit is conventionally defined by the International Committee for Weights and Measures. A better definition of the unit, achieving a standard, is never achieved. By pragmatism, standards provided by big companies are generally trusted.

The definition of a physical quantity provided by the measuring instrument interacts directly with the *manner* of the observation of this measure. In mechanical probing systems, either optical or capacitive, we usually expect different results. This is noticed when correcting students' workpieces resulting from machine tools. A coordinate measuring machine (CMM) is by far the device that provides the most accurate results.

In metrology, we classify the possible errors in two or three broad categories. Some metrologists retain two categories, but, in fact, it would be easy to distinguish them into three categories. We discuss further on, for example, random errors. It is always possible to decompose the error into systematic error and random error.

According to the VIM, the random error is a "result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatable conditions." This is expressed as:

$$\text{Random error} = \text{error} - \text{systematic error}$$

Because only a finite number of measurements can be made, it is possible to determine only an estimate of random error.

1.4.6. *Random errors*

These are non-reproducible errors that obey certain statistical laws. Let us again consider the quantity to be measured X . Its measurement was performed several times under apparently identical conditions with measurements independent of one another. Despite these precautions, we notice that results are different. Therefore, these are called measurement faithfulness flaws. The latter are manifested by the non-repeatability of results.

Among the many causes of random errors, we discuss the faithfulness deficiency of the instrument. Faithfulness flaws lead to random errors; hence, the statistical treatment of the results allows estimating the uncertainty. These flaws have several possible causes listed in Table 1.3 [PRI 96] which is not exhaustive.

On reading the position of a pointer against the graduations, it is not necessary to consider more than half a division. The sight of the operator does not change the quality of the measuring instrument. Before starting the measurement of a quantity, we determine the required precision of an instrument and an appropriate method to meet the goal, thus considering the possibility of accuracy flaws for the instrument and the method itself. Never forget to make the necessary adjustments to eliminate or minimize the accuracy flaws.

Errors due to the apparatus characteristics	Examples in conventional dimensional metrology
Reading error: it depends on the experimenter skills but also on the sharpness of graduations	Micrometer or caliper
Errors due to environmental factors	Environmental factors are not considered during the experimental phase (e.g. change in temperature or pressure)
Standard mobility (mobility errors): below a certain value, changes in the measurand will no longer be detected	Uneven displacement of slats of a digital P to C or a slider below the distance between two turns of a coil that do not affect the final result
Standard resolution/(resolution error): is the combination of the two types of previous earlier errors	Graduation from P to C or considering the last digit of a digital device
Hysteresis error	Dilation of a material after machining
Errors due to spurious signals: classic cases of the recorded spectral density functions	A surface appears flat to the naked eye. Under the microscope, this would be different. Thermal agitation provokes a noise floor causing voltage fluctuations. This noise is superimposed on the wanted signal

Table 1.3. Major errors linked to the measurement device characteristics

Ultimately, we intend to make a correction of the obtained results if the accuracy defects are inherent to the applied method. This correction will clearly remain low compared to the result; otherwise, the method will be disqualified. Therefore, we can achieve either properly reducing or correcting systematic errors. The measurement accuracy is always limited by the *measurement faithfulness shortcomings causing random errors or sensitivity of the instrument*. As a first approximation, we may assume that a low-sensitive instrument appears to be faithful (repeated measurements of the same quantity give the same result), whereas a more-sensitive instrument may reveal faithfulness flaws due to the instrument itself, to the quantity intended to be measured, and to the external conditions during the measurement.

The aim of any modest and pragmatic metrologist is to provide a result, the closest possible to the true value. For this, he or she must reduce errors as much as possible. Yet, to reduce errors, especially random ones, he or she repeats the measurements and tries to reduce systematic errors by applying appropriate corrections. Within our various laboratories, we have recorded the common errors. The chart trend helps better understand and track the type of error while handling measurement equipment. Most of the features are involved in the evaluation of the measurement uncertainty:

- repeatability;
- reproducibility;
- linearity;
- sensitivity;
- precision;
- resolution;
- faithfulness;
- correctness;
- accuracy.

We present their definitions according to ISO [VIM 93].

1.4.6.1. *Repeatability (minimum value of precision) according to ISO 3534-1 and ISO 5725-1*

Repeatability is the dispersion of independent measurements obtained on identical samples by the same operator, using the same equipment and within a short-time interval. This is the first characteristic to assess because the significance of other factors is tested based on the repeatability. It is evaluated on the domain studied at k concentration levels by repeating n measurements for each one. From equation [1.7], the standard deviation of repeatability S_r can be written as:

$$S_r = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad [1.17]$$

1.4.6.2. *Reproducibility (internal)*

Note that repeatability should never be confused with reproducibility.

At least one factor varies with respect to repeatability, often the *time* or *operator* (internally) factor. The effect of the factor studied is estimated through the analysis of the variance s_p^2 . Figure 1.7 shows an example using MathCAD software.

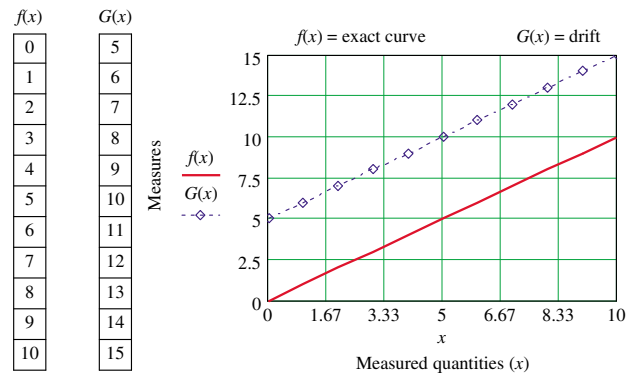


Figure 1.7. Zero error (offset adjustment)

It can be clearly seen that by repeating the same measurement tests, the variability is then highlighted, hence the approximation (estimate) by the expression of variance s_r^2 .

Scale error depends linearly on the measured quantity. Over time, aging softens the components to what is termed the “drift” as shown by curve $G(x)$ in Figure 1.8. Here is a simulation example.

In addition to the impact of the various factors, we should, in the metrology of sensors, assess the aging components by expressing the latent variation of its output signal versus time (in hours, months, or years), which is defined by the drift.

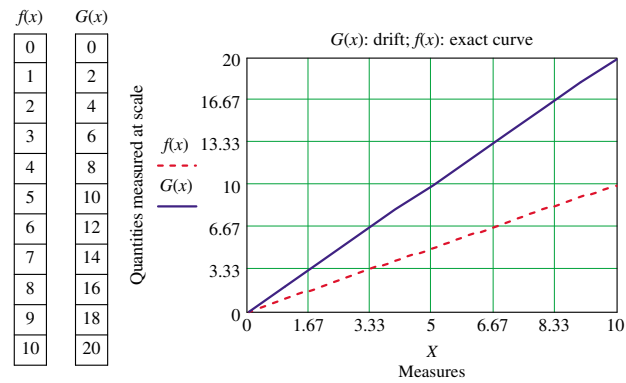


Figure 1.8. Scale error

1.4.6.3. Linearity error

Linearity error expresses the univocal and linear relationship between the results obtained in the entire field of knowledge concerned with measurement and the corresponding properties of the material. A nonlinear relationship is usually eliminated by correction using a nonlinear calibration function. In practice, this is achieved through a calibration curve wherein linearity is approached. To determine the line, the least squares method can be implemented. The linearity error reflects a non-straight line feature as shown in Figure 1.9.

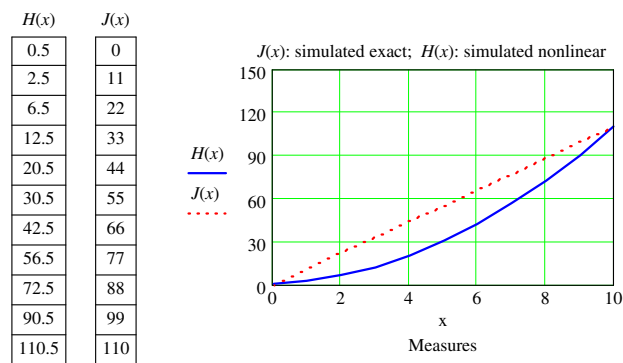


Figure 1.9. Linearity error

Hysteresis error occurs when the measurement result depends on the anterior conditions during the earlier measurement. This is often noticed when measuring an incremental, using a projector profile. Hysteresis is also known as reversibility, which characterizes the capability of the device to give the same indication when it reaches the same value of the measured quantities by increasing or decreasing values (Figure 1.10). It is clear that this is a deviation of the real curve dashed from the ideal continued curve.

Mobility/displacement error has the characteristic of being jagged. This error is often due to a signal digitizing (CMM, potentiometer, and so on).

Measurement range is important in almost all disciplines associated with metrology. It is defined as “a set values of measurands for which the error of a measuring instrument lies between specific limits. The maximum value of the measurement range is called full-scale.”

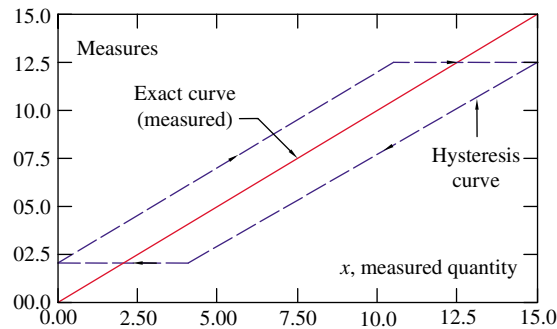


Figure 1.10. *Hysteresis error*

A device indicating measurements may, sometimes, have a graduated dial in units of quantity to be measured; its measurement range is not always confounded with the scale range. In mechanical manufacturing, the range is understood as the tolerance imposed or given to the final dimension relative to the measure of the nominal dimension, for example, $\varnothing\frac{1}{2} \pm 0.001$ in. Of course, the instrument used (micrometer) goes beyond the nominal dimension, which is $\varnothing\frac{1}{2}$.

Rangeability is defined as the minimum ratio of the measuring range to the full scale (Rank = minimum measuring range), which is formalized as

$$\text{Rangeability} = \frac{\text{Rank}}{\text{Full scale}} \quad [1.18]$$

Calibration curve is specific to each device. It converts the raw measure into the corrected measure. It is obtained by subjecting the instrument to a true value of the quantity to be measured, which is provided with a standard apparatus and accurately reads the raw measure given.

1.4.6.4. Sensitivity

Sensitivity is the quotient of the increase in the response of a measuring instrument and the corresponding increase in the input signal.

This definition of the VIM [VIM 93] applies to apparatus and devices of various signals. In other words, it is a parameter that expresses the variation of the output signal of the measurement of a device based on the variation of the input signal. A device is deemed more sensitive, as a small change in the quantity being measured causes a greater change to the measuring device. If the input value is of the same kind as the output value, the sensitivity is called *gain*. Let us see how can this be tackled, by the following reasoning.

Let X be the quantity to be measured, and x be the indication or the signal provided by the device. For all values of $X \in$ measuring scale, there is a corresponding value of x such that $f(X) = x$. The sensitivity around the value X is the quotient q such that $q = (dx/dX)$. If the function is linear, the sensitivity of the device is constant so that $q = (\Delta x/\Delta X)$. Sensitivity expresses the smallest Δx amount that can be measured for a given value x of the measured quantity. This sensitivity may be constant along the scale.

Sensitivity is higher when dx is small. When x and X are similar, q , which is dimensionless, can be called GAIN, in dB . If I is the indication given by trial and α , a quantity to be measured.

S is the sensitivity amount close to a given value U of the sensitivity quantity α to be measured:

$$S = \frac{\delta U}{\Delta \alpha} \quad [1.19]$$

It is generally assumed to be the slope of the graduation curve within an interval. The average sensitivity can then be expressed as follows:

$$\alpha_{\text{mean}} = \frac{\Delta U}{\Delta \alpha} \quad [1.20]$$

Figure 1.11 is shows an example using MathCAD software.

For $\alpha = (1, \dots, 10)$, $f(\alpha) = 20 \times \text{Log}(\alpha)$, the gain is $f(\alpha) = 20 \times \text{Log}(\alpha)(dB)$

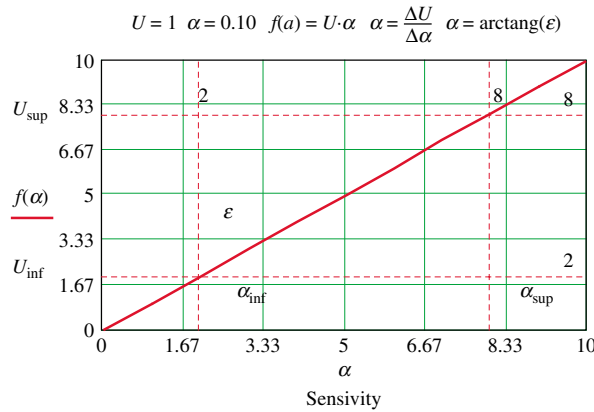


Figure 1.11. Example of the determination of the sensitivity line

Case of a linear apparatus: equation [1.20] makes sense only if the device is linear along the measurement interval. That is to say, if the output is proportional to the input. Consistency then becomes the capability of a measuring instrument to maintain constant its metrological characteristics over a defined period. Nevertheless, sensitivity should not be confused with resolution (or resolving power). Resolution is the smallest variation in the quantity intended to be measured that is perceptible by the device. For example, a dial gauge indicates 100 mm. A variation of 0.1 mm moves the comparator's needle, whereas a variation of 0.05 mm does not move the needle. Hence, the resolution of the comparator is 0.1 mm. When accuracy is limited by the sensitivity of the instrument, it is assumed that any measure X gives an estimate for the true value of quantity X and we should simply guard against coarse errors (errors in reading, for example) by making two successive measurements X_1 and X_2 .

Let R be the resolution of the instrument. The results are considered consistent if $|X_1 - X_2| < R$; the following is then adopted as a result of the measure:

$$X = \frac{x_1 + x_2}{2}, \text{ with } \{x - R \leq X \leq x + R\}$$

Relative uncertainty will be (R/X) , and the precision of the measurement is written as $(R/X) \cdot 100$ in percentage. Note that if two successive results x_1 and x_2 do not match, a third test x_3 should be done. If x_3 is consistent with one of the first two tests, for example, x_1 , then x_2 will be rejected as an aberrant value, and finally, the measurement result would be $X = (x_1 + x_3)/2 \dots$

So, if x_3 is inconsistent with any of the first two results, this may be a sign of a precision that is limited, not by the sensitivity of the device, but rather by measurement faithfulness flaw. *Measurement faithfulness flaws limit the precision of the measurement.*

When precision is limited by the measurement faithfulness flaws, uncertainties are random, and we can derive maximum information from experimental results using statistical methods. This second scenario is the most common in practice. It is attempted to implement methods for optimal measuring accuracy.

Ultimately, *resolution* is used for digital display devices. It expresses the smallest value that can be displayed. *Robustness* expresses the resistance or insensitivity to the effects of some influencing variables. *Selectivity* is the ability to correctly measure despite the impact of *interferences*, such as the capability of the method to differentiate two objects having approximately similar properties (qualitative).

1.4.6.5. Precision

Precision expresses the degree of concordance of the characteristics of independent quantity values resulting from the application of a measurement process under specified conditions. Precision describes the closeness of agreement between quantity values; the results obtained by measurements of the quantity. The term “accuracy class” means the class of measuring apparatus that corresponds to the value (in percentage) of the ratio of the greatest possible error to the measurement range. Here is a succinct formulation:

$$\text{Class} = \left(\frac{\text{The greatest measurement error}}{\text{The measurement range}} \right) \times 100 \quad [1.21]$$

The measuring device is characterized by a number called the class index. Manufacturers usually assign a class to each produced instrument. A class represents the upper limit of the absolute intrinsic error in hundredths of the greatest indication provided by an instrument. For example, a digital micrometer with precision class 0.25 is an instrument whose intrinsic absolute error does not exceed 0.2% of its maximum indication when used under normal conditions. If the instrument has 100 divisions, this absolute intrinsic error 0.25 is then $\leq (0.25/100) \times 100 = 0.25$ division. The measuring device is as precise as the measurement results that it indicates coincide with the *true value* that is sought to be achieved by the measurement.

Precision is more easily defined by the precision error. It is expressed in units of magnitude (absolute error) or percentage (relative error). Beyond the operating conditions, the precision of the device is mainly related to two types of characteristics: accuracy and faithfulness. A device is deemed valid if it is both *accurate* and *faithful*. The precision degree of the measuring device can also be negatively affected by external causes such as operational error; error due to environmental factors, temperature and pressure; the reference or calibration error; hysteresis error; and/or finesse error.

In practice, precision is a factor that sets the overall maximum error ($\pm$ in), which may occur during the measurement. It is usually expressed as a percentage of a full scale. The measuring device provides more precision (in relative values) at maximal values on the scale. If the value x characterizes the measurand, the precision of the instrument will be equal to the ratio (dx/x) of the total error represented by dx and x . This, in fact, characterizes the quality of an instrument in terms of errors. As previously explained, precision is deemed higher when the indications are closer to the true value (that is to say, dx is small) (Figure 1.12).

$$\text{For } \mu = 1, \sigma = 1/2, k = 1, \text{ and Ave} = (\mu - 4.5\sigma, \mu = 4.4\sigma \dots \mu + 4.5\sigma)$$

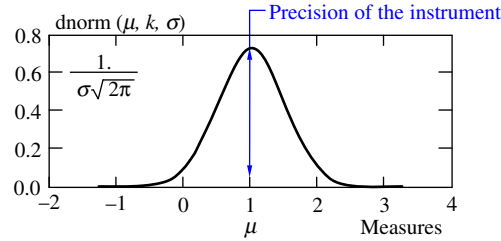


Figure 1.12. Illustration of the precision curve: true value (arrows), hence a precise measuring instrument (apparatus)

1.4.6.6. Resolution

The resolution is a quantitative expression that represents the smallest interval between two elements so that they can be separated by an observation instrument. The resolution is the smallest difference significantly perceptible in the corresponding indication (displaying device or records). When the meter is a digital device, resolution is defined as follows:

$$\text{Class} = \frac{\text{measurement range}}{\text{number of measurements points}} \quad [1.22]$$

The resolution of measuring instrument with digital display is a source of uncertainty. Indeed, if ε is the quantification of the instrument, the value of the quantity lies within the interval $[-\varepsilon/2, +\varepsilon/2]$ with a constant probability throughout the interval. Therefore, it is a rectangular probability distribution.

Later we present a numerical example in this regard. Errors lead to a dispersion of results at repeated measurements. Their statistical treatment allows estimating the most probable value of the quantity being measured and determining the limits of uncertainty.

When measurement of the same quantity X was repeated n times, giving the results, $x_1, x_2, \dots, x_n$, the average value is defined by its mean μ . An indication of the dispersion of these results is given by, respectively, the variance σ^2 or the standard deviation σ . When the random errors affecting the different measurements are independent, the probability of occurrence of different outcomes usually meets the normal law as shown in Figure 1.13.

Normal distribution of a random number creates a vector of random numbers from a normal distribution with the mean μ and standard deviation σ . Here is the mathematical approach to follow:

Number of random deviations: $n = 800$, mean = 0, and standard deviation $\sigma = 2$

Number of classes for the histogram: bin = 17

Vector of random deviation: Newton = anorm(n, μ, σ)

Frequency law: lower = full (min (Newton)), upper = ceiling (max (Newton))

$$h = \frac{\text{upper} - \text{lower}}{\text{bin}}; j = 0 \dots \text{bin}; \text{Mean}_j = \text{lower} + h \times j$$

$$f = \text{hist}(\text{Mean}, \text{newton}); \text{Mean} = \text{Mean} + 0.5 \times h$$

$$\text{Adjustment function: } F(x) = n \times h \times \text{dnorm}(n, \mu, \sigma)$$

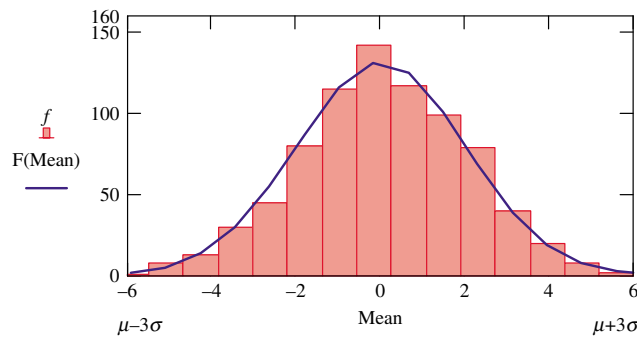


Figure 1.13. Chart representing normal distribution (Gauss-Laplace)

The most probable value is the mean value of measurements, $E(X) = \mu$. In general, we consider an uncertainty to be equal to three times the standard deviation ($\delta x = 3\sigma$).

1.4.6.7. Faithfulness

According to the VIM, faithfulness – in metrology – is “the ability of a measuring instrument to provide very close indications during repeated applications of quantity measurements under the same measurement conditions.”

Measurement faithfulness is the capability of the measuring device to provide consistent indications (results), measures which are free from accidental (fortuitous) errors for the same quantity being measured. This generally reflects the case of the measuring device inducing small errors. Measurement faithfulness is represented as follows (Figure 1.14):

For $\mu = 1$; $\sigma = 1/2$; $k = 1$ and mean = $(\mu - 4.5\sigma, \mu = 4.4\sigma \dots \mu + 4.5\sigma)$ ***

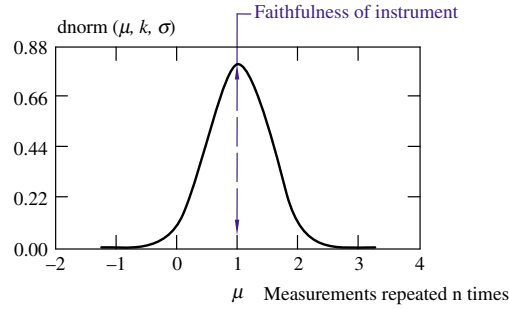


Figure 1.14. Faithfulness and true value (dashed hence the measuring instrument is faithful)

Faithfulness defines the dispersion of results. If only one measurement is performed, precision would be the probability that it is representative of the average result. The latter is obtained by carrying out an infinite number of measurements. The standard deviation is often considered the repeatability error. If we make a set of measurements on the quantity G , we would obtain the maximum value $V_{\max}$ and the minimum value $V_{\min}$. The limiting errors of faithfulness are expressed as:

$$E_{\max} = -\left[\frac{V_{\max} - V_{\min}}{2}\right] \text{ and } E_{\min} = +\left[\frac{V_{\max} - V_{\min}}{2}\right] \quad [1.23]$$

For example, if a and b were two measurement results of the gauge block that is assumed to measure 2 in.:

$$a = 2.0504 \text{ in.}, b = 2.0023 \text{ in.}, \text{ then from } E = \pm(a - b)/2: E = \pm 0.024 \text{ in.}$$

1.4.6.8. Trueness

Measurement trueness characterizes the capability of measuring apparatus to provide indications equal to the true value of the quantity being measured, that is, the value that is not flawed by systematic errors. The instrument is more likely true when its average value is close to the true value. According to the VIM, the trueness is defined as “The ability of an instrument to provide information free from bias; the closeness of agreement between the average value obtained through numerous replicate measurements and an accepted reference quantity value.”

The average result is itself likely to include trueness error. Let us consider the following pertinent question:

The trueness of the result $U = u(t)$. Is this a true result in metrology?

This cannot be affirmed because of the presence of disturbing factors such as using a model based on physics laws or the local experimental approach. The impact of such causes can be predicted, that is, the estimation of the value of error component U . The variability of results is inconstant; hence, this consequence on the measurement error is equal to $(U - u)$.

Furthermore, the trueness error is the overall error resulting from all causes for each measurement results separately. In multiple measurements, trueness error would be the difference between the average result and the true value, which is formalized as:

$$D = \mu - V_{\text{true}} \quad [1.24]$$

where μ is an arithmetic mean of a large number of measurements and V_{true} is the true value (known conventionally as *true* because it is about the gauge block).

In a two-dimensional representation, considering the true value as the origin, trueness error would be considered the centroid of all measures. To evaluate the trueness, we should have *references*.

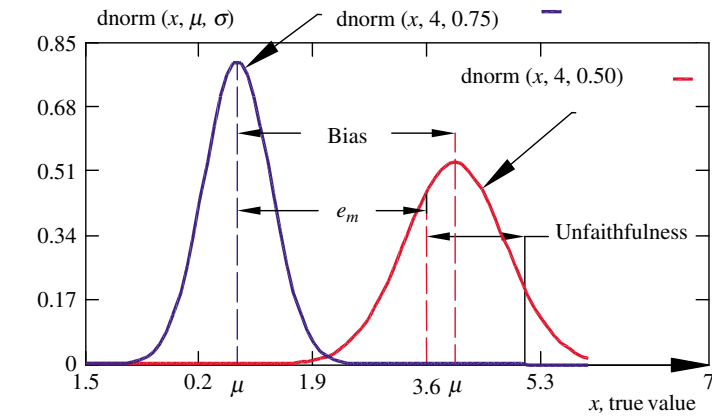
Reference values that are accepted in *analysis methods* may have various origins, namely, the reference material, the value provided by a reference method, and the value stemming from another aptitude testing (inter-laboratory: if the reference value is *traceable* to the SI units). According to ISO 3534-1 and ISO 5725-1, the reference value accepted is the value deemed conventionally true. Therefore, starting from this fact, the VIM excludes the term “precision” in this context.

The term bias (or systematic error) is involved in the assessment of the measuring trueness, which in turn results from systematic errors. The bias is then described as the difference between the expected value and the accepted reference value.

For the reference value $X_{\text{reference}}$ with the mean μ , the bias is expressed as follows (Figure 1.15):

$$\text{Bias} = \mu - X_{\text{reference}} \text{ and } E_{\text{relative trueness}} = + \left[\frac{(\mu - X_{\text{reference}})}{X_{\text{reference}}} \right] \times 100 \quad [1.25]$$

Systematic errors are due to the lack of trueness in the implemented measurement methods. Human (operator) errors may also occur.



Unfaithfulness: random error in repeatability metrology (or reproducibility)
 e_m : error on the measurement or accuracy
 bias: systematic error

Figure 1.15. Graphic illustration of bias

1.4.6.9. Accuracy

Accuracy (not precision) reflects the degree of concordance between the measured value and the true or expected value. The accuracy in French: exactitude (which should not be used for “*précision*”) is the closeness of agreement between the measurement result and the theoretical true value (conventional) responsible for an accepted *reference value* = qualitative. Figure 1.16 statistically illustrates this.

For: $\mu = 1$; $\sigma = 1/2$; $k = 2$ and mean = $(\mu - 4.5\sigma, \mu = 4.4\sigma \dots \mu + 4.5\sigma)^{***}$

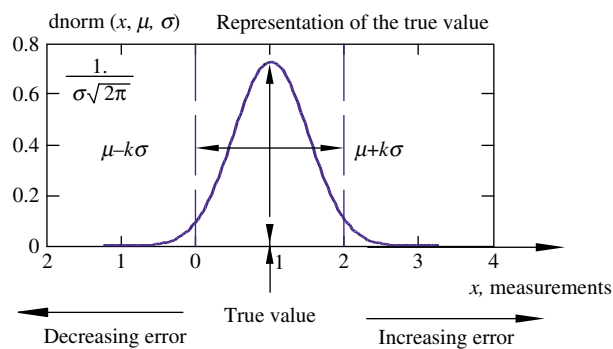


Figure 1.16. Statistical illustration of the error and the true value

DISCUSSION.— Contrary to what has been agreed in the general culture of people, *the measured value is the ultimate value of truth*. It turns out that this fact is not so implicit or even wrong. The measured value is *not* the true value. We could say that the industry of legal metrology tends toward “honesty”, and industrial metrology must establish a guaranteed functionality, starting first by talking to people, the same language to describe concepts if long been accepted as an immutable truth. After that, we would involve the principles of calculations of mathematical formalism for modeling errors and uncertainties. That is we could imagine the *enigmatic true value* (Figure 1.17).

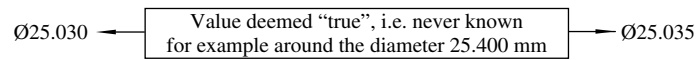


Figure 1.17. Pictorial expression of true value

NOTE.— The true value does not necessarily signify absolute mean. The true value is never known. In fact, it is a set of values that could reasonably be attributed to the value of the bore under consideration, in this case.

1.5. Applications based on errors in dimensional metrology

1.5.1. Absolute error $|\delta| = E_a$

Absolute error is inherent to the measurement of the actual dimension (true value of the measurand). For example, for an actual dimension of 50 ($\forall$ standard) and a measured dimension of 50.015, the absolute error $|\delta|$ is calculated as:

$$\text{example 1: } |\delta| = 50.015 - 50 = +0.015 \text{ mm;}$$

$$\text{example 2: } |\delta| = 79.982 - 80 = -0.018 \text{ mm.}$$

1.5.2. Relative error $\delta = E_r$

The relative error, δ , is reported as a percentage as it reflects the ratio of the absolute error to the corresponding actual dimension. For the first two examples, the relative error is then:

$$\left\{ \begin{array}{l} \delta = 0.015 \times \left(\frac{100}{50} \right) = 0.03\% \text{ and} \\ \delta = (-0.018) \times \left(\frac{100}{80} \right) = -0.023\% \end{array} \right.$$

1.5.3. Systematic error

This is the algebraic difference between the average measured values and the actual dimension. Assuming, for example, five replicated measurements carried out on the measurand having a real dimension 50:

50.02 – 50.01 – 50.02 – 50.01 – 49.99 → the systematic error is therefore written as:

$$\delta_{\text{systematic}} = + \left(\frac{50.02 + 50.01 + 50.02 + 49.99}{5} \right) - (50.00) = +0.01 \text{ mm}$$

According to the VIM, the bias or systematic error is the difference of average that would ensue from an infinite number of replicated measurements of the same measurand carried out under repeatability conditions and the true value of the measurand.

In practice, we calculate the ratio “measurand value/known value”; the measured values are obtained by a sufficient number of repeated measurements on the reference materials or standards whose values are well known.

1.5.4. Accidental error (fortuitous error)

As defined in section 1.4.1, accidental error, also known as fortuitous, is expressed by the algebraic difference between the individual measurement result and the average of measured values. The relative accidental error δu at the fifth measurement is $49.99 - 50.01 = -0.02$.

In our example case, the measurement uncertainty is set by assigning to the confirmed error, compared to the reference dimension, double sign ($\pm$). For example, the fifth measurement of uncertainty of the standard operating procedure is ± 0.01 .

1.5.5. Expansion effect on a bore/shaft assembly

WARNING.— The concept of the following example is inspired, partly, from the technical literature [CAS 78]. Data, tests, and nuances of these materials are appropriate to our real practice in laboratory. We fully generated them using MathCAD software while keeping the spirit of the authors’ original pedagogical approach [CAS 78]. In fact, this is a classic problem that we reprocess as follows.

The measurement environment has a great impact on the *accuracy* of results. Errors originated by possible change in temperature, as small as they may be, could significantly affect the overall uncertainty through the various errors defined

previously. We are constantly facing this laboratory reality in our metrology workshops and when machining light alloys. With this in mind about workshops, we present three case studies.

1.5.5.1. Problem set #1

Consider a $\varnothing 30.005$ -mm steel shaft with a linear expansion coefficient of $\lambda_{\text{steel}} = 1.2 \times 10^{-5}$, contained in a bronze bore $\varnothing 29.99$, with $\lambda_{\text{bronze}} = 1.8 \times 10^{-5}$ (for Figure 1.18(b): $\lambda_{\text{aluminum}} = 2.3 \times 10^{-5}$).

Under a temperature deemed ideal in the laboratory (i.e. $\tau = 20^\circ\text{C}$), the ensuing clearance corresponds therefore to:

$$\text{Clearance} = 30.005 - 29.99 = +0.015 \text{ mm}$$

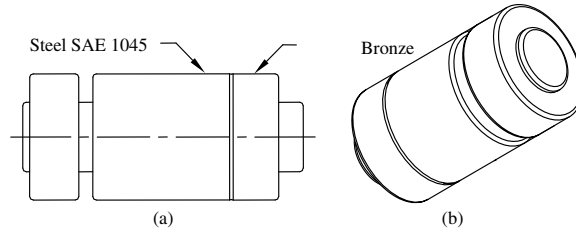


Figure 1.18. Shaft/bore systems (a) and (b)

For a given temperature deviation, the variation in length for steel is, from [1.16] we deduce:

$$\Delta L_1 = L \times \lambda_1 \times \tau \quad [1.26]$$

Similarly, for the bronze, we consider the following:

$$\Delta L_2 = L \times \lambda_2 \times \tau \quad [1.27]$$

For the same nominal dimension L , the bronze, based on physical considerations, extends by the length ΔL , which is expressed as:

$$\Delta L = \Delta L_2 - \Delta L_1 = \tau \times L \times (\lambda_2 - \lambda_1) \quad [1.28]$$

The simple clearance of 0.015 initially calculated will be insignificant compared with the temperature difference τ such that the expression of temperature [1.28] can be deduced as follows:

$$\tau(L) = \frac{\Delta L_2 - \Delta L_1}{L \times (\lambda_2 - \lambda_1)} = \frac{\Delta L}{L \times (\lambda_2 - \lambda_1)} \quad [1.29]$$

Consider a $\varnothing 30.005$ -mm steel shaft with the linear expansion coefficient $\lambda_{\text{steel}} = 1.2 \times 10^{-5}$, contained in a bronze bore $\varnothing 29.99$, with $\lambda_{\text{bronze}} = 1.8 \times 10^{-5}$ (for Figure 19(b): $\lambda_{\text{aluminum}} = 2.3 \times 10^{-5}$).

For $\Delta L = 0.015$; $\lambda_1 = 1.2 \times 10^{-5}$ (steel) of $\varnothing 30.005$; $\lambda_2 = 1.8 \times 10^{-5}$ (bronze) of $\varnothing 29.99$; at length $L = 25$ mm, $\tau = 100^\circ$; at $L = 50$ mm, $\tau = 50^\circ$; and so on, here is a sample calculation generated using MathCAD software for L varying between 0 and 100 mm (Figure 1.19).

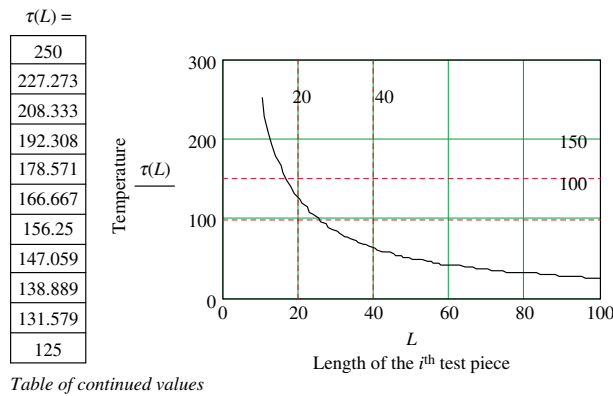


Figure 1.19. Impact of material dilatation on measurements

DISCUSSION.— Taking into account the room temperature, which is normally 20°C (experiment done by ourselves in heat treatment workshop), we note that local heating in the assembly gives it a heating temperature that is higher than:

$$\tau_{\text{heating}} = 83.333^\circ\text{C} + 20^\circ\text{C} = +103.333^\circ\text{C}$$

Thus, at 103.333°C , the system is blocked or even deteriorated in terms of materials. If the shaft had been in bronze and the bore in steel, that is, the steel part is located inside the bronze, the temperature decreases by 83.333°C (case 2) on the assembly:

$$\tau_{\text{contraction}} = 20^\circ\text{C} - 83.333^\circ\text{C} = -63.333^\circ\text{C}$$

The last case is often adopted because the friction is responsible for warming the parts in contact. This is a typical case analyzed in the course of analysis and material processing and heat treatments.

1.5.5.2. Problem set #2

The following problem is a real case studied in the workshop using tools and equipment of dimensional metrology laboratory. The machined alloy is aluminum-grade 6061, assembled at a construction steel grade SAE 1045. Often, when working

with light alloys such as the 6061, we may underestimate the expansion (or the contraction) of the material. The previous example is edifying. In this regard, a project case benefiting the Dental Hygiene Department (Cégep) led us to propose the analysis and the results from the viewpoint of dimensional metrology.

We wanted to make this montage possible to work with both Saharan and Nordic temperature *ranges*, that is, temperatures ranging from -05°C to $+50^{\circ}\text{C}$. The parts were computer numerical control (CNC) machined and controlled under conditions of temperature at 25°C for the 1045 container and 80°C for the 6061 container. The implemented metrological instrument used in first analysis is a max–min caliber.

After that, the control was performed twice on CMM and on form projector under normal conditions in terms of temperature ($20^{\circ}\text{C} \pm 2^{\circ}\text{C}$) and hygrometry ($65\% \pm 5\%$). The quality of the assembly according to CSA and ISO provides a good agreement. We then read (Figure 1.20):

$\varnothing 40 \text{ H7/u6}$

Under normal conditions, the above calculation by the functional dimensioning leads to reading², respectively, limiting values = min 0.035 and max 0.076. The average value = 0.0555. The dimension of the steel container is measured at 25°C using the gauge at $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$ induces an error of $+0.0019$ because when it falls back to 20°C , it shrinks by 0.0019.

Results provided by the Autodesk Inventor Pro 2009.

Fit symbol (ANSI)	1.574803 in.	H7/u6
Minimum diameter of the hole	Inches	1.57480 in.
Maximum diameter of the hole	Hole _{min}	1.57580 in.
Upper tolerance of the hole	Hole _{max}	0.00100 in.
Lower tolerance of the hole	Hole _{upper}	0.00000 in.
Minimum diameter of the shaft	Hole _{lower}	1.57780 in.
Maximum diameter of the shaft	Shaft _{min}	1.57840 in.
Upper tolerance of the shaft	Shaft _{max}	0.00360 in.
Lower tolerance of the shaft	Shaft _{upper}	0.00300 in.
Minimum interference diametric value of the fit	Shaft _{lower}	0.00200 in.
Maximum interference diametric value of the fit	Interference _{min}	0.00360 in.
Distance between midpoints of tolerance zones	Interference _{max}	-0.00280 in.

The results are expressed in inches (ANSI B4-11967 (R1974)), whereas the nominal diameter is expressed in millimeters (mm) (ISO 286-1:1988)

² To read the normalized values of adjustments, the reader may refer to tables presented in ISO 286-1 (1988) and ANSI B4.1. 1967 (R1974). See also the main tables presented in [OBE 96], Appendices of Chapter 2.

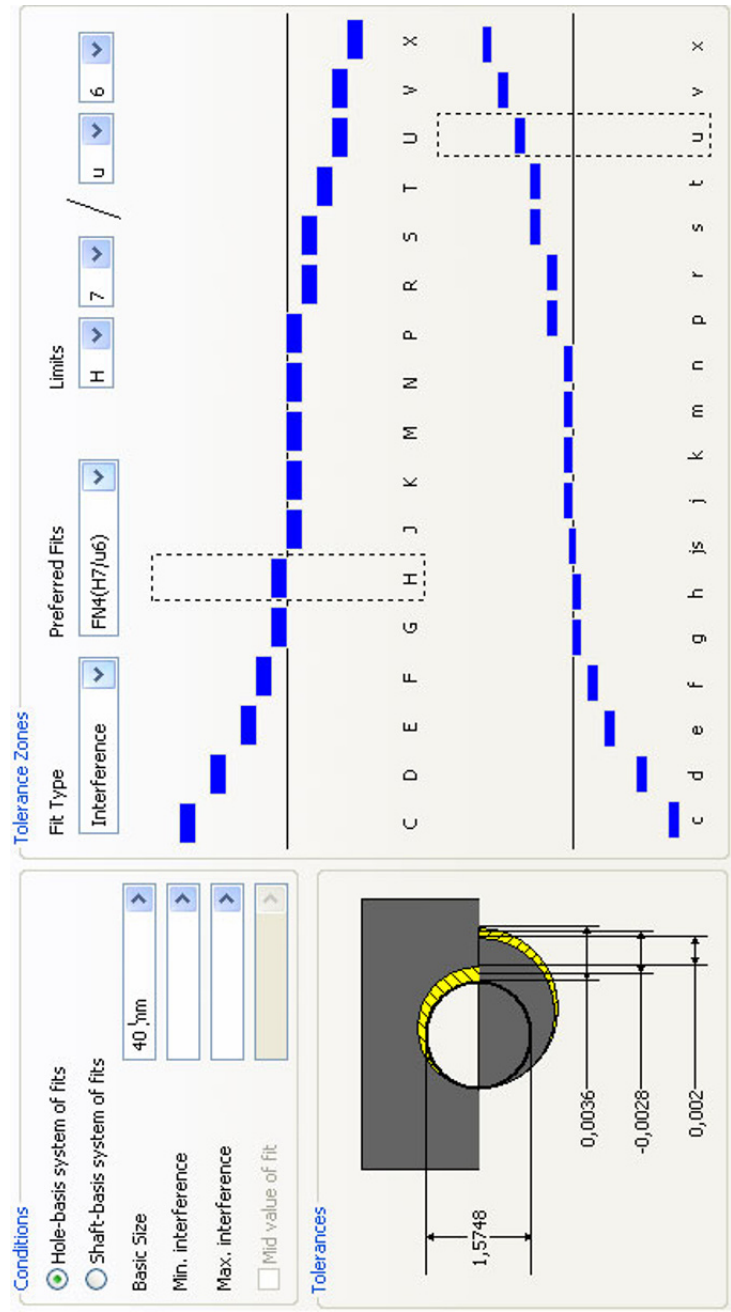


Figure 1.20. Result of a shaft-bore assembly: $\varnothing 40$ H7/h6 (ISO) or FN4 (CSA, Canada and ANSI, USA)

The dimension of the aluminum alloy content is measured at $80^{\circ}\text{C} \pm 2^{\circ}\text{C}$ (after heat treatment). The gauge, at 20°C , shows an error of $+0.0441$, and it contracts by 0.0441 .

First scenario: By reading Table 1.4a of adjustments quality (ISO), it shows $\varnothing 40$ H7/u6.

Corresponding real dimensions	Assembly at $\tau = -5^{\circ}\text{C}$
Container: steel 1045	Max = $40.0231 - 0.0096 = +40.0135$ Min = $40.9981 - 0.0096 = +39.9885$
Container: aluminum 6061	Max = $40.0319 - 0.0184 = +40.0135$ Min = $40.0159 - 0.0184 = +39.9875$
Max → Dimensions differences: Min →	Max = $40.0135 - 39.9975 = +0.016 \rightarrow$ clearance Min = $39.9885 - 40.0135 = -0.025 \rightarrow$ clamping
Conclusion on the assembly state at temperature $\tau = -5^{\circ}\text{C}$	Uncertain adjustment According to ISO 286-2

Table 1.4a. Uncertain adjustment of the shaft/bore assembly (see Appendix 1)

We notice that the *adjustment is uncertain* because the result is presented with a max clearance and min clamping.

Second scenario: From the assembly $\varnothing 40$ H7/u6, the adjustments shown in Table 1.4b occurs.

The corresponding real dimensions	Assembly at $\tau = 20^{\circ}\text{C}$
1045 steel container	After a contraction of 0.0019 Max = $40.025 - 0.0019 = +40.0231$ Min = $40.000 - 0.0019 = +39.9981$
6061 aluminum container	After a contraction of 0.0441 Max = $40.076 - 0.0441 = +40.0319$ Min = $40.060 - 0.0441 = +40.0159$
Max → Difference between dimensions: Min →	Max = $40.0231 - 40.0159 = +0.0072 \rightarrow$ clearance Min = $39.9881 - 40.0319 = -0.0338 \rightarrow$ clamping
Conclusion on the state assembly at temperature $\tau = 20^{\circ}\text{C} \pm 2^{\circ}\text{C}$	Uncertain (normal), according to ISO 286-2
Corresponding real dimensions	Assembly to the $\tau = 20^{\circ}\text{C}$

Table 1.4b. Calculating a normal fitting: shaft/bore (see Appendix 1)

Third scenario: From the assembly $\varnothing 400$ H7/u6, the adjustments are shown in Table 1.4c occurs.

Corresponding real dimensions	Assembly at $\tau = 50^{\circ}\text{C}$
1045 steel container	Max: $40.0231 + 0.0115 = + 40.0346$ Min: $39.9981 + 0.0115 = + 40.0096$
6061 aluminum container	Max: $40.0319 - 0.0184 = + 40.0540$ Min: $40.0159 - 0.0184 = + 40.0380$
Max $\rightarrow$ Difference between dimension: Min $\rightarrow$	Max: $40.0231 - 40.0159 = + 0.0072 \rightarrow$ clearance Min: $39.9881 - 40.0319 = - 0.0338 \rightarrow$ clamping
Conclusion on the assembly state at temperature $\tau = +50^{\circ}\text{C}$	Clamping (tendency to clamping) Heat treatment by banding

Table 1.4c. Calculating a clamp adjustment, shaft/bore (see Appendix 1)

DISCUSSION.— If we make a graphical representation of the three cases studied, we would find that these three cases of assembly do not meet the specifications $\varnothing 400$ H7/u6. This is true both under temperature 20°C and -5°C . Also the assembly does present the clamping only between temperatures ranging from $\tau_2 \cong 35^{\circ}\text{C}$ to 50°C . We note also that even this type of clamping is insufficient. Also the assembly with a low torque, at temperature 50°C , will have a clearance that may reach 0.016. This value is very important when temperatures fall to -5°C . We have not verified this in practice. This is the deduction from our simulated calculations.

1.6. Correction of possible measurement errors

When experiments were carried out on the shaft/bore assembly, the temperature in the metrology laboratory was maintained at normal ($\tau = 20^{\circ}\text{C} \pm 2^{\circ}\text{C}$). We ensured the coincidence of the temperatures of parts with the instruments and apparatus (micrometer and CMM). These experiments were conducted and repeated five times, under conditions deemed unchanged. The dimensions, materials, processing conditions on CNC; conventional, geometric constraints; and the ensuing heat treatments were observed in line with the characteristics of materials (SAE 1045 and 6061).

We found that there was no significant dimensional error on the assembly and that three days after deburring and heat treatment for standardization, the pieces were assembled with a slight tightening. The micrometer used for measurements is made of hardened steel. Other experiments were conducted without regard to the expansion coefficients. The temperature inside the metrology laboratory had been deliberately changed ($\tau > 20^{\circ}\text{C}$). This approach aimed at bringing potential corrections to the theoretical measures to be achieved; consider:

- τ_1 and $\lambda_{\text{instrument}}$ are, respectively, the temperature and expansion coefficient of the measuring instrument (micrometer stainless steel);
- τ_2 and λ_{piece} are, respectively, the temperature and expansion coefficient of the test piece (mild steel);
- L_m is the length resulting from the measurement by a micrometer.

Under normal conditions ($\tau_1 = 20^\circ\text{C}$), the instrument indicates the same reading. We emphasize this fact because the instrument contracts or expands without affecting reading. However, we should recall that this fact is limiting and concerns parts that are not included in precision engineering. This hypothesis is devised for the validity of the explanation on the expansion phenomena and their impact on errors and uncertainties involved.

We also note that the workpiece and the instrument do not have the same length at normal temperature ($\tau_1 = 20^\circ\text{C}$), because there is an intentional error. Let $L_{\alpha 20}$ be the length of the instrument at 20°C , and L_{p20} be the length of the workpiece at 20°C . We formulate the expression of the length of the instrument and length of the workpiece, respectively, as follows:

$$L_{\text{instrument}} = L_{\alpha(20^\circ)} \times \{ 1 + \lambda_1 \times (\tau_1 - 20^\circ) \} \quad [1.30]$$

$$L_{\text{piece}} = L_{\text{piece}(20^\circ)} \times \{ 1 + \lambda_2 \times (\tau_2 - 20^\circ) \} \quad [1.31]$$

In measuring, these lengths are equal, which allows us to write the following:

$$L_{\alpha(20^\circ)} \times \{ 1 + \lambda_1 \times (\tau_1 - 20^\circ) \} = L_{\text{piece}(20^\circ)} \times \{ 1 + \lambda_2 \times (\tau_2 - 20^\circ) \} \quad [1.32]$$

Using the formula for calculating the error, we consider:

$$\delta = L_{\alpha(20^\circ)} - L_{\text{piece}(20^\circ)} \quad [1.33]$$

For a given measured length ($L_\delta = L_m$), we set the expression of the error depending on the temperature difference and the length of the instrument $L_{\alpha 20}$. Substituting the respective equations [1.30] and [1.31] hence:

$$\begin{aligned} \delta &= (L_{\alpha(20^\circ)} - L_{\text{piece}(20^\circ)}) \times \left[\frac{\{ 1 + \lambda_1 \times (\tau_1 - 20^\circ) \}}{\{ 1 + \lambda_2 \times (\tau_2 - 20^\circ) \}} \right] \\ &= \delta \times \left[1 - \frac{\{ 1 + \lambda_1 \times (\tau_1 - 20^\circ) \}}{\{ 1 + \lambda_2 \times (\tau_2 - 20^\circ) \}} \right] \end{aligned} \quad [1.34]$$

NUMERICAL APPLICATIONS.— You are asked to calculate the error δ and the corrected length $L_2 = L_{p20}$ at 20°C. Calculate both the error δ due to temperature deviations and L_{p20} , the length of the piece at 20°C. We follow this assumption:

- $\tau_2 = \text{temperature of the aluminum piece in } ^\circ\text{C}; \lambda_{\text{aluminum}} = 2.3 \times 10^{-5};$
- $\tau_1 = \text{temperature of the measuring instrument in } ^\circ\text{C}; \lambda_{\text{instrument}} = 1.2 \times 10^{-5};$
- $L_\delta = L_m = \text{measured length in millimeters or in inches.}$

Explanatory approach to the numerical application: we now use equation [1.34] to simulate three scenarios that support our previous reasoning. For this, we use the MathCAD software.

We assume that the micrometer is made of special steel with an expansion coefficient $= \lambda_{\text{instrument}}$ and also the parts are in aluminum alloy (1, 2) and bronze. Expansion coefficients would be, respectively, $\lambda_{\text{aluminum}(1)}$, $\lambda_{\text{aluminum}(2)}$, and λ_{bronze}

Case 1: $\lambda_{\text{aluminum}(1)} = 2.3 \times 10^{-5}; t_2 = 30; t_1 = 25; L_1 = 5 \dots 25; \lambda_{\text{instrument}}(\text{steel}) = 1.2 \times 10^{-5}$

$$\delta(L_1) = L_1 \times \left[1 - \frac{\{1 + \lambda_{\text{instrument}} \times (\tau_1 - 20^\circ)\}}{\{1 + \lambda_{\text{aluminum}(1)} \times (\tau_2 - 20^\circ)\}} \right] \text{mm} \Rightarrow L = 20 \text{ mm}, \delta(L) = 3.399 \times 10^{-3}$$

The expansion relative to $L = 20 \text{ mm}$, $\delta(L)$ is approximately 3.399×10^{-3} . Though this variation is not significant to the naked eye, it should be reported.

Case 2: $\lambda_{\text{aluminum}(2)} = 2.3 \times 10^{-5}; T_2 = 30; T_1 = 18; L_1 = 5 \dots 25; \lambda_{\text{instrument}}(\text{steel}) = 1.2 \times 10^{-5}$

$$\delta(L_1) = L_1 \times \left[1 - \frac{\{1 + \lambda_{\text{instrument}} \times (\tau_1 - 20^\circ)\}}{\{1 + \lambda_{\text{aluminum}(2)} \times (\tau_2 - 20^\circ)\}} \right] \text{mm} \Rightarrow L = 20 \text{ mm}, \varepsilon(L) = 4.599 \times 10^{-3}$$

The expansion relative to L , $\varepsilon(L)$ is approximately 4.599×10^{-3} . Even if this deviation is not significant to the naked eye, it should be reported.

Case 3: $\lambda_{\text{bronze}(1)} = 1.8 \times 10^{-5}; \tau_2 = 30; \tau_1 = 20; L_1 = 5 \dots 25; \lambda_{\text{instrument}}(\text{steel}) = 1.2 \times 10^{-5}$

$$\delta(L_1) = L_1 \times \left[1 - \frac{\{1 + \lambda_{\text{instrument}} \times (\tau_1 - 20^\circ)\}}{\{1 + \lambda_{\text{bronze}} \times (\tau_2 - 20^\circ)\}} \right] \text{mm} \Rightarrow L = 20 \text{ mm}, \Delta(L) = 4.079 \times 10^{-3}$$

The extension relative to $L = 20$ mm, $\Delta(L)$ is approximately 4.079×10^{-3} . Even if this deviation is not significant to the naked eye, it should be reported.

We note that the error due to the expansion effect increases proportionally with the temperature, the material, and the length of the piece:

$$\text{Error} = \text{length of the piece after dilatation} - \text{length of the piece at } 20^\circ\text{C}$$

Figure 1.21 shows the result of L ranging between 5 and 25 mm.

1.6.1. Overall error and uncertainty

In brief, we note that the errors of different natures are combined. It is often difficult to delineate their distribution to deduce the overall error. It is suggested to group similar errors; however, sometimes we infer an overall error, which considers all the errors. This overall error is called algebraic sum of the constituent errors. Consider the following example:

- δ_a , error due to the apparatus: +0.0005;
- δ_r , error due to reading: –0.0010;
- δ_h , error due to handling the device: –0.0060;
- δ_t , error due to differences in temperature (t°): +0.0160 (calculated).

$$\delta_{\text{total}} = \sum_{i=1}^n \text{Errors} \quad [1.35]$$

$$\delta_{\text{total}} = \sum_{i=1}^{n=4} \text{Errors} = (+0.0005) - (0.001) - (0.006) - (0.016) = 0.0095 \text{ mm}$$

The correction to be made would be of opposite sign (–0.0095) calculated value.

The uncertainty will then be $\pm 0.0005 \pm 0.001 \pm 0.006 \pm 0 = \pm 0.005$.

$L_1 =$	$\delta(L_1) =$	$\Delta(L_1) =$	$\varepsilon(L_1) =$
5	$8.498 \cdot 10^{-4}$	$1.02 \cdot 10^{-3}$	$1.15 \cdot 10^{-3}$
6	$1.02 \cdot 10^{-3}$	$1.224 \cdot 10^{-3}$	$1.38 \cdot 10^{-3}$
7	$1.19 \cdot 10^{-3}$	$1.428 \cdot 10^{-3}$	$1.61 \cdot 10^{-3}$
8	$1.36 \cdot 10^{-3}$	$1.632 \cdot 10^{-3}$	$1.84 \cdot 10^{-3}$
9	$1.53 \cdot 10^{-3}$	$1.836 \cdot 10^{-3}$	$2.07 \cdot 10^{-3}$
10	$1.7 \cdot 10^{-3}$	$2.04 \cdot 10^{-3}$	$2.299 \cdot 10^{-3}$
11	$1.87 \cdot 10^{-3}$	$2.244 \cdot 10^{-3}$	$2.529 \cdot 10^{-3}$
12	$2.04 \cdot 10^{-3}$	$2.448 \cdot 10^{-3}$	$2.759 \cdot 10^{-3}$
13	$2.209 \cdot 10^{-3}$	$2.652 \cdot 10^{-3}$	$2.989 \cdot 10^{-3}$
14	$2.379 \cdot 10^{-3}$	$2.855 \cdot 10^{-3}$	$3.219 \cdot 10^{-3}$
15	$2.549 \cdot 10^{-3}$	$3.059 \cdot 10^{-3}$	$3.449 \cdot 10^{-3}$
16	$2.719 \cdot 10^{-3}$	$3.263 \cdot 10^{-3}$	$3.679 \cdot 10^{-3}$
17	$2.889 \cdot 10^{-3}$	$3.467 \cdot 10^{-3}$	$3.909 \cdot 10^{-3}$
18	$3.059 \cdot 10^{-3}$	$3.671 \cdot 10^{-3}$	$4.139 \cdot 10^{-3}$
19	$3.229 \cdot 10^{-3}$	$3.875 \cdot 10^{-3}$	$4.369 \cdot 10^{-3}$
20	$3.399 \cdot 10^{-3}$	$4.079 \cdot 10^{-3}$	$4.599 \cdot 10^{-3}$
21	$3.569 \cdot 10^{-3}$	$4.283 \cdot 10^{-3}$	$4.829 \cdot 10^{-3}$
22	$3.739 \cdot 10^{-3}$	$4.487 \cdot 10^{-3}$	$5.059 \cdot 10^{-3}$
23	$3.909 \cdot 10^{-3}$	$4.691 \cdot 10^{-3}$	$5.289 \cdot 10^{-3}$
24	$4.079 \cdot 10^{-3}$	$4.895 \cdot 10^{-3}$	$5.519 \cdot 10^{-3}$
25	$4.249 \cdot 10^{-3}$	$5.099 \cdot 10^{-3}$	$5.749 \cdot 10^{-3}$

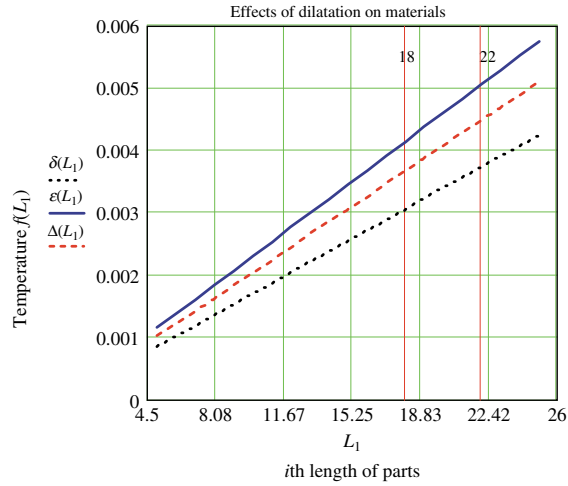


Figure 1.21. Effect of dilatation on materials

1.6.2. Uncertainty due to calibration methods

Measurement uncertainties are parameters characterizing the dispersion values during the measurement. Therefore, the study of uncertainties [MUL 81, TAY 05] aims to determine the capabilities of measuring means. When setting up a calibration method, we should proceed to the metrological qualification of the method. This operation is based on technical tests and on objective analysis of the causes of uncertainty. This uncertainty is determined from several components, particularly those of assembly standards, involved instruments, and environmental factors. Uncertainty related to the instrument to be calibrated is determined based on the characteristics of this instrument including measurement *faithfulness* and *reading error*.

In general, measurement uncertainty comprises several components. Some of these errors may be estimated based on the statistical distribution of the series of measurements results and can be characterized by experimental standard deviation. The estimation of other factors can be based only on the experimentation or other information that the mathematical formalism may refer to as *a priori* functions.

“Determining the measurement uncertainty allows estimating the likelihood of reporting a non-conforming product as conform or a conform product as a bad one.”

1.6.3. Capability of measuring instruments

This notion is important as it provides information on the degree of agreement that links the performance of a measuring device to the value of tolerance to be verified. The term “capability” is equivalent to the French term “capabilité.” Some works define the “capability” as “adequacy between the tolerance interval and the overall uncertainty of measurements.” Capability is also a concept used in Quality Control. Methods of *capability* have been developed by car manufacturers (AIAG) in the United States. They are applicable to other sectors of the industry and do not contradict the GPS standards.

We know, based on experience, that the choice of high-performance measuring means inevitably induces prohibitive cost and thus an *over-quality*. For example, we do not systematically use a means of control by CMM when a simple micrometer would be sufficient. The opposite is also true because if we do not choose the adequate means of control, there is a risk of unacceptable discard. Some dimensions stemming from definition drawing with strict tolerances are difficult to achieve in practice, so they are *de facto* discarded during the control. The choice of the device is therefore subject to the tolerance test. In mechanical manufacturing, the capability index is formulated as follows:

$$C_p = \left\{ \frac{T_U - T_L}{6 \times \sigma} \right\} \quad [1.36]$$

T_U is the upper tolerance, and T_L is the lower tolerance. Knowing σ is the standard deviation of the series of produced pieces and μ is the arithmetic mean, the coefficient of capability C_{pk} is calculated by

$$C_{pk} = \left\{ \frac{T_U - \mu}{3 \cdot \sigma} \right\} \text{ and } \left\{ \frac{T_L - \mu}{3 \cdot \sigma} \right\} \quad [1.37]$$

In metrology, the index of capability C_{mm} of measuring means is written:

$$C_{pk} = \left\{ \frac{TI}{6 \cdot UG} \right\} \quad [1.38]$$

where TI is the interval of tolerance and UG is the overall uncertainty. Because of this overall uncertainty, we develop an approach to explain it and try to show how to calculate it depending on various cases.

1.7. Estimation of uncertainties of measurement errors in metrology

1.7.1. Definitions of simplified equations of uncertainty measurements

The culture of dimensional metrology has long been associated with the caliper and micrometer, as well as the gauge block. We should quickly get out of this restrictive culture where, for a long time, metrology was almost confined only to machine shops. To attain the goal of quality, appropriate measurements should be carried out to achieve the objectives. The teaching of metrology must not only respond to measurements without adequate explanations of errors and formalism of uncertainties. In the following, we attempt to model the uncertainties of measurement errors and then support them with examples of application in the form of workshops. Measurement uncertainty denoted by U is a parameter associated with the measurement result that characterizes the dispersion of values that can reasonably be attributed to the object responsible for the measured value. The measurement uncertainty is the result of the combined effects of components, sources of uncertainty, commonly known as influence quantities. In this matter, the ISO/IEC 17025 standard mentions the estimation of uncertainty. According to the GUM [NIS 94], the overall uncertainty includes, in principle, all the factors that affect the result. Compound uncertainty results from the calculation of compound uncertainties according to the approximate propagation law:

- the *standard uncertainty* is usually expressed as a standard deviation (S , *standard deviation*);
- the standard deviation is the dispersion of n measurements results, based on the same measurand, around the arithmetic mean $\bar{x}$ of n results. x_i is the result of the i th measurement.

The coefficient of variation S_R designates the standard deviation divided by the mean (S/μ). It is, therefore, the standard deviation given as the relative value (%) and not absolute in the specified unit of measurement. S_R refers to the coefficient of variation for *repeatability* and *reproducibility*.

Repeatability is the closeness of agreement (usually expressed as standard deviation) between the results of successive measurements based on the same measurand and carried out under identical measurement conditions. The measurements are performed under the same conditions are termed repeatability, that is, same process, same operator, same apparatus (instrument) used under the same conditions, the same place, at the same time, and with the same quantitative expressions of the results. Furthermore, the measurements are replicated over a short period of time.

Reproducibility is the closeness of agreement (usually expressed as standard deviation) between the results of successive measurements based on the same measurand and carried out under different measurement conditions (to be specific:

principle, method, operator, device, apparatus, reference material (standard), place, conditions, time (date), and quantitative expression of the result).

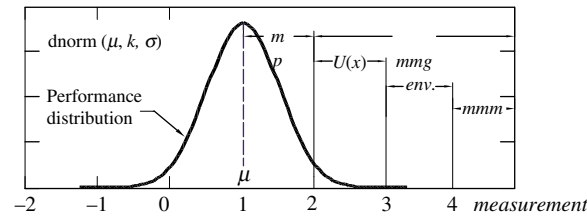


Figure 1.22. Illustration of measuring specifications

In Figure 1.22, mp is the margin of production, $U(x)$ is the measurement uncertainty, $env.$ is a deviation due to the environment, mmm is the certain flexibility degree in metrology, and mmg is the overall flexibility.

Note that the results shown here are assumed to be already corrected.

Expanded uncertainty is a quantity that defines an interval around the measurement result that can be expected to include a high proportion of the distribution of values that could reasonably be attributed to the quantity being measured. It is actually a multiple of the standard deviation S or the overall uncertainty U , via a coverage factor k . For example, using a factor $k = 2$ means, statistically, that the value “reasonably attributable” to the measured object is found with a probability or “level of confidence” of about 95% within the interval more or less double the amount S responsible for double U amount around the measured value. With a factor $k = 3$, the confidence level is approximately 99.7% on a normal distribution. The coverage factor k is a multiplier of the compound standard uncertainty to obtain the expanded uncertainty.

1.7.2. Issue of mathematical statistics evaluation of uncertainties in dimensional metrology

While measuring, various uncertainties emerge. The latter have already been defined earlier. When overall uncertainty combined together, it leads to the overall compound uncertainty (IGC). The latter is written as $u_c^2(y)$. An IGC is a polyfactorial overall function, which is actually a result revealed with all input quantities X_i . It is actually the mathematical model that defines and reflects the measurand, the measurement method, and the operating procedure. Note that in many cases, the measurand is not derived directly but rather determined from n polyfactorial input quantities of the function Y [DIX 51, NIS 94, GUI 00, PRI 96]:

$$Y = f\{X_1, X_2, X_3, \dots, X_n\} \quad [1.39]$$

For example, equation [1.23] did not only represent the physical law but also the measurement process. This results in expressing all the quantities that contribute significantly to the result uncertainty, by the equation $Y = kx$ (unit). The simplified form of the expression derived from $u_c^2(y)$ in the practice of dimensional metrology is reduced to simple forms hereinafter explained.

Let A denote the sum of X_i quantities multiplied by constant factors a_i to give:

$$Y = a_1X_1 + a_2X_2 + a_3X_3 + \dots + a_nX_n \quad [1.40]$$

Results of various measurements are expressed as:

$$y = a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n \quad [1.41]$$

Each of these measurements is affected by uncertainties. The compound standard uncertainty is then expressed as:

$$u_c^2(y) = a_1^2 \cdot u^2(x_1) + a_2^2 \cdot u^2(x_2) + a_3^2 \cdot u^2(x_3) + \dots + a_n^2 \cdot u^2(x_n) \quad [1.42]$$

The equation of measurements is expressed as the product of quantities X_i exponentiated to $a, b, \dots, q$, and multiplied by a constant A :

$$Y = \text{Cst}(A) \times X_1^a \cdot X_2^b \cdot X_3^c \cdot \dots \cdot X_n^q \quad [1.43]$$

The IGC can, in its turn, be written as:

$$y = \text{Cst}(A) \times x_1^a \cdot x_2^b \cdot x_3^c \cdot \dots \cdot x_n^q \quad [1.44]$$

Compound standard uncertainty is written as:

$$u_{\text{cr}}^2(y) = a^2 \cdot u_{\text{cr}}^2(x_1) + b^2 \cdot u_{\text{cr}}^2(x_2) + c^2 \cdot u_{\text{cr}}^2(x_3) + \dots + q^2 \cdot u_{\text{cr}}^2(x_n) \quad [1.45]$$

In this case, $u_r(x_i)$ expresses the relative uncertainty of x_i and is defined by the ratio:

$$u_r(x_i) = \frac{u(x_i)}{|x_i|} \quad [1.46]$$

where $|x_i|$ is the absolute value of the $x_i \neq 0$. $u_{cr}(y)$ is the compound relative uncertainty defined as:

$$u_{cr}(y) = \frac{u_c(y)}{|y|}, \text{ with } |y| \neq 0 \quad [1.47]$$

1.7.3. Uncertainty range, coverage factor k and range of relative uncertainty

If the distribution probability characterized by the measurements result y , and $u_c(y)$ standard deviation and its compound standard uncertainty are approximated by the expression of the Gaussian, and if $u_c(y)$ is estimated, then the interval $\{y - u_c(y)\}$ to $\{y + u_c(y)\}$ contains approximately 68% of the distribution of the true value. Understandably, within this 68% confidence interval, Y is greater than or equal to $y - u_c(y)$, and it is less than or equal to $y + u_c(y)$, usually written as [GUI 00, NIS 94, PRI 96]:

$$Y = y \pm u_c(y) \quad [1.48]$$

The range of the compound uncertainty u_c (*expanded uncertainty*) expresses the uncertainty of results of various measurements and their regularity as for consistency of materials and structures [GRO 94]. In this case, the range of uncertainty U is obtained by multiplying $u_c(y)$ by a factor called the coverage factor k :

$$U = k \cdot u_c(y) \quad [1.49]$$

with $Y \geq \{y - U\}$ and $\{y + U\}$, where Y is:

$$Y = y \pm U \quad [1.50]$$

Coverage factor k is usually chosen according to the desired confidence interval, with respect to the confidence interval defined by:

$$U = k \cdot u_c \quad [1.51]$$

Coverage factor k is of order 1, 2, and 3 when the normal distribution and u_c are consistent estimators of the standard deviation of y . For example:

$U = 1 \times u_c$ ($k = 1$) is defined within the confidence interval of 68%;

$U = 2 \times u_c$ ($k = 2$) is defined within the confidence interval of 95%;

$U = 3 \times u_c$ ($k = 3$) is defined within the confidence interval greater than 99%.

By analogy with the range of relative uncertainty u_r and the standard uncertainty previously defined by the expression $u_c^2(y)$, the range of the uncertainty regarding measurements y is calculated as:

$$U_r = \frac{U}{|y|}, \text{ with } y \neq 0 \quad [1.52]$$

It should be noted that the Gaussian distribution is not always valid to model any process. We cite, for example, the accidental random events where the Weibull law [GRO 95] would be more appropriate in continuum mechanics. The aim of the measurements, subject to terms of this type, is to obtain an estimate of x_0 as accurately as possible; the *true value* perpetually sought. We realize, by examining the bell curve, that the estimate will be as much accurate as the parent distribution $P(\mu \text{ and } \sigma)$ is tight, that is to say that σ is smaller.

The measurement method, the equipment used, and the experimenter's skill contribute to the magnitude of σ . The symbolism used in the context of this book complies with those generally used in both English and French handbook. This choice is deliberate and intended to avoid confusion.

As these quantities are used to express the components of uncertainty, we opted for the following expressions:

$$\left\{ \begin{array}{l} \text{variance } (X) = V(X) = u^2(X); \\ \text{standard uncertainty } u(x) = \sqrt{u^2(x)}; \\ \text{compound uncertainty} = u_c(y); \\ \text{extended uncertainty } U = k \cdot u_c(y), (k, \text{convergence coefficient}); \\ \text{range of uncertainty } Y = y \end{array} \right.$$

DISCUSSION.— It has been succinctly demonstrated that the range of uncertainty depends strongly on the overall compound uncertainty. The simplified form of the expression driven from $u_c^2(y)$ in the practice of dimensional metrology is reduced to simple forms. However, it is always beneficial to follow succinct steps. Nevertheless, we should be bear in mind that measurement uncertainty generally includes several components.

Some uncertainties can be easily calculated using an appropriate method. Other features can be estimated through their respective standard deviations. Finally, modeling the measurement process remains essential to properly facilitate the analysis, or even the understanding of physical realities and multiple influence factors.

1.8. Approaches for determining type A and B uncertainties according to the GUM

1.8.1. Introduction

According to the VIM, the ISO standard describes the uncertainty of a measurement as a “parameter associated with the result of a measurement, which characterizes the dispersion of the values that could reasonably be attributed to the measurand” [GUI 00, NIS 94].

Various guides including the GUM [GUI 00] clearly describe how to:

- assess, separately, the contribution of each source of uncertainty;
- combine the various contributions;
- report the uncertainty of the measurement result.

The objective through measurements under the “uncertainty” approach is not to determine the *true value*. Yet, this approach recognizes that the information obtained during the measurement gives only an *interval of values attributed to the measurand*. Whatever the accuracy of the measuring process, it cannot reduce the interval to a single value because of the inherently finite amount of details. A definitional uncertainty imposes a lower limit to any measurement uncertainty. The interval can be represented by one of its values, called the *measured value*. In the GUM, the *definitional uncertainty* is assumed to be negligible with respect to the considered measurement uncertainty.

The measurand can then be represented by an *essentially unique* value. The objective of the measurement is to establish probabilities that the specified measured values are consistent with the definition of the measurand, based on these measurements. Hence, the GUM defines two methods for estimating uncertainty: type A uncertainty uses statistical means with repeated measurements and calculating standard deviations and type B uncertainty uses other anterior calibration data, repeatability, reproducibility, intercomparisons, published constants, and so on. Data are expressed in terms of standard deviation values or intervals – *a priori* law. The compound uncertainty is then calculated by combining the uncertainties under the propagation law of the computational function of the result [1.45]. The compound uncertainty $U(y)$ is the square root of the sum of the partial derivatives squared (correlations excluded).

$$U_{\bar{y}}^2(\text{compound}) = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \cdot U_{x_i}^2 \quad [1.53]$$

Partial derivatives $\partial f / \partial x_i$ are called *sensitivity coefficients* expressed by the dispersion (*variance*). The compound uncertainty is in fact the expression of the standard deviation formulated hereinafter:

$$U_{y(\text{compound})} = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \cdot U_{x_i}^2} \quad [1.54]$$

We now suggest some properties of simple functions of the compound uncertainty.

1.8.2. Properties

- For $Y = A + B$, we write $U_c(y) = \sqrt{U_A^2 + U_B^2}$.
- For $Y = A - B$, we write $U_c(y) = \sqrt{U_A^2 + U_B^2}$.
- For $Y = A \times B$, we write $U_c(y) = \sqrt{B^2 \cdot U_A^2 + A^2 \cdot U_B^2}$.
- For $Y = A / B$, we write $U_c(y) = \sqrt{U_A^2 / B^2 + A^2 \cdot U_B^2 / B^4}$.
- For $Y = k \times A$, we write $U_c(y) = k \times U_A$.

It is convenient to use the absolute values of U for such dimensional sensitivity coefficients. However, relative values are used in expressions incorporating sensitivity coefficients (typically C unit = 1, i.e. dimensionless). The uncertainty assessment is made using the rules of error propagation or by default according to appropriate formulas (*a priori* laws). The GUM approach is succinctly summarized in Table 1.5 [DIX 51, GUI 00].

It is unrealistic to evaluate separately the impact of each factor on uncertainty. Evidence has shown that uncertainties resulting from the analytical method are often too low. Therefore, it does not seem useful, for testing laboratories, to do the calculation of the uncertainty based strictly on the procedural approach of the GUM, summarized from 1 to 8. Replicated measurements are used for each influence quantity via equations that may be complex. The parameter can be, for example, a standard deviation (or a multiple of it) or half-width of an interval having a stated confidence level. Above all, it is useful to sum linear uncertainties, that is, to apply the propagation law considering that all uncertainties are correlated.

Based on the foregoing, the task consists of determining each quantity x_i along the standard uncertainty $u(x_i)$ associated with it. Therefore, the law of propagation of uncertainties allows us to calculate the compound variance $u_c^2(y)$. Hence, the compound standard deviation is $U_c(y)$. The expanded uncertainty (U) is then defined

by equation [1.51], which is obtained by multiplying the compound standard deviation by the coverage factor k . The value of the coverage factor is linked to the desired probability as presented earlier.

No.	Analytical approach of the GUM (in eight distinct stages)
01	Formulate the result y as a function of input quantities $(x_1, x_2, \dots, x_n)$ $y = f(x_1, x_2, \dots, x_n)$
02	Identify input data/quantities (measurements, data specifications)
03	Determine the uncertainty on each input data/quantity of type A and/or B
04	Identify covariance, that is, the correlations between the effects of different sources of uncertainty on data/quantity, by default. Correlations are generally neglected, which may distort calculations of uncertainty
05	Calculate the result $f(x_1, x_2, \dots, x_n)$ of the measurement based on the input quantities $(x_1, x_2, \dots, x_n)$
06	Calculate the compound uncertainty with data in point 3, correlations excluded. For simplicity, they are calculated according to the law of uncertainty propagation based on the mathematical formula of the result
07	Calculate the expanded compound uncertainty with $k = 2$, (e.g. two times the compound uncertainty)
08	Deliver the result of the measurement y with the expanded compound uncertainty. Indicate $k = 2$, for a “confidence level” approximately 95% for the Gaussian, for example, $k \cdot U_c(y)$

Table 1.5. *An eight-stage summarization of the GUM approach*

As just demonstrated, uncertainties are evaluated based on their different components, because in the process of evaluating measurement uncertainty, we have to estimate the standard uncertainties $u(x_i)$ or the variance $u_c^2(x_i)$ corresponding to each of the components that intervene in the evaluation of compound uncertainty $U_c(y)$. It is important to consider each case as being specific to the laboratory in which measuring tests were performed. It was judicious to propose, in the manual [GUI 00] ISO/CEI/OIML/BIMP, some of the different components of uncertainty. In addition to clearly defining imperfect realizations of the measurand, this guide addresses the following issues:

- May the measured sample not represent the defined measurand?
- Is knowledge of the effects of environment on the measurement procedure correct?
- Is the resolution of the instrument consistent?
- Are the values and reference materials assigned to standards?
- Are the assumptions of the measurement methods and procedures appropriately approximated?

– Are the conditions underlying repeated observations of the measurand identical?

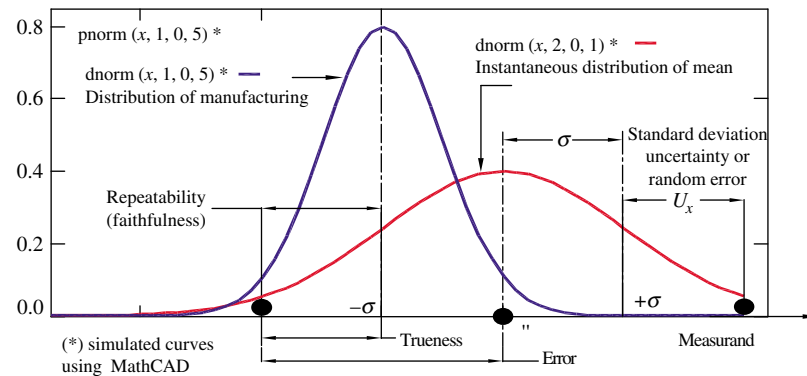


Figure 1.23. Illustrative diagram for methods A and B application

It is accepted that various sources of uncertainties are not all independent. Some of them contribute to the dispersion of observations and repeated measurements. Each of these uncertainties contributes proportionately to the compound uncertainty. When calculating the measurement uncertainty, one of the troubles the metrologist may face is to identify all the components x_i that have an impact on the measurement result and quantify their standard uncertainty. To evaluate the numerical value of the latter, the International Bureau of Weights and Measures (IBWM) suggests two frequently used methods: type A and type B (Figure 1.23).

Type-A methods are based on the variability of the measurement result hence the need of statistical approach to analyze them (*repeatability conditions and reproducibility conditions earlier explained*). It is a comprehensive statistical approach of n measurements. The mean $\mu = E(x)$ represents the measurement result, and the standard deviation is the uncertainty u_x of a series of n replicated measurements of $(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$ input values. Ideally, n tends to ∞ . We therefore have practical test samples to analyze statistically using a representative law.

Type-B methods do not involve statistical approaches, essential for causes that do not induce variation in terms of multifactorial result. An assessment of errors, for example, systematic errors (such as parallax error and the zero adjustment of the device) or random errors (such as reading error and apparatus error) is carried out. Under this method, we apply the physical study or the hypothesis of distribution laws, called *a priori* laws (e.g. uniform law). A single measurement provides the measurement value, then an analytical approach to uncertainty sources: measurement conditions, calibration of apparatus, implying the understanding of the physics of measurement. In fact, this method covers all that is not linked to

statistics (specification, calibration certificates, influence factor, and so on). In the following sections, we provide a simplified presentation of type A and B methods. Then, we present workshop examples.

1.8.3. Brief description of type-A uncertainty evaluation method

Under the same experimental conditions, the repetitions give rise to mathematical dispersions on numerical values of the measure repeated n times. Assume that the measuring method has a good resolution. The arithmetic mean μ calculated based on individual values x_i can give a good estimate of the expectation value of the population under consideration, with n independent values v_{ij} . As discussed earlier, the repetition of n measurements is approximated by the arithmetic mean μ . This means that each of the values x_i , as a random value, constitutes a random variable. We calculate the variance on the arithmetic mean, by applying the law of propagation of uncertainties, earlier formulated and which can be expressed as [NIS 94, PRI 96]:

$$U_c^2(\mu) = \left(\frac{1}{n}\right)^2 \cdot u^2(x_1) + \left(\frac{1}{n}\right)^2 \cdot u^2(x_2) + \left(\frac{1}{n}\right)^2 \cdot u^2(x_3) + \dots + 2 \cdot \sum_i \sum_j \left(\frac{1}{n}\right) \cdot \left(\frac{1}{n}\right) \cdot \{u(x_i \cdot x_j)\} \quad [1.55]$$

Let R be the correlation coefficient between the two values x_i and x_j :

$$u(x_i, x_j) = R \{u(x_i), u(x_j)\} \quad [1.56]$$

Note that in non-correlation, that is, independent probabilities, the Monte Carlo simulation [GRO 94, GRO 95, GUI 00] would be most appropriate to our study. If $u(x_i) = \sigma^2$ is the variance of the population of the series of experiments, after simplification, we obtain:

$$u^2(\bar{x}) = \left\{ \left(\frac{1}{k}\right)^2 \times k \cdot \sigma^2 + k(k-1) \times \left(\frac{1}{k^2}\right) \times \sigma \cdot R \right\} = \left\{ \frac{\sigma^2}{k} + \left(\frac{k-1}{k}\right) \times (\sigma^2 \cdot R) \right\} \quad [1.57]$$

Note that when observations on the measurements are independent ($R = 0$), then equation [1.57] becomes simply the following formula:

$$u^2(\bar{x}) = \left\{ \sigma^2 \frac{1}{k} \right\} \quad [1.58]$$

When observations on the measurements are completely correlated, that is, $R = I$, then equation [1.58] would be:

$$u^2(\bar{x}) = \sigma^2 \quad [1.59]$$

The estimator is therefore expressed as the mean formula:

$$x_j = \frac{1}{k} \cdot \sum_{i=1}^k x_{ij} \quad [1.60]$$

The experimental variance estimator is then expressed by:

$$S_{xj} = \sqrt{\left(\frac{1}{k-1}\right) \cdot \sum_{i=1}^k (x_{ij} - \bar{x}_{ij})^2} \quad [1.61]$$

Standard deviation formula is represented as follows:

$$u(x_i) = S(\bar{x}) = \sqrt{\left(\frac{1}{k-1}\right) \cdot \sum_{i=1}^k (x_{i,k} - \bar{x}_i)^2} \quad [1.62]$$

In a laboratory scenario, several experiments $\{k_1, k_2, \dots, k_i\}$ have been conducted and the corresponding estimates $\{S_1^2, S_2^2 \dots S_i^2\}$ were calculated; the expression of the variance of the total population can be obtained by combining these different estimators:

$$S^2 = \left\{ \frac{(k_1 - 1) \times S_1^2 + (k_2 - 1) \times S_2^2 + \dots + (k_i - 1) \times S_i^2}{(k_1 - 1) + (k_2 - 1) + \dots + (k_i - 1)} \right\} \quad [1.63]$$

By introducing the number of degrees of freedom (λ_i : *d.d.l*) so that $\{\lambda_i = k_i - 1\}$, the estimator is written:

$$S^2 = \left\{ \frac{\lambda_1 \times S_1^2 + \lambda_2 \times S_2^2 + \dots + \lambda_i \times S_i^2}{\lambda_1 + \lambda_2 + \dots + \lambda_i} \right\} \quad [1.64]$$

This method allows us to calculate the repeatability component $u^2(\mu)$ by:

$$u(\bar{x})^2 = \frac{S^2}{k} \quad [1.65]$$

During the measurement process, the operator carried out a single measurement, $u(\mu)^2$ then becomes

$$u^2(\bar{x}) = S^2 \quad [1.66]$$

Equations [1.65] and [1.66] indicate that the repeatability of the measurement process must be estimated by several tests prior to commissioning. We find under the foregoing that the type-A method is based on the statistical approach because of the multiplicity of input values.

1.8.4. Type-B uncertainty methods

Type-B methods quantify the uncertainties of the various components involved in the measurement process. Under this aspect, we may cite the uncertainties on corrections of *calibration and even those related to the environment*. *Type-B methods* are often used to exclude the statistical approaches. It is for this reason that many laboratory operators, reluctant toward mathematics, lean to this method. It is based on the experience gained by operators as it also relies on tests and knowledge of physical phenomena [NIS 94].

To study these methods, we therefore confine ourselves to a concrete *experimental case* on the measurement process. We must first recall that for each variable x_j involved in the measurement process, we formalize the corresponding standard uncertainties, with the features outlined earlier.

Beforehand, we should know the law of distribution of the measured variables and the range of each of which. In fact, some literatures suggest that type-B methods are not really experimental. This method just ignores the statistical methods themselves while using the “theoretical” results of the latter.

In general, if the measuring device manufacturer provides the standard uncertainty, then it should be used directly. For example, if the tolerance of a ruler of 30 mm is 0.025 mm, then uncertainty will be $U = U_r$ for the type B error: from [1.52]:

$$U_r = \frac{U}{|y|}$$

Assuming a uniform distribution (law):

$$U_r = \frac{\Delta c}{\sqrt{3}} = \frac{0.025}{\sqrt{3}} = 0.014$$

If the error is approximated by the Gaussian, which is very common (cumulative errors), then we apply the normal distribution to calculate the uncertainty U : for $\Delta c = 0.025$ (Table 1.6) for Gaussian law:

$$U = \frac{\Delta c}{3} = \frac{0.025}{3} = 8.333 \times 10^{-3}$$

In fact, the type-*B* method exploits the statistical data provided by the manufacturer through a shifty singularity of the experiment rather than that of the measuring instrument based on which the manufacturer provides us with a typical and specific uncertainty. If we attempt, in a measurement process, to make correction, the latter x_i would be *de facto* unknown. We only know the limits within which it would fall, that is, between the dimension C_{ll} : lower limit and the dimension C_{ul} : upper limit.

We know from the literature [ACN 84, GUI 00, PRI 96] that the value of the correction will be estimated by the expression of x_i :

$$x_i = \left\{ \frac{1}{2 \times (C_{\text{lower limit}} - C_{\text{upper limit}})} \right\} \quad [1.67]$$

The corresponding variance estimator is therefore written for an *a priori* uniform distribution (or rectangle) (Table 1.6) as:

$$S^2(x_i) = \left\{ \frac{1}{12 \times (C_{\text{lower limit}} - C_{\text{upper limit}})^2} \right\} \quad [1.68]$$

Let $TI = 2C_i$ be the difference between the two limits (upper and lower):

$$S^2(x_i) = \left\{ \frac{1}{3 \times (C_{\text{upper limit}})^2} \right\} \quad [1.69]$$

We notice, in fact, that statistics formulas remain essential, even in the type-*B* method. If we use the language and representations of GPS [CHA 99], we can express the standard uncertainty on x_i .

We now present a summary table on three laws usually used in the calculation of standard uncertainty. Table 1.6 provides a structured reading of the distribution of methods *A* and *B* [GUI 00, NIS 94].

In metrology, the law derived from *Arc sine* calculates the uncertainty on quantities around the *true value*. It is, as such, used in the temperature set between the instrument and the test object.

Why does Table 1.6 provide four other laws while specifically discussing this law? The reason is that the mathematical literature often uses the *Arc Sine* Law and *not its derivative*. In the field of metrology, it is wise to use the derived law.

Furthermore, on seeing Table 1.6, we notice that the latest calculations correspond to the rectangular distribution. This means that x_i has the same opportunity to take any value between $\{C_{ll}$ and $C_{ul}\}$. For example, after calibration,

a standard mass indicates on the calibration certificate a deviation from its nominal value and a calibration uncertainty U (in gram) along with the coverage factor $k = 2$. The standard uncertainty on the correction will be simply estimated using equation [1. 51], and we deduce that for $U = 0.00573$; $k = 2$ (confidence level 95%):

$$\Rightarrow u(C_e) = \frac{U}{k} = 2.865 \times 10^{-3} \text{ g}$$

Range 2 and mean $\mu = 0$, for each of the three of approximated distributions Measurement uncertainty calculated and reduced depending on the statistical distribution			
Appropriate statistical distribution laws	Diagram of the statistical distribution around the value x_i	Variance σ^2	Standard deviation σ
<i>Normal distribution (Gauss-Laplace)</i> If the uncertainty or standard deviation is given in expanded form, the uncertainty will be divided by the factor k: 99.73% ($k = 3$) $C = 3\sigma U(x_1) = \text{range}/k$		$\frac{C^2}{9}$	$\frac{C}{3}$
Uniform distribution or rectangle function used for statistical calibration and digital displays. Extreme values $(c)+$ and $(c)-$ are given and probable = general case, selected by default when in doubt = “the worst case.” Divided by $\sqrt{3}$		$\frac{C^2}{3}$	$\frac{C}{\sqrt{3}}$
Derivative of the function: <i>Arc sine</i>. It is used for periodic disturbances to analyze the effect of influence quantities ranging between two extreme points in a substantially sinusoidal pattern, for example, local temperature of a workshop. If temperature variations t° are indicated by $\pm C$, then $\{u = C/2^{1/2}\} \equiv \{u = C/1.414\}$		$\frac{C^2}{2}$	$\frac{C}{\sqrt{2}}$
<i>Triangular probability distribution</i>: often used for parallelism. The measured values are generally closer to the “center” than the extreme. For rarer cases, divided by $\sqrt{6}$		$\frac{C^2}{6}$	$\frac{C}{\sqrt{6}}$

Table 1.6. Summary of certain common laws to calculate the standard uncertainty $u(x)$: measurement uncertainty calculated and reduced by the statistical distribution

1.8.4.1. Influence of temperature on the dimensions

Depending on the desired precision and the size of the test piece, we should be sure, at best, about the homogeneity of ambient parameters around the test piece. Thermal balance must be assured between the test piece and the equipment used for the test. The derivative of temperature over time and the temperature gradient in the space at the workstation must remain within limits such that this balance is maintained during the test. It is important to master the stabilization time during control in machining processes or other heat treatment. It depends on the volume of the piece and its nuance as shown in Figure 1.24 [CAS 78, CAL 05].

The stabilization time is approximately 30 times greater for the large cylinder than for the plate and the small cylinder (of same material), under the same thermal conditions (initial and final). The $T_{\text{stabilization}}$ depends on the following ratio:

$$\text{Time}_{\text{Stabilization}} = \frac{\text{Volume}}{\text{Surface}} \quad [1.70]$$

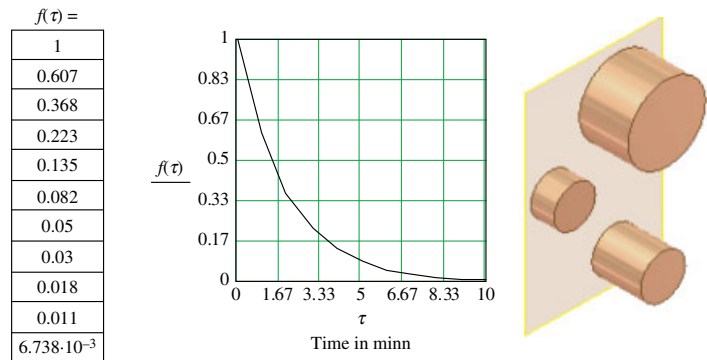


Figure 1.24. Highlighting the stabilization time as a function of the surface

Using a simple simulation (Figure 1.25), here is a numerical and graphical illustration of the influence of exchange surfaces on thermal stabilization duration under the same, both initial and final, conditions, temperature: 20°C, atmospheric pressure: 101,325 Pa (1,013.25 mbar), and humidity hygrometry: 65%. It is worth mentioning that the rate of humidity affects mainly the size of pieces made of rubber, plastic, and granite, as the vibrations can introduce errors of records (devices measuring roughness and circularity such as CMMs).

We must also ensure the absence of magnetic field that influences the results of electronic measurements (CMM). The weight of the piece can also cause an elastic deformation of the measuring instrument. If the measurement is made with a nonzero force measuring, the result should be corrected.

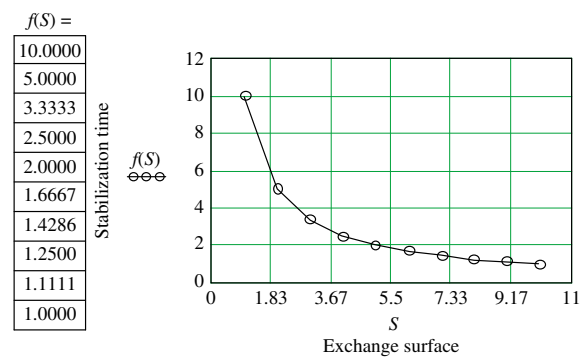


Figure 1.25. Thermal relationship as a function of the exchange surface

This correction is not necessary for comparative measurements made with the same means of comparison with the same force measurement, between similar elements of the same material and same surface condition.

EXAMPLE 1.1.— To estimate the *repeatability*, a series of measurements on the gauge block is performed under the same conditions. The length measured is 25.30 mm.

The mean, the standard deviation, and other statistical criteria from the statistical treatment [DIX 51] of calibration are determined based on a series of 14 experiments (Table 1.7):

- enter the first number in the series: *start*:= 25.000;
- enter the last number in the series: *end*:= 25.020;
- enter the spacing between points: *incr*:= 0.0015;
- vector creation function: vector **V**.

Experiments	1	2	3	4	5	6	7
Measurements	25.000	25.002	25.004	25.006	25.008	25.010	25.012
Experiments	8	9	10	11	12	13	14
Measurements	25.014	25.016	25.018	25.001	25.003	25.000	25.015

Table 1.7. Experimental data on 14 measurements

We calculate the mean and standard deviation on a sample as follows:

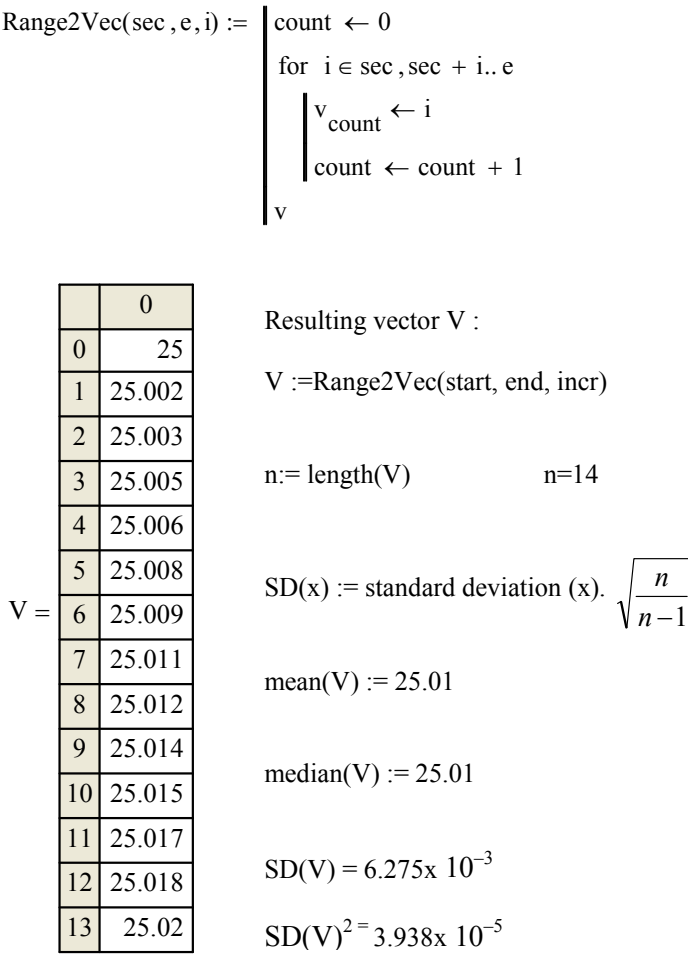


Figure 1.26 shows the graph of the curve and the mean of calibration:

$i = 0 \dots n - 1 \quad h_i = mean(V) + SD(V) \quad h_0 = mean(V) - SD(V)$

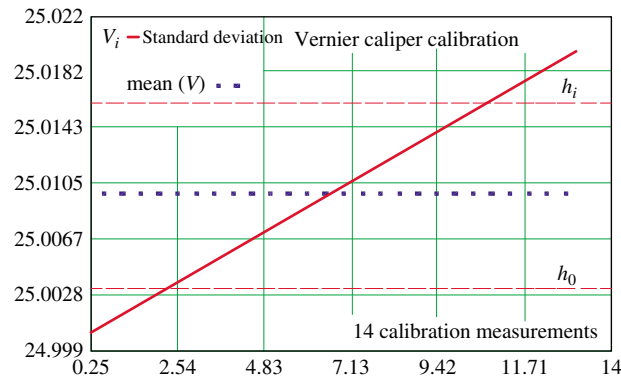


Figure 1.26. Calibration curve (simulated) with a vernier caliper (14 tests)

APPLICATION EXAMPLE 1.2.— In the following, we focus on simple cases from dimensional metrology. As such, using a digital micrometer {0–2 in.}, 10 measurements are done on the pitch diameter of the standard gauge (Mitutoyo; Figure 1.27) [CAT 00].



Figure 1.27. Measurement of pitch diameter (inches) using the wire method (Mitutoyo)

The instrument is inspected by the manufacturer stating that the *bias* error was defined by a class of $\pm 10 \mu\text{m}$. For example, let us measure the pitch diameter of the measuring standard, such as “go” and “no go”. Consider a series of measurements to

be repeated many times under the same experimental conditions. Then, let us calculate the mean μ , the standard deviation ($s = \sigma$), and the density of probability $P(x)$ assuming, *a priori*, that the distribution follows the Gaussian.

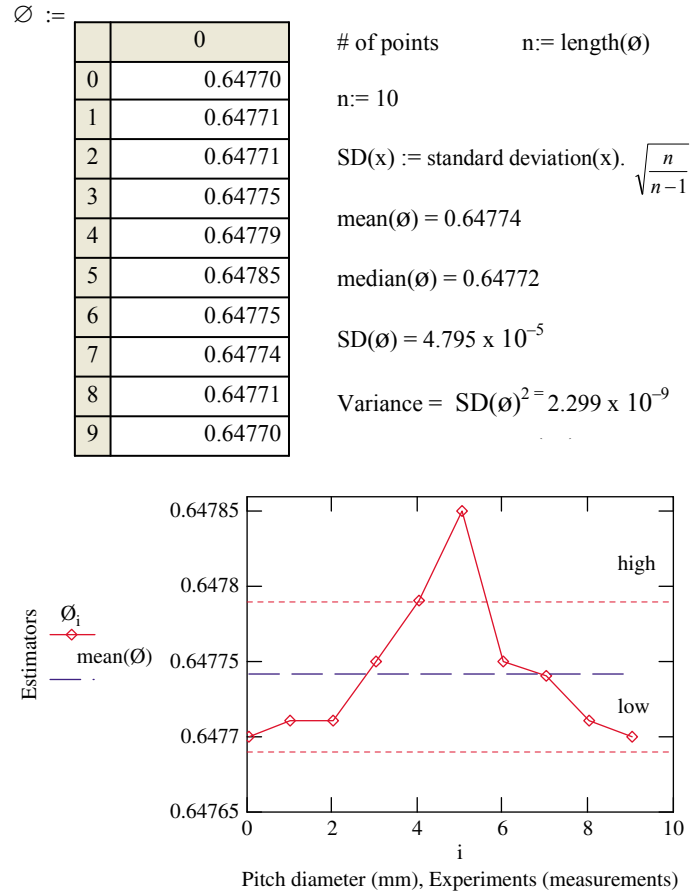


Figure 1.28. Pitch diameter ($\emptyset_{pitch}$) of a thread

Readings of the measurements and results are made using the MathCAD software. For $i = 1 \dots 10$ and $N = 10$, enter a vector of data to be analyzed. Consider the pitch ($\emptyset_{pitch}$) of a thread, and i is the number of the experiment (Figure 1.28). N is the total number of experiments. The probability distribution function (y_j) will be:

$$x_j = \frac{j}{10} \quad y_j = \left(\frac{1}{s\sqrt{2\pi}} \cdot e \right)^{-\frac{(x_j - x_{bar})^2}{2s^2}} \cdot N \quad \text{where } x_j = 1 \dots 1,000$$

Then, draw y_j as function of x_j . This helps chart a distribution supposed *a priori* rectangle as shown in Figure 1.29.

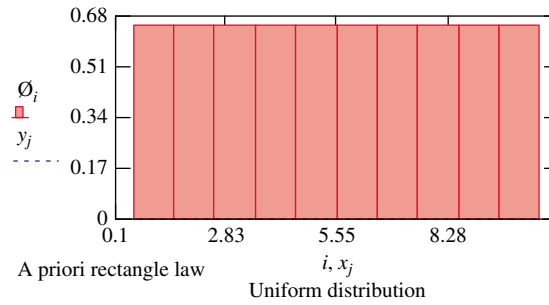


Figure 1.29. Graphic plot of y_j ; function *a priori* rectangle

By comparing the graph shown in Figure 1.29 with that shown in the second last row of Table 1.6, we find that the appropriate distribution in our case is a rectangular distribution (QED).

$$\begin{aligned} &\text{normc}(-1) = 0.159 \\ &\text{Verification for} \\ &\mu := 0 \quad \sigma := 1 \end{aligned} \quad \int_{-1000}^{-1} \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}} dx = 0.1586552537$$

If we plot the graph $f_i = (x_i)$ of pitch diameters measurements, we would find that the probability distribution is actually closer to the Gaussian distribution with mean $= 0$ and $\sigma = 1$.

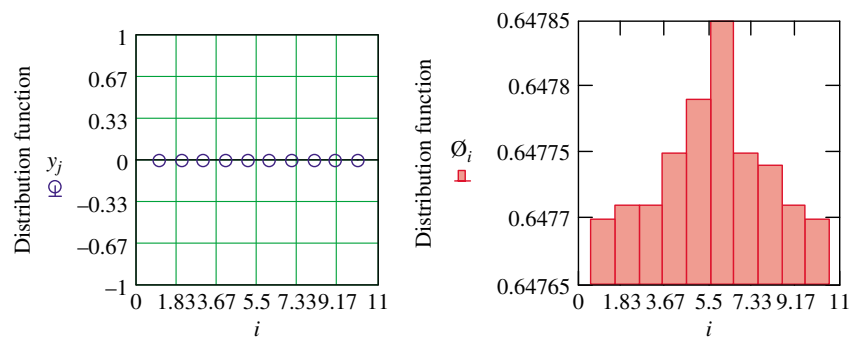


Figure 1.30. Plot the curve of probability distributions

$$\left\{ \begin{array}{l} \text{For } \mu = 0.647741 \text{ and } \sigma = 4.794673 \times 10^{-5} \\ \text{we will get } \rightarrow \int_{-1000}^{-1} \left(\frac{1}{\sigma \times \sqrt{2\pi}} \right) \times e^{-\frac{(x-\mu)^2}{2\sigma^2}} \end{array} \right.$$

By plotting the simulated normal distribution and comparing it with our own data (bar chart) in the graph shown in Figure 1.30, we find that the reality of our experiments corresponds to a distribution of rectangular type. We cannot make this distribution follow the Gaussian, since it is a uniform law. We calculate the standard uncertainty as to the veracity of the data hence the interval $\{-C; +C\}$ of a uniform law. ($x_{\text{bar}} = \mu$) is the measurements (observations) mean and C_j the bias correction ($C_j = 0$ corresponds to the best of our knowledge). The model of the measurement process is expressed by the formula:

$$y = \bar{x} + \{C_j \text{ corrected}\} \quad [1.71]$$

Assuming a_1 and a_2 unit (dimensionless = 1), based on the law of uncertainty propagation, we consider the following:

$$u_c^2(y) = a_1^2 \times u^2(\bar{x}) + a_2^2 \times u^2(C_i) = u^2(\bar{x}) + u^2(C_j \text{ corrected}) \quad [1.72]$$

Equation [1.72] also takes the simplified form in which S represents the experimental standard deviation of a series of n measurements. As of [1.14] and [1.57], consider equation [1.63] with $\mu = 0.647741$ and $s = 0.000048$, which are calculated earlier. Applying [1.50] to our case of thread gauge in inches, the law of propagation of uncertainties $U = u^2(\mu)$ on the mean (μ) provides:

$$\text{For } s = 4.794673 \times 10^{-5} \text{ and } N = 10, u = \left(\frac{s^2}{N} \right) = 2.299 \times 10^{-10}$$

$u^2(C_j)$ will be estimated using the type-B method. We consider that the best value correction, $u^2(C_j) = 0$, and the correction have the same probability of taking any value within $\{-10 \mu\text{m and } +10 \mu\text{m}\}$. The distribution is rectangular, as we have just seen; thus, the standard uncertainty corresponds to the distribution within $\{-C \text{ to } C\}$ and therefore $u = (C/1.732)$ (see Table 1.6, line 2):

$$\text{For a correction } C = 0.01, u = \left(\frac{C}{\sqrt{3}} \right) = 5.774 \times 10^{-3} \text{ then } u^2 = 3.334 \times 10^{-5}.$$

Finally, we calculate the standard uncertainty (from its variance) as follows: $u^2(y) = u^2(\mu) + u^2(C_j)$:

$$u^2(y) = 0.001 \times 10^{-5} \text{ mm}^2 + 3.334 \times 10^{-5} \text{ mm}^2 = 3.335 \times 10^{-5} \text{ mm}^2$$

$$\text{Verification: } u = \sqrt{u_c(y)} = 5.775 \times 10^{-3}$$

$$5.775 \times 10^{-3} \text{ mm} = 0.05775 \text{ m} = 0.2273615 \text{ in.} = 0.2273615 \text{ } \mu\text{in.}$$

$$u_c^2(y) = 0.2273615 \text{ } \mu\text{in.}$$

The final result of the measurement will thus be written as $Y = \mu \pm k \cdot u_c^2(y)$, for $\{k = 2\}$.

$$Y = 0.647741 \text{ in.} \pm 2 \cdot [0.2273615 \times 10^{-6}]$$

$$[0.647741] + [0.2273615 \times 10^{-6}]^2 \cdot 2 = 0.647741\mathbf{45}$$

$$[0.647741] - [0.2273615 \cdot 10^{-6}]^2 \cdot 2 = 0.64774\mathbf{055}$$

We note that the error is totally insignificant to the point where it is sought in the seventh decimal place, which does not count because the results are rounded to three decimal places. We note that the arithmetic mean remained equal to 0.6547740. The quantified uncertainty is then:

$$\{-C; +C\}, \text{ that is } \{-0.64774055; +0.64774145\}$$

1.9. Principle of uncertainty calculation: types A and B

When performing measuring experiments on mechanical parts, they should be repeated several times under the same conditions in the hope of obtaining a good trend. Unfortunately, there are always dispersions, and for this reason, we fall back on statistical modeling. Thus, we consider a mathematical expectation of the first order, which is the mean μ , and then, we calculate the variance σ^2 and a standard deviation σ on a sample of size n . We then carry out mathematical statistics approaches [NIS 94, PRI 96, TAY 94].

From a practical measure for true value x_i , we have to make calculations of the average of n samples, the values being, obviously, assumed identical. The ensuing systematic errors may however be reduced by applying corrections. This appeals to a sense of analysis that the operator is, unfortunately, not always expected to master. The metrologist should be rigorous to obtain a good correlation between physical measurements and figures that are expected to reflect his or her numerical representation. Knowledge of the measuring process and the fundamental principles from physics is one of the best guarantors of the conduct of the metrology project. In practice, errors are not discussed quickly. Various errors occur during the measurement. We recall a few of them:

- temperature and pressure;
- precision of instruments and position of the feature being measured;
- deformation of the mechanical part (we discuss this in Chapter 3);
- disruption of the quantity measured by the presence of the measuring instrument;
- error due to the measurement method itself;
- error due to the operator, and so on.

The errors and their causes being enumerated, we now attempt to deal with the propagation law and the required corrections. Rigorous work of good metrologists means thinking about the errors not yet identified. Those already identified will be subject to potential adjustments to compensate. While these adjustments would be judiciously realized, there is still doubt on the value of the correction, and this is where a rigorous mind is needed. Among the various corrections, three categories should be defined: *calibration, environmental, and standardization corrections*:

- Calibration corrections [CAL 05] are determined and represented in the calibration certificates (in Canada, *Calibration Laboratory Assessment Service* (CLAS)).
- Environmental corrections [CAS 78, CAL 05] offset the impact of environmental factors such as pressure and temperature. To take this into account, it is necessary to know the sensitivity coefficients to different physical states.
- Standardization corrections (CAN3-85-Z299) tend to reduce the results under standard conditions, that is, under normal conditions (standards) at $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$.

It is mathematically evident for the measurand Y to be bounded by the function of measurable quantities ($X_0, X_1, \dots, X_k$) by the direct measurement method. From equation [1.23], we define $Y = F(X_1, X_2, \dots, X_k)$. Each of these quantities ($X_0, X_1, \dots, X_n$) will be subjected to corrections offsetting the alleged systematic errors. The respective corrections take the following designations: ($C_1, C_2, \dots, C_k$). The overall correction to be applied following a Taylor expansion for corrections to made on ($C_0, C_1, \dots, C_k$) because they are too small compared with the measured quantities values ($X_0, X_1, \dots, X_k$):

$$C_y = \left(\frac{\partial F}{\partial x_1}\right) \cdot C_1 + \left(\frac{\partial F}{\partial x_2}\right) \cdot C_2 + \left(\frac{\partial F}{\partial x_3}\right) \cdot C_3 + \dots + \left(\frac{\partial F}{\partial x_k}\right) \cdot C_k \quad [1.73]$$

We model the measurement process to properly control the law of propagation of uncertainties. This approach, in our view, facilitates the understanding of facts inherent to measurement. However, we should not expect the mathematical model to

reflect the full phenomena related to the process. The model [GUI 00, MUL 81, NIS 94, PRI 96, TAY 05] certainly constitutes one way to read easily under the mathematical formalism that it suggests. However, it might, in many respects, offer a simplistic view of the metrology function. There is no formula suitable for all cases of measurement as there is no single measurement procedure. Ultimately, we must argue that modeling the measurement process is an important step to estimate the uncertainties. If $y = f(x)$ is the result of the measurement process, we consider:

$$f(x) = \bar{x} + \{ (C_{\text{standard}}) + (C_{\text{environment}}) \} \quad [1.74]$$

where $\bar{x}$ is the average value of the gross measurements, C_{standard} is the correction due to calibration, and $C_{\text{environment}}$ is the correction due to the environment.

In multiplicative and additive corrections, the model then makes the following mathematical form:

$$f(x) = \left(\sum_{i=1}^k C_{\text{standard}} \right) + \left(\prod_{p=1}^m C_{\text{environment}} \right) \times X \quad [1.75]$$

where X is the raw reading of the measurement instrument, C_{addi} represents the additive corrections, and C_{mult} represents the multiplicative corrections.

An important factor, however, continues to be missing in our model. Unfortunately, we cannot quantify it because this is *doubt*. In each of the factors depending on the model, there is a doubt on the value assigned to the component. Is it not the major concern for metrologists?

Usually, we significantly repeat our readings. The dispersion of values is always so relieved. Succeeding all measurement tests to obtain the same physical quantity value would be a pure fluke. We use the arithmetic mean to represent the *best value* while doubt still remains. There is also a doubt in the value of the corrections. For calibration, the correction is probably indicated by its own uncertainty. We know that physical constants are calculated using methods hardly questioned by the metrologists. But these are sophisticated users of the measure and inherent constants. Often, laboratory operators do simply content themselves with the implementation of recommendations which, incidentally are reliable, but unfortunately, are not exhaustive.

1.9.1. Error on the repeated measure: calculation of compound standard uncertainty

If we tried, through one measurement, to obtain the true value x_0 of a physical quantity, we would find that the replicated measurements of this measurand would

lead, each time, to different results $x_1, x_2, \dots, x_n$. These are the results of n measurements performed under identical conditions as explained earlier. Experts in metrologists call this doubt: *uncertainties*. We must not believe that the uncertainties are a dump where we put the most inaccurate concepts. It is quite opposite to this. The uncertainties reflect the image of rigor and research, from crescendo, of the true value. These uncertainties (*doubts*) are expressed numerically by amounts that reflect the phenomenon of dispersion, either the standard deviations $U(x_i)$ (*standard uncertainty*) or variances $U^2(x_i)$.

The corrections in the model where the value is unknown must appear. The reason is that these corrections even equal to zero have variance and standard uncertainty. Each of the quantities $x_1, x_2, \dots, x_n$ is being affected by an uncertainty $u(x_1), u(x_2), u(x_3), \dots, u(x_n)$. These quantities are often expressed as standard deviation and that is why they are called the standard uncertainties. The components of the compound uncertainty of measurement results y , represented by $u_c(y)$, are calculated by the expression of the estimated variance: $u_c^2(y)$. Sometimes, we use the variance whose mathematical expression [1.53] takes the following form:

$$u_c^2(y) = \sum_{i=1}^k \left(\frac{\partial F}{\partial x_i} \right)^2 \cdot u^2(x_i) + 2 \cdot \sum_{i=1}^{k-1} \cdot \sum_{j=i+1}^k \left(\frac{\partial F}{\partial x_i} \right) \cdot \left(\frac{\partial F}{\partial x_j} \right) \cdot u(x_i, x_j) \quad [1.76]$$

This equation is based on the Taylor series approximation of order one, very small values of measures taken into account by the equation of error propagation of uncertainty where x_i and x_j are the estimators of X_i, X_j , and $u(x_i, x_j) = u(x_j, x_i)$ is the estimated covariance x_j, x_i . The degree of correlation R between x_i and x_j is characterized by the estimator of the correlation coefficient $R(x_i, x_j)$, which is written according to the properties presented earlier:

$$R(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i) \cdot u(x_j)} \quad [1.77]$$

$R\{x_i, x_j\}$ is often obtained by mathematical interpolation or by using the method of least squares. In the context of processing data from our laboratory, we use MathCAD software. $R\{x_i, x_j\}$ is included between $\{-1$ and $1\}$. Of course, if x_i and x_j are independent, so $R(x_i, x_j)$ is not zero ($\neq 0$), equation [1.76] $u_c^2(y)$ then becomes:

$$u_c^2(y) = \sum_{i=1}^k \left(\frac{\partial F}{\partial x_i} \right)^2 \cdot u^2(x_i) + \dots + \dots + 2 \cdot \sum_{i=1}^{k-1} \cdot \sum_{j=i+1}^k \left(\frac{\partial F}{\partial x_i} \right) \cdot \left(\frac{\partial F}{\partial x_j} \right) \cdot u(x_j) \cdot u(x_j) \cdot R(x_i, x_j) \quad [1.78]$$

Partial derivatives $(\partial F/\partial x_i)$, consistent with the measured values X_i are valued at the point which reflects the standard uncertainty associated with estimates of the position x_i ; $u(x_i, x_j)$ is in fact the covariance associated with x_i and x_j . The higher the number of measured values n , the closer we get to the true value of the quantity under consideration x_0 .

When we combine various uncertainties, we obtain a sort of law distribution called the law of propagation of uncertainties. The classical form for an appraiser to the measurand Y is after several measurable quantities observed. In our process of dimensional metrology, x_1 represents the arithmetic mean of a series of measurements x_2 temperature for determining the environmental correction, x_3 correcting the calibration of the micrometer and constant x_i , and so on. In fact, the expression of the compound uncertainty $u_c^2(y)$, when x_i and x_j are independent and $R(x_i, x_j)$ is different from zero but included between $\{-1$ and $+1\}$, takes the formula ensuing from the mathematical interpolation applied to $u_c^2(y)$. This latter relationship may seem a little complicated if we are not familiar with some mathematical developments to which it refers. Ultimately, equation [1.72] is simplified as follows:

$$y = f \left(\begin{array}{l} \text{unique and repeated observations} \\ \text{additive and multiplicative corrections} \\ \text{taking into account physical constants if controlled} \end{array} \right) \quad [1.79]$$

$y = f(x)$ is the result of the measurement process. In this expression, it will be a simple measuring instrument on which it is applied; a calibration Co_e is another correction of environment. As μ is the arithmetic mean, assuming the covariance values are all equal to zero, we accordingly calculate the compound uncertainty of $y = f(x)$, which takes the following explicit form:

$$u_c^2(y) = u_c^2(\mu) + u_c^2(Co_{\text{environment}}) + u_c^2(Co_{\text{additive}}) + u_c^2(Co_{\text{Cst\_physical}}) \quad [1.80]$$

NOTE.— The physical constants are usually very rarely corrected. From the year 2008 to 2009, time is delayed only by 1 second (see clock – NRC Ottawa, Canada – at CsV (Cesium 5), Chapter 3, Figure 3.2). Would Metrologists take this into account? We think so.

In the exercises herein, we neglect the term $u_c^2(Co_{\text{Cst\_physical}})$. In terms of standard deviation, equation [1.80] takes this form:

$$u_c(y) = \sqrt{u_c^2(\mu) + u_c^2(Co_{\text{environment}}) + u_c^2(Co_{\text{additives}})} \quad [1.81]$$

$u_c^2(x)$ represents here the variance of the average gross measures. It is sometimes estimated from the sample of the series of measures. In other cases, it is estimated from tests that are used to estimate the variance of repeatability of the measurement process. It is in this latter case that metrology laboratory operators find themselves confused. The fact is that we have more data and information to estimate the variance of repeatability.

1.9.2. Applications on the laboratory calculations of uncertainties

Before presenting some applications, it is interesting to summarize the approach advocated in seven distinct steps:

- definition of the quantity being measured;
- analysis of the causes of error;
- finding ways to compensate for these errors (corrections if any);
- modeling measurement process;
- estimate the standard uncertainty on the outcome;
- expression of the result and its expanded uncertainty;
- conclusion.

The measurement uncertainty is in our opinion, one of the best indicators, if not the best quality of a measurement or calibration. It therefore becomes imperative to carefully quantify the uncertainty in favor of metrology. Examples of problems that we deal here are actual cases. To avoid mistakes, it is strongly recommended to use SI units, that is, *mm*, *second*, *degree Celsius*, and *radians for angles*. In Canada imperial units are still being used. We discuss some cases in imperial units, but we reiterate our concern to make sure to use the SI units whenever we choose is “offered.” Nevertheless, we should strongly avoid using two systems of unit within a single problem. We did it as part of the problem in the gauge block in inches and interval correction in SI. This choice is deliberately made to show unnecessary hardship to add to a simple problem of dimensional metrology. Expression of experimental uncertainties is also dependent on the proper method of modeling.

We have discussed the two methods used by metrologists: type A and B methods. However, there is no simple correspondence between the ranking in both categories, nor as to the “random” nor systematic. According to the recommendations of the IBWM INC-1 [ACN 84, NIS 94 VIM 93], the term “systematic uncertainty” is likely to lead to misinterpretations. Therefore, it becomes important to have the vigilance language added to the formalism of mathematics to try to avoid the pitfalls of some confusion suggested by the softer tone. Any

description of uncertainty should include a complete list of its components and indicate, for each, the method used to assign a numerical value.

Components of *type A* are characterized by variances s_i^2 or estimated by standard deviations s_i . Components of *type B* should be categorized by $(u_j)^2$ and can thus be considered by approximations of the corresponding variances whose existence is admitted. The terms $(u_j)^2$ and u_j can be used as variances and standard deviations, respectively. The compound uncertainty should be characterized by the value obtained by applying the usual method of combining variances. The compound uncertainty and that these components should be expressed in the form of “standard deviations.”

1.9.3. Simplified models for the calculations of measurement uncertainties

The approach presented above is exhaustive. It does not answer the five questions it suggests. Sometimes, it happens that an entry is not necessary, and for this reason, we proposed the same succinct streamlined five-step approach:

- definition of the quantity measured;
- measuring conditions (environment, atmosphere, and so on);
- analysis of the causes of errors;
- determination of any corrections;
- modeling the measurement process and proper application of the law of uncertainty propagation according to the international guide ISO/IEC.

We have already seen how we came to the variance. If there is no covariance, we proceed to estimate the *repeatability* of the measurement method variance (s^2) formulated below:

$$\begin{aligned} u_c^2(y) &= \left(\frac{\partial F}{\partial x_1}\right) \cdot u^2(x_1) + \left(\frac{\partial F}{\partial x_2}\right) \cdot u^2(x_2) + \cdots + \left(\frac{\partial F}{\partial x_k}\right) \cdot u^2(x_k) \\ &= \sum_{i=1}^k [|\text{Co}_i| \cdot u^2(x_i)]^2 \end{aligned} \quad [1.82]$$

EXERCISE 1.3.– Samy prefers to use a digital caliper to measure an average diameter of 1 in. as in Figure 1.31. The instrument is inspected by the manufacturer Mitutoyo, which states that the bias error was defined by a class of $\pm 2 \mu\text{m}$, with a confidence of 95% hence $k = 2$. The calibration of the instrument is viewed as a correction C_e to which an expanded uncertainty U is associated. So:

$$C_e = 7 \text{ mm and } U = \pm 2 \text{ mm.}$$

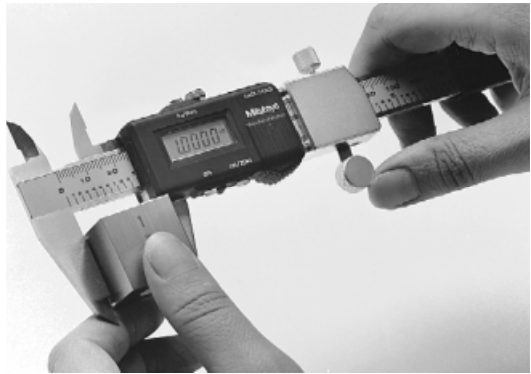


Figure 1.31. Image control compliance of a P to C on a 1-in. standard gauge block

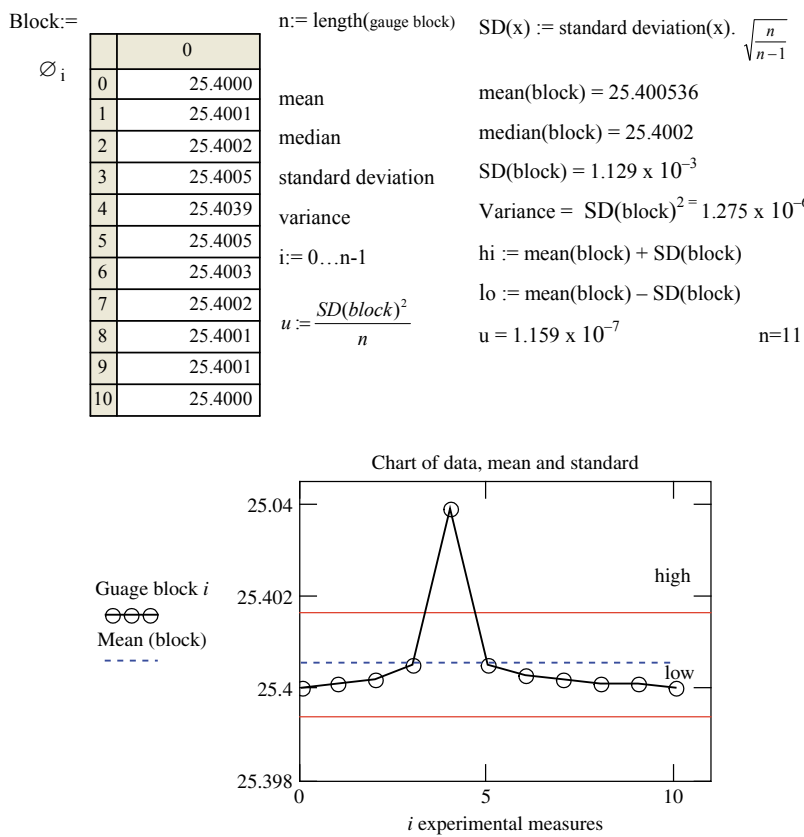


Figure 1.32. Gaussian measures of the gauge block with 1 in.

Samy described his method of measurement and has determined the standard deviation of repeatability S_r so that $S_r = 0.006$ mm. He makes a new series $n = 11$ of $i = \{1 \text{ to } 11\}$ measures f_i and reads them in metric. Measurements of the gauge block to be checked give the results shown in Figure 1.32.

The calculated mean (μ) = 25.400536 mm (1 in.). The model of the gauge block measuring process is given by the simplified formula [1.59]. For the correction of calibration, $C_e = 7 \mu\text{m}$ and the mean of measures $\mu = 25.400536$ mm (1 in.). Therefore, equation [1.74] can be written as:

$$y = m + \text{Co}(\text{with Co} = C_e \text{ a correction of calibration from } P \text{ to } C)$$

For $C = 0.007$, we obtain $y = 25.400 + 0.007 = 25.408$ mm $\approx$ 1 in.

Using the law of uncertainty propagation: $y = \mu + C_e$:

$$\text{from [1.64]}, u_c^2(y) = u_c^2(\mu) + u_c^2(\text{Co}_{\text{calibration}}) + 0 = \frac{S_r^2(y)}{n}.$$

By involving the standard deviation of repeatability $(S_r)^2 = 0.006$ calculated in the qualification of the measurement process, we find:

$$\text{Numerical application on the } P \text{ to } C: \frac{S_r^2(y)}{n} = \frac{0.006^2}{11} = 3.273 \times 10^{-6} \text{ mm}^2$$

The standard uncertainty on the calibration correction of P to C is then:

$$U = \pm 2 \mu\text{m} (k = 2 \text{ at confidence level of } 95 \%); u(C_e) = 0.001 \text{ mm}.$$

Therefore, $u_c^2(y) = 3.273 \times 10^{-6} \text{ mm}^2 + 1 \times 10^{-6} \text{ mm}^2 = 4.273 \times 10^{-6} \text{ mm}^2$.

Using the formula $u_c(y) = y \pm S_r$ (where $k = 2$), $u_c(y)$ will be calculated as follows:

$$u_c(y) = 25.400 \text{ mm} \pm 0.006 \text{ mm (at confidence level of } 95\%); \text{ thus, } u_c(y) = 25.406 \text{ mm}$$

Conclusion: the gauge block has indeed approved the calibration of P to C used by Samy. The approach that guided him in this laboratory is shown in Figure 1.33.

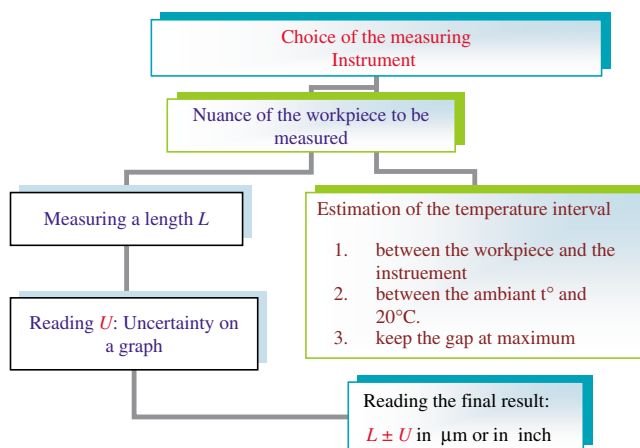
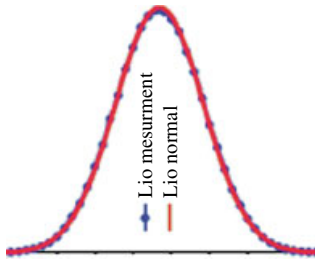


Figure 1.33. Flowchart on the approach of the choice of the measurement instrument (P to C)

Using the software GUMic [GUM 08], we attempt to assess the compound uncertainty via the GUM and Monte Carlo simulation on the gauge block used earlier to calibrate the P to C.

Final result: $Y = 25.40054 \pm 0.00031 \mu\text{m}$, with $k = 2$ at confidence level 95 %			
Measurand Y	GUM method order 1	Monte Carlo method	Curves
Mean	254005.43×10^{-4}	254005.43×10^{-4}	
Compound uncertainty $U_c(Y)$	1.57×10^{-4}	1.57×10^{-4}	
Number of the effective degrees of freedom ν_{eff}	∞ (infinite)		
The results are identical for both methods so the measurand is almost linear along its uncertainty interval			
Standard uncertainty Unit of quantity: $U(x_i)$	Sensitivity coefficient $\frac{\partial Y}{\partial x_i}$	Standard uncertainty Unit of measurand $\frac{\partial Y}{\partial x_i} \times u(x_i)$	Number of the effective degrees of freedom ν_{eff}
$1.57 \times 10^{-4} \mu\text{m}$	$1 \mu\text{m}$	$1.57 \times 10^{-4} \mu\text{m}$	∞
$1.57 \times 10^{-6} \mu\text{m}$	$1 \mu\text{m}$	$0.02 \times 10^{-4} \mu\text{m}$	∞

Measurand distribution
 Normal distribution

DISCUSSION.— For validation purposes, we used GUMic software. The two methods we implemented are the GUM method and Monte Carlo method. The results are absolutely the same, with $u_c(y) = 1.57 \times 10^{-4}$. The measurand, that is, the value subject to measurement (P to C calibrated related to 1 in. (25.4 mm)) follows a normal distribution. The function (mean) derived from the first method is $y = 25.400536$ mm compared with that issued by the software is $Y = 25.400540 \pm 0.00036 \approx 25.400$ (rounded to the nearest thousandth).

1.9.4. Laboratory model of dimensional metrology

Table 1.8 is presented as a model. It could be modified according to individual interests.

Title of laboratory	Instruments and/or apparatus	Conditions and environment	Uncertainty type: $U(x_i)$	δ_{relative}	δ_{absolute}
Exact title of laboratory	Metrologic appellation (VIM)	Temperature and humidity	Mathematical expressions	Formula	Formula
Observations:					
Experiments					
01					
02					
03					
Ni					
Discussions, graphical plotting, and possible future extensions Conclusion					

Table 1.8. Model of laboratory table on uncertainties

1.9.5. Measurement uncertainty evaluation discussion

The measurement uncertainty is evaluated by the GUM analytical approach published by ISO in 1993, which has become an international standard. Is this the only method? No, there are other methods, particularly when it becomes difficult to formalize into a single equation all factors leading to the expanded uncertainty.

In these cases, we fall back on methods based on inter-laboratory comparisons, showing strong proficiency testing. We summarize two main approaches [PRI 96] in Figure 1.34.

The major requirement is to have adequate means to quantify the various factors of uncertainty and not to multiply the same contributions. This would distort the calculation. In other words, it is the quantification of the repeatability ensued by various means (control charts, among others) that requires a 10-fold increase in observations.

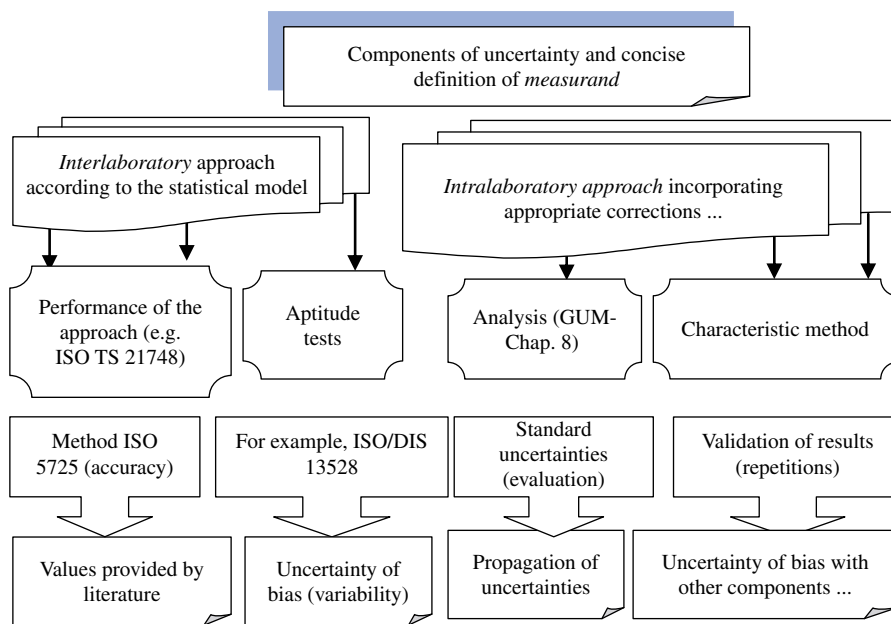


Figure 1.34. Summary of methods for calculating the uncertainty

Let us go back a bit on the language issue: in our view, the *measurand* should be defined rigorously. *A priori* any faithful method has its concepts and should be used to assess uncertainties.

The methods intra: the GUM approach is *analytical*. It is based on the expression of the measurand physical model as $y = f(x_1, x_2, x_3, \dots, x_n)$. We have seen in this chapter how the GUM describes the approach to evaluate these types of uncertainties of input variables $(x_1, x_2, x_3, \dots, x_n)$. It would be illogical in a core of “uncertainty mania” to apply the GUM approach to all areas. It is already apparent that in the medical field, for example, this analytical approach, as it is, remains unrealistic.

1.9.6. *Contribution of the GUM in dimensional metrology*

We can say that the GUM clearly defines the conduct, definition, and calculation of measurement uncertainties. In the past, almost everyone had their own approach. Technical and scientific literature proposed many approaches. This fact opened the door to many interpretations, sometimes unreliable.

Nowadays, the ISO Guide best locates uncertainties that must be clearly identified as either types of uncertainty or expanded uncertainties. The definition of measurement uncertainty provided by the VIM is a parameter mathematically related to the measurement result. It has been defined in this chapter that this uncertainty characterizes the dispersion of values that can be attributed to the measurand (that is, the quantity intended to be measured). Also, we discussed that the standard uncertainty is an uncertainty that is consistently associated with the standard deviation. The expanded uncertainty is a multiple of the standard uncertainty that defines an interval in which we can hope to find a large number of values that can reasonably be attributed to the measurand. We have deliberately inspired this “language” by the VIM to ensure consistency with the international language of metrology in this matter.

The study on uncertainties results therefore from a formalized mathematical reflection, hence the equations of measurement called measurement model. In some metrology laboratories, it is not encumbered with mathematical formalism. This remains a simplistic description of the mathematical description of the process expected to be used to determine the input parameters of the model. Are we allowed to continue saying that we make metrology laboratories? The answer is no. Metrology is not this.

The final uncertainty – usually known as compound uncertainty – is the result of the propagation of all the elementary components through a linear mathematical approximation resulting from the measurement model. It is nevertheless clear that to make an uncertainty calculation in line with the GUM, it is necessary to use the model of the measurement. In the old school physics lessons [CAS 78, FRI 78], we calculated the absolute and relative uncertainty based on the predetermined quotients. We made a brief presentation of these quotients at the beginning of this chapter. This school methodology was accepted earlier because of the standard teaching tool at this stage of learning. It is no longer sufficient.

Can we continue considering uncertainties as a secondary “thing” in metrology? No! This would not be serious when metrology is deemed to be uncertainty. Many metrology laboratories unfortunately ignored this aspect. This makes no sense in the eyes of the GUM. We know that some components represent uncertainties resulting from random errors and other from systematic errors. A lot of “chatter” in the implicated departments had free rein. Some laboratories included the systematic errors. Others corrected them but sometimes blatantly disregarded their effects on

the outcome of uncertainties. Often, these applications (ways of doing rather) are formed by adding at the end “in accordance with ISO”

Nowadays, the ISO Guide related to the GUM [TAY 94] has the undeniable merit of clarifying the situation: we must look for systematic errors, and each measurement result must be corrected for these errors; then, uncertainties corrected for systematic errors should be included in the calculation of uncertainties. The ISO Guide also reminds us that there is no correlation between *type A* and *type B* uncertainties as there are none between the *random* and the *systematic error*. Metrology consists of uncertainties. Throughout this chapter, we have attempted to show how and why we calculate uncertainties.

1.10. Summary

In this chapter, we have highlighted the importance of errors and uncertainties, starting from making a distinction between them in terms of definitions based on the VIM. We have also proposed some examples of numerical calculations. These examples have been simulated and in many cases performed on laboratory. It is clear that for dimensional metrology, the uncertainty calculations are essential. Some books suggest that metrology is the uncertainty. This is not without merit, and we agree with this vision. We have also emphasized the ability (*skill*) of metrologist during the measurement process. The same goes for calibration and respect of the work environment.

As we have demonstrated, there are various ways to estimate and quantify uncertainties. First, we proposed simple methods used in ordinary work practices, and then, we end by discussing type A and B methods. The latter are applied in many cases of engineering school. Of course, method A is unfortunately less used in the Cégep due to programs of statistics. Method B is, in turn, more widespread because it gets rid of some statistical methods, Taylor expansions, and calculus. At university level, these two methods are easily affordable. Equipment quality control (Statistical Process Control) is increasingly equipped using computers and more of these methods of computing capabilities. Finally, we find that the causes of uncertainty are many and can be function of measurement *repeatability*, as well as influence factors inherent to the *environment*. Under real conditions, the uncertainties depend on the following:

- instruments and measuring device;
- the workpiece to be measured, its roughness and nuance, that is, the material;
- the work piece mounting during the test;
- deformations due to the lock of the workpiece and the skill of the operator, and so on.

The idea of uncertainty remains, unfortunately, stuck to the laboratory. In workshop practice, the idea of the instrument resolution remains associated with that of the ensuing uncertainty. The interests of the reflection on the uncertainties are summarized as follows:

- considerable cost savings in terms of litigations that might be avoided;
- a necessary step to demonstrate the aptitude...;
- a know-how validate for a reassuring and an unequivocal judgment on the measure.

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www.login-entreprises.com/index.html (go to GUMic software). “Uncertainties calculation using the GUMic software.”

Canadian Standard: CAN3-Z299.1-85.

See also the following sites:

National Measurement Standards:

http://www.nrc-cnrc.gc.ca/randd/areas/measurement_f.html.

R & D Ottawa, Canada: http://inms-ienm.nrc-cnrc.gc.ca/com/main_f.php.

Calibration Laboratory Assessment Service (CLAS) Ottawa, Canada (Accreditation):
http://inms-ienm.nrc-cnrc.gc.ca/clas/clas_f.html.

Chapter 2

Fundamentals of Dimensional and Geometrical Tolerances According to ISO, CSA (Canada), and ANSI (USA)

2.1. Introduction to geometrical products specification

The concept of geometrical products specification (GPS) was proposed by the International Organization for Standardization (ISO) to specify and organize the verification of dimensional and geometrical products. The specification is mainly concerned with dimensional metrology and, as such, ISO/TR 14638 of 1995 deals with the definitions of functional geometric features of interrelated parts. The macro-geometric characteristics considered here are the surfaces of the parts at different stages of their transformation aiming tolerancing. The selection of method, instrument, and its calibration is based on these parameters. There are various manuals, which address GPS in different aspects. It reminds the laudable work [CHA 99], which, however, would be insufficient when the univocity of the language is not followed. The emphasis on the uncertainties and tolerancing in metrology has made GPS an inevitable function in metrology applied to manufacturing.

Generally, we might be forced to *tolerate* the breach of language, but in metrology, showing blur would taint greater uncertainties, and thus, the purpose of metrology is to master them [PAU 70]. Many disciplines impose this linguistic partitioning to properly accommodate the mathematical rigor that supports them. Weak vocabulary in metrology leads to misinterpretations. Neither art nor business can permit breaches in vocabulary. The companies concerned with this fact have already started training their technicians to use common language [VIM 93].

In Quebec, this problem is still more acute because the translation has made from English to French (sometimes by awkwardly francizing English terms) without worrying about the content of the subject. The new industrial issues that are due to reduction in time in terms of product design and implementation have created, in recent years, a special interest in GPS. Tolerancing is the perfect support for product-integrated design. It is also a manufacturing process. This leads to the following:

- reducing the constraints of precision and thereby the product cost;
- making the quotation just enough to fulfill the demands; and
- using an unequivocal and concise language that avoids misunderstandings.

Computer-aided design/computer-aided manufacturing (CAD/CAM; in French: Conception et Fabrication Assistées par Ordinateur (CFAO)) tools offer integrated functions of tolerancing in their geometrical models. This allows checking the syntax and the relevance of topological tolerances specified on a given design. They also allow transmitting this information, not as a simple text, but as coherent data, directly usable by another application. Tolerancing is part of a general design approach. Associated with tools of functional analysis and hierarchization of features, it helps in reducing the “gap” between the design department and the process department. The latter improve their manufacturing processes so that they better fulfill the requirements that are clearly specified. Although ISO standards bring real progress, they nevertheless have shortcomings and contradictions for which the international standards committees have decided to provide some solutions. Thus, a master plan (Tables 2.3 and 2.4) is put forth for the work of technical committees to find common solutions to recurring problems.

In the ISO/TR14638 report concerning dimensional metrology, it is said that any element *deemed ideal* requires, to properly fit the assembly, an accurate dimensioning. The element must have a *perfect shape*, that is, a *perfect plan* or a *perfect surface of revolution*. Any piece resulting from mechanical manufacturing is *never perfect*. Therefore, it is necessary to define the set of acceptable actual geometries to ensure that the piece is functional, which is the main aim of GPS. This chapter shows how important it is, in metrology, to define: *the shapes, the dimensions, and the surface characteristics that ensure optimal performance of a piece*. Ultimately, we consider:

Definition of the nominal dimension $\equiv$ dimensioning + surface states [2.1]

In metrology, we also define the *dispersions* around the (what was called) *optimal* for which the function is always satisfied. This gives the so-called functional dimensioning:

Functional dimensioning $\equiv$ dimensioning + tolerancing [2.2]

Tolerances are described by the word *tolerancing*, but in fact it is important to separate the nominal dimensioning [2.1] from the functional dimensioning [2.2], because the *tolerancing* identifies *geometrical deviations* allowed around the nominal, whereas the *dimensioning* sets the perfect or nominal geometry. In technical drawings, the dimensioning is compulsory. Figure 2.1 shows one example of dimensioning.

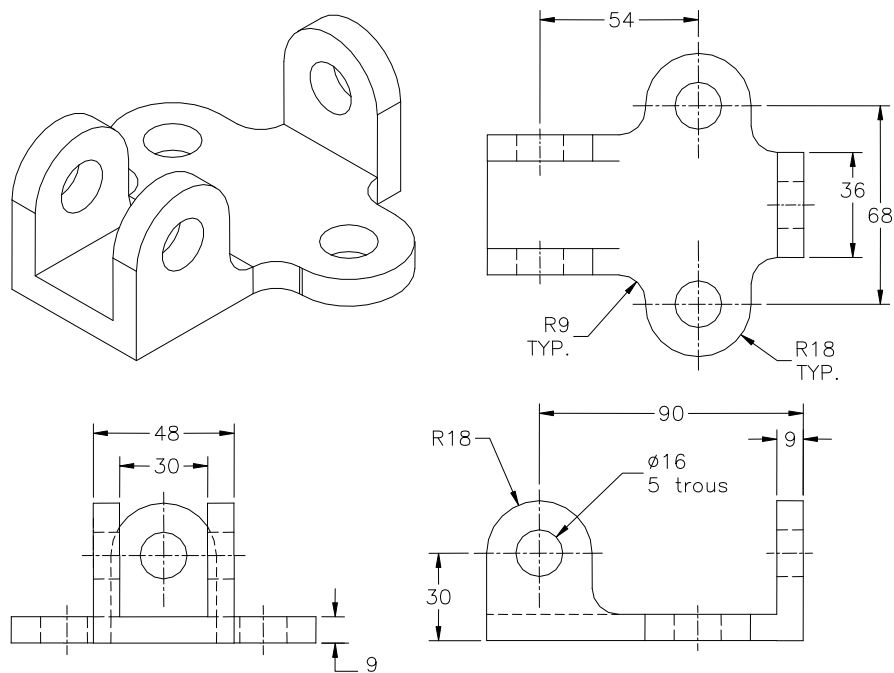


Figure 2.1. Dimensioning of technical drawing parts

Dimensioning with tolerances follows universal rules and is governed by standards such as ISO 129 and conventions [OBE 96]. A piece consists of a set of elementary surfaces. The parts/pieces are assembled to each other, in *contact* with different surfaces, based on their respective planes.

In dimensional metrology, dimensioning aims at defining, by a drawing (two-dimensional (2D) or three-dimensional (3D)), the shape, size, and position relative to other surfaces, of each of the elementary surfaces of the piece (Figure 2.2).

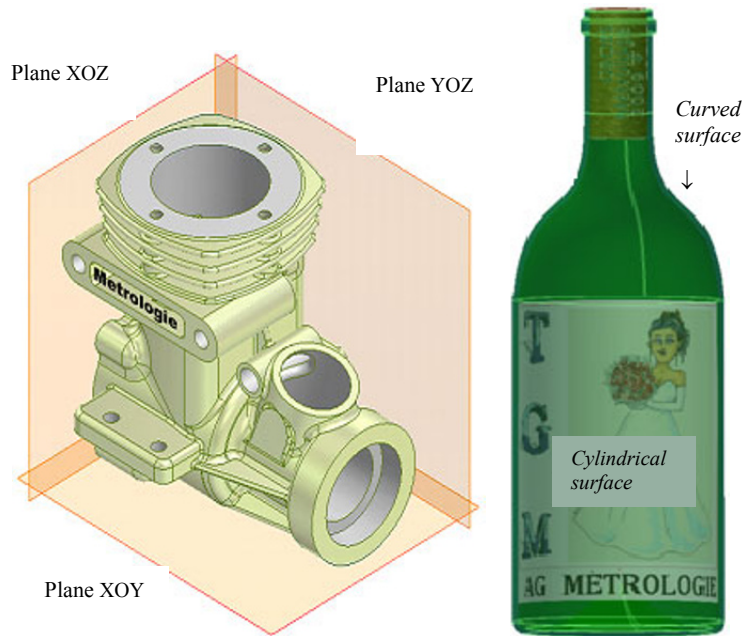


Figure 2.2. Examples of geometry modeling of parts and planes

According to ISO 8015-1985 on the Fundamental Tolerancing Principle, each geometrical or dimensional requirement specified on a drawing must be considered separately, that is, *independently* of other requirements unless a particular relationship is annotated on the drawing to specify it. Some parameters (angles or distances) are implicitly quoted by the drawing to explain the fact of the geometrical constraint. In this context, the axis features are involved in the definition of the nominal quotation. There are cases where a quotation is given independently of other dimensions of the drawing. This is conventionally called the dimensioning *independency*. In technical drawing, the proposed dimensioning on a drawing does not go beyond the description of the size, but rarely constraint (dimensional or geometrical). But in the context of GPS, for example, there are other special relationships which are as follows:

- The requirement of the envelope $\textcircled{E}$ defines the relationship between the size and the shape.
- The requirement of maximum material $\textcircled{M}$ defines the relationship among the size, shape, and position. Similarly to condition [2.1], the requirement of $\textcircled{M}$ is also very important during machining or assembly drawing for machining.

2.2. Dimensional tolerances and adjustments

Toleranced dimensioning is based on the interchangeability of the manufactured components in the series [CHE 89]. For example, a machined part must be mounted on a machine whose components have the same drawing conventions regardless of location. Manufacturing processes cannot reproduce, with repeatable accuracy, components with the same dimensions perfectly. The min or max deviations of actual surfaces compared with nominal surfaces are defined using tolerances. Across borders exchanges have led to the creation of an international system of tolerances and adjustments, witness ISO 286. The latter sets the foundations for communication in the form of values, symbols, terminology, and definitions of the assembly system whose classical description is illustrated in Figure 2.3.

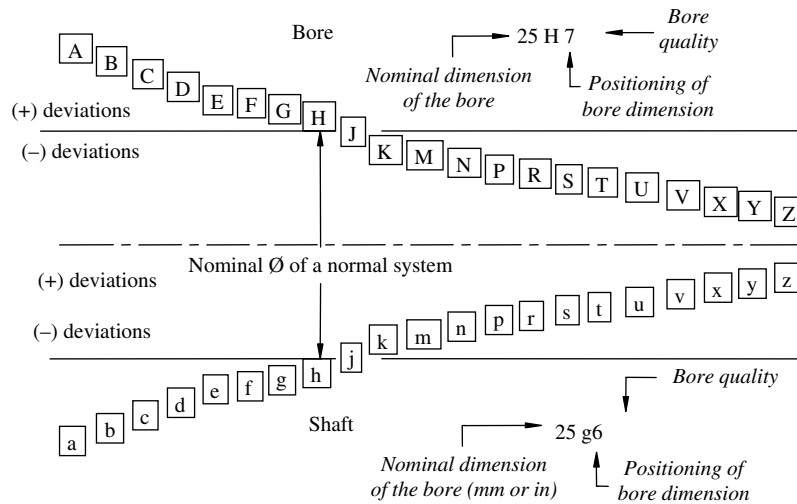


Figure 2.3. Designation of shaft/bore adjustments system (SI and CSA)

Metrology instruments that are used in dimensional control are calibrated according to standards consistent with ISO 286. Geometry deals with smooth pieces with cylindrical surfaces or parallel faces. An imposed dimension will be more easily achieved if it could vary between two limiting values, that is, a maximum dimension and a minimum dimension. The difference between the two dimensions is conventionally called the “tolerance interval (TI)”. Of course, the higher the required accuracy, the smaller the tolerance should be.

Toleranced dimensioning. This is the dimensioning of a piece with allowable deviations ($\pm$) around the nominal, for example, $\varnothing 20 \pm 0.025$.

Nominal dimension. This is a dimension serving as a reference for the identification and registration on the drawings. It serves as a reference when calculating or even reading up on the tables for ISO standard adjustments or CSA B97.3.70 [GIE 82, OBE 96] (see all the tables in Appendix 1).

Figure 2.4 summarizes the terminology used on a shaft/bore system.

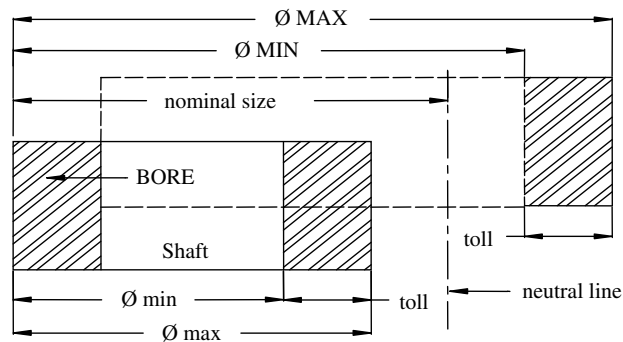


Figure 2.4. Terms from universal tolerancing for a shaft/bore assembly

Tolerance or TI. It is a permitted variation (tolerated, permissible) of the actual dimension of the piece. Deviations ($\pm$) from the nominal dimensioning [2.1] constitute what is internationally termed as *tolerance*. For circular pieces (shapes):

$$\text{Higher deviation (HD)} \rightarrow \text{HD} = \varnothing_{\text{max}} - \varnothing_{\text{nominal}} \quad [2.3]$$

HD is equal to the difference between the maximum allowable dimension and the nominal dimension:

$$\text{Lower deviation (LD)} \rightarrow \text{LD} = \varnothing_{\text{min}} - \varnothing_{\text{nominal}} \quad [2.4]$$

LD is equal to the difference between the minimum permissible dimension and the nominal dimension.

Toleranced dimensioning: adjustments case. Around the nominal, the deviation between the upper value and the lower one is represented by TI. The manufacturing mode remains specific to each workshop; however, the dimensioning and the surface state obey norms that are presumed to have been the “international consensus”. For CSA and ANSI standards, we should refer to the *Machinery’s Handbook* [OBE 96].

ISO standardized adjustments. These adjustments are categories of tolerated and standardized dimensions. They are used for welding two pieces that are usually cylindrical or even prismatic (CAN3B78.1-M83). Under the ISO 286-2, the readings are carried out in tables [OBE 96] for shafts and bores separately. These unrestricted universal tables give directly the higher and the lower deviations depending on the nominal diameter. By referring to the tables in ISO 286-2 or the *Machinery's Handbook* [OBE 96], we derive the following.

2.2.1. Adjustments with clearance: Ø80 H8/f7 (see Figures A1.1(a)–(c) in Appendix 1)

- Clearance min = 0.030 mm and
- Clearance max = 0.106 mm.

The difference, between the dimensions of parts before the assembly, is *positive*. The tolerance zone of the bore (TI A) is entirely above the tolerance zone of the shaft (TI a). The condition *J* (clearance) is *always positive*.

2.2.2. Adjustments with uncertain clearance: Ø80 H7/k6 (see Figures A1.1(a)–(c) in Appendix 1)

- Clearance max = 0.009 mm and
- Clamping max = 0.002 mm.

The difference, between the dimensions of parts before the assembly, is *negative*. The tolerance zone of the bore (TI A) is entirely below the tolerance zone of the shaft (TI a). The condition *J* (clamping) is *always negative*.

2.2.3. Adjustments with clamping or interference (see Figures A1.1(a)–(c))

For example, for the assembly Ø80 H7/p6, reading qualities H8/h7 helps writing:

- clamping min = 0.002 mm and
- clamping max = 0.051 mm.

The difference, between the dimensions of parts before assembly, can be either *negative* or *positive*. The tolerance zones of the bore (TI A) and of the shaft (TI a) overlap, which we call as interference. The condition $J_{\max}$ (clearance) is *positive* and the condition $J_{\min}$ is *negative*. A schematic illustration for the three cases of adjustment is shown in Figure 2.5.

IMPORTANT NOTE.— In sections 2.2.1–2.2.3, adjustments are expressed in metric units (mm), whereas in Figures A1.1(a)–(c) in Appendix 1, equivalent results are deliberately expressed in imperial units. To do this, we used the software *Auto DESK Inventor Pro*.

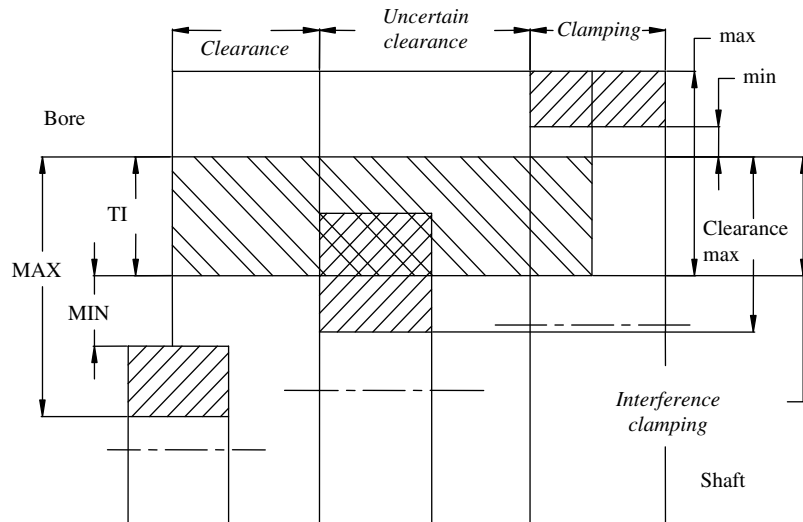
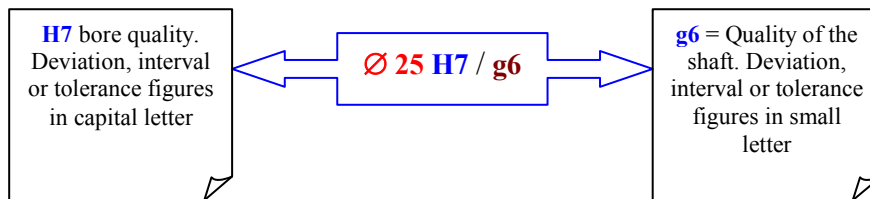


Figure 2.5. Schematic model of the three main cases of adjustments (SI)

ISO and CSA (standards) have similar designations. The following figure shows the output of LC5 (according to ANSI, CSA, and ISO = H7/g6 for ISO):



The CSA B97.3.70 suggests a North American additional designation. Some tables have been added, but the basic definition of the international system has not been affected (see all the tables in Appendix 1).

Entries are encrypted by their respective quantities in millimicron. For the CSA, attention should be paid to the unit of measurement used. Sometimes, imperial units are used in many cases. It is highly recommended not to mix “micrometer (μm)” with “microinch (μin)” (Figure 2.6).

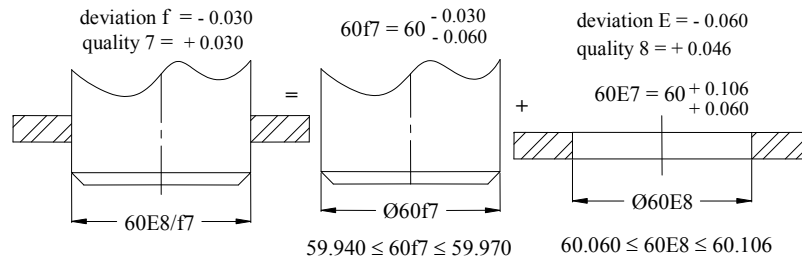
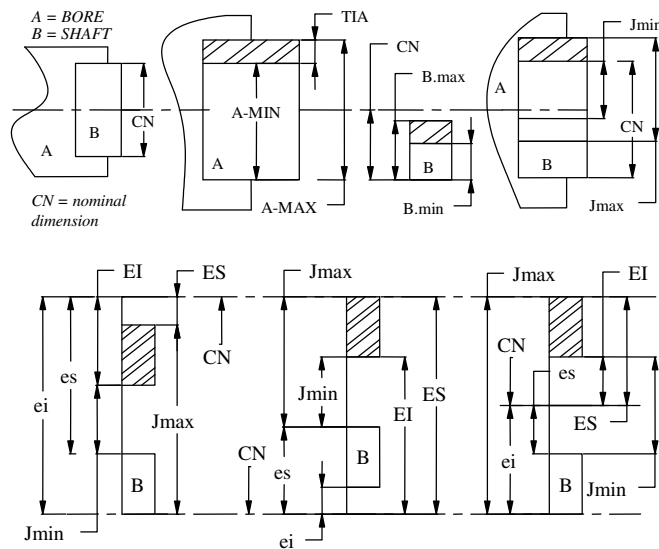


Figure 2.6. Examples of shaft/bore adjustments system (see also Appendix 1)

2.2.4. Approach for the calculation of an adjustment with clearance

Figure 2.7 is deliberately proposed to popularize reading assembly systems (shaft/bore). We limit ourselves to the case of an assembly with clearance H7/g6, but the calculation procedure and reading are the same for the other two cases (interference and clamping). In all cases, we resort to tables of ISO standard. Figure 2.7 illustrates that calculations are made relative to the nominal diameter represented by the CN. Clearances are function of deviations, and the ensemble gives what is called the condition (+). Figure 2.7 shows the designation of shaft/bore adjustments system.



Figures 2.7. Designation of shaft/bore adjustments system (SI)

$J_{\max} = HD + (-ld) = HD - ld$ $J_{\min} = LD + (-hd) = LD - hd$	Case 1: (HD and LD) > 0; (hd and ld) < 0
$J_{\max} = HD + (-ld) = HD - ld$ $J_{\min} = LD + (-hd) = LD - hd$	Case 2: (HD and LD) > 0; (hd and ld) > 0
$J_{\max} = (-ld) - (-HD) = HD - ld$ $J_{\min} = (-hd) - (-LD) = LD - hd$	Case 3: (HD and LD) > 0; (hd and ld) > 0

The three above-mentioned cases reflect the approach of calculations of clearances (max/min) allowing the assessment, the assembly type. Calculations adjustments by force or by dilatation are simple as evidenced by the terms of the TI of clearance as well as the average/mean

$$TI J = J_{\max} - J_{\min}$$

$$= HD - ld - LD - hd$$

$$= HD - ld - LD + hd$$

$$= HD - LD + hd - ld$$

$$= TI A + TI B$$

$$J_{\text{mean}} = \frac{J_{\max} - J_{\min}}{2} \quad [2.5]$$

2.2.5. Dimensioning according to ANSI and CSA [OBE 96]

The CSA standards and ANSI B97.3.70 [OBE 96] opted for the designations RC, LC, LT, LN, and FN. We shall just explain them without applying them to practical cases since the logic of reading and the logic of exploiting the tables are analogous to that proposed earlier by ISO.

As for the North American representation case, we propose an illustration to show what was agreed to be admitted in dimensioning according to ANSI B4.1-1967, R 1987. This standard is consistent with the *American–British–Canadian* (ABC), which is grouped into five categories as follows:

- RC: *running or sliding clearance fit* is the equivalent of ISO H7/g6,
- LC: location clearance fit,
- LT: transition clearance, interference fit,
- LN: location interference fit, and

- FN: force or shrink fit.

There are nine (9) categories of RC tolerances: *RC1*, *RC2*, ..., *RC9*, each of which corresponds to a specific assembly. By referring *Machinery's Handbook* [OBE 96], we can find all complementary information on this subject. There are five categories of *FN* assemblies: *FN1*, *FN2*, ..., *FN5*, which are grouped by ANSI. Later, we present some dimensioning cases recommended by the ANSI standard and accepted by the ABC.

Mechanical manufacturing technologies are becoming more important in the fields of education. The formative virtues of these technologies contribute positively to adapt education to the level of professional applications. The key themes of measuring are based on GPS standard. In dimensional metrology, we often measure shapes, positions, and lengths. It is then easier to define them, before measuring them. Geometrical constraints impose specific requirements on the *shape*, *position* or *surface state*. We present each of the geometrical constraints separately according to *ISO 1101-1983*. We summarize the descriptions of the geometrical and dimensional constraints by assigning them an appropriate symbol:

- a geometrical tolerancing of form, orientation, location, and run; and
- a generalities, definitions, symbols, and indications on drawings.

Form tolerances are universally grouped into six categories. We present them based on their symbols, and then we try to offer some application examples using the appropriate symbolism.

Geometrical and Dimensional Constraints					
<i>Form tolerances</i>					
Rectitude —	Flatness 	Circularity 	Cylindricity 	Profile line 	Profile surface 
<i>Orientation tolerances</i>			<i>Tolerances of position</i>		
Parallelism //	Perpendicularity ⊥	Tilt ∠	Coaxiality 	Symmetry ≡	Localization 
<i>Beat tolerances</i>			<i>Dimensional tolerances</i>		
Simple beat 	Total beat 		Example H7/g6 (ISO 282-1); LC or RC 5 according to the standard ANSI B4.1-1967 (R1974) and CSA B78.		

Table 2.1. Tolerances symbols (e.g.  means concentricity and coaxiality)

2.2.6. Definition of geometrical form constraints

The geometrical items include size, shape, and surface condition. Their form is of particular importance in the case of adjustments. Geometrical shapes are described mathematically. No primary standard is associated with them. Determining, as precisely as possible, deviations from simple shapes, such as a circle, a cylinder, a line, or a plane, is one of the tasks assigned to the dimensional metrology. This means that they are able to perform measurements of devices, which allows reproducing these forms without referring to their materialization.

2.2.6.1. Rectitude

Typically, the calibration of straightness standards, such as rules made of granite or steel, or even angular standards, is performed using a precision air bearings slide. Thus, it is *possible to achieve uncertainties in the straightness deviations of the order of 0.1*, which is possible to obtain measurement uncertainties of approximately $0.1\text{ }\mu\text{m}$ for linearity deviations over a measuring range of up to 800 mm. Conventionally, a surface is probed in a perfectly straight line or a perfect circle and then the rectitude is deduced.

2.2.6.2. Flatness

The quantity that measures the flatness is mainly embodied by optical flat for small dimensions and by marble for larger ones. A flatness interferometer, with phase-shifting measurement and image processing of the interference fringes, is used for calibration of optical flats (see Chapter 3). Generally, the flatness of the marbles is measured by electronic levels.

2.2.6.3. Circularity

This constraint applies to a cylinder or a cone in which all surface points are placed at equal distance on a perpendicular plane to the axis of that cylinder or cone.

2.2.6.4. Cylindricity

The area is bounded by two toleranced coaxial cylinders by the imposed tolerance. All points of the concerned surface must be within the tolerance zone. In dimensional metrology, the control of these forms is performed in various ways including the use of dial indicators and the method is called as comparison.

2.3. International vocabulary of metrology

2.3.1. Local nominal dimensions according to ISO/DIS 14660-1996

Language issues are one of the GPS problems. Despite the recommendations of the international vocabulary of metrology (VIM), many people still believe that the axis is a physically touchable (tangible) item. This reflects, in our view, a deficit of *rigor*. The axis – we should specify which axis is in question – is a wonderful creation in the spirit of the surveyor. As for the metrologist, he or she needs a physical entity to position, to refer to and, to measure and control. But the axis is far from being a tangible materialization since it is a theoretical entity. And, as such, ISO 14660-1996 suggests the following:

- an unambiguous expression of functional requirements;
- a truly international and unique code (this will not be easy!); and
- an overall consistency of all standards, thus avoiding “oversights,” useless duplications (redundancy), and especially multiple interpretations.

In addition to their separate respective definitions, Figure 2.8 provides a pragmatic understanding [CHA 99] *of the axes* and the local nominal dimension. We would note that the derivative element may be – among others – a center, a centerline, or even a median surface. The nominal element may be a cylindrical or a circular item.

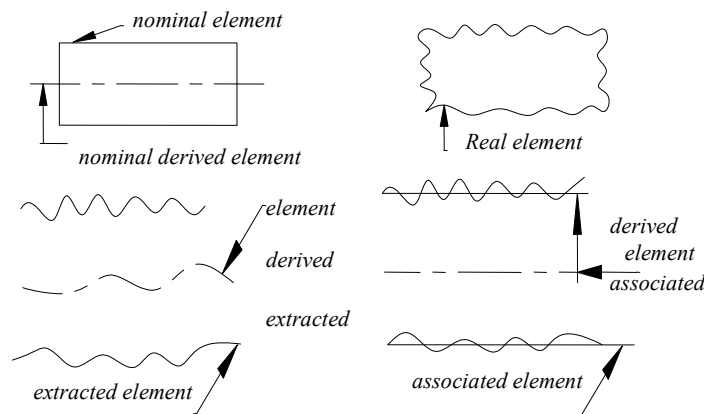


Figure 2.8. Nominal local dimensions. Nominal axis according to ISO/DIS 14660-1996

According to the metrologist, the nominal element is palpable. Then, we associate it with a derived component that is extracted from the element subjected

to measurement. This “extraction” is based on elements of mathematical analysis (geometry), which sheds light on the axis and the theoretical shape of the element (cylinder, circle, etc.), which would be deemed to be qualified as such.

2.3.2. Definition of the axis extracted from a cylinder or a cone

The *extracted axis* is derived from all circles having the *same center*. Mathematical methods applied to “situate” it are diverse, but the aim is to find a single entity known as the *extracted axis* by performing geometrical constructions for this purpose. Debate about the probing of the axis would lapse when observing what metrologists do, as shown in Figure 2.9 (courtesy of Mitutoyo).

Let us ask a *rigorous* technician metrologist to physically probe an axis; he or she will ignore our inquiry because he/she pertinently knows that he/she can only probe a generator. To demonstrate that these two probed generators are aiming the coaxiality in common for the two cylinders, we would seek to prove that the two probed cylinders would admit the *same axis of rotation*. If our technician has done his/her machining work properly, and so the assembly, then these two cylinders would admit the same axis. Only at that moment, could we theoretically probe the axis and never before that.



Figure 2.9. Inspection of the concentricity of local nominal dimensions to extract a “nominal axis” as defined in ISO/DIS 14660-1996 (courtesy of Mitutoyo Canada)

The Mitutoyo metrologists did not probe the axis because, obviously, they would *never* find it. They probe generators, which are in good compliance with ISO 1460. These generators:

- associate a cylinder (or a cone) with the extracted surface (e.g. using the least squares criterion);
- extract lines from the nominal surface after several cutting planes perpendicular to the associated axis, and associating a circle on each extracted line; and
- all the centers of circles give the extracted axis.

The extracted derived element (*extracted axis*) refers to the entity to be controlled (cylinder, cone, etc.), which is shown in Figure 2.10.

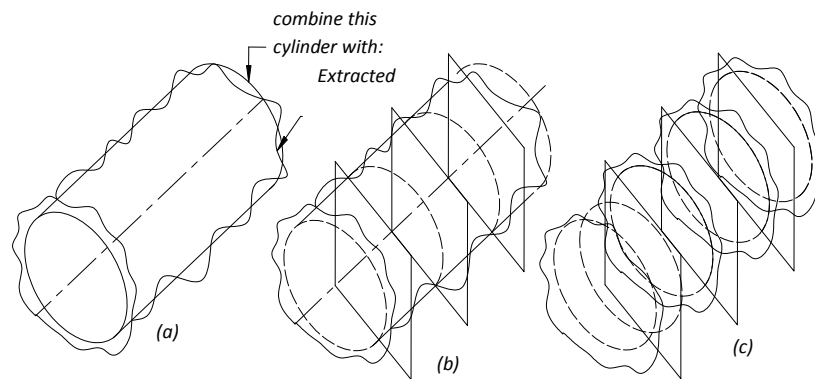


Figure 2.10. Definition of the extracted axis of a cylinder or a cone

2.3.3. Definition of the local size extracted from a cylinder

The size extracted from a cylinder gives the outline idea of a circle called *associated*. The size is calculated by mathematical regression. In each section perpendicular to the *associated axis*, we measure the distance between two opposite points belonging to the actual line along a straight line passing through the center of the associated circle (Figure 2.11).

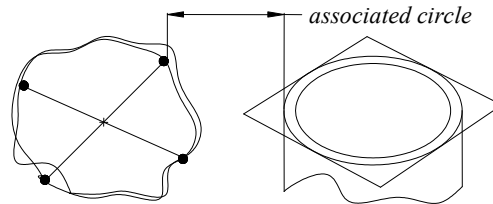
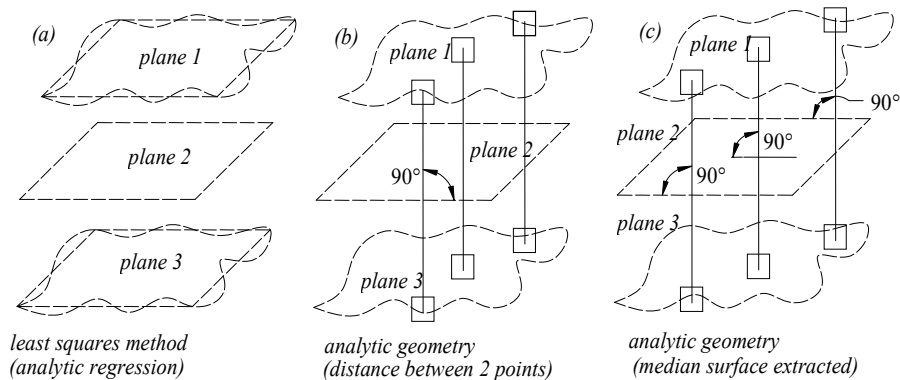


Figure 2.11. Definition of the local size extracted from a cylinder

2.3.4. Definition of local size extracted from two parallel surfaces

The local size extracted from two parallel surfaces is important when using a 3D measuring machine (Figure 2.12). This consists of:

- associating two parallel planes to the extracted surfaces (least squares criterion and distance between the variable planes);
- measuring the distance between two points belonging to the opposing surfaces along a straight line perpendicular to the associated planes; and
- identifying the extracted median surface.



Figures 2.12. Definition of local size extracted from two parallel surfaces

The local size extracted from two parallel surfaces is the set of mid-points of pairs of points belonging to the opposing surfaces along straight lines perpendicular to the associated median plane (Figure 2.12(c)). Dimensional and geometrical specifications, given on a drawing, characterize the geometric elements and their

positioning relative to each other. Geometric elements may be, for example, surfaces as a “morsel” of plane (cylinder, cone, etc.), or lines (line segment, circle, etc.), or even points. The *perfect elements* are defined and positioned in space relative to each other constitute what is currently called the *geometric model of definition*. The two types of control (verification of compliance with specifications) are the control by attribute and the control by measuring.

Control by attribute [NRC 00, NRC 02]. In this case, the obtained result is directly binary, that is, the characteristics of the real part are either compliant or non-compliant with the specifications. It is possible to use a shaft caliper, a dial indicator, etc. For example, for a control by pad, the smallest side must penetrate and the biggest one must not. This is the classical principle “Go,” “No Go”. In this case, we only control and do not measure. We quantify indirectly through the attribute of the chosen standard. We are *de facto* dependent in terms of measurement.

Control by measuring. A size (length or angle) is associated with the specified physical quantity. This value has to be identified whether it is included with or beyond the tolerable deviation. It is possible to use conventional instruments, a profile projector, or a coordinate measuring machine (CMM). We recall that the issue is not mathematical. It is rather physical and *linguistic*. Based upon the works of the VIM, the latter is about to be mastered.

2.3.4.1. Specified element and reference element

The specifications given on the drawing of definition are intrinsic, that is, a single geometric element is concerned. These are also the specifications of relative position, which position an element (specified element) compared with another one (reference element). The reference element and the specified element should be identified when the specification is directed. Reference element is identified by a blackened triangle, whereas the specified element is identified by an arrow. When it is impossible to distinguish between the reference element and the specified element, the elements are then considered successively as reference and specified (Figure 2.13).

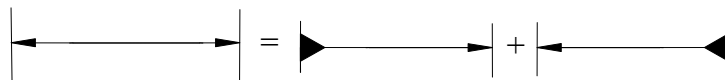


Figure 2.13. Identification of the reference element and the specified element

2.3.5. Notion of simulated element and associated element

The perfect elements, under admitted defects, may “represent” the nominal elements. This means a “simulated” surface, such as for example, a workpiece surface expected to be flat and which may be represented by the surface of a

“marble” on which it rests. “Mathematically perfect” elements may “represent” nominal elements, by mathematical construction based on “palpated” points (including CMM from a “point cloud”). This refers to “associated” elements stemming from the method of least squares:

- The reference surface is not palpated but simulated (contact marble, filler gauge, vee, contact on jaw instrument, etc.). We assume that the surface is perfectly polished, that is, deemed “ideal” and calibrated as such.
- Intermediate components are not built by calculation, but simulated (e.g. square for squareness).
- The specified surface is palpated or simulated.

Manufactured parts are never perfect. This is noted through our experience in mechanical manufacturing by machining and molding. It is utopian to believe that a piece is perfect at the end of its manufacturing stages. It is just good or bad, that is, approved or rejected, according to the jargon used in quality control. In view of the foregoing, it is therefore necessary to define all the nominal geometries that are acceptable, from the functional point of view (e.g. see Figure 2.14).

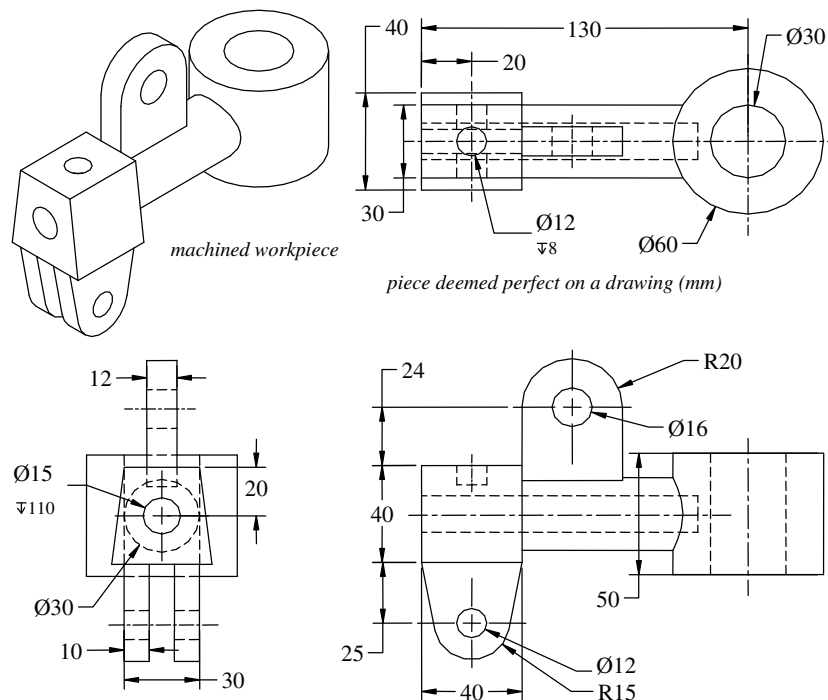


Figure 2.14. Piece “deemed perfect”, that is, defined by a dimensioned drawing

2.4. GPS standard covering ISO/TR14638-1995

The metrological concepts have certainly evolved, and standards have done likewise. The disparate ways, in which measurements are performed in different locations, are sometimes unreliable: Which language should be adopted? How to read and correctly interpret the tolerancing? How to choose characteristics that are concretely observable and in good-fit with the function? A generous and laudable work [CHA 99] was conducted by a team of expert instructors in France, entitled *Operation of GPS concept and standards* (French: “*Exploitation du concept G.P.S et de normalisation*”). This work provides extensive clarifications on GPS. The dispersion around the nominal is optimal, for which the function is always satisfied. Tolerancing is the act of determining deviation amounts (HD, LD and hd and ld) around the nominal to dimension a physical nominal entity (piece). In fact, it is important to separate the two operations:

- The definition of the perfect or nominal geometry deals with the functional dimensioning.
- The definition of geometric deviations tolerable around the nominal deals with tolerancing.

Dimensioning aims at defining, via a 2D drawing or a 3D model, the form, the size, and the position relative to each one of the elementary surfaces of the piece. *Tolerancing* aims at defining, via tolerances, the maximum deviations that nominal surfaces may support compared with nominal surfaces. The nominal geometry is defined by a drawing, which shows:

- the form of the surfaces making up the “skin” of the piece;
- the size dimensions of surfaces as well as the distance or angle between them;
- certain parameters of distances and angles between the surfaces are implicit; listed via the drawing: perpendicularity, parallelism, zero-distances, etc.; and
- features of axis are involved in the definition of the nominal.

2.4.1. Principle of independency according to ISO 8015-1985 (classic case)

The concept of Independency Principle is an explanation about the subject (Figure 2.15). It should be understood that the geometric or dimensional specification covers only a *single entity*. It does not address the surrounding constraints unless a contrary indication is clearly specified (written). In other words, each geometric or dimensional requirement specified on a drawing must be considered separately

(independently of the others), unless a particular relationship is specified. Special relationships are as follows:

- the requirement of the envelope $\textcircled{E}$, that is, the relationship between *size* and *form* and
- the maximum material $\textcircled{M}$ principle, such as a relationship among *size*, *form*, and *position*.

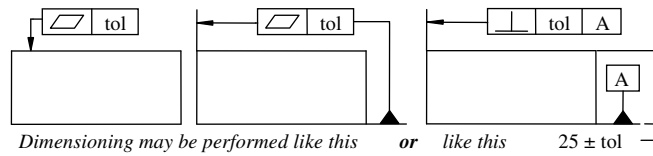


Figure 2.15. Principle of independency of geometrical constraints dimensioning

Defects that are known as “dimensional” are local defects, which result from measuring the actual distance between any two nominal points, or by the angular measurement between two identified straight lines (ISO 2692). In both cases, we limit the defects by indicating a maximum and minimum limiting value (Figure 2.16).

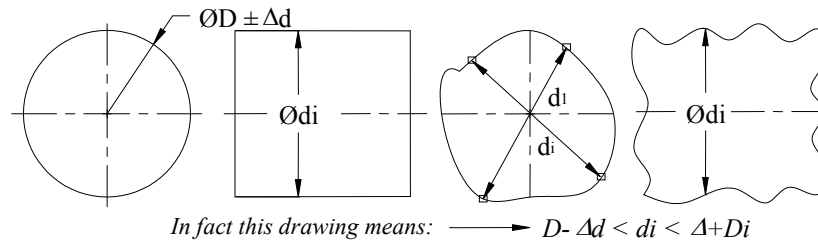


Figure 2.16. Example on the principle of independency of geometrical dimensioning

2.4.2. Envelope requirement according to ISO 8015

Used to measure parts dedicated to ensure a precise connection, it is noted with the symbol $\textcircled{E}$ after the dimension. This requirement establishes a relationship between the dimensional and the intrinsic specification of the surface (defects of forms). The surface of the cylindrical element must not exceed the envelope of perfect form with the dimension $\textcircled{M}$. The maximum material $\textcircled{M}$ is obtained when the shaft is at its maximum diameter and the bore is at its minimum diameter.

Figure 2.17 shows the significance of the envelope and maximum material requirements.

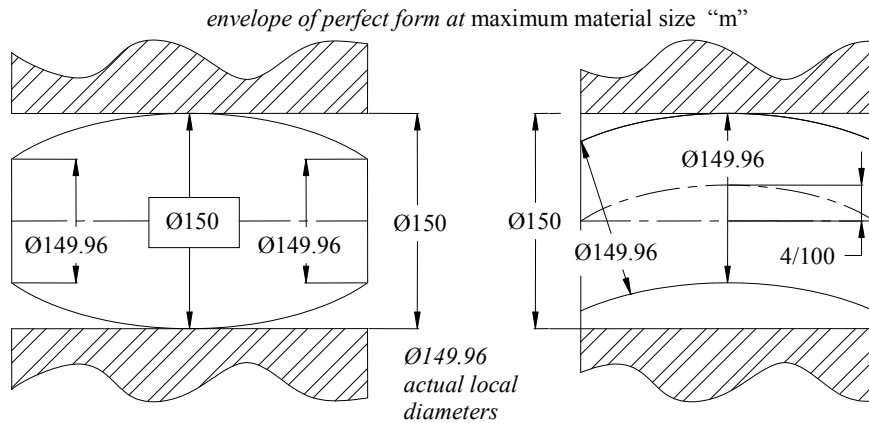


Figure 2.17. Envelope requirement and maximum material requirement

Figure 2.17 shows that no actual local size will be lower than the diameter d . Thus, it implies that the workpiece should meet the following requirements:

- The actual local diameter of each shaft shall remain within the dimensional tolerance of 0.04; let d_1 , d_2 , d_3 , and d_i be actual local diameters. They shall be included between Ø150 and Ø149.
- The whole shaft shall remain within the limit of the perfect cylindrical envelope of Ø150.

It follows that the shaft must be perfectly cylindrical when all local diameters are at $\textcircled{M}$, that is, ØD. At $\textcircled{M}$, for the shaft and the bore and at the maximum of perpendicularity defects, the clearance becomes zero. At the minimum material $\textcircled{L}$, for the bore and the shaft and at the minimum of defects, the resulting clearance is of 0.05 mm.

Thus, in the second situation, at the minimum material it is possible to have a greater perpendicularity defect without disrupting the functioning of the assembly. Figure 2.18 illustrates the assembly of piece at maximum material.

In controlling the shaft and the bore, two templates may be used to check whether the bore and the shaft mate or not. The templates being by definition *elements of control*, they are supposed to be *geometrically perfect*.

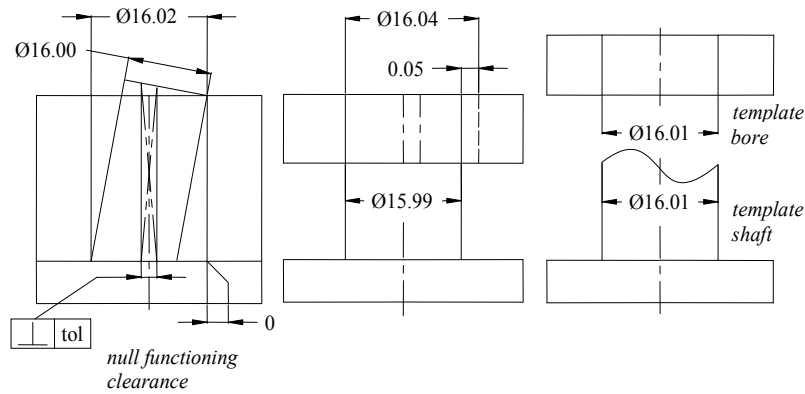


Figure 2.18. Illustration of the assembly of piece at maximum material

2.4.3. Maximum material principle according to ISO 2692-1988 (classic case)

The purpose of this principle is to ensure assembly at the *lowest cost*. The use of the principle of maximum material facilitates manufacturing, *without affecting* the free assembly, elements for which there is *interdependency* between size and geometry. The symbol $\textcircled{M}$ indicates that the tolerance was chosen taking into account the limits on the maximum material of the element or elements. The geometrical tolerance can be extended to the difference between the actual size of the finished part and the dimension corresponding to $\textcircled{M}$. According to the cases studied, the principle $\textcircled{M}$ applies to the toleranced element, the reference element, or both. The principle of maximum material applies to a cylinder, a cylinders group or two vis-à-vis parallel planes. The principle of independency does not because the geometrical tolerance depends on the actual size and also referred as the maximum material principle, but the term requirement is preferable. Tolerance could be zero, if it is associated with the condition at $\textcircled{M}$, as shown schematically in Figure 2.19.

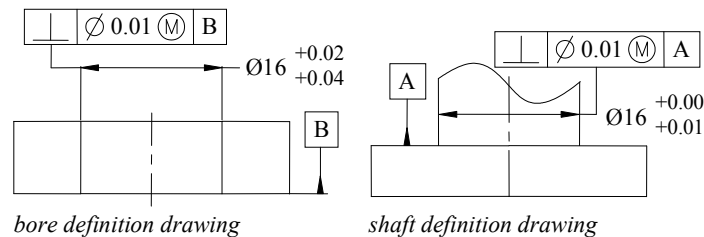


Figure 2.19. Designation of piece assembly at maximum material

The requirement of maximum material (*virtual states*) consists of *linking the geometrical tolerance to the dimensional tolerance* so as not to exceed a *virtual state* set by the two tolerances. The virtual state is obtained by adding (for a shaft) or subtracting (for a bore) the geometrical tolerance to/from the maximum material. Figure 2.20 shows the designation of the principle of maximum material.

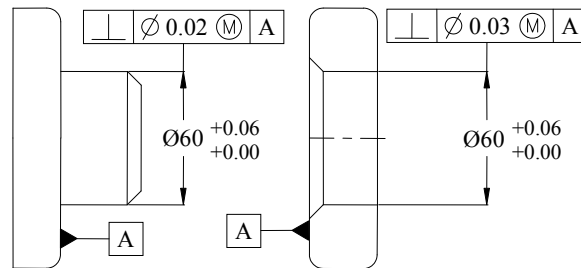


Figure 2.20. Designation of the principle of maximum material (*virtual states*)

The example in Figure 2.20 shows the following:

- size of the shaft at maximum 60 (– 0.06) at 0;
- size of the shaft at virtual state: 60 (–0.06) to (+0.02);
- size of the bore at maximum material: 60 + 0; and
- size of the bore at the virtual state, that is, $60^{-0.03} \rightarrow$ minimal clearance (*assembly support on A*) $\rightarrow j_m = 0.01 = 1/100\text{th}$.

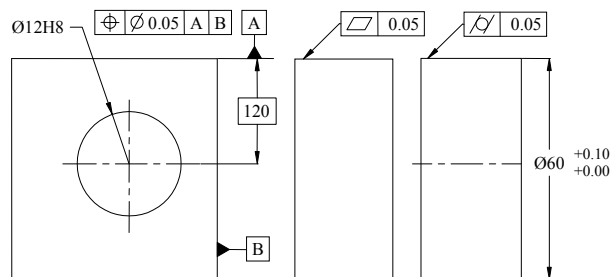


Figure 2.21. Examples of location, roundness, and flatness constraints

This function (maximum material) is probably better suited to conserve material during the machining processing (Figure 2.20). Experience shows that more examples are needed for students to raise awareness of respecting standards, in

the context of manufacturing analysis classes (three examples of dimensioning are shown in Figure 2.21).

These examples do not reflect all the geometrical constraints.

2.4.4. Form tolerances

The shape constraint could be applied diverse definitions such as the righteausness, the cylindricity, the circularity, or the flatness.

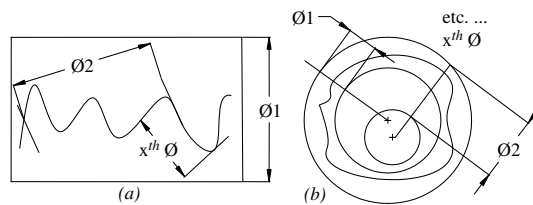


Figure 2.22. Significance of form tolerance

To measure form imperfection, the orientation and position of the *form tolerance zone* are such that they minimize the defect (Figure 2.22). Both the choice of the instrument and the measurement technique are based on the tolerance indicated on the drawing.

The arrow clearly shows the element to be tolerated. All points on the indicated surface should be on the theoretical 3D profile at the specified tolerance zone. The profile can be oriented or non-oriented relative to the reference. It is the same for the position. The dimension must clearly indicate that the element to be tolerated should be relative to the drawing it indicates.

Figure 2.23 shows an example of element to be tolerated according to ISO 8015-1985 standard. In this case, the inspection chain is carried out through CMM.

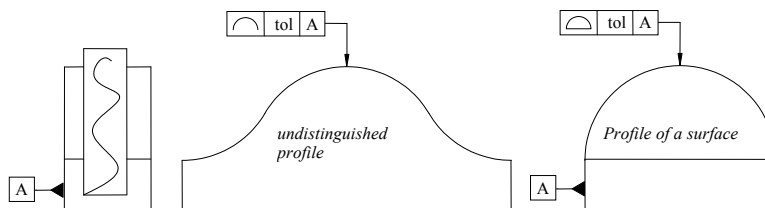


Figure 2.23. Profile and surface tolerance relative to the position

2.4.5. Flatness tolerances

The flatness is not ambiguous because of the non-circularity of the entity. It means that all points on the surface area must be on one plane at the specified tolerance zone (Figure 2.24).

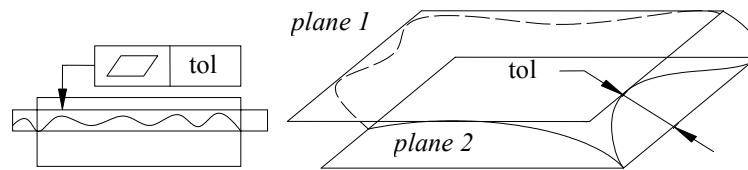


Figure 2.24. Flatness constraint and its conventional inspection

Tolerance area including the tolerated element is always expressed as a whole, unless the two previous specifications are mentioned. Also, a mesh (preferably square) is expressed based on constant pitch and the workpiece is planed by zeroing three distinct points (Pt_1 , Pt_2 , and Pt_3). Later, in Figure 4.52, we will present a brief procedure.

2.4.6. Straightness tolerance

All points on the surface area shall lie on a line in the direction indicated by the specified tolerance zone. When we place a tolerance in a given entity to a specific TI, the drawing becomes important because a deviation in the attention may distort the understanding of tolerance.

For example, in Figure 2.25(a), the tolerated element is the axis of the cylinder because the arrow that points to the framework indicates the tolerated straightness on diameter indicates the whole cylinder materialized by its axis. This is the result of the symbol $\varnothing$ that prioritizes axis and not the generators of the cylinder.

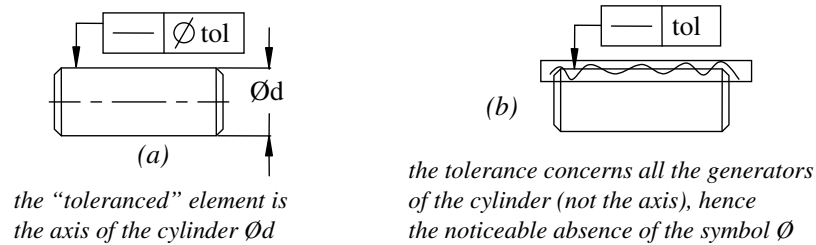


Figure 2.25. Toleranced element on a Øtol . Meaning of straightness constraint. Conventional control (by comparison) of straightness (courtesy of Mitutoyo)

Note that the axis of the test piece is parallel to the plane of measurement. The comparator support (touch knife) moves in accordance with the rule. The measurement is repeated on, at least, three generators to get an accurate result. As is traditionally done on machine tools, the test piece should be turned to eliminate the so-called preferred positions:

- *Case of maximum material principle.* A control ring (cylinder envelope) is to control the piece (*go, not go*). The piece must fully penetrate the ring gauge.
- *Example of deficiency of the ISO standard on straightness.* This example goes without comments. Let us simply examine designations as shown in Figure 2.26.

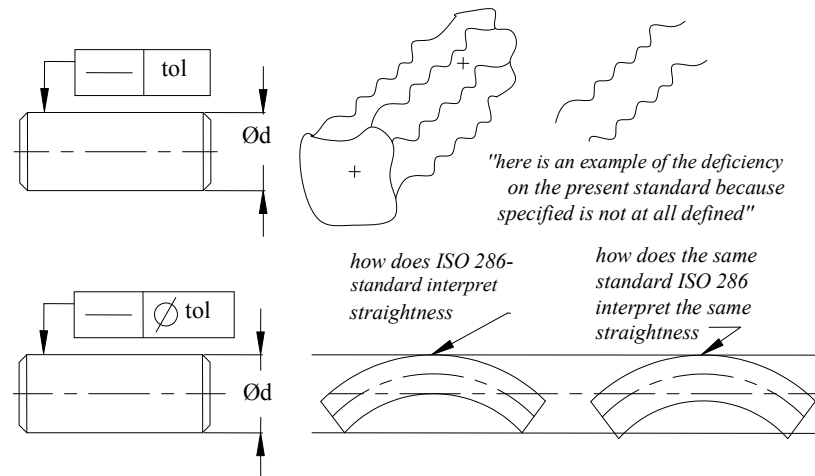


Figure 2.26. Illustrative examples of the “deficiency” in the ISO standard for straightness

2.4.7. Roundness

If the designated surface is cut by any plane perpendicular to the axis, the result that ensues gives a *perfect circle* at the specified tolerance zone. The roundness, as shown in Figure 2.27, covers the section planes (*Plane₁*, *Plane₂*, ..., *Plane_i*). This simple description cannot mislead because the symbol is clear and the arrow points directly to the circle (or a point of the cylinder that belongs to a circle).

We know that the cylinder is composed of infinity of circles perfectly aligned along the nominal axis. There may be an infinite number of circles around the *nominal axis* and the tolerance zone concerns *each section plane* which is shown in Figure 2.27.

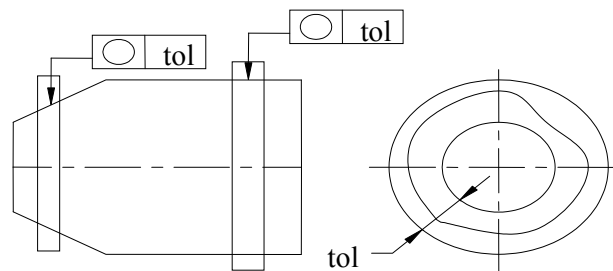


Figure 2.27. Defects of the constraint of circularity and their meanings

2.4.7.1. Channels (means) of inspection in conventional metrology

We present and comment hereinafter a series of means of checks (traditional means).

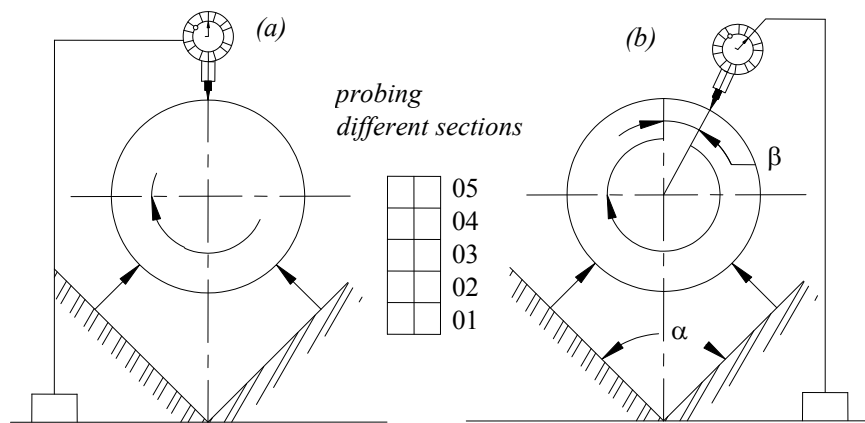


Figure 2.28. Channels of conventional inspection of circularity constraints

Careful choice of angles α and β in Figure 2.28(b) would allow preventing certain defects due to lobed parts and pairs. The control assembly called asymmetric (Figure 2.28(b)) would be better suited and is dependent on the angles α and β . This involves a circularity measuring instrument if not the CMM. In this case, the axis of rotation is of very high precision. Punctual assembly is performed via sliding and punctual pivot. The measurement speed is consistent with the response time depending on the measuring equipment being used.

2.4.8. Cylindricity

All points on the surface area must be on a perfect cylinder around the central axis at the specified tolerance zone (Figure 2.29).

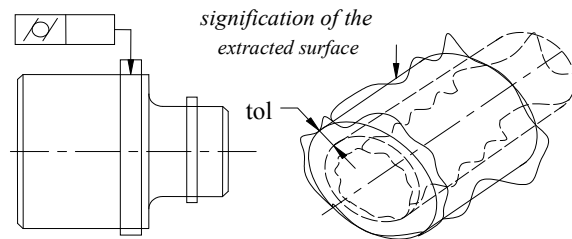
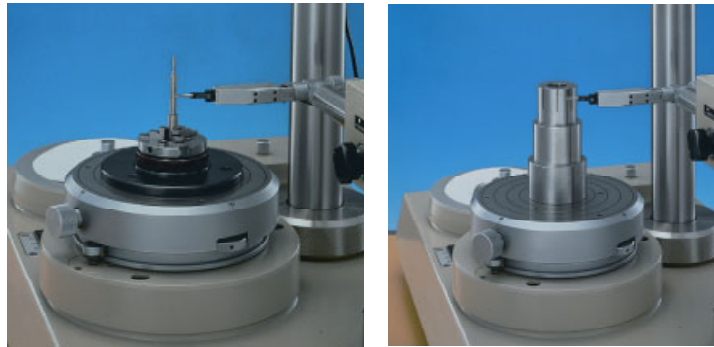
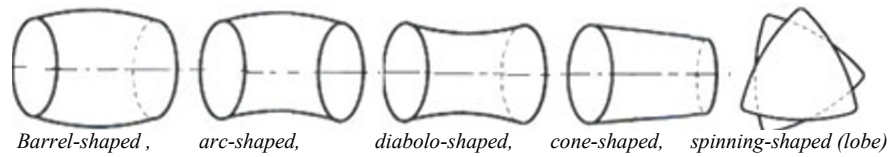


Figure 2.29. *Cylindricity imperfections. Cylindricity constraint and its meaning. Conventional controls respective of the cylindrical form and roundness (courtesy of Mitutoyo Canada)*

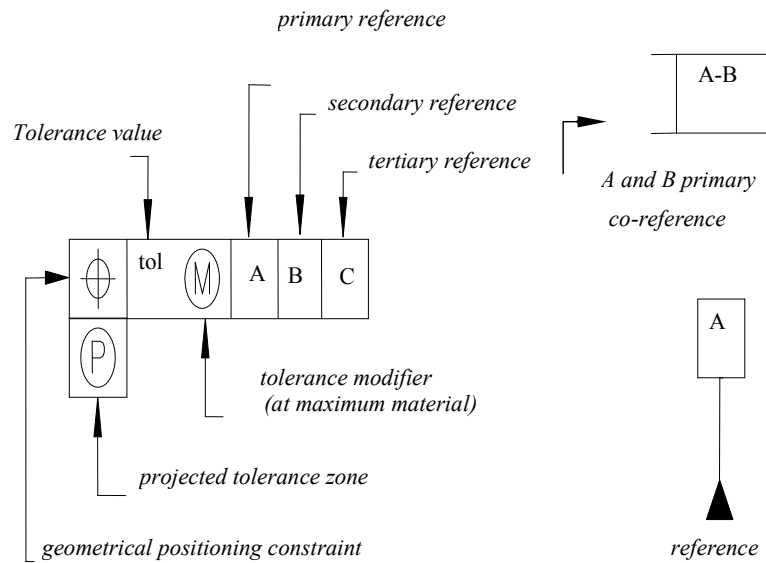
IMPORTANT NOTE.— The techniques applied to the cylindricity deviations, stemming from traditional metrology called marble and comparator, are given only based on discrete elements from the piece surface deemed cylindrical, that is, by statements of straightness or even roundness. Of course, on CMM, we probe at least eight points on two extracted circles sufficiently spaced. For example, for a $\varnothing 50$ mm lying over an actual length $l = 200$ mm, the technical literature advises to probe at least 20 points.

2.4.9. Orientation tolerances

– Parallelism tolerance //;

- perpendicularity tolerance $\perp$; and
- tolerance of inclination $\angle$.

Each orientation tolerance must relate to a reference element.

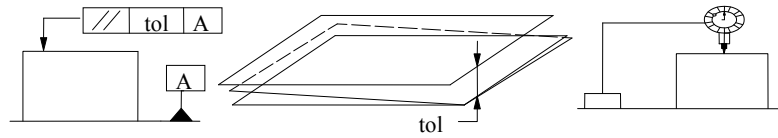


An orientation tolerance limits the orientation defect as well as form defects. Orientation tolerances are explained in the following sections.

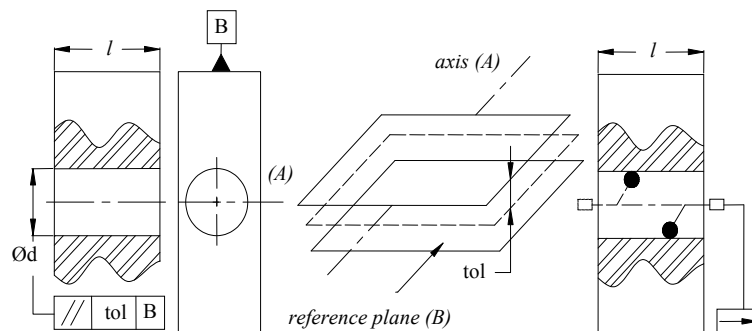
2.4.10. Parallelism (straight line/straight line)

All points (or central axis) lying on the designated surface are located on a single plane parallel to the plane (or axis) designated by the reference surface (A) at the specified tolerance zone.

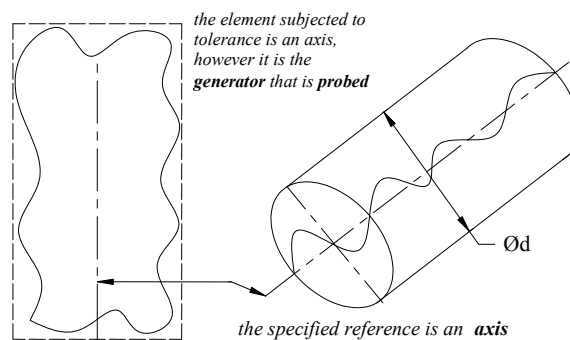
Parallelism is also measured relative to two palpated straight lines. This presupposes that the reference is designated on one of the two items to be controlled. The tolerated element is, in fact, an *actual* axis and the reference (A) concerns the axis of the first palpated item.



The comparator is calibrated at zero on a point belonging to the surface to be inspected. It will be displaced over that surface.



The piece should be planed and the whole axis should be parallel ($//$) to the rule. Then the higher generator is inspected. This method has limitation because the comparators accessibility is limited. The same is applied for a CMM since the probes are not long enough to access the entire probing surface.



Parallelism concerns relative entities, that is, a straight line relative to another straight line, a straight line to a plane, or a plane over another (Figure 2.30).



Figure 2.30. *Significance of a parallelism (straight line/straight line) inspection (Mitutoyo Canada)*

2.4.11. Parallelism plane/plane (plane/straight line) on CMM

In the case of a plane/straight line, we would first probe a plane called reference (A) with at least three points on the first surface, and then control the parallelism with respect to a Gaussian preference straight line. In the plane-to-plane case, we probe first a plane Pl_1 called reference (A) with at least three points on the first surface and then controls the parallelism with respect to the other plane Pl_2 as shown in Figure 2.31.

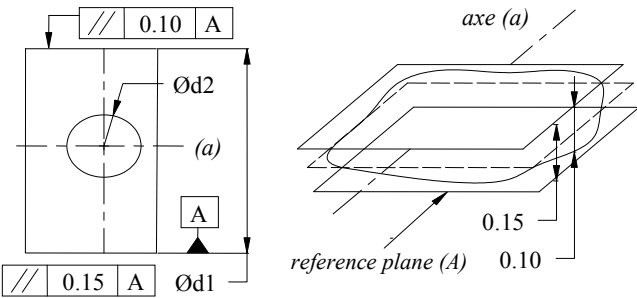


Figure 2.31. *Exhibit of control of “plane-to-straight line and plane-to-plane” parallelism*

2.4.11.1. The perpendicularity straight line/straight line (squareness)

All points and all lines on the surface are designated on the only plane at 90° relative to the reference (A) located within the specified tolerance. The approach is

similar to that previously explained. Figure 2.32 illustrates the perpendicularity constraint and thereby suggests the instrument and the mounting technique to make the measurement. Actually, the tolerated entity in Figure 2.32 is the *nominal axis* relative to a reference specified (A). As we cannot physically reach the axis, we “think” illustrating it as in Figure 2.32.

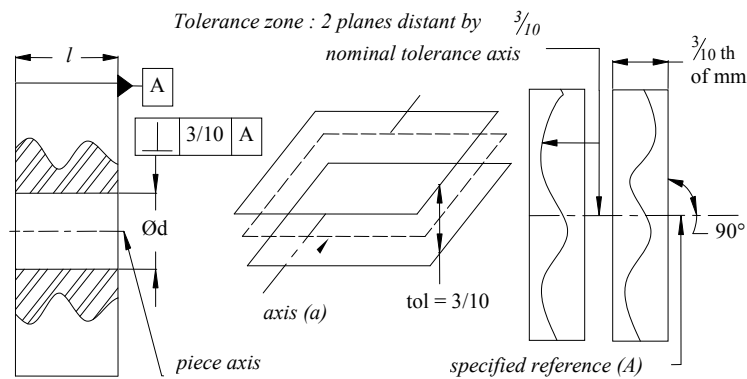


Figure 2.32. Exhibit of control of a perpendicular (straight line/straight line)

2.4.11.2. Perpendicularity plane/straight line and straight line/straight line

In Figure 2.33, the perpendicularity is measured on a plane relative to a straight line or a plane from another plane as shown in Figure 2.33(a).

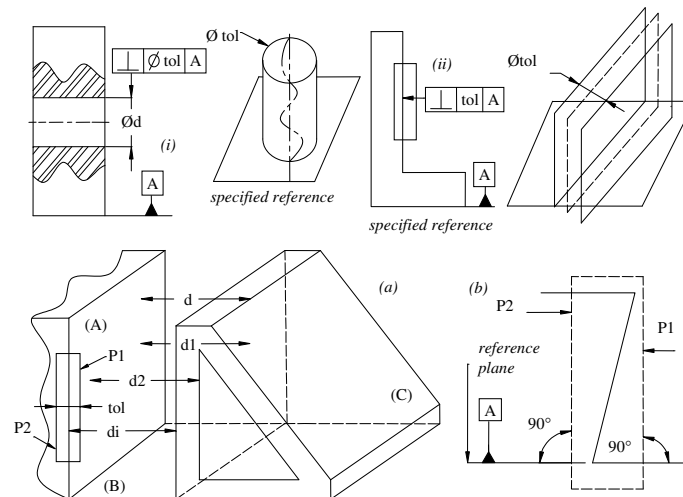


Figure 2.33. Perpendicularity control (plane/straight line and plane/plane)

2.4.12. A workshop exercise on dimensional metrology

Check the squareness of the mounting bracket according to the specifications on drawings (i) and (ii). The dimensions must match the measurements that result from our respective workshops set squares. It is recommended to use traditional metrology devices and instruments, that is, marble and granite, comparator, measuring blocks, and gauge blocks – if necessary.

2.4.12.1. Workshop and laboratory exercise on dimensional metrology

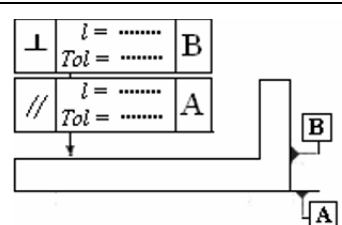
Definition drawing		Problem statement		
		Execute, according to CSA, ISO, or ANSI standards, the indications of geometrical tolerances specified below. The workpiece machined by high-precision surfacing or by flat grinding, depending on the workshops. We are asked to inspect the constraint of parallelism and perpendicularity (GPS).		
Test machine, for example, machining a block	Manufacturing conditions	Controls in workshop/laboratory	Apparatus and instruments of control	Allowable errors, uncertainty with $k = 2$ at the <i>threshold</i> 95%
Conventional machining (or CNC): surfacing or surface grinding of T-shaped block	For example, how to fix the part subject to control (test piece?)	Control of geometrical constraints// and (b) with a precise description of the faces (support/surface)	MMT: – If workshops, traditional metrology; – if laboratory, CMM	Error must be <0.005/300 mm
TIs will be strictly respected on the length (l) and the cross-section (b)				
Experimentation 01				
Experimentation 02				
Experimentation n				
The value				

Table of values read in accordance with the definition drawing and machining range	
Value read via the longitudinal displacement →	In mm and in.
Value read via the transversal displacement →	In mm and in.
Dimensions (range) of the piece to be machined →	In mm and in.
Tolerance provided in longitudinal direction (l) →	Report the tolerance on the drawing + drawing
Tolerance provided in transversal direction (t) →	Report the tolerance on the drawing + drawing
Results of calculations of uncertainties (section 2.1)	$K = 2$ at the threshold 95% (according to GUM)
<i>Equipment of the checkpoint in metrology workshop and laboratory</i>	
Reference surface, if workshop → for example Reference surface, if laboratory →	Table and gauge block rectified high precision granite marble
Amplification of the precision, if workshop → Amplification of the precision, if laboratory →	
Comparator support, if workshop → Comparator support, if laboratory →	

2.4.13. Angularity

All points on the designated surface or the axis (nominal) must lie on one plane, in accordance with the angle exactly specified on the reference of plane (A) or the nominal axis at the specified area tolerance zone. As with other geometric constraints, angularity requires first a reference origin point. Here, it is assumed to be (A). For a conical cylinder, using CMM, it is noted that four points should be probed around the circle distant from another circle in which probing is exercised, at least, four times to obtain the desired angle and conicity (Figure 2.34).

2.4.14. Positioning tolerances

Based on orientation tolerances, we explain the following three position tolerances, namely:

- symmetry,
- coaxiality (sometimes called concentricity), and
- positioning.

Note that a position tolerance not only *limits the positioning defect* for the tolerated element, but also limits the *form and orientation* defects. Each of these tolerances must relate *to one reference at least*. We present two symmetry cases in the following sections.

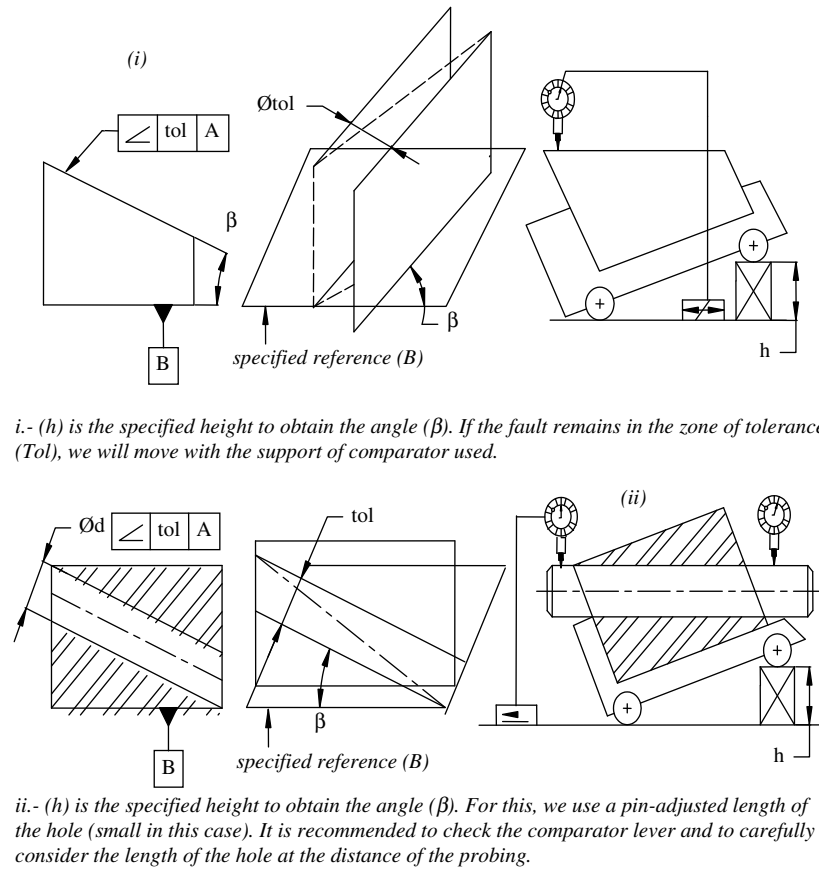


Figure 2.34. Exhibit of an angularity constraint control

2.4.14.1. Symmetry

2.4.14.1.1. Symmetry I

For the case of symmetry, it is easier to consider two cases of designation. The first is known as symmetry I. The second case is in fact noticeably identical. The difference lies in reading the reference specified relative to the *actual median surface* as exhibited in Figure 2.35.

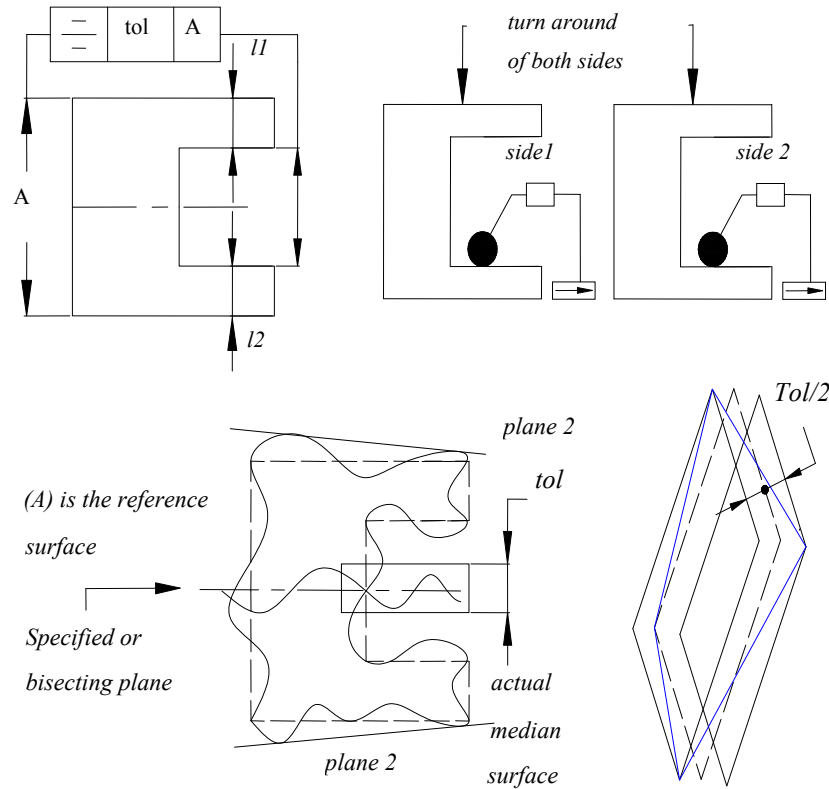


Figure 2.35. Symmetry I constraint (example of a clevis)

The fault is deliberately exaggerated. This is done by the method known as “zero,” that is to say, put on zero in (1) thereafter deflect the comparator in (2) to verify that the tolerance meets the measurement condition (called direct): $|I_1 - I_2| \leq \text{tolerance}$.

For example, the symmetry (I) is designated by a *nominal median surface* located on the generator of the cylinder and thus the reference surface (A) is affected. This is explained by the fact that the generator of the “actual” medial surface should be probed. This is physically possible. It is also easy to control this constraint using another mechanical or optical device (profile projector). Control by mechanical comparison, rather than optical, seems to be more relevant for geometry for space or accessibility reasons.

2.4.14.1.2. Symmetry II

Symmetry (II) is designated by an “actual median surface” located inside the generator of the cylinder, and the reference surface (A) that is designated to it refers to the cylinder, as for symmetry (I). The “actual” median surface points within the path of key (e.g. the key housing groove). In courses on design for mechanical manufacturing, we performed a snowmobile cam (Ski-Doo); here are the results (Figure 2.36).

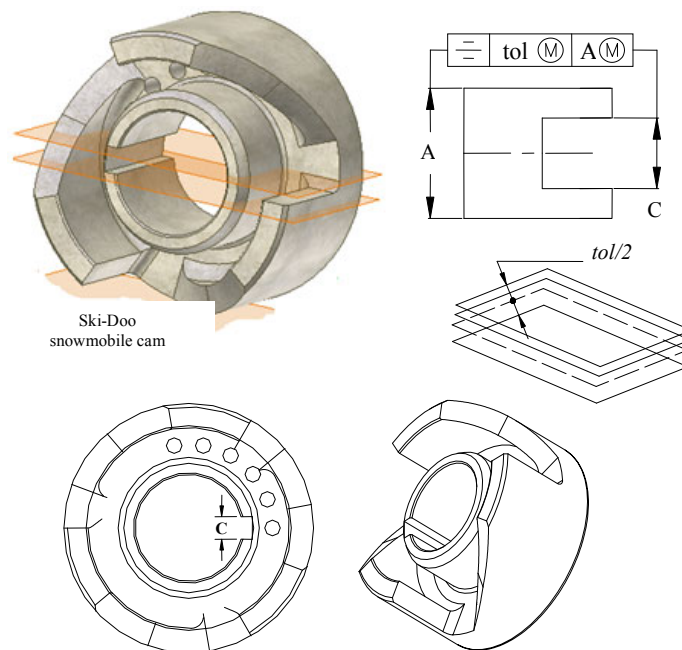


Figure 2.36. Example on the designation of the symmetry II constraint

2.4.14.2. Coaxiality or concentricity

If the designated area by the cutting plane is perpendicular to the reference surface (A) (or to its “nominal” axis), then every central cut surface must lie on the referenced axis, at the cylindrical tolerance area. This also controls the rotational equilibrium. Coaxiality (sometimes referred to *concentricity*) is a constraint that imposes a requirement in relation to the “nominal” axis. The dual use of vocabulary is, in fact, not by chance. We know that the axis has a very specific connotation. Around this mathematical entity, things are spinning. The center does not necessarily refer to the movements. It may be either static or dynamic.

In the case of moving objects, the term axis is preferred to the term center. We talk about the nominal axis of the cylinder not its center because this would be less rigorous and evasive. A center is often considered as a point (material).

An axis is rather considered similar to a line (straight line or generator). But let us be consistent, that is, the *nominal axis* that we are talking about is not the simulated one. Here, we may refer to axis when it comes to specify a reference (A in our case). This does not mean that the metrologist will find the *illustrious imaginary axis* to install its comparator or probe. He simply deduces, if the audit is consistent with its design definition that the *material axis* used in rotation was well aligned. What does “well aligned” mean? It means to well align the tailstock and tip in the chuck. Figure 2.37 clearly illustrates the mechanical metrology comparison, and mounting the marble is controlled using two dial gauges.



Figure 2.37. Control of the concentricity (coaxiality) by mechanical comparison (Mitutoyo)

In the mechanical metrology comparison, two dial indicators are used. If the information shown on the dial gauge is identical (just agreement with tolerances), then we conclude that the cylinders have a common axis, which is a nominal one (the axis).

It would be explicit to support our explanation with a drawing (Figure 2.38).

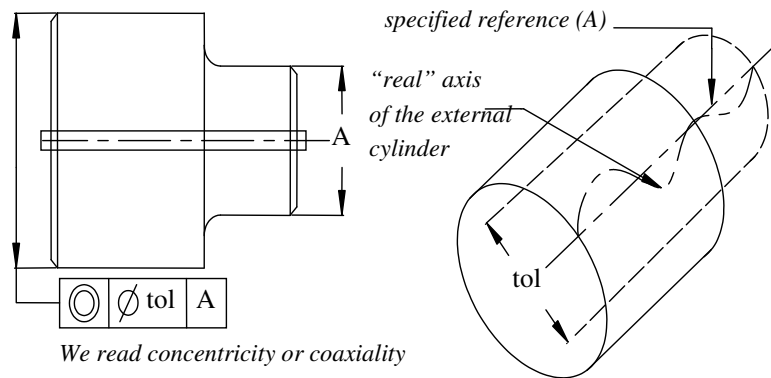
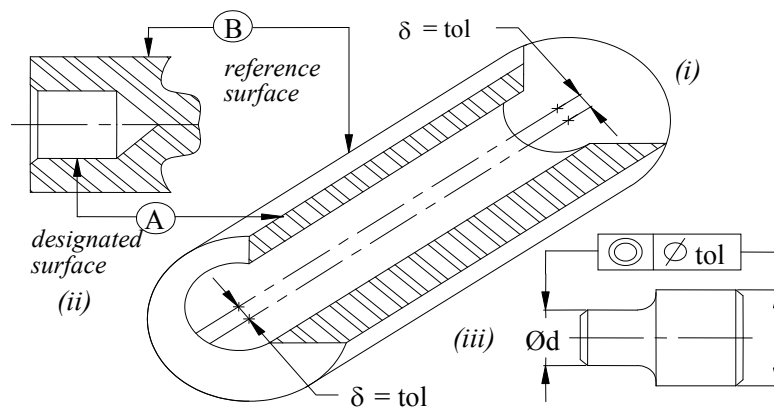
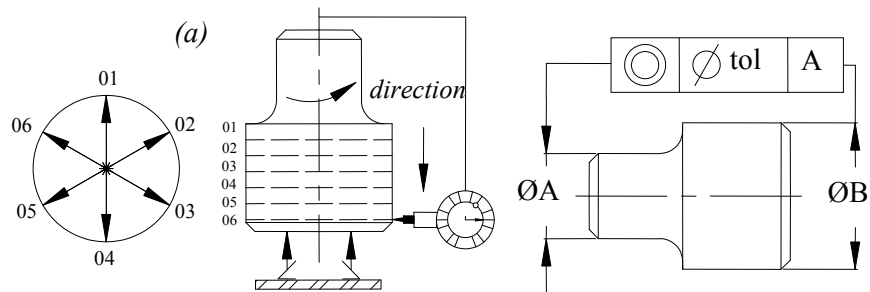


Figure 2.38. Design and designation of concentricity constraint

The intent of the assembly is to demonstrate, within the tolerancing limits imposed by the design, that the two palpated entities have one and only one axis, which is common to them. This is simply expressed in terms of analytic geometry. In practice, this requires specific ways in addition to a proven skill.



Adjustable mounting on V-shaped block. Probing will be carried out on two extreme sections following, at least, six angular positions. We could also use a pin that should penetrate without forcing.



We use a high-precision spindle in the above figure (a). The test piece is placed such that the axis of (B) will coincide with the axis of the spindle. At least one statement on six should be realized. The result is excellent, but requires high skill.

In Figure 2.39(a) and (b), the piece will be placed on a V-shaped block. Many sections (Si) are then probed before to come up with a meaningful calculated mean.

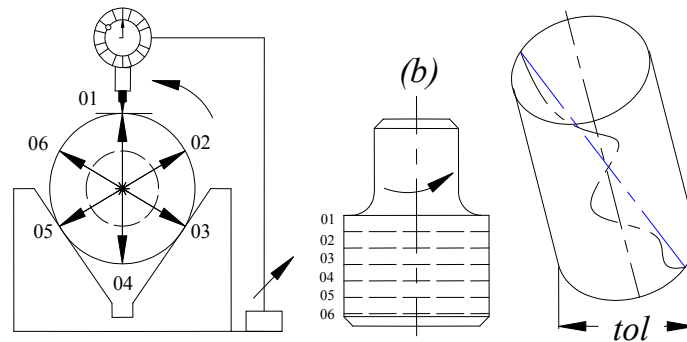


Figure 2.39. Control of concentricity by mechanical comparison

2.4.14.3. Positioning

Positioning (localization in French) means that the entity you control is localized by coordinates on the reference (x , y , and z). For this, it is imperative to choose at least two reference surfaces (A) and (B) and to verify, given the limits permitted by the TI, that the coordinate of the hole is (x and y). If this is the case, we confirm that the permitted positioning defect is verified, if not we disprove it and a future solution will be chosen (Figure 2.40).

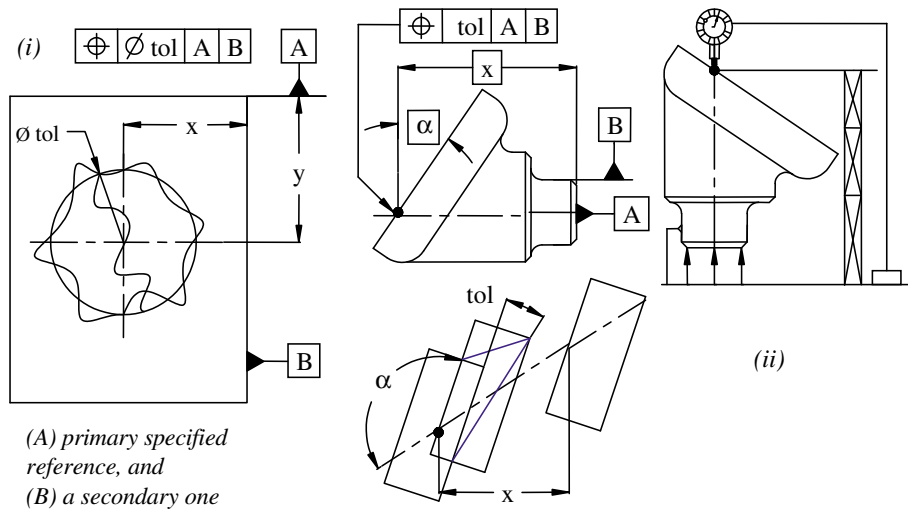


Figure 2.40. Exhibit of positioning constraint (selected cases)

Here we apply a differential method. The axis of the probe passes through the axis of the (cylinder) B. The support of the comparator is well fixed. Generator (1) is palpated first, then the piece is turned and generator (2) is palpated (looking for the cusp). It is a differential method called by comparison to a gauge block (i.e. high accuracy) height contained within the framed dimension (x) and (y). Of course, there are other ways (channels) to investigations to be explored in metrology. For example, we could use a control pin that would return without straining to control in the way “Go” and “No Go.”

Sometimes positioning replaces symmetry. Here, the entity designated by the axes shall be localized within the zone of tolerance of its *true theoretical* position and correctly oriented compared with the relative reference indicated by the plane or axis.

It is known that the zone of tolerance may relate to the functions listed earlier. The corresponding ISO standard is clear. We should refer to it to conduct an accurate dimensioning and make the interchangeability becomes really concrete. Continuing to *systematically* point toward the axis, each time a drawing is presented to tolerance dimensioning, is simplistic and even erroneous in many cases. The tolerance zone can be only one of the following types:

Form tolerance $\rightarrow$ *orientation tolerance* $\rightarrow$ *position tolerance*.

- A position tolerance limits defects in terms of positioning, orientation, and form.
- An orientation tolerance limits defects in terms of orientation and form.
- A form tolerance limits only form defects.
- The same surface can be tolerated depending on the three characteristics combined together (positioning, orientation, and form).

2.4.15. Tolerance of single radial flap (radial runout)

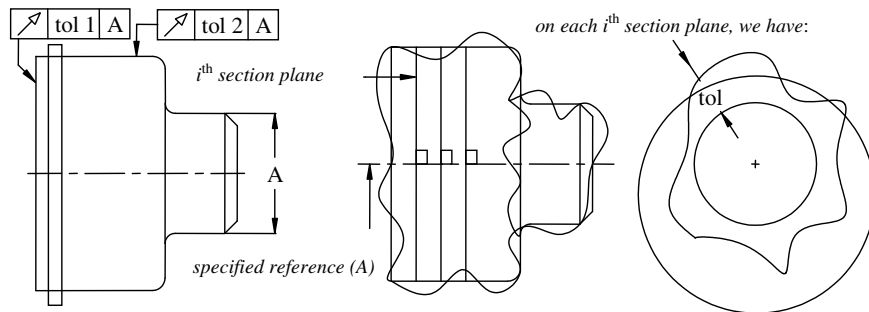


Figure 2.41. Tolerance of single radial flap and its significance

It should be understood that each circular element of the indicated surface is allowed to deviate only upstream of its shape and its orientation rotated 360° from the reference designated by the central axis. The beat comes in two categories. The simple beat and beat up. The reference in question is the central axis, but physical referencing is made on the specified surface (A). Figure 2.41 shows the meaning of beating. It is a precise assembly where the measurement is repeated in several sections.

2.4.16. Tolerance of single axial flap (axial runout)

Figure 2.42 shows the significance of tolerance of simple axial runout. However, it should be noted that the metrological control of beats is important for moving parts. Of course, it would be easy only to control beats because other factors (vibration, questionable editing, etc.) will distort tolerance check imposed by the previous plot.

The reference surface should be pointed toward the generator of the cylinder and the symbols arrow (TI and reference surface) to the entity orthogonal to the surface assumed rotating. If the comparator indicated a value within the limits required by the drawing, then the beat will be controlled in this type of control. It will not be bound by the movement for reasons inherent in the machine, the conditions of movement and skill of editing/assembly.

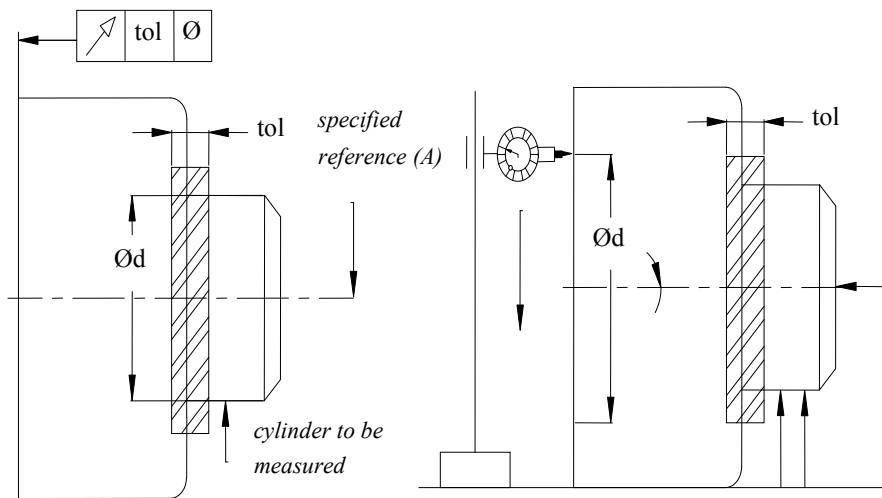


Figure 2.42. Significance of tolerance of simple axial runout

For one revolution of the workpiece, the value indicated by the comparator does not exceed the tolerance (Tol). For each cylinder subjected to measurement, diameter varies. It is also feasible to use a clamp and a pin of high precision. If measurement direction is not implicitly normal to the surface, it should be indicated by a framed dimension.

2.4.16.1. The total radial flap (total runout)

The total beat (called double) can be either radial or axial. The entire surface shown is allowed to deviate only upstream of its theoretical form and its orientation during a 360° rotation from the reference designated by the central axis (Figure 2.43).

Assembly is realized on Vee + axial thrust-bearing. The maximum (total) deviation of reading on the comparator is of Tol.

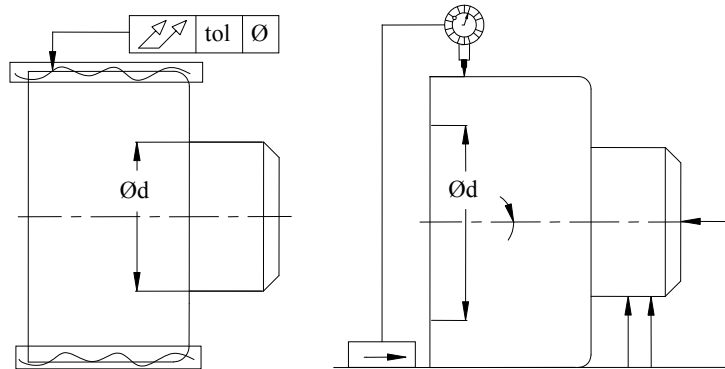


Figure 2.43. Control and description of the total runout constraint

2.4.16.2. The total axial runout

In this case, we prefer to, theoretically, refer to the axis. The reason is obvious. The part that revolves around is called an axis of rotation. For forklift, a rotation movement should be imposed around this axis, which generates an infinite number of generators that are palpable, thus materialize control and measurement (Figure 2.44).

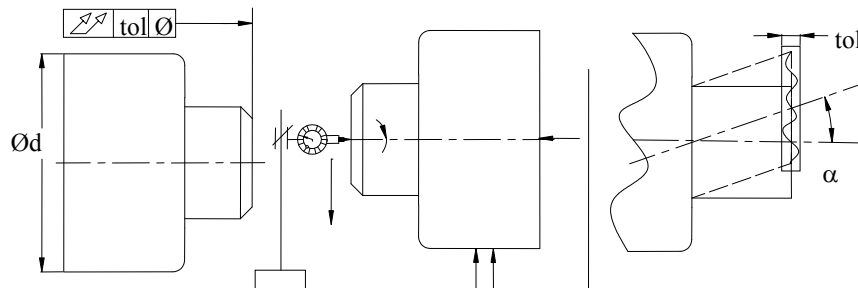


Figure 2.44. Control of the total axial stress beat

The maximum deviation (total) of reading on the comparator is *Tol*. The assembly is performed on a V-shaped block + (axial) thrust-bearing.

All the cases already presented cover all the entity to be inspected, that is, the whole part is assumed as homogeneous and deemed perfect. Next, we deal with an “isolated” fraction of the part.

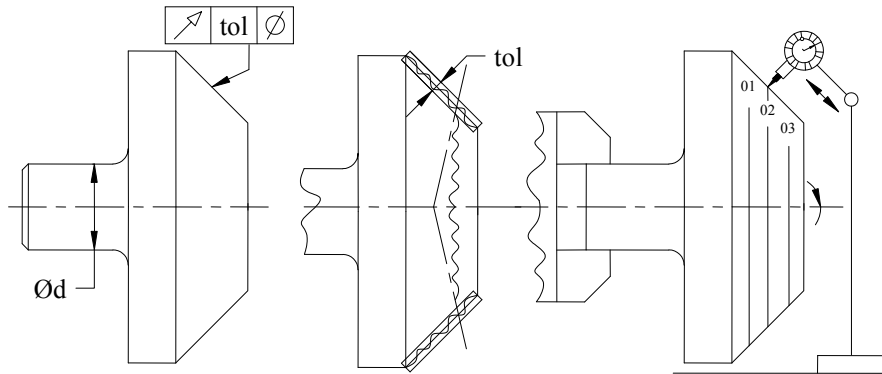


Figure 2.45. Method of control of the simple oblique beat (pin and clip)

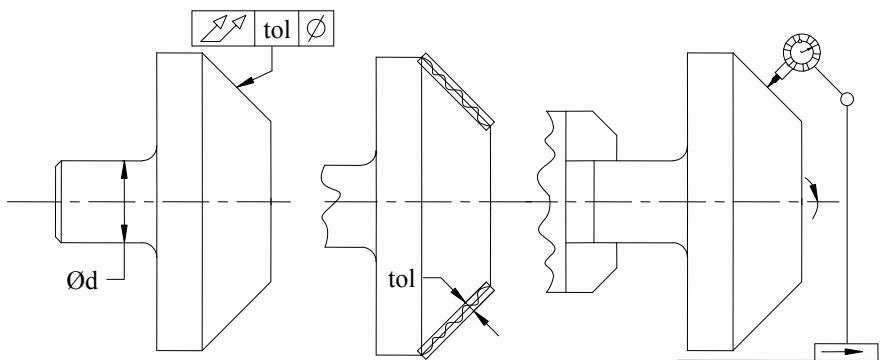


Figure 2.46. Control method of total angle flap (pin and clip)

2.4.17. Zone of tolerance applied to a restricted portion of the piece (as in // and in)

The tolerance zone is applied over the entire length of the tolerated element (by default). However, it is possible for functional reasons, to restrict a tolerance zone to a portion of the tolerated element. ISO 1101 allows doing so. In the following example, we propose the designation and the significance of these constraints on the direction of the specified reference area (Figure 2.47).

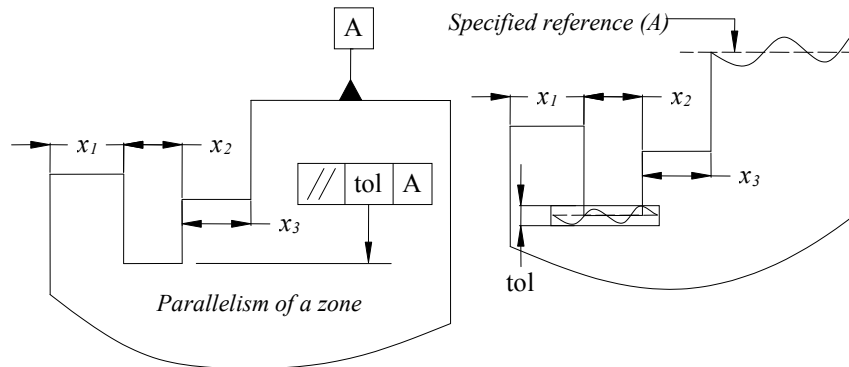


Figure 2.47. Designations of parallelism tolerances of a “zone”

For a well-specified zone, we can control the flatness as shown in Figure 2.48.

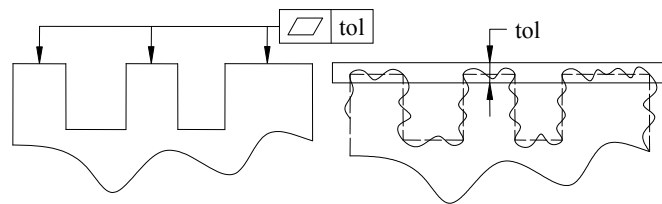


Figure 2.48. Designation of a flatness tolerance of a specific “zone”

If, by tolerancing the whole feature, another tolerance similar but smaller and restricted to a limited length is added, the latter shall be entered below the first one. When a common tolerance zone is applied to separate features, the requirement must be indicated by the term “common zone” over the tolerance limits.

2.4.18. Projected tolerance zone according to ISO 10578 (classic case)

A classic example of tolerancing zone, known as *projected*, deals with fasteners and bolts (Figure 2.49). In fact, we project on a designated length, the value of a dimension (here for example, the positioning of the screw head) and denote by (P) followed by the value on which the projection takes place (here, 25 mm) compared with the reference surface (A). The clearance between the screw body and the bore of the piece (b) is 0.15. Partial dimensioning of (a) is given on the right of the screw drawing. The question of *projected tolerance zone* is encountered in the case of mounting screws into threads, or pins in a bored house. To do this, we should make

sure that the bolt or the screw is inserted along the “nominal” axis and that the projection of the thread is collinear following the rest of the implanted rod. Otherwise, we say that the editing is done on horseback. This means the axis of the threaded rod does not follow the one supposed to continue on the head or the protrusion and thus we consider the projected tolerancing zone.

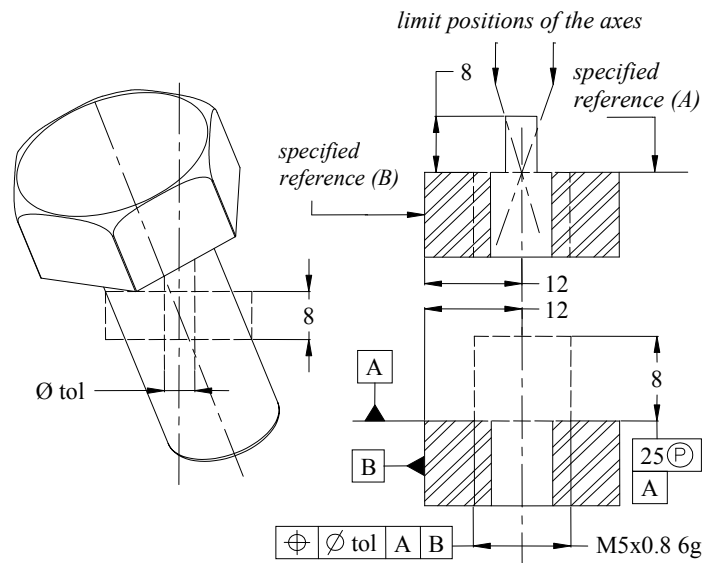


Figure 2.49. Projected tolerance zone according to ISO 10578

2.4.18.1. System based on simple references

We are dealing here with tolerancing reference systems but not with axis systems as Cartesian, cylindrical, or spherical references.

When a specified reference system is established by two or more elements, that is, by multiple specified references, their letters of reference are set out in the third box and the boxes following the tolerance limits, respecting the order of specified references.

2.4.18.2. System consisting of multiple references

The order of specified references has a considerable influence on the result. The reference, in this case, concerns the reference surfaces to which we refer to confirm or refute the tolerancing required by the drawing. We have the freedom to choose them and vary them (Figure 2.40). Next, we present explicit examples of complete technical drawings.

2.4.18.3. System consisting of double references

A typical example that we encounter in the industry is that of an entity like the bent as in Figure 2.50. The secondary reference is constructed using the primary perpendicular reference as shown in Figure 2.50. We note that the reference (A) is perpendicular to (B). First, we designate the *nominal axis* of the part and trace the two orthogonal axes. It is on these two areas that we should work on the references (A) and (B). The reference (A) is a primary reference and (B) is a secondary reference. There would be a third one, which would be designated as a tertiary reference (C).

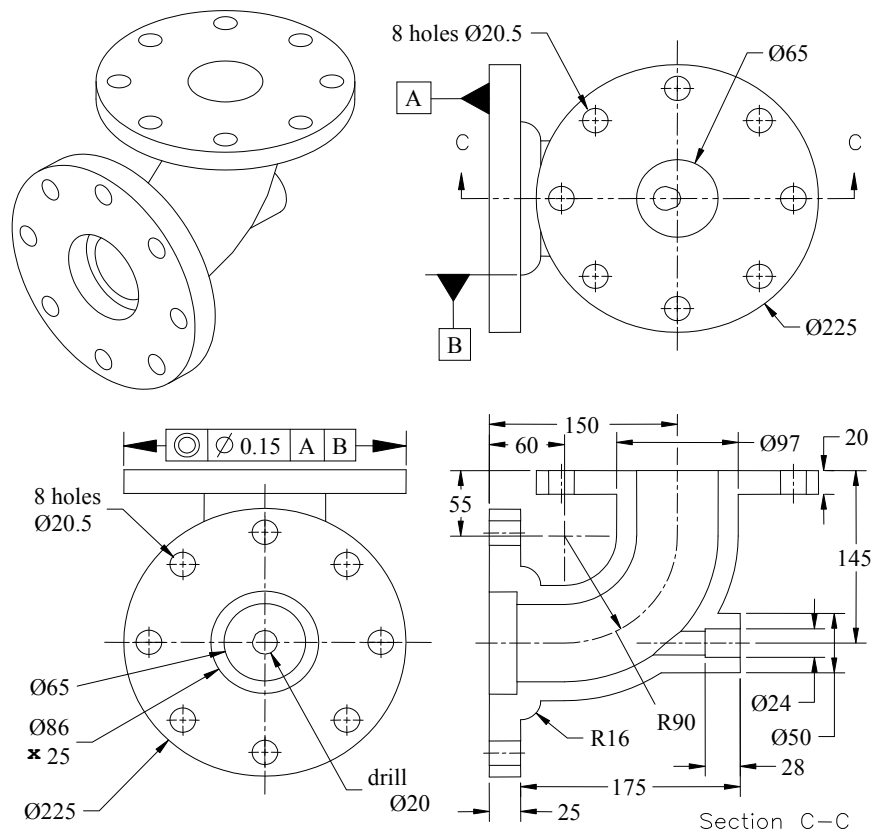


Figure 2.50. Designation of reference systems on a 2D drawing

2.4.18.4. System consisting of tertiary references

Tertiary reference is built using primary and secondary (perpendicular) references. The “simple” reference system to be operated hereinafter covers the three reference surfaces (A), (B), and (C).

Obviously, it involves all three major planes based on which the reference surfaces are selected. To explain this, it would be good to present a drawing (3D), three orthogonal planes on a Cartesian coordinates.

In fact, it is easy to locate the three orthogonal planes where we can easily put our three reference surfaces as shown in Figure 2.51.

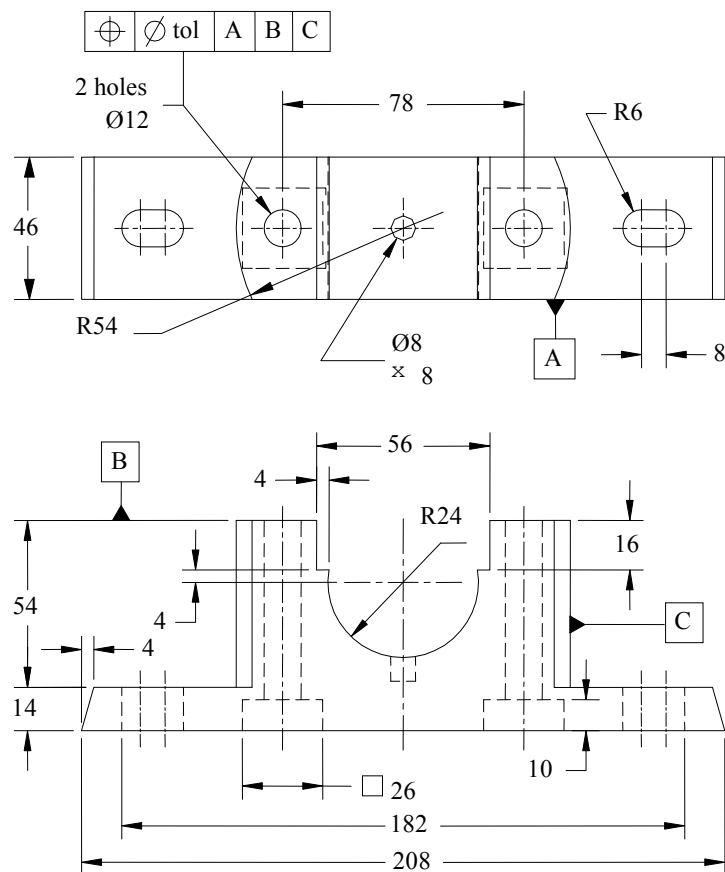


Figure 2.51. Designation of reference systems on a 2D drawing

The exercise on dimensional and geometric tolerances is applied to Figure 2.52.

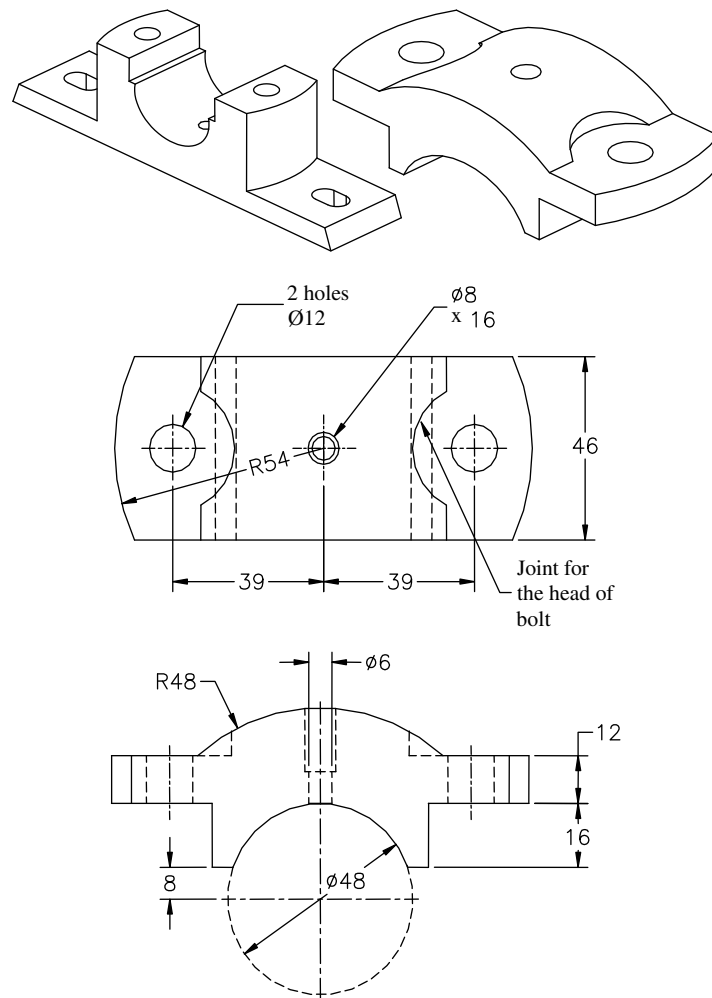


Figure 2.52. Example of exercise on the reference systems on a drawing definition

2.5. Conicity according to ISO 3040-1990

Conicity is defined as the ratio between the difference of diameters of two sections of a cone and their distance. The conical form is shown in Figure 2.53.

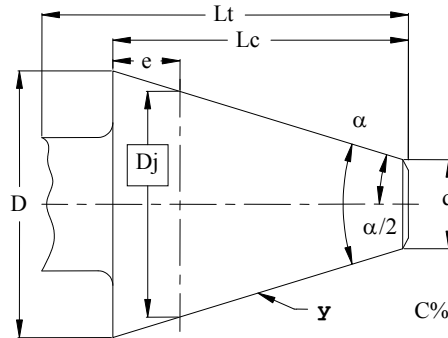


Figure 2.53. Representation of the conicity

The following parameters characterize conicity, which is shown in Figure 2.53:

- D is the diameter of the cone or truncated cone, in mm or inches;
- d is the small diameter of the cone or truncated cone, in mm or inches;
- D_j is the diameter gauge in mm or inches (theoretical $\varnothing$, therefore without tolerance);
- L_c is the length of the cone or truncated cone, in mm or inches;
- L_t is the total length of the piecework, in mm or inches;
- α is the angle of inclination of the generator from the axis, in degrees;
- $\alpha/2$ is the apex angle of generators, in degrees;
-  is the conicity given in percent or number (example, 15% or 0.15);
- $\angle$ is the slope expressed by $\tan(\alpha/2)$ by consideration of the drawing; and
- e is the limit of penetration in mm.

In dimensional metrology, as in the rest of the technical activities, the calculation of the conicity (C) is given by the following formula:

$$\text{Conicity} = \tan\left(\frac{\alpha}{2}\right) = \left(\frac{D-d}{L_c}\right) \quad [2.6]$$

The technological functions of the cones allow:

- to align and position two features (tapered pin and the spindle nose);
- to realize the orthogonal transmission components (bevel gear)
- to connect the different diameters by eliminating shoulders. Therefore, boot failure is due to concentrators' constraints (stress);
- to ensure the regulation of flow, in hydraulics (cone fitting); and
- to ensure water-tightness by contact in some cocks valve, valve.

In the case of machine tools, a low conicity ensures adhesion of two elements (inner and outer Morse cones). Among the different types, four principle types of cones could be distinguished (Figure 2.54):

- Cone shank (pin nose) is used in tooling for machine tools. These cones are usually around 7/24 without jamming and their conicity of approximately 29.2% (for numbers of cones ranging from 30 to 60).
- Cone wedging taper $\leq 5\%$.
- Taper cone metric: 1, 2, 5, 10, and 20%. They are used mechanically.
- Walruses taper cones around 5% (0.05) according to their number.

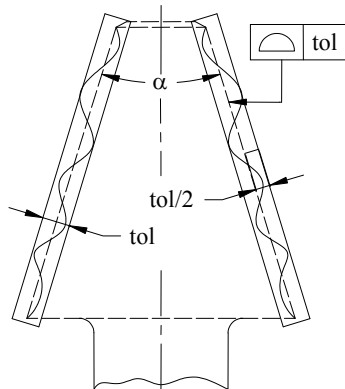


Figure 2.54. Interpretation of a cone form tolerancing (courtesy of Mitutoyo)

Conicity control is done in various ways. Among many others, there is CMM, traditional metrology, and profile projector.

Form tolerancing for a cone. The cone is toleranced to the manner of a skewed/curved surface. The symbol resembles an undistinguished form. Figure 2.55 shows the designation of a cone.

Tolerancing of its axial position. The axial position of a cone is given relative to a plane gauge (L), which is shown in Figure 2.55.

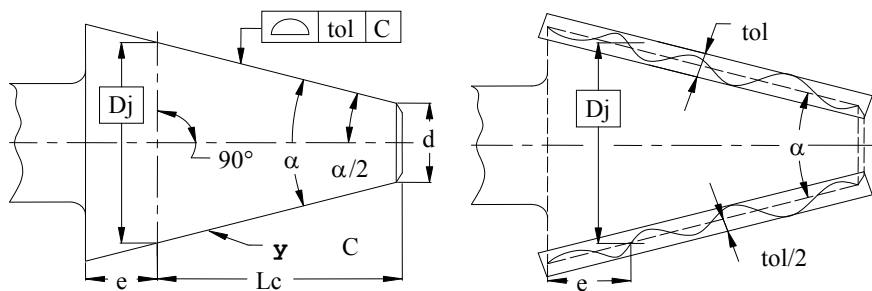


Figure 2.55. Interpretation of axial position tolerancing of a cone

2.5.1. Conicity calculation: slope, $\tan(\alpha)$, large and small diameter

Using a spreadsheet (Excel) or mathematics software, it is possible to model, by programming, all the formulas on the conicity (Table 2.2).

Usual formulas			
Conicity (C)	Conicity (C) in %	Conicity (C)	Conicity (C)
$C = \left(\frac{D-d}{L} \right)$	$C\% = \left(\frac{D-d}{L} \right) \times 100$	$C = \left(\frac{\text{Slope}}{2} \right)$	$C = \left(\frac{\tan(\alpha)}{2} \right)$
Knowing d , the conicity and the conical length L , thus we calculate D		$D = \left(d + \frac{C}{L} \right)$ mm or inches	
Knowing D ; the conicity and the conical length L , thus we calculate d		$d = \left(D - \frac{C}{L} \right)$ mm or inches	
Knowing D ; the conicity and d , nous thus we calculate L		$L = \left(\frac{D-d}{C} \right)$ mm or inches	

Table 2.2. Calculation formulas of conicity

2.5.1.1. CASE 1: Calculation of (d) and $\tan(\alpha)$

For a conicity of 15%; $D = 75$ mm and $L = 35$ mm, we find:

$$d = \left(D - \frac{\text{Conicity}}{L} \right) = 74.996 \text{ mm and } \tan(\alpha) = \left(\frac{\text{Conicity}}{2} \right) = 0.075 \text{ or } 4.297 \text{ deg}$$

2.5.1.2. CASE 2: Calculation of D and $\tan(\alpha)$

For a conicity of 7%; $d = 54$ and $L = 25$, we find:

$$D = \left(d + \frac{\text{Conicity}}{L} \right) = 54.003 \text{ mm and } \tan(\alpha) = \left(\frac{\text{Conicity}}{2} \right) = 0.035 \text{ or } 2.005 \text{ deg}$$

2.5.1.3. CASE 3: Calculation of L and $\tan(\alpha)$

For a conicity of 25%; $D = 54$ mm and $d = 25$ mm $\rightarrow L = 116$ mm:

$$L = \left(\frac{D-d}{\text{Conicity}} \right) \text{ and } \tan(\alpha) = \left(\frac{\text{Conicity}}{2} \right) \text{ then } L = 116 \text{ mm and } \left(\frac{\text{Conicity}}{2} \right) = 0.125$$

2.5.1.4. CASE 4: Calculation of D and the conicity C . P is the slope

For $D = 77$ mm; $L = 60$ mm; $\alpha = 2^\circ$, since $\tan(\alpha) = \text{slope} = 0.125$, we find:

$$C = 2 \times \tan(\alpha) = 0.25, \quad d = \left(D - \frac{C}{L} \right) = 76.996 \text{ mm, Slope} = \left(\frac{C}{2} \right) = 0.125 \text{ (QED)}$$

2.6. Linear dimensional tolerances

The designer's role is fundamental in design and manufacturing. The functional score contributes greatly to the efficiency of manufacturing. This cannot be done without metrology before and after machining. This chapter is written in accordance with Canadian standard CAN/CSA-B78.2-M91. Also, it is reflected in its entire spirit of ISO 8015-1985. The concept of geometric specification or GPS, as proposed by the ISO, aims to provide coherent standards. The main standard relevant to GPS is ISO/TR 14638.

According to ISO 8015-1985, dimensional tolerancing means that linear tolerance limits only local actual sizes (measured between two points) of an element but not its form deviations. An element consists of a *cylindrical surface* or two *parallel flat surfaces* as shown in Figure 2.56.

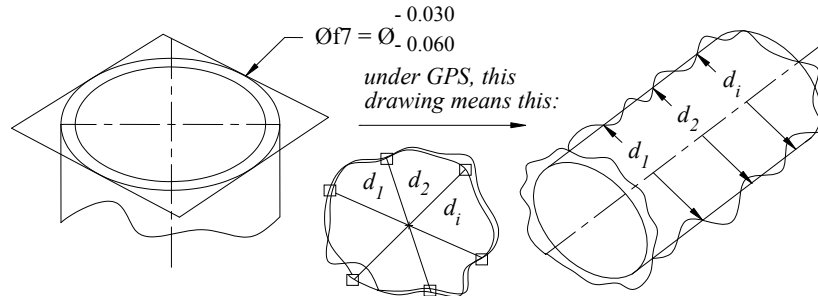


Figure 2.56. Exhibit of linear tolerance to cylindrical surfaces

The meaning is graphically shown in Figures 2.56 and 2.57.

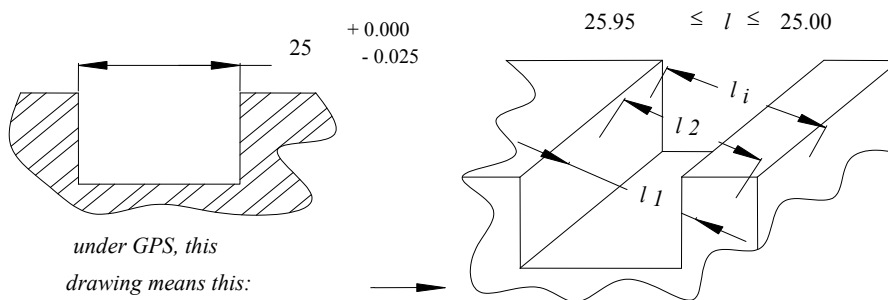


Figure 2.57. Tolerance for linear parallel planes

Dimensional tolerances may be either for lengths or angles. Currently, dimensional tolerance is defined within the standards for two geometric figures known as dimensional features as described earlier:

- a cylindrical nominal surface (Figure 2.56) and
- two nominal surfaces flat and parallel (Figure 2.57).

Direction of bipoints is not explained in the standard (see ISO 8015-1985). The draft of standard ISO 14660 provided dimensional entities such as cylinders, spheres, two opposed parallel surfaces, cone, and wedge.

- Linear tolerance and its principle of independency expressed by dimension $\pm$ Tol.
- Linear tolerance and the envelope requirement are expressed by dimension $\pm$ Tol (E).

– Angular tolerance limits only the general orientation of lines or linear elements surface. Note that the angular tolerance is partially defined (ISO) for two lines in a dihedral. The plane in which the two lines lie and the criterion of association of straight lines with actual lines is not defined. According to ISO 2768-1, the angular tolerances are expressed as dimension $\pm$ Tol.

The above-mentioned specific sizes are explained in the following sections.

2.6.1. Consequence: “size” tolerancing

Linear tolerances can be used only to tolerance “sizes,” that is to say:

- the diameter of a cylinder or sphere (Figure 2.56) and
- the width between two parallel planes in vis-à-vis (Figure 2.57).

Indeed, for the other dimension (e.g. distances dimensions), *actual local sizes* are not defined and what to avoid is illustrated in Figure 2.58.

Let us control, for example, the toleranced dimensioning in the “nominal” part as indicated on the drawing. We take a chosen reference plane. We consider an example of tolerancing of “sizes” to infer what it is important not to do. In fact, it is an ambiguous distance control or even not in accordance with the ISO rating, GPS, CSA, or ANSI.

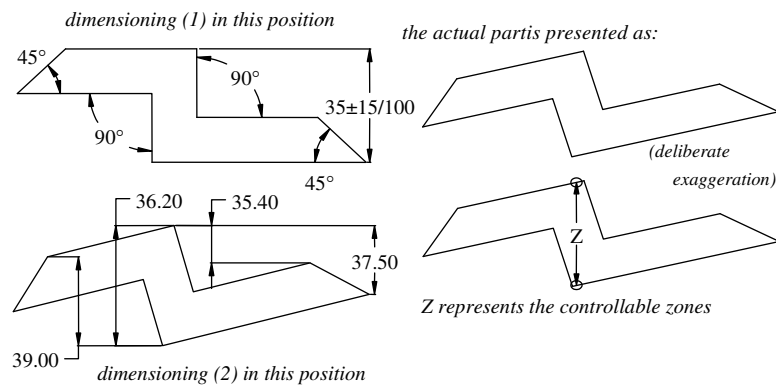


Figure 2.58. Graphical solution of the example of dimensional tolerancing

2.6.2. Consequence: independency with regard to the form

A linear dimensional tolerance does not limit the form default of the element (this allows to respect the principle of independency) indicated in Figure 2.59.

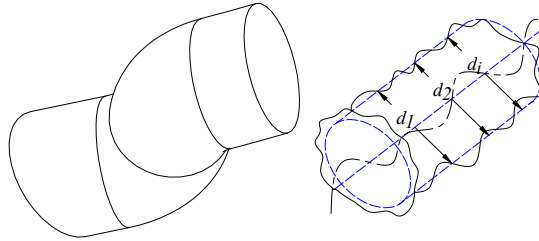


Figure 2.59. *Independency from the form*

The differences in form should be limited (ISO 2768) by the following criteria:

- form tolerance indicated individually by $\text{⌀} 0.05$ or $\text{⌡} 0.05$ and
- a requirement of the envelope (E).

An envelope requirement may be added following a linear tolerance as shown in Figure 2.60.

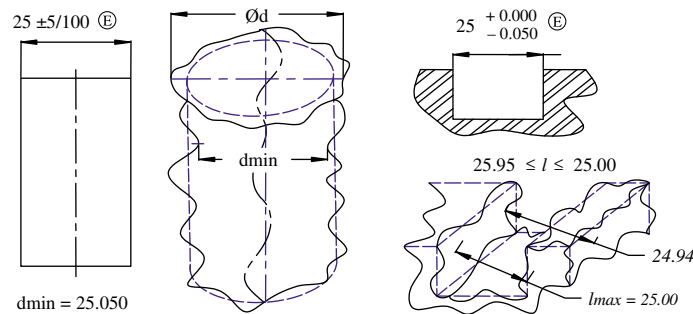


Figure 2.60. *Schematic on principle of the envelope requirement (E)*

The envelope requirement means that, in addition to conditions in actual local dimensions, the surface must not exceed an *envelope of perfect form* at *maximum material size* of the element. The envelope requirement limits form defects as shown in Figure 2.61.

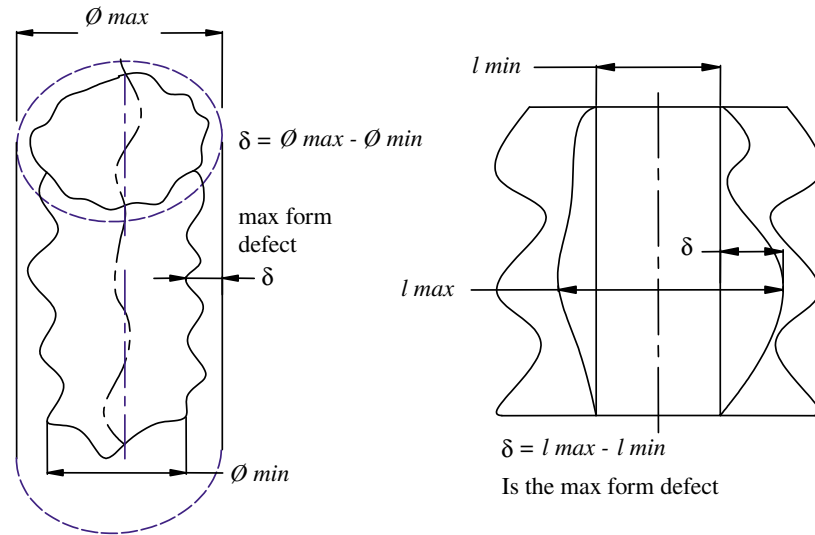


Figure 2.61. Symbolization of the envelope requirement to limit a form

2.7. Positioning a group of elements

Since 1988, the GPS standard has been more interesting because it provides succinct answers to the industry, witness ISO 5458-1988: GPS (*geometrical tolerancing – positional tolerancing*). For example, in mechanic manufacturing, every slot is strictly separate from its coast and its tolerance. We must add the number of times (e.g. 6x) and this dimension is repeated on the drawing, assuming that all forms are identical. However, if the tolerances were different, it would avoid confusion. Some would say that the tolerated dimension/COTE applies to all forms deemed identical: Under which would it be well understood? The localization of a group of holes is always relative to a reference.

We propose, for example, a dimension for the following drawing in accordance with ISO 5458. This rating/dimensioning is also used in the representations of North American designs (see CAN/CSA-B78.2-M91).

EXERCISE 2.1.– In Figures 2.62 and 2.63, it is proposed to draw the necessary views and to proceed with the dimensional and geometrical dimensioning in accordance with the standards:

- provide the geometrical positioning constraint with a GPS sense;
- comment this choice by justifying it in accordance with the GPS (ISO) standard;

- impose additional geometric and dimensional constraints, on the drawing (avoid choices that would impose any geometric or dimensional transfers); and
- control the dimensions of both pieces using the conventional means of metrology (i.e. calipers, micrometers, comparator, wedges, and marble in workshop).

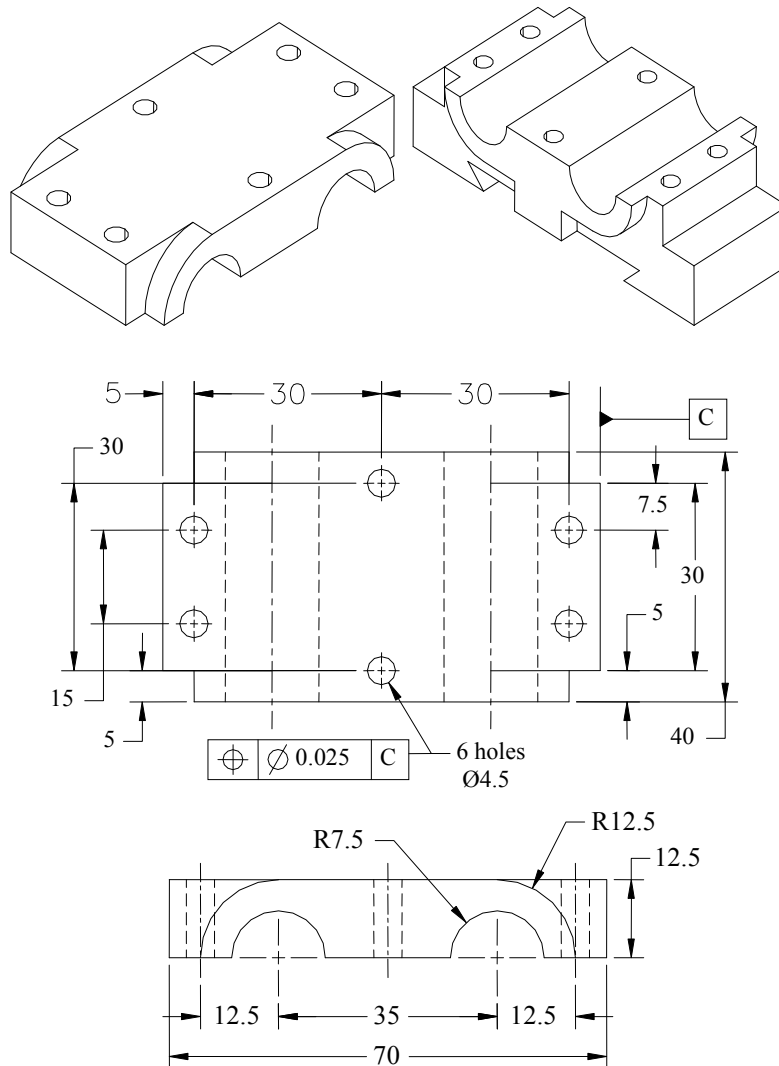


Figure 2.62. Example (1): Positioning of a group of features on two 2D drawings

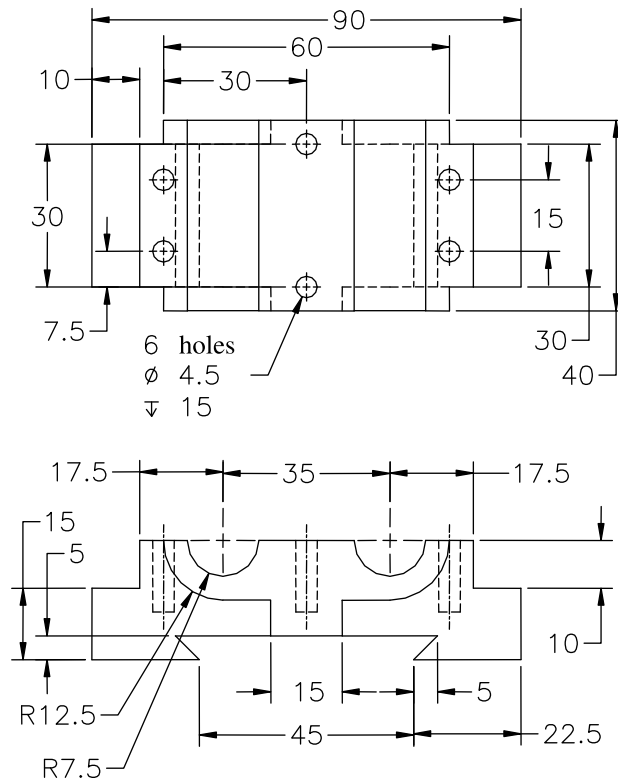


Figure 2.63. Example (2) positioning of a group of elements on two 2D drawings

2.8. GPS standards according to the report CR ISO/TR14638 of 1996

The summary of the main ISO standards written by the group of harmonization “adjustment”, “dimensioning and tolerancing”, and “metrology and surface conditions” to structure the GPS standards following the master plan are presented in Table 2.3. The fundamental GPS standards is ISO 8015, 14638 → Master plan GPS.

In the case of general GPS standards, we should adopt only the standards that establish the rules indicated on the drawings and verification of geometric characteristics are organized in the General GPS matrix (Table 2.3) presenting the different geometrical characteristics and the six links in the chain of standards on each characteristic. The matrix of general GPS standards is the goal of “chain of standards” which is to link unambiguously the indication on the drawing in the SI unit.

Matrix of general GPS standards							
Link number		1	2	3	4	5	6
Characteristics of the element							
Subcharacteristic of the element or parameter							
Indication in documentations of the product – codification							
Tolerances and theoretical definitions and values							
Definition of characteristic or parameter on real workpiece							
Determination of the Workpiece deviations. Comparison with tolerance limits							
Requirements to measuring equipment							
Calibration requirements and calibration standards							
Size							
Distance							
Height (pitch)							
Distance between elements							
Actual element							
Derived element							
1101(R) 2692(R) 10578	1101(R) 2692(R)]	129(R) 406	?	?	?	7863, *10360-1 ...	?
1101(R) 5458(R) 10578	1101(R) 5458(R)	129(R) 406	?	?	?	463(R) *10360	?
*14660-1 *14660-2	?	?	?	?	?	?	?
5460	5460	?	?	?	?	?	?
*10360	463(R) *10360	?	?	?	?	?	?
?	?	?	?	?	?	?	?

Table 2.3. Matrix of general GPS standards (Source: ISO/TR14638-1996) [CHA 99]

For complementary standards that address the field of machining and other technological processes, we include tolerance standards depending on the process

(ISO 2786) and the standards that define the geometry of specific products such as threads, gears and grooves.

For standardizers, this matrix, showing the missing or redundant standards is now used for standardization. For users, it allows an unambiguous understanding to the reader of the drawings.

2.9. Rational dimensioning for a controlled metrology: indices of capability and performance indices statistical process specification

It is known that the competitiveness of enterprises depends on various components of the *value*, namely the cost, time, and especially the *quality*. We know that we need to preserve and improve quality; therefore, it is essential to ensure proper measurement, its means and its method.

Given the multiplicity of input variables in the measurement function, the statistical tool is becoming increasingly inevitable. It is even as essential as the use of indicators of performance and capability of control means depends on it during the operating results. The statistical process control (SPC) manuals for the instruments used in dimensional metrology are not fortuitous. They are there to ensure the performance of production and they concern, therefore, the serial measurement.

Two kinds of indices may be differentiated on the SPCs and which link the tolerance limits to the true results obtained via processes (Cosmos of Mitutoyo, Ford and Chrysler, etc.). These indices are designated by Pp and Ppk ; Cp and Cpk . The index Cpm is more used and is expected to become widespread although a bit awkward in its interpretation. Each of these indices “*” are defined in Table 2.4.

Symbols	Indices	Interpretations
Pp Ppk	The performance Indicator of the quality delivered	Translates process capacity to produce over the long term. Incorporates the effects of assignable causes
Cp or Cpk	Intrinsic capability of the process	Reflects the possibilities of the method in the absence of assignable causes
Cm Cmk	Short-term capability Machine capability	Reflects the capability of the means of production from a spot test
Cpm^{**}	Short-term capability	Global index incorporating the mean and standard deviation

Table 2.4. Coefficients and indices of capabilities (*) This table was prepared by the inspiration of SPC manuals of Mitutoyo in addition to SPCs from Chrysler Corporation, Ford Motor Company, and General Motors Corporation). (**) In quality control, we use this index of capability (source: documentation of quality control on Mitutoyo-Cosmos software)

- The indices, Pp and Ppk , of process performance are calculated from the results plotted on a control chart, allowing verification whether the process performances are sufficient. They represent the yielded quality.
- The process capability indices, Cp and Cpk (*process capability*), are calculated from the results plotted on a control card, to verify the intrinsic capability of the process regardless of assignable causes, possibly. These indices may be representative of the quality only if the process is stable in average and dispersion. This is done under control but with no assignable causes.
- The indices of machine capability, Cm and Cmk , are not retained by the documentation published by the manufacturers (Mitutoyo, Starrett, etc.) and can be used in spot tests.
- The capability index, Cpm , is unique because it takes into account the concepts of quality loss function of Professor Tagushi (Genichi Tagushi). This index takes into account both the dispersion and the standard deviation of mean compared with the target (toleranced dimension to be controlled in precision dimensional metrology).

In metrology applied to mechanical production, it is customary to set tolerance limits but it can happen only when the target value is controlled, regardless of geometrical or dimensional tolerance, to assess compliance with a drawing. In these cases, the performance indices (Pp and Ppk) and capability indices (Cp , Cpk , Cm , and Cmk) are important tools for verifying compliance with tolerance. We should not believe that all the “slot” of tolerance is useable in mechanical manufacturing. This is also due to this hazard as it tries to set a tolerance, because we already know that it will not be “wholly” exploitable. But when controlling in metrology, we accept the parts that are dimensionally included within the tolerances limits. We will try to show how the tolerances and the dispersions may cause loss of quality. Typically, in dimensional metrology also, one binds the so-called functional tolerances and chooses the means of control capable to determine the dimension’s exact value, which is illustrated in Figure 2.64.

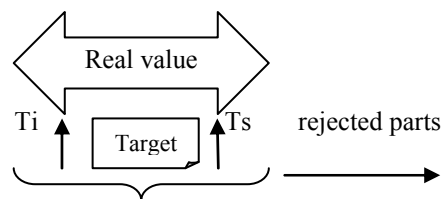


Figure 2.64. Target value (accurate) to be controlled in the toleranced zone by a TI

In unit production, this is valid, less expensive in time control but in serial production, the problem may be complicated because it proceeds by statistical approach. It means that we use the rules of the SPC that have been advocated by W.E. Deming and W.A. Shewhart [SHE 82]. However, the SPC chart does not have main objective respect/compliance with tolerances, but rather the objective of the command of processes, as shown in Figure 2.65.

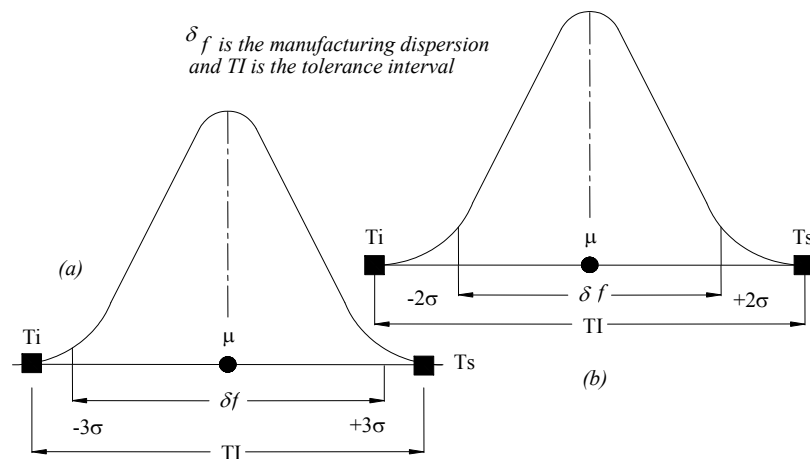


Figure 2.65. Tolerance intervals and manufacturing dispersion

We note that by expanding the TI, thus increasing the dimensional tolerance (b), we can obtain a result acceptable by the client. In fact, in production workshops, we manufacture parts in combination with machining ranges. In production, these are the indicators of performance and capability that verify compliance, in an indirect manner. The metrologist would be interested in the true dimension and the uncertainty with which it returns the result. Does this uncertainty translate the truthfulness of the tolerance? If we rely on the SPC rules, it will not be correct because the actual dispersions concerning the metrologist (client) are not translated.

The metrologist is not always involved in production, but measures what is received from the workshop, choosing the appropriate means for its measurement and this in line – the best possible – with the tolerance listed on the drawing. Metrologist works on uncertainties but unfortunately it has little influence on the production workshop. If control is done by sampling, the metrologist can claim his uncertainties as shown in Figure 2.66.

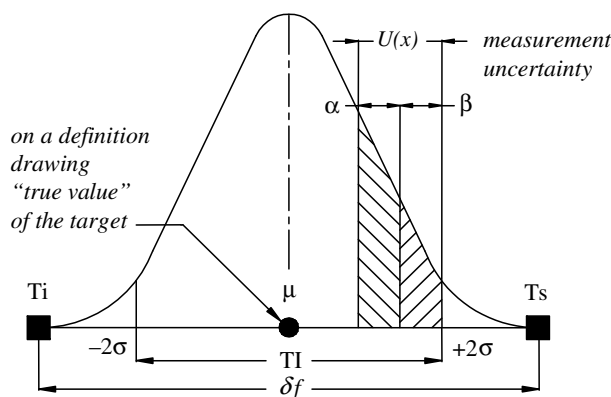


Figure 2.66. Tolerances and control of parts

- The dimensions of the workpiece, located in the zone (α), are out of tolerance but can be accepted because of measurement uncertainties.
- The dimensions of the workpiece, located within the zone (β), are within tolerances but may be refused because of measurement uncertainties.

The problem is not just a problem of lower tolerance limit as the *capability* is sufficient within this zone. Also note that we are not allowed to translate into tolerances what we would desire because the uncertainties are there to disprove it. Other factors influence the *true value*. Note the *repeatability* and *reproducibility errors* of the metrologist's control means. We note that there are various factors that influence the measurement and their optimization requires a lot of constraints. In fact, metrologists should be equipped with additional means of analysis, in addition to mastering the mechanical manufacturing tool. It is necessary, for the metrologist, to express a measurement result with its *uncertainty*. In fact, it is even imperative to do so. We know that two simple measurements performed under the same conditions by two different operators may lead to a significant difference, sometimes problematic and subject to questioning (caution).

Some would wonder: is uncertainty manipulated by the metrology?

The answer is: yes, sometimes metrology manipulates uncertainty.

Before answering this question, let us first say: yes, the metrologist needs to be formal, while *doubting* objectively. For example, the second is the duration of 9,192,631,770 periods of transition from the cesium atom and thus the metrologist watchmaker has the right to add an uncertainty of 4/10th second. Are metrologists so purist to seek precision, sometimes incredible?

One thing is certain: as long as international competition fuels their work, metrologists will continue to pay more attention in terms of precision. This approach will affect all areas of performance and metrology will end up officially at the center of science and techniques applied to arts and crafts.

Yes, metrology sometimes manipulates uncertainty.

The uncertainty and instrument calibration function is there to prove it, step by step. The metrologist provides a quantified estimate of the entity that he handles with various means of measurement. The continuum mechanics and relativity prefer the reliability first. The standards also run out.

By calibrating standards (by qualified laboratories) with the aid of other standards, uncertainties would “swell up.” Some would wonder: is calibration mandatory? Everyone has the right not to calibrate as they also have the right to measure “faux.” It will lose its credibility and this is not *nothing*, in a world that aims to crescendo performance... We believe that sound management of manufactured geometry means also better formalizing a need before seeking the accurate answer necessary for dimensional metrology.

In short, managing manufactured geometry is, above all, defining and quantifying the target by apprehending the “nominal” and then react and correct any discrepancies that metrology alone cannot *recover*.

For example, the chain of dimensions ensuring the adequacy to the request (BE) of the research department is the response of the methods department (BM). All leads of course to defining the mission of the factory owner before the qualified metrologist intervenes. In what follows, we will try to provide a familiar explanation to this process through an example based on a vise.

Why has this choice been decided?

On the one hand, all users (students, laboratory technicians and trainers) are not using the same means of control in conventional machining and, on the other hand, unitary machining ranges are manufactured for educational purposes leading sometimes to abstraction of clamping... In some cases, the inadequacy between machining apprenticeship and metrology is obvious. Machining techniques are first taught through *approximative metrology*, followed by a training on *dimensional metrology*, and then the quality control is done afterwards. This *aberrant sprain* cunningly crippling the trainer because of the limits imposed by the training frame-plan known as standard (in Quebec).

Dimensioning placed on such manufacturing drawings is often approximate if it does not involve dispersions even less to supported master of metrology instruments and even less adequate considerations of the functional rating. It is a sad statement of educational incoherence.

To machine such a part, it is first useful not to rely on a classical dimensioning, which gives the solution to the very beginning. The identification of this piece, for machining purposes, is done as shown in Figure 2.67.

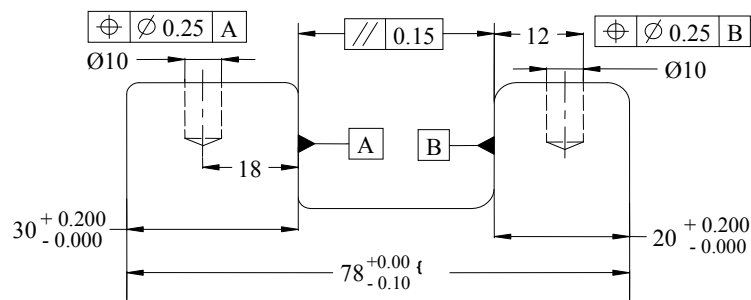


Figure 2.67. Drawing with tolerances of the workpiece by the BE (case of our base conventionally machined in the workshop)

DISCUSSION - For metrologists, the problem may not arise in terms of classical dimensioning and rational dimensioning, as the metrologist measures the dimensions, which he is supposed to return faithfully with the best possible accuracy. Yet classic dimensioning, in our view, is questionable because it leads to thinking in terms of solution ahead of the problem. This question deserves to be elucidated by the designers of machining lines. This is not surprising given the real and difficulty in making a workshop technician understand that we should not continue to put systematically quantified dimensions on preparedness contracts before performing a machining simulation [AGU 00].

In contrast to the conventional approach [PAO 75], which starts by the solution and then validates following the mentality: “dimension as you machine”; the rational approach tends to first analyze the issue in terms of need. Dimensioning of contracts of phase will define, at all stages, the expected results because they are needed. At each stage, we will express just what is necessary with the minimum possible chip. This objective is normally assigned to the task of the factory owner, machinist. It is only after dealing with this optimization problem [AGU 00] that the task of metrologist becomes accessible, with the least amount of errors on TI. That being proved, we can now begin work on the monitoring arrangements and procedures for conducting measurements. Dimensioning known as a profession

allows the analysis of tools and adjustment as illustrated by the example of the vise base (Figure 2.68).

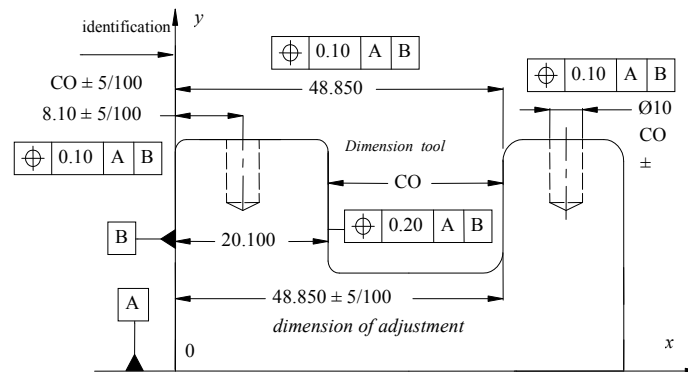
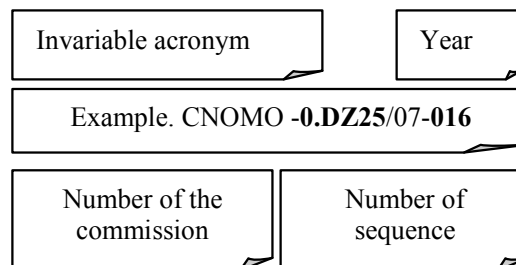


Figure 2.68. Analysis of tools and phases adjustments setting in dimensioning

In metrology and quality control, it is important to determine a *Cpk* (process capability index = coefficient of dispersion and position of the manufacturing according to standards), or a *Cma* (coefficient of means ability, Standardization Committee for Manufacturing and equipment (CNOMO; in French: Comité de Normalisation des Moyens de Production) on prospective means for parts control. The standard CNOMO was revised in December 2003. Since January 1, 2004, the new features introduced are:

- the term “homologation” is now replaced by the term “labelization”;
- the length of the label CNOMO is now limited to 5 years;
- products approved for a label CNOMO are included into a repertoire of labeled products, available at website: www.cnomo.com; and
- the right to use the label is subject of a bill for each approval for label according to the CNOMO scale in use.



We will not develop this aspect of the (CNOMO) standard. We continue our reasoning by using capability indices. Knowing the mean μ , the Cpk is thus calculated:

$$Cpk = \frac{\text{The smallest margin}}{3 \cdot \sigma_0} \quad [2.7]$$

$$\{\text{Lower margin} = \mu - L_1\} \text{ and } \{\text{Higher margin} = L_h - \mu\} \quad [2.8]$$

The Cpk is measured relative to the limits of acceptance around the standard deviation (SD σ_0) that incorporates the dispersion for any wear and misalignment. In dimensional metrology, we are made to ask ourselves the following question: How can we use the quotation to set thresholds for Cpk and Cma indices? Is this listing job that involves setting or quotation rational i.e. optimized [AGU 00]? We first present the approach for calculating the Cma (under the Canadian standard CNOMO [CLA 00, 02, NRC 00, 02] or the European [E41.32.110N] and then we try to answer for the earlier questions. The Cma is calculated as follows:

$$Cma = \left\{ \frac{TI}{D_i} \right\} \quad [2.9]$$

where TI is the tolerance interval and D_i , instant dispersion = $6 \sigma_i$; with σ_i the standard deviation intrinsic of the mean.

All the phenomena of realignment will be ignored. This is relative to the control means assigned by the metrologist, knowing all the steps outlined here.

The following simulated plotting (Figure 2.69) presents an assessment of the capability of metrological instrumentation.

Digital implementation. Let us ask again the previous question and try to get numerical results. How to use dimensioning to set thresholds for Cpk and Cma indices? Is it the dimensioning profession which involves adjustments or rather, rational dimensioning, that is, the optimized dimensioning? In other words, you are asked to estimate the Cma and Cpk , given the following measuring data:

$$\begin{aligned} \mu &= 25; L_1 = 0.0125; L_h = 0.015; \sigma = 0.00025; \\ (\sigma_i &= 0.0020 \text{ is the SD of the mean}) \end{aligned}$$

Solution:

$$\text{lower margin} = \mu - L_1 = 24.988 \text{ and higher margin} = L_h - \mu = -24.985$$

$$TI = L_h - L_l = 2.5 \times 10^{-3}; D_{\text{dispersion}} = 6 \times \sigma = 1.5 \times 10^{-3} \text{ and}$$

$$Cma = \frac{TI}{D_{\text{dispersion}}} = 1.667$$

The Cma is comparable to Canadian standards CNOMO (1.66 for the ideal). Our reasoning is tenable:

$$\text{For } \sigma_{\text{intrinsic}} = 0.0020, \quad Cpk = \frac{\text{lower margin}}{3 \times \sigma_{\text{intrinsic}}} = 4.165 \times 10^{-3}$$

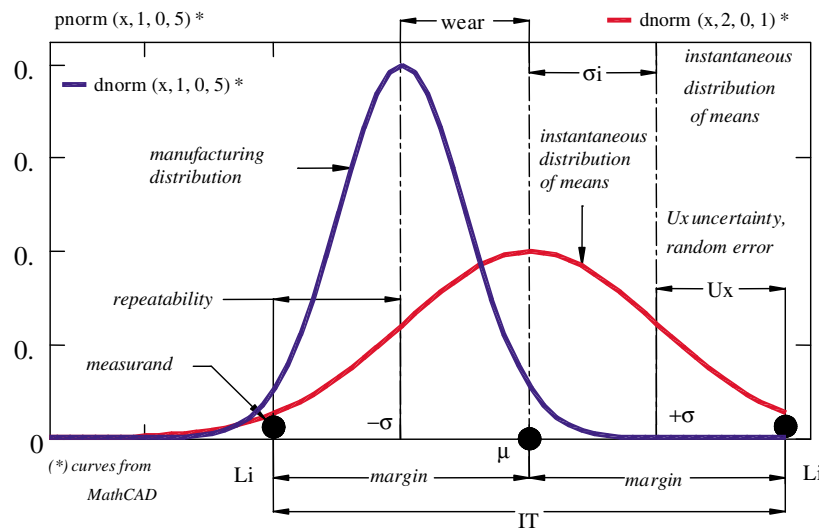


Figure 2.69. Assessment of the capability of metrological means

If the dimensioning craft was imposed, it might be concluded as a non-conformity because it would intervene on the machine tool to modify the conditions of execution. Dimension tool (48.85) is out of tolerance for $[48.85 \pm 5/100]$ (Figure 2.68). But this tool is a simple means of manufacturing. It is therefore necessary to make the connection between the means of production and the dimensioning to properly coordinate the task for both the manufacturer and the metrologist: I will machine so I must thinking of measuring instruments and appropriate controls: hence the dimensional metrology.

The work presented here does not ignore the dimensioning craft (tools dimensions, machine dimensions, apparatus dimensions, etc.) in favor of rational

dimensioning. First, the habits are there and we should realistically explain that rational dimensioning is more profitable in terms of optimizing manufacturing. This is also due to time management and priorities.

Rational dimensioning will not perform the same work twice, and will allow the metrologist testing the part of the design to do so in an objective manner. He will do so within the assigned tolerance limits, following the feasible correction ratings, and without doing it twice. Admittedly, the rational number is causing technicians to demonstrate mathematical formalism and analysis. Therefore, this is already more complicated than the said job of dimensioning.

The process of trading requires a rational deduction hypothetico-deductive. It expresses the need. The trading business leads, in some cases, to zero control. It is undeniable that the listing of contracts rational phase induces intellectual conflict between the designer and shop technician. One thing that is desirable is that it is appropriate to avoid empiricism, sometimes rambling, of business (shop) trading and move to the reality that mathematical optimization can bring. The culture of mechanics and factory workers can be enriched and well defined by this. Is this not one of the benefits to rational trading to enable control and avoid repetition.

In this section, we shall not repeat the main definitions and descriptions of the adjustments set out above. We limit ourselves to specific designations with the Canadian standard CSA B97.3-M1982, which regulates the precise interpretation of the limits and tolerance. On the whole we can say that it is inspired by ISO and ANSI Standard incorporates Limits and Fits (ANSI B4.1-1967 R, 1994).

What makes us to say that the CSA is the same as ANSI? A reading of the two standards in Chapter Standard Limits and Fits (see *Machinery's Handbook*, 26th edition, p. 642–682), where it says on page 642 that “this standard is in accord with the recommendations of American-British-Canadian (ABC) conferences up to a diameter 20 inches.” In principle, more than 250,000 combinations of adjustments are feasible as the total number of positions and qualities of tolerances involved in any system CSA, ISO, or ANSI. As in the ISO system, the Canadian system offers to do one of the three main functions that describe the three common types of adjustments, that is, turning adjustments, adjustments and positioning adjustments with force. Positioning adjustments ensure an assembly shaft/bore with clearance, with clamps or in many cases, an uncertain adjustment.

Moreover, the Canadian standard has added five other types of adjustments and this, to facilitate calculations in the imperial system of measurements (inches). The bores and shafts of these adjustments are shown on the appropriate tables which will be found easily in the *Machinery's Handbook* [OBE 96] 25th edition, p. 648–655, and in CSA B97.3-M1982.

When selecting an adjustment fitting one of our applications, we start with the nominal dimension then determining the type of category-adjustment using assigned equipment. The required adjustment will be used for calculations purposes as we can go through a designation. For example, for a turning adjustment, we write $H8/f7 = RC4$ 110 mm on the drawing of definition (or manufacturing). For a force adjustment FN4, we find the international equivalent in $H7/u6$, etc.

We offer some explanatory tables that refer to the tables of qualities to be sought on the CSA B97.3 standard or by referring to the international tables proposed by the literature [OBE 96].

Tables 2.5–2.10 are explained in accordance with the standard ANSI B4.1-1967, R1994. The CSA standard incorporated them. For further details, the reader should either refer to the standard itself or the *Machinery's Handbook*, 26th Edition [OBE 96] (English version).

$\varnothing$ nominal 110 mm		Fundamental deviation Calculation dimension		Quality	Tolerance (TI)	Limits
ACNOR (CSA)	ISO (SI)			[OBE 96]	[OBE 96]	
RC4	H8	LD = 0	110	8	+0.054	$110^{+0.054}_{-0.000}$
	F7	HD = – 0.036	109.964	7	–0.035	$109.964^{+0.000}_{-0.035}$
FN4	H7	LD = 0	110	7	+0.035	$110^{+0.035}_{-0.000}$
	u6	LD = + 0.144	$[110 + ld + TI]$ 110.166	6	–0.022	$110.166^{+0.000}_{-0.022}$

Table 2.5. Sample entries: RC4 or (H8/f7) and FN4 or (H7/u6) in terms of adjustments

Designation	Bore/shaft	Description
RC1	H5/g4	Reserved to slippery precision adjustments. They are used for assemblies without clearance but always for high precision (e.g. calibers)
RC2	H6/g5	Reserved to slippery precision adjustments. Having a maxi clearance higher than the previous (RC2). Even manually, parts can run but this adjustment does not tolerate continuous rotations. There is a risk of seizure for large parts rotating at low temperature
RC3	H7/f9	Compared with RC2, this adjustment is almost the most moderately clamped because of the continuous rotation it tolerates. It is often used in low-pressure bearing (oil lubricated). It is suitable for invariable temperatures

Table 2.6. Rotating (or sliding) adjustments (ANSI)

RC4	H8/f7	The clamped turning adjustment is primarily intended for bearings lubricated with grease or oil. They are found on machinery at low pressure. This adjustment is not suitable at varying temperatures
RC5 RC6	H8/e7 H9/e8	These turning adjustments are suitable for high speeds and/or high-pressure conditions. The temperature may vary
RC7	H9/d8 H9/d9	Free turning, this adjustment is used when accuracy is not essential and where the temperature can vary
RC8 RC9	H10/c9 H11/ (RC9)10 H11/c11	They are called adjustments bizarrely turning loose. They are best suited to commercial tolerances; for example, the layer of shafts and other cold-rolled pipes, etc.

Table 2.6. (continued) Rotating (or sliding) adjustments (ANSI)

Designation	Bore/shaft	Description
LC1 LC2 LC3 LC4	H6/h5 H7/h6 H8/h7 H10/h9	They are called normal shaft/bore because they have a minimal clearance zero. They are suitable for stationary positioning of parts and pins. Note that the categories LC1 and LC2 are also used for rolling adjustments
LC5 LC6	H7/g6 H9/f8	These adjustments have a low minimum set and adjustments are provided for tight positioning of the parts still
LC7 LC8 LC9 LC0 LC11	H10/e9 H10/d9 H11/c10 H12/(LC10)11 H13/(LC11)12	These adjustments have clearances and tolerances progressively larger (e.g. bolted assembly). They are useful for various clearances such as RC8 or RC9

Table 2.7. Positioning adjustments with clearance (ANSI)

Designation	Bore/shaft	Description
LT1 LT2	H7/js6 H8/js7	These adjustments include a small amount of clearance. They are used for fitting if the maximum clearance must be lower than that of LC1–LC3 adjustments. A slight clamping is tolerated for the assembly by pressure
LT3 LT4	H7/k6 H8/k7	These adjustments are blocked and do not provide nearly any clearance. Sometimes, they are indicated on certain clamps to eliminate vibration. They are used for running tracks for the ball and keyway
LT5 LT6	H7/n6 H8/n7	These adjustments provide a slight interference, although a significant assembling effort is required in the extremes limits

Table 2.8. Uncertain positioning adjustments (ANSI and CSA)

Designation	Bore/shaft	Description
LN1 LN2	H5/n5 H7/p6	These are adjustments to the press with a very low minimal clamping. They are suitable for parts such as straight studs, which are assembled with an arbor press in steel, cast iron, or brass. Clamping is too low for satisfactory adjustments in elastic materials or light alloys
LN3	H7/r6	This adjustment is suitable as an adjusted to hard press in the steel and brass, or as an adjustment to the press in light alloys (aluminum and copper)
LN4 LN5 LN6	H8/(LN4)7 H9/(LN5)8 H10/(LN6)9	In the United States and Canada, LN4 is used for assemblies in steel. These adjustments are intended as adjustment to the press for flexible materials

Table 2.9. Positioning and adjustments with tightening (ANSI)

Designation	Bore/shaft	Description
FN1	H6/(FN1)5	It is a slightly clamped adjustment. A slight assembly pressure is provided for assemblies called permanent. It is suitable for thin sections
FN2	<i>H7/s6</i>	It is used in (special) adjustments in average friction. It is suitable for materials assembled by shrinking. In terms of the ISO standard, these adjustments are not highly recommended
FN3	H7/t6	Adjustment suitable for assembly by shrinking and dry friction parts. These adjustments are infrequent
FN4 FN5	<i>H7/u6</i> <i>H8/x7</i>	These adjustments are frequently used for forced (blocked) assemblies. They are suitable for pieces subjected to high stress. The assembly is done by thermal expansion

Table 2.10. Adjustments with force or by dilatation (first choice adjustments are indicated in italics in this table) (ANSI)

2.10. Summary and discussion

It has been briefly noted that the language problem begins to be increasingly better identified, as the matrix of GPS shows. Without an understandable language by the community of *arts and crafts* in alignment with the schools, ambiguity would face many years ahead; *stubbornness* too. If, for example, we continued to confuse the *nominal axis* with the *simulated or fictive one*, there would be problems at school. If ambiguity persists, dimensioning would be cheated. Yet the purpose, in dimensional metrology, is to facilitate the reading of drawings of definition. The idea of the axis is a great idea of mathematicians. Theorizing the idea of the axis is certainly accommodating more than one title.

We have seen that the actual dimensions of mechanical parts *never* correspond *exactly* to those specified in the design phase. When solving problems related to tolerance, the design becomes mainly central, witness the growing diversity of software systems solutions integrated to the CMA. From research departments on production, passing through the applicable methods and control, a crucial problem arises: the unification of languages and harmonization of approaches taken along the stages of design, manufacturing, and quality control of products. The values characterizing the *tolerance zones* are determined by experience, that is, by similarity with other mechanisms and by calculation. This is generally a heavy and sometimes tedious process because of complex mathematical concepts. In practice, the values of tolerance are allocated to some optimal solutions, because of the limited data available, during the manufacturing process. It is usually taken to evaluate the accuracies to be achieved in manufacturing, with metrology in its infancy. Tolerances are given between the surfaces, even before the production processes are known.

Often students face cases of dimensional and geometrical transfers. Now if we knew the importance of “clearance” that directly affects the performance of assemblies, he/she would avoid many free *gesticulations*, in workshop. Through a particular choice of tolerance, we might restrict the possible choices for a range of assembly (e.g. case of clamped adjustment). Similarly, these choices will affect the metrological means implemented for qualifying products. We should recognize that tolerancing is a compromise between precision necessary to meet the functional requirements on the one hand, and opportunities, manufacturing costs constraints, and control on the other hand. Long tolerancing management has only been fragmented. The problems associated with a misallocation of tolerance appeared during manufacture where any change is costly.

Nowadays, tolerancing is based on the use and interpretation of a common language. It has emerged over time as the international standard: ISO language relating to tolerancing. This is actually a set of graphic rules for specifying the actual geometry of parts by an annotation of nominal models. It is clear that the current ISO language related to tolerancing has a number of important limitations. Despite willingness to make an unequivocal international language, the use of ISO language for tolerancing is sometimes a source of ambiguity. This gives rise to errors of interpretation that are questionable. As for a text, standards include syntactic and grammar indications to make sense. However, it appears that certain standards for tolerancing advocate different interpretations of identical annotations. This is the case with such a glaring contradiction in standards ISO ISO.8015 and ISO 286 (Figure 2.26).

There is an ambiguity, among others, due to poor definition of the standard applied to tolerances on cone and angle. The complexity of the used language

constitutes in many cases, a disability. The large number of symbols are special tolerances (symmetry, concentricity, etc.). The ISO is currently trying to solve these problems by introducing the GPS. To ensure consistency of the system, the “normalizer” suggests a classification of standards based on two axes: the first corresponds to the geometrical characteristics of an object, allowing characterizing it in terms of *size, shape, and orientation*. The second deals with the stages of *dimensional and geometric dimensioning* of products, allowing characterizing geometric elements, assessing dimensional deviations, defining and controlling measuring instruments. All being recorded in a double entry table named (Table 2.3) Matrix GPS.

Furthermore, GPS does not always define a clear method of tolerancing. It identifies examples. It is up to the designer to obtain solutions to a given problem. In addition, there arises a size limitation: neither the ISO nor the GPS define rules for 3D tolerancing. The most common practice to date is still in the coating of 2D-planes issued from CAD – witness the 3D exhibits presented in this handbook via *Inventor Pro* software. A reminder on tolerancing issues is needless, yet they are crucial in terms of competitiveness for industrial products. In Quebec, however, experience (College) shows that expertise reports difficulties encountered in mechanical engineering technology. These problems of *inelegant misunderstanding*, moreover, are all obstacles to good management of tolerancing.

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Chapter 3

Measurement and Controls Using Linear and Angular Standards

3.1. Key dimensional metrology standards

Solids, usually made of treated steel, are used as standards with the utmost accuracy in metrology. Their surfaces are finely polished before using them to define a linear or angular dimension of a material. We already know that upon heating, metals do expand. We have already addressed this principle in Chapter 1. This is the principal reason for using the dimensions of standards at an average conventional temperature of 20°C. Prototype standards include the meter for linear dimensions and the angle for angular dimensions. In fact, the quality and precision of the measurement depends essentially on standards, measuring instruments, and measurement conditions. We specifically elaborate these three factors as follows:

- etalon standards define physical references. They must be unaffected by physical outer attacks and, more often, universally accepted;
- measuring instruments involve different processes that determine the quality; and
- measurement conditions have an undeniable influence in the final analysis.

In this chapter we are primarily concerned with the issue of standards and conventional instruments as well as other gauges with reference to measurement, usually by comparison, of parts ensuing from mechanical manufacturing. Geometrical description of parts requires dimensional knowledge; i.e. shape and surface condition. The shape, e.g. is of great importance in the case of adjustments. Geometric shapes are often described mathematically and no primary standard is

necessarily associated with them in this case. Dimensional metrology can precisely determine the deviations from simple and common shapes like a circle, a cylinder, a line, or a plane.

Measurements, performed under the best experimental conditions, generally provide precisions of the order of $1\ \mu$ [10^{-6}] or even a nano [10^{-9}]. The developments in arts and crafts and international exchanges have led to the general consensus on an international system (SI) of units. The International Bureau of Weights and Measures (IBWM) studies, develops, and retains the original international standards, popularly called primary. It offers, on request, faithful reproductions – “secondary standards” – the “standards of length, mass, and density”. Basically, it uses the density of a cubic decimeter of water at 4°C , (about $1\ \text{kg dm}^{-3}$) and which varies with the isotopic composition of water. The density of mercury being more stable ($1,354,588\ \text{kg dm}^{-3}$) at 20°C and under normal atmospheric pressure, the accuracy is of one millionth (nearly). This is the reason why we rely upon IBWM in this regard.

In Canada, the Institute for National Measurement Standards (INMS) [CNR 05] is in charge of the dimensional metrology program. It provides comprehensive services for calibration of dimensional measurements with the highest accuracy that can be connected to the SI unit of length. The dimensional metrology program provides technical assistance and assessment of laboratories upon request for a calibration laboratory assessment service (CLAS) [CLA 00]. The program also publishes and distributes documents dealing with specific issues affecting the dimensional measurements.

3.1.1. Time and frequency standards [CNR 05]

Time is considered as a fourth dimension which is involved in all activities. Yet, time has a considerable influence on the aging of materials. For this reason, we must admit the statement “(to) give time to time”. Sometimes, we do not count time and the consequences are highly significant. In October 1967, Caesium-133 (Cs) was chosen to define the second as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Caesium-133 atom.

The reference instruments of International Atomic Time (IAT) are established by the Bureau International de l’Heure (or the International Time Bureau). They display the time with exactitude in the order of [10^{-12}] from which the civil time is derived. Traditionally, a clock contains an oscillator, that is to say a pendulum, a spring with a mechanical winding or with a quartz crystal. As for the atomic clock, it has a quartz crystal whose frequency f is locked on the frequency f_{at} of the transition

between an atom's two energy levels. As such, to measure time, the quartz crystal (oscillating exactly at f_{at}) is coupled to electronic devices acting as frequency dividers. Periodic pulses are thus generated (one high-accuracy pulse per second). Since the length is calculated according to the number of velocity, which is $V = 299,792,458 \text{ m s}^{-1}$, we hence deduce that the meter derives from the indication given by the Caesium-133 atomic clock. Other crystals would be used as the transition element between the needles and the time mechanism. But why specifically the Caesium-133 atom?

The answer lies in the technique of crystallography. We know that Cs is an alkaline that has the lowest melting point and the highest vapor pressure. This allows it to obtain an atomic beam at lower temperatures. There is only one stable isotope for Cs. We also know that the ground state of Cs is divided only into two hyperfine levels on which the atoms are also distributed at room temperature. Transition frequency is part of the field of microwaves (hyperfrequency) which can be detected using electronic systems. This prevents problems of separation or mixing up of isotopes. It is plausible that future advances in terms of frequency standards would lead to the choice of another atom or ion with more advantages than the Cs. We present a little further the model V of a cesium clock CsV, designed by National Research Council (NRC)-INMS Ottawa.

3.1.2. *Force and pressure standards*

In dimensional metrology, gravimetry (mass and related quantities such as force, pressure, flow, viscosity, etc.) is used throughout the measurement of force and pressure. Even if we consider weighing itself is not important, it is obvious that the force exerted on a wedge affects pressure. We elaborate on this perspective in more detail in section 3.2. Based on this observation, the latter becomes unavoidable in metrology. Traditionally, force measurements are compared to the Newtonian classical force of gravity $F = m \times g$, where (m) is the body mass and g is the acceleration of gravity (9.81 m s^{-2} ...). There is a worldwide network of gravimetric stations allowing determining the value of g up to a millionth. The most precise measurements are performed by comparison with the pressure exerted by a column of mercury (Hg). The highest accuracy ever achieved by the BIMP is on the order of 10^{-8} .

3.1.3. *Electrical standards*

These are mostly standards of resistance and electromotive force (reels, scales, capacitors, etc.). For these standards also, the BIMP ensures uniformity to the millionth level. However, we do not discuss in detail the electrical standards in this chapter.

3.1.4. Temperature standards [CNR 05]

Temperature is a physical quantity of thermodynamics. The value 273.16K (Kelvin) is selected as the temperature of “triple point of water,” i.e. the point at which water vapor, liquid water, and ice coexist in equilibrium. We know that the freezing point of water under normal atmospheric pressure is 273.15K and its boiling point is 373.15K. The temperature difference is 100K. The temperature conventional scale, with simple and accurate benchmarks, for all temperatures used was adopted in 1927. After being revised in 1968, it was named the international practical temperature scale (IPTS). Its fixed points serve to measure constant-standard instruments such as platinum resistance thermometers, thermocouples, platinum rhodium, and optical pyrometers.

3.1.5. Photometric standards

These standards consist of incandescent filament lamps powered electrically. These are standards of luminous intensity in a specified direction. As for the material standard, it is the mole, i.e. the amount of material contained in 12 g of carbon 12, that is to say $6,022,098 \times 10^{23}$ particles.

In Canada, the institution in charge of maintaining the primary length standard, establishing, and conserving the Canadian standard meter, according to its international definition in the SI, is the INMS. The INMS maintains the helium–neon laser locked to an iodine-stabilized atomic resonance at a wavelength of 633 nm. The INSM program maintains and develops facilities for the dissemination of accurate units of length for scientific and industrial purposes across Canada. Calibration [CNR 05] is performed there on a 633 nm wavelength of helium–neon stabilized lasers which are in use in dimensional metrology and in various instruments such as gravimeters. The bars and the gauge blocks are calibrated via interferometric calibration of gauge blocks of the NRC which accepts gauge blocks and rods up to 1 m long. The uncertainty of calibration can be of the order of $20 + 0.4 L$ nm with L , the length of the standard in millimeter.

3.1.6. Measurement, comparison, and control

Instruments of enumeration are usually simple and safe. They are increasingly used when the phenomena being measured are converted to digital signals. Their absolute accuracy is that of the size of the elementary quantity.

Instruments using a direct comparison with a standard similar in nature to the quantity to be measured are the most traditional ones. Their low accuracy,

in general, is limited by the resolving power of the eye that determines the thickness of the graduations of measuring scale. This resolving power is amplified by optical magnification.

Analog and digital instruments transform the phenomenon to be observed into magnitudes that are easy to apprehend in a different platform (electrical or optical). To this end, we may have recourse to sensors and transducers. The quality achieved in the realization of the latter, provides an analogy that may exceed 10^{-6} and accuracy of the same order. The causes of errors are numerous. The problem is to first reduce them to acceptable values and, second, to delineate the scope. Common errors are due to usual defects of the instruments, the observer, and variations in environmental conditions.

The measurement methods are designed to obtain the best possible result for a given instrument, by an experimenter. We must distinguish those imposed by the design of the instrument and those intended to reduce the influence of environmental variation. Distinction is also to be made between methods to compensate for defects in the instrument and those designed to reduce defects on the part of the observer. In the previous chapter, we saw that defects in equipment manufacturing resulting in a reproducible systematic bias can be measured and corrected, as for example, when simple clocks spring forward. The random errors, such as hysteresis, lack of trueness, faithfulness, or precision, could be reduced by repeating measures.

Errors due to the observer are also divided into systematic errors – generally difficult to detect, and therefore not possible to correct – and random errors that can also be reduced by measurement repeatability.

In this vast field of dimensional metrology, we distinguish:

- manual measurement (parts), which is done either through variable instruments such as rulers, graduated squares, calipers, micrometers, etc. Also, measurement is performed using instruments with fixed sizes such as buffers, gauges, shims, etc. This measure is generally used for verification of serial parts in a workshop;
- automatic measurement is typically part of control systems for machine tools (digitally operated machine) or production chains. It uses sensors acting directly on the machinery orders via computers to centralize information and process it;
- the “essence” of a measurement standard is to allow, at any given time, the most accurate and most precise measurements. The same unit can be linked to successive and different standards. Furthermore, the standard should be as inalterable as possible (univocal) and make use of repeatable and verifiable phenomena.

3.2. Meter, time, and mass

3.2.1. The meter

Historically, the units of length were primarily related to living organs such as the foot and thumb. These units are still in use in the United States, the United Kingdom, and Canada. Their main disadvantages are the subdivisions of the unit which are not decimals (1 fathom = 6 ft, 1 ft = 12 in., 1 in. = 12 lines, 1 line = 12 points). In addition, from one country to another, the value of the unit changes: Is the Canadian pound (mass) significantly different from the English one? Yes. However, 1 kg is the same regardless of the country. In 1791, the meter was defined as the 40,000,000th parts of the terrestrial meridian whose prototype was a platinum bar of 20 mm wide and 4 mm thick, maintained at 0°C to preserve a length of 1 m. In 1875, the International Convention of the Meter, adopted the standard as the prototype made of platinum–iridium with a more rigid X-shaped section (Figure 3.1). The length of the standard meter was then reported on the bar with two lines engraved in the metal. In 1903, the meter had become a conventional unit although the most accurate measurements indicated that it lacked 0.2 mm to represent the 40,000,000th parts of the terrestrial meridian.

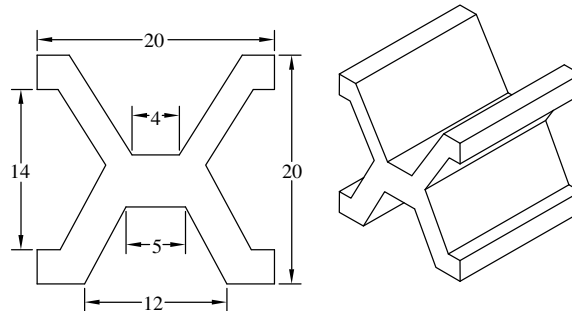


Figure 3.1. Classic profile of the standard meter 1875

In 1960, the meter was defined as 1,650,763.73 times a wavelength, in vacuum, of orange radiation emitted by krypton 86. Since 1983, with improved techniques, and following effective emergence of the laser, a new definition of the meter has been adopted. “The meter is then the length of the path traveled in a vacuum by light during $1/299,792,458$ of a second.”

Nowadays, the physical realization of the unit of length is performed using a He–Ne laser (helium–neon) of known and very stable frequency during an atomic transition, i.e. an absorption line of iodine gas. They are regularly compared with each other or with other lasers equally stabilized and belonging to various

calibration laboratories. The relative uncertainty of the realization of the meter using He–Ne laser stabilized to iodine is in the order of $[2.5 \times 10^{-11}]$, which corresponds to a length of 1 mm from the Earth’s circumference. Interferometry enables the counting and interpolation of wavelengths of laser light on a known path along a static measure, extending, as such, the domain of measurement of fractions of nanometers to several tens of meters.

After this brief history of the meter, we now discuss some measuring or control devices. The objective is to familiarize the reader regarding the instruments used in measurement and their limits in terms of accuracy, in order to determine which instrument is most appropriate to the corresponding cases:

- to acquire a better understanding and mastery of each instrument;
- to learn how to correctly use significant figures while taking into account the uncertainties on the various measurements carried out; and
- to determine the possible causes of errors when using an instrument.

3.2.2. Time

The second was originally defined as 1/86,400 of a mean solar day; then, for various reasons, it was redefined in the same way to 1/86,400 of the mean solar day 1900 January 1. As previously explained, the most convenient definition [CNR 05] of the second is now accepted as 9,192,631,770 vibrations of a certain radiation of Cs-133; see Figure 3.2, Model 5 of the clock (CsV) NRC Ottawa (Canada).



Figure 3.2. Atomic clock INMS (source: NRC, Ottawa, Canada). Model of a clock (CsV) in Cesium, built by the NRC in Ottawa (Canada). In the photo Mr. Rob Douglass (crouching) and Jean-Simon Boulanger, NRC, Ottawa. Frequency and Time Research Group, NRC-INMS

3.2.3. *Mass*

Historically, the kilogram was defined as the mass of a liter of water at 4°C. Following the practical difficulties with this definition in 1889, it was redefined as being equal to the mass of the cylinder of platinum–iridium deposited at the IBWM. This is the international standard kilogram. From these definitions of measuring instruments, we conclude that the measures are still affected by uncertainty, with time and environment. This is due to on the one hand to the measuring device bias (systematic error) and on the other to the experimenter, that is to say, we ourselves (random error). In turn, these errors usually result from:

- reading error;
- error on the zero reading (reference);
- parallax;
- size of the divisions on graduations;
- the irregularity of the divisions;
- the width of the needle;
- interferences (dust, debris, chips, etc.); and
- the method of the experimenter, etc.

For example, the micrometer is more accurate than a vernier caliper. It is useful for measuring small lengths. Some instruments are more accurate than others, but are suitable only for specific circumstances. We shall, therefore, choose the most appropriate instrument for each measure with the objective of realizing a combination of both convenience and accuracy. Among the many available standards, we are particularly interested to discuss: the gauge block, the cylindrical wires, and balls standards “spheres.”

3.3. Deformations and mechanical causes of errors

The standards (blocks and others), even if not in use, are likely to be imperfect with time (aging, fatigue, etc.). In this section, we discuss some examples of deformations that sometimes tend to be wrongly overlooked.

3.3.1. *Quantitative assessment of gauge blocks [CHE 64]*

It is important to distinguish the quality of the workshop from the so-called laboratory quality. In both types, there are reduced tolerances including, therefore,

defects in terms of size and shape (quality in workshop) hereafter expressed by the tolerance interval (TI). The term $f(\mu)$ is a function of the TI in micrometer or microinch:

$$IT = f(\mu) = 0.01 \times \mu + \left(\frac{2 \cdot L}{10^6} \right) \quad [3.1]$$

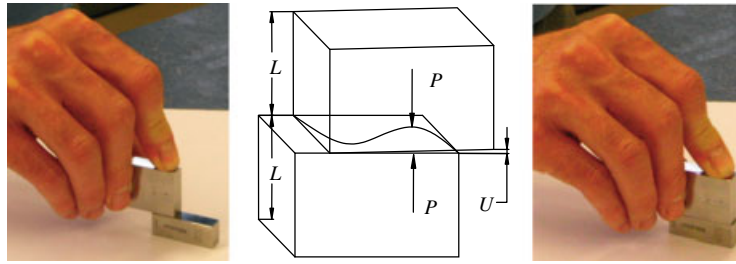


Figure 3.3(a). Adhesion of two wedges

The sway (U in micrometer or microinch) creates the area between the two wedges (Figure 3.3). Here, L is the length of the gauge block (μ in micrometer or microinch), and P is a slight manual pressure (in daN).

NUMERICAL APPLICATION 1.— For the convenience of calculations using MathCAD software, let us consider $\mu = 0.1$, $L = 2$, and from [3.1] we consider $TI = f(\mu)$ as follows:

$$TI = f(\mu) = 0.01 \times \mu + \left(\frac{2 \cdot L}{10^6} \right) = 1.004 \times 10^{-3}$$

For laboratory quality, the expression [3.1] becomes:

$$TI = f(\mu) = 0.05 \times \mu + \left(\frac{2 \cdot L}{10^6} \right) \quad [3.2]$$

NUMERICAL APPLICATION 2.— For $\mu = 0.05$ and $L = 2$, from [3.1] we set $TI = f(\mu)$ as follows:

$$TI = f(\mu) = 0.05 \times \mu + \left(\frac{2 \cdot L}{10^6} \right) = 2.504 \times 10^{-3} \text{ mm}$$

where L is the length of contact of the work piece and TI is the tolerance interval in μ :

Let: $a = 0.01$; $b = 0.01$; $L = (0 \dots 25.4)$, and consider $TI_1(L) = (a \times L + b)$;

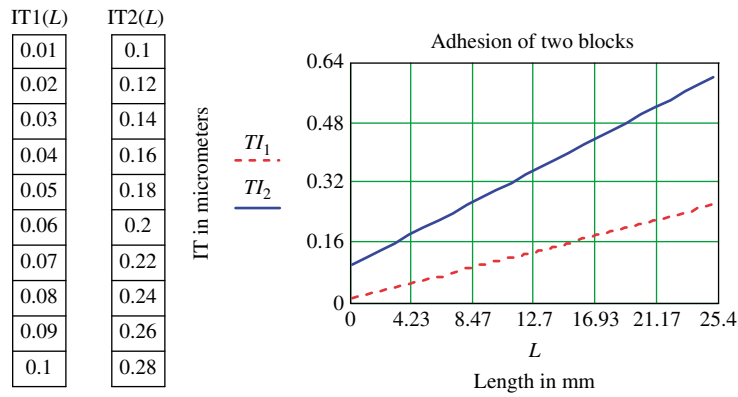


Figure 3.3(b). Result of a simulation based on the adhesion of two blocks

3.3.2. Assessment of cylindrical rod and ball gauges (spheres). Local crashing of cylindrical rods K_1 [CHE 64]

From Figures 3.4 and 3.5, we observe that P is a finger pressure and U is a tacking motion. The crashing of pads or cylindrical rods is calculated empirically by the following equation:

$$K_1 = 0.000092 \times \left(\frac{P}{L}\right) \times \left(\sqrt[3]{\frac{1}{\Phi}}\right) \quad [3.3]$$

where P is the contact load in Newton (N) (usually it is between 1 and 3 N), L is the length of contact with the part in millimeter or inch; and Φ is the diameter in millimeter or inch.

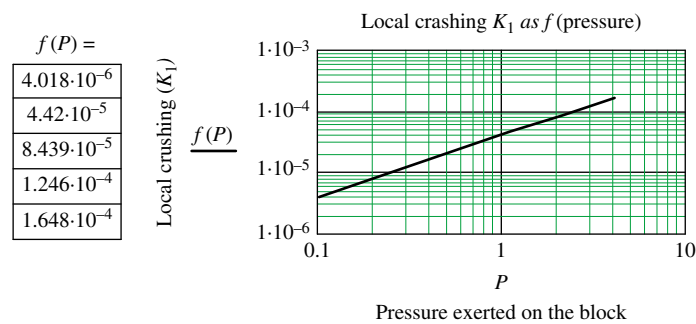


Figure 3.4. Local crashing of blocks (Example 1)

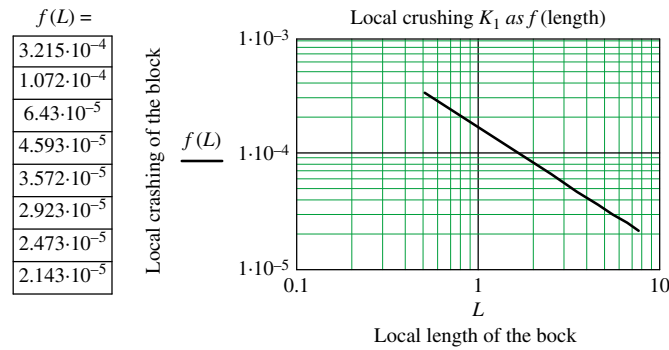


Figure 3.5. Local crushing of blocks (Example 2)

NUMERICAL APPLICATION 1.— Let $L = 2$ in.; $\Phi = 1.5$ in.; and $P = 0.1$ to 5 N. From [3.3], we consider $K1 = f(P)$.

NUMERICAL APPLICATION 2.— Let $L = 0.5$ to 5 in.; $\Phi = 1.5$ in.; and $P = 2$ N. From [3.3], we consider $K1 = f(L)$.

3.3.3. Recommendations for correct block staking [MIT 00]

The steps illustrated in Figure 3.6 are based on the recommendations of the manufacturer Mitutoyo. We have performed them, in the laboratory of dimensional metrology, on blocks of the same manufacturer and the results derived there from are largely inconclusive. Here is how to join:

- a set of two thick blocks;
- a thin block with a thick block; and
- two thin blocks.

We propose the steps to follow as recommended by the manufacturer – Mitutoyo – in this case (Figure 3.6).

COMMENT ON FIGURE 3.6.—

➔ For phase (i), here are the recommended steps:

1. Bring the two surfaces in to contact at right angles.
2. After applying pressure, turn the blocks 90° from each other.
3. Drag one of the blocks so that the surfaces coincide.



Figure 3.6. *Explanatory steps for blocks adhesion*

➔ For phase (ii), here are the recommended steps:

1. Place one of the two blocks at the end of the other.
2. After applying pressure on the entire surface, drag the thin gauge block on to the other one while maintaining a light adhesion by pressure.

➔ For phase (iii), here are the recommended steps:

1. To shield blocks from any possible distortion, put the first thin gauge block on a thicker shim.
2. Rub the second thin block on the first.
3. Remove the thick block.

3.3.4. Punctual contact (spherical buttons, beads, and thread flanks of a thread buffer) K_2° [CHE 64]

The theory of contact is simply applied in this case to assess the intensity of the flattening, K_2 , through the following simple experimental formula:

$$K_2 = 0.0014 \times \left(\sqrt[3]{\frac{P^2}{r}} \right) \quad [3.4]$$

where P is the contact load in N or daN (N or lb in North America), and r is the radius of the ball in millimeters (inches).

Over time and especially due to constant usage, there is a compression (flattening) that is calculated empirically by the following equation:

$$K_3 = \frac{P \times L}{E \times S} \quad [3.5]$$

where P is the contact load daN, L is the length of contact with the work piece in millimeter, S is the square millimeter section, E is Young's modulus of elasticity (in MPa or N mm⁻²).

NUMERICAL APPLICATION.— Let $P = 3$ N; $S = 300$ mm²; $L = 200$ mm; and $E = 220,000$ Pa. From [3.5], (the) flattening (compression): $K_3 = 0.00001$ mm or 0.01μ . If $S = 80$ mm²; $P = 3$ N; $L = 120$ mm; and $E = 2.10^6$ MPa, then using:

$$\text{Initial data: } S = 80 \text{ mm}^2; \quad r = \sqrt{\frac{4 \cdot S}{\pi}} = 10.093 \text{ mm}$$

$$\text{For } P = 3; \quad r = 10.093; \text{ using [3.4], consider } K = 0.0014 \times \sqrt[3]{\frac{P^2}{r}} = 1.348 \times 10^{-3} \text{ mm}$$

$$\text{For } P = 3; \quad L = 1120; \quad E = 2 \times 10^6; \quad S = 80 \text{ using [3.5]; and } K_3 = \frac{P \cdot L}{S \cdot E} = 2.250 \times 10^{-6}$$

3.3.5. Total flattening of cylindrical gauges (k_p) [CHE 64]

Total flattening (compression) (k_p) is calculated by taking the sum of flattening values. Of course, we must be careful not to consider the flattening (compression) [CHE 64] for a cylindrical shape with the spherical one and vice versa. Thus, the complete compression (k_p) takes the following form:

$$K_p = K_1 + K_3 \quad [3.6]$$

Using [3.3] and [3.5] in [3.6], we obtain [3.7]:

$$K_{\text{cylinder}} = 0.000092 \times \left(\frac{P}{L} \right) \times \left(\sqrt[3]{\frac{1}{\Phi}} \right) + \left(\frac{P \cdot L}{S \cdot E} \right) \quad [3.7]$$

3.3.6. Total flattening of balls (spheres) K_{sph} [CHE 64]

The approach is analogous to the previous one in terms of calculation of the total flattening compression of spherical shapes. The term K_{sph} is the mathematical expression of the flattened (spherical flatness) ball, whereas K_{cyl} is the mathematical expression for the cylindrical flattening:

$$K_{\text{spheres}} = K_2 + K_3 \quad [3.8]$$

By replacing [3.4] and [3.5] in [3.8], we obtain [3.9]:

$$K_2 = 0.0014 \times \left(\sqrt[3]{\frac{P^2}{r}} \right) + \left(\frac{P \cdot L}{S \cdot E} \right) \quad [3.9]$$

NUMERICAL APPLICATION.— Let $P = 0.1 \dots 10$; $r = 10$; $E = 2 \times 10^6$; $L = 100$; and $S = \pi \cdot r^2 = 314.159$. Now, [3.7] gives the results of Figure 3.7.

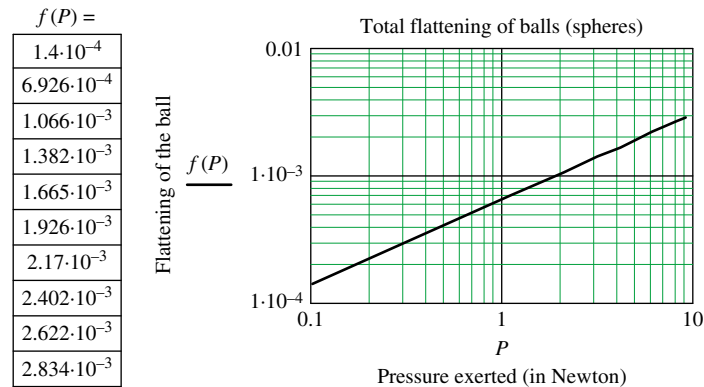


Figure 3.7. Total flattening of balls

3.3.7. Measurement and precision with micrometer [MIT 00]

3.3.7.1. Airy points

The points of support, known as Airy and Bessel points [MIT 00], allow for support, in a horizontal position, the standards of cylindrical measurements or tubular inside micrometers in order to obtain specific flexure conditions. Airy points are used for two-point support and ensure the parallelism of both end faces, as in Figure 3.8.

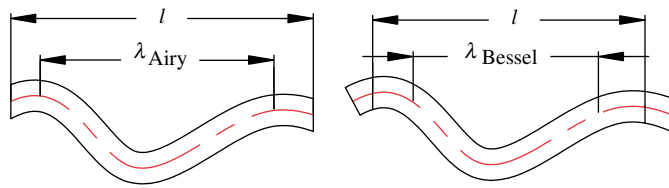


Figure 3.8. Schematic of the principle of AIRY points and BESSEL points

$$\text{Airy points: } \lambda = 0.557 \times l \quad [3.10]$$

where λ is a released length and l an actual length.

3.3.7.2. Bessel points [MIT 00]

Bessel points are used for two-point support, and minimize the change in the overall length.

$$\text{Bessel points: } \lambda = 0.5590 \times l \quad [3.11]$$

3.3.7.3. ABBE's principle

ABBE's principle [MIT 00] states that "the maximum accuracy may be obtained when the standard scale and the workpiece being measured are aligned along the line of measurement". When the contact points of a micrometer are far from the axis of graduations, as shown in Figure 3.9, measurement error will become significant.

In this case, we especially pay attention to the applied measuring force using the following formula:

$$\varepsilon = (l - L) = R \times \tan(\theta) = R \times \theta; \quad \theta \text{ is very small, thus } \tan(\theta) = \theta \quad [3.12]$$

where ε is the difference of the lengths ($l - L$), i.e. the deformation in millimeters; r is the distance between nozzle/drum marked in millimeter; l is the opening deformed in millimeters; L is the standard opening, when measuring in millimeters; and θ is

the angle of deformation (degree). When the argument of the function θ is very small, the function $\tan(\theta)$ equates mathematically to this argument and hence $\tan(\theta) \cong \theta$. This is a simulated example to calculate the angle of deformation of a micrometer.

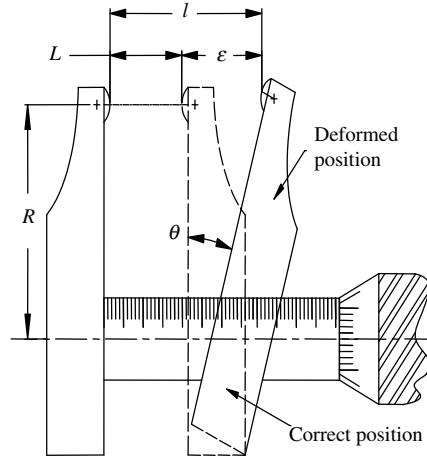


Figure 3.9. Schematic illustration of the principle of ABBE

NUMERICAL APPLICATION.— If $l = 8$ mm, $L = 7$ mm, and $R = 8$ mm, then $\epsilon = 1$ mm and $y = \theta$. Now

From [3.12], consider $y = \frac{\epsilon}{\theta}$ since $y = \theta$. Then, $\tan(\theta) = \frac{\epsilon}{\theta}$.

If $y = 0.125$, then $\tan(\theta) = 0.125$ or $\theta = 0.125$ (because $\tan(\theta)$ assimilated to θ). In other words, $\theta = \tan(0.125)^{-1} = 8 \rightarrow \theta(\text{in degrees}) = 0.14^\circ$.

Verification: $\epsilon = R \cdot \tan(\theta) = 8 \tan(0.125) = 1$.

3.3.7.4. Hooke's law

Hooke's law demonstrates the relationship between applied force and the resulting deformation. In the limit of elasticity, the deformation of a solid is directly proportional to the force applied to it. Hence the following relationship:

$$\sigma = \epsilon \times E \quad [3.13]$$

where σ is the applied stress (constraint) in $\text{Mpa} = \text{N mm}^{-2}$; ϵ is the relative deformation $= \Delta l/l$; and E is the modulus of elasticity in $\text{N mm}^{-2} = 1.96 \times 10^5$ (in Pa or N m^{-2}).

3.3.7.5. Theory of contact in the formula of Hertz

The formula of Hertz [MIT 00] gives the result of deformation within the limits of elasticity when two surfaces (faces) (spherical, cylindrical, or flat) are pressed against each other under a certain force (Figure 3.10).

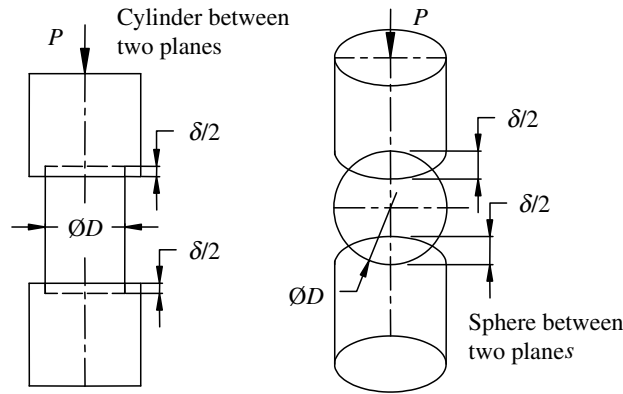


Figure 3.10. Flattening a sphere and a cylinder between two planes

Compressed surfaces [MIT 00] against each other result in deformation (δ_1 and δ_2) which is given in each case as follows:

(i) spherical and plane surface (contact point):

$$\delta_1 = (0.82) \times \left(\sqrt[3]{\frac{P^2}{D}} \right) \quad [3.14]$$

(ii) cylinder and plane surface (contact point):

$$\delta_2 = (0.942) \times \left(\sqrt[3]{\frac{1}{D}} \right) \times \left(\frac{P}{L} \right) \quad [3.15]$$

where δ_2 is the value of the deformation micrometer; D is the diameter of the ball (sphere) in millimeters; L is the length of the cylinder in millimeters; and P is the applied load in N.

We now elaborate on the points related to instruments used in dimensional metrology.

3.4. Marble, V-blocks, gauge blocks, and dial gauges

3.4.1. Control of flat surfaces [CAS 78, CHE 64] on marble

Strictly speaking, it is not about measuring instruments but about instruments of control. A marble is a reference surface, made of cast iron or granite, planed and ground so as to present an optimum flatness and devoid of deformation under stress (compression, heat, etc.). A marble is a control surface and not a work table. Control of flat surfaces is, mostly, done in machine tool laboratories such as milling machines. The control is based on the following features:

- the dimensions of the flat surface or its extent;
- the frequency of use of the surface and the level of correction required;
- the method of obtaining the surface and (as well as) the correction level of the machine and the tool of its physical generation.

We do not consider here the constraint of correctness (straightness) when a small and narrow area is considered. Figure 3.11 represents a classic case of control of a small (reduced) area. As shown in Figure 3.11(a), for a machine tool, we retain control of flatness that implies a sufficient impact on correctness.

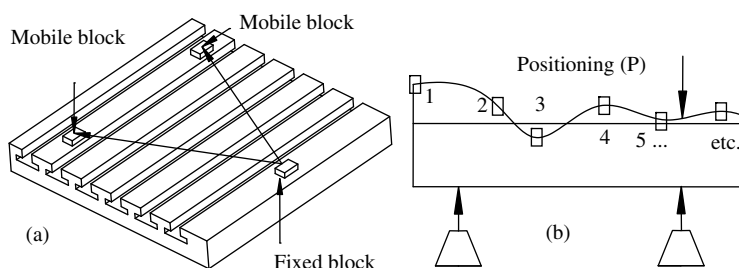


Figure 3.11. Verification (control) of the plane with ruler and blocks.
Two fixed blocks (a) and one mobile wedge (b). Example of representation
of straightness and flatness of a marble

3.4.2. Measurement by comparison of small marble surfaces

A marble is the physical configuration of a plane. It is also a reference plane which serves to support any side of a piece intended to undergo measurement of its correction. It is imperative that the marble be free of any distortion. Whatever the nature or the origin, the deformation (internal tensions, faulty timing, work piece weight, etc.) is totally excluded. To avoid distortion, we provide enough time for the marble till it becomes stabilized.

- With a mobile comparator, the work piece is based on three adjustable gauge blocks (height H) (Figure 3.12).
- With a fixed comparator, it is the piece (the work piece) that slides on the marble.

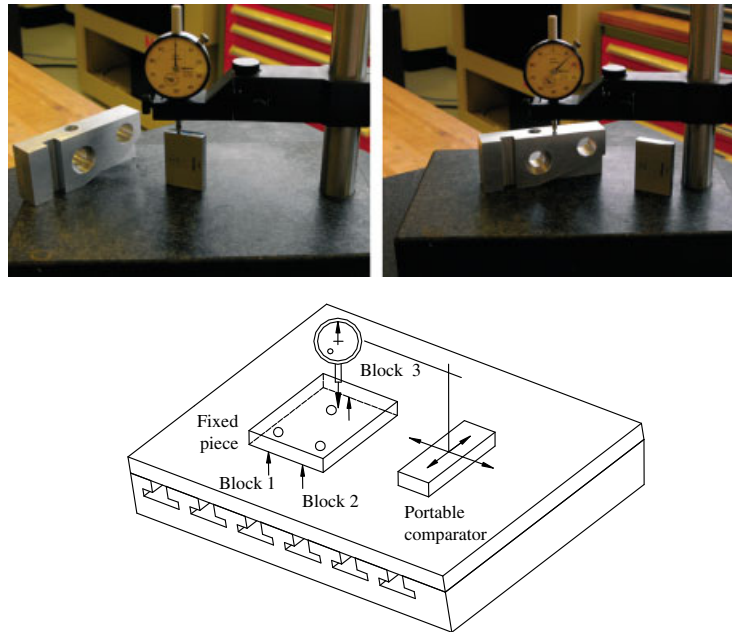


Figure 3.12. *Checking the flatness with a gauge/comparator (fixed and mobile)*

The current marbles, with a non-adjustable four-foot support, are almost never perfect on the bench which supports them as shown in Figure 3.13.

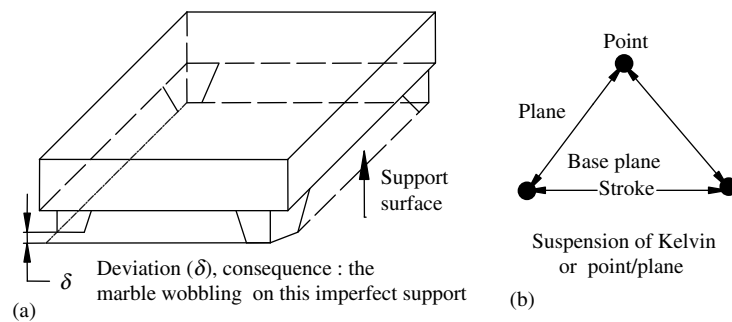


Figure 3.13(a). *Classic suspension of a marble*

Three feet are in contact with the support but the fourth is more or less deviated depending on the load-carrying capacity of the marble. It is, therefore, absolutely necessary to block it horizontally.

The line-point-plane system, called Kelvin, enables the marble to remain in an unchanged position. The base, very rigid and perfectly supported, bears a conical milling machine at 120° , a V-shaped groove aligned with the cone and a small-area plane. The marble has three feet shaped ball screw jacks. One of those feet is centered in the recess. The second foot is localized freely in the V-shaped groove while the third one is localized freely on the plane of the base. These three supports do not, however, provide any support allowing for free expansion. We note that nothing prevents the deformation of the marble under the action of its own weight or the load it would carry. There are various ways to control and measure flatness of the marbles.

Conventional control of flatness: A surface is considered flat or planar when the variation of the distance of its points relative to a geometric plane, which is parallel to the general direction of the plane to be controlled, is less than the value of δ given as:

$$(d_{\max i} - d_{\min i}) \leq \delta$$

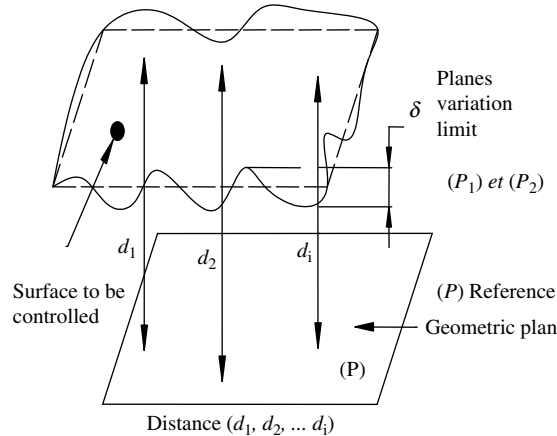


Figure 3.13(b). Conventional designation of flatness defect

3.4.3. V-shaped block

The V-blocks are support pieces with similar qualities to those of a marble. We use Vs put on a marble in order to control centering, distortion, and run-outs of

some mechanical parts. The use of V-blocks on a different support may distort the accuracy of the measurement. The test procedure consists in placing the rotating piece on V-blocks, and then fixing the dial gauge on the marble. By rotating the piece, we determine the round bottom, as shown in Figure 3.14.



Figure 3.14. *Mounting on V-blocks*

3.4.4. Parallel blocks

The parallel gauge blocks are among the most indispensable means to calibration in the traditional measuring instruments in the workshop. These are essential tools because they are the ultimate reference entity. They are always ground, thermally treated, and honed to prevent premature aging and adverse events related to thermal expansion. They can be made of steel with treated zirconium or in ceramic. Although expensive, they offer dimensional stability and are also light. The wear (in microns) is based (as a function of) on several factors but the most influential is (still) loading [MIT 00].

Like the stop of micrometers, gauge blocks are of various geometrical and dimensional shapes. Their role is very specific and analog to that applied to parallel gauge blocks. Like stops of micrometers, gauge blocks can sometimes show flatness or parallelism defects, if not both at once. The efficient means of control of major defects remains the flat glass. If, as shown in the graphic illustration (Figure 3.15), interference fringes appear, we deduce that there is one of these defects or both depending on the configuration of the appearing interference fringes due to the

experiment called the air wedge. We discuss this method in detail in Chapter 5 that is dedicated to interferometric metrology.

It is clear from Figure 3.15 that the interference fringes are visible and are all parallel. This means that the gauge block does not show significant defects in terms of flatness or straightness.

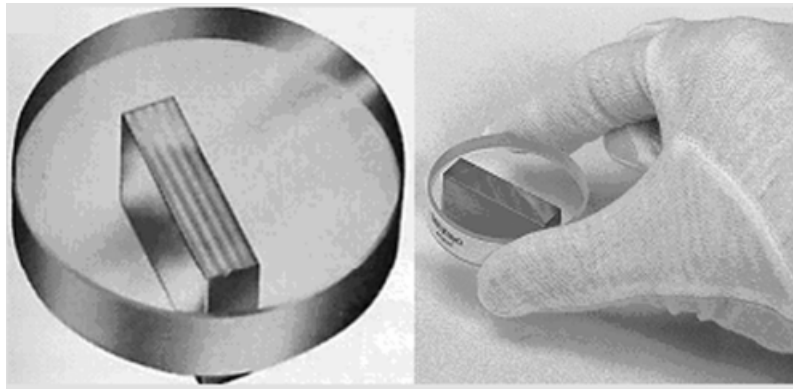


Figure 3.15. *Parallel-plane glasses inspecting the flatness (Courtesy of Mitutoyo)*

3.4.4.1. *Guide [MIT 00] of the measurement and the precision of gauge blocks*

Gauge blocks, whether they are parallel or angular, are essential in workshops and laboratories. In addition to their regular maintenance and care regarding their use, the aspect of adhesion during setting is of immense importance. For this purpose, the manufacturer Mitutoyo recommends the following approach:

1. Use fewer gauge blocks to constitute the required dimension:
 - (a) use the gauge blocks as thick as possible; and
 - (b) select the blocks starting from an integer or as close as possible to an integer value and terminate stacking with the gauge block having the largest decimal.

Aspects such as maintenance and inspection of defects are noteworthy as they are the best guarantors of the good use of blocks. It is recommended to:

2. Clean the gauge blocks with a suitable product.
3. Check that the gauge block is free from defects (see Figure 3.15). To do so, we may use a plane-parallel glass with interference to inspect the flatness of the block. The procedure is described as follows:

- (a) carefully clean the surface to be measured;
- (b) bring into contact the optical plate with the surface to be verified;
- (c) carefully adhere the optical plate on the surface to be inspected – the interference fringes will appear.

Three types of checks (verifications) may be required and are indispensable. They are as follows:

First inspection: If the fringes do not appear, we may believe that a dust or a smudge may contaminate the surface to be inspected, hence the importance of cleaning with an appropriate product. By pressing carefully the glass on the surface being inspected, the interference fringes will appear.

Second inspection: If the fringes disappear, we understand that there is a flaw to be corrected.

Third inspection: If the fringes disappear partially, we should rub and press the optic against the surface. If the interference fringes appear in the same place on the surface being inspected, there is burr on the gauge block. If the interference fringes appear in the same location on the surface of the glass, then the glass has a flaw.

In the latter cases, the burr on the surface to be inspected should be removed as per the below procedure:

4. Apply and spread a little oil on the surface of the block. Then wipe the surface film. Low viscosity grease, gear oil, or even Vaseline may also be applied.
5. Join the two sides of the block depending on the size to be adhered.

3.5. Dial gauge

A dial gauge is a device used to appraise a surface condition or unevenness (clearance between gears, axial clearance, centering, out-of-round, overrun, flatness of a surface, cut-outs, etc.) with an accuracy of one hundredth (100th) of a millimeter (Figure 3.16). It resembles a watch with a revolving dial and a mobile index. The reading is easy and precise as it is done directly on the dial. A mobile key changes the needle's position with a reference sprocket and is reverted to its original position by a system of spiral spring (elastic potential energy). The mobile dial allows to manually adjust the setting of the graduation to "zero" when in contact with the workpiece. A comparator with a non-inclinable sliding rod is used to make hard-to-access measurements, or to adjust machine tools. It is also used for comparative measurements using a marking gauge or a base table. The main precautions to be taken when using comparators are:

- the gauge shall be perpendicular to the contact surface in order to avoid measurement errors;
- gauge shall be “armed” with two or three turns to avoid measurements at the stroke end;
- the comparator is a delicate instrument which must be handled with care. Hence, regular calibration is recommended.

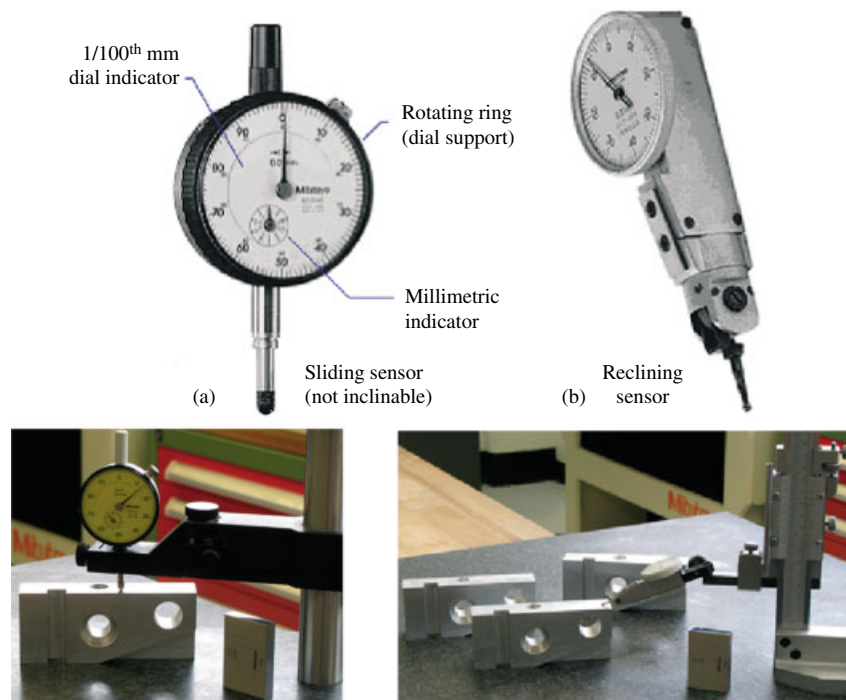


Figure 3.16. Dial indicator with non-inclinable sliding rod (a) and with inclinable touch-probe (b)

Indirect measurement: The scale (dimension/size) to be measured is compared to a similar size, a known value; somewhat different from that of the measurand (we measure the difference between the two quantities).

Measurement by comparison: This is done using a comparator. A stack of wedge equal to the average dimension to be controlled is done under the contact of the comparator. The zero of the dial is set to the needle. Gauge blocks are replaced by the piece. The value of the measured quantity will equal the value of the stack of the gauge blocks $\pm$ the difference between the zero of the dial and the needle gauge.

There are two kind of the comparator readings: classical reading based on needle-gauge and the digital-one. The angle of inclination is important for accuracy as shown in Figure 3.17.

Precautions: In order to obtain better results, and thus good performance, and a significant lifetime for the comparator, the following precautions should be taken.

Angle of contact point: Adjust the point of contact so that it becomes parallel to the surface of the workpiece being inspected (see Figure 3.17(a)).

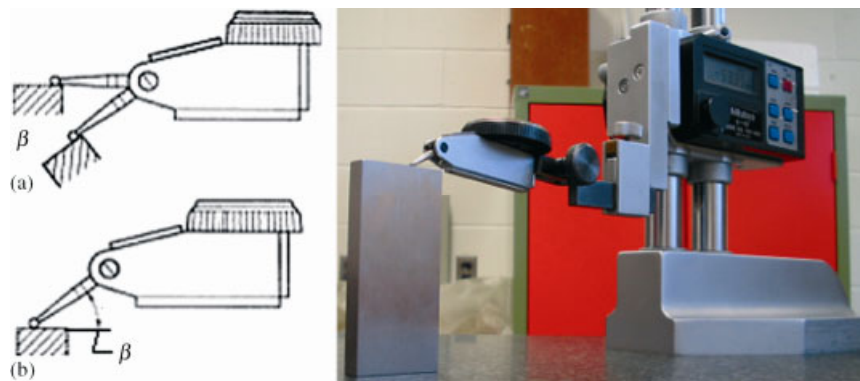


Figure 3.17. Setting a comparator on walnut dovetail with respect to an angle of about 15° to the horizontal formed with the gauge block (or the test piece)

If the contact point were on a specific angle with the surface being measured, as shown in Figure 3.17(b), adjustments should be made on the basis of the following equation:

$$\text{Actual value (mm)} = \text{Reading}(l) \times \text{Correction factor}(k) = l \times k \quad [3.16]$$

Table 3.1 indicates the choice of (k) depending of the measured angle (β°).

Angle (β°)	Correction factor (k)	Angle (β°)	Correction factor (k)
10°	0.98	40°	0.76
20°	0.94	50°	0.64
30°	0.86	60°	0.50

Table 3.1. Correction factor as a function of the angle of inclination of the needle relative to the surface of control (gauge block or part) (source: Mitutoyo Canada [MIT 00])

Example of application: Assume an angle (β) of 30° and an effective reading (l) of the indicator of 0.05 mm. Using equation [3.16], the actual value would be: $0.05 \times 0.86 = 0.043$ mm.

Setting the comparator: In order to avoid measurement errors due to deflections, the dial indicator shall be fastened to a stable base support. Ensure that the dial indicator is fixed firmly, even if a clamping nut with dovetail or any other clamping option was used.

Probe (sensor) length: The length of the probing arm is specific to each dial gauge model. If it is not “correct,” significant measurement errors may result. In many cases, corrections are necessary, without which the overall uncertainty is affected.

3.5.1. Mechanical dial gauges with inside and outside contacts

These comparators can be handled manually (Figure 3.18). Thus, users face the difficulty of maintaining adequate pressure from one measurement to another. Such a method raises the problem of the random pressure exerted by the hand of the operator as shown below. However, we can notice that the difficulty to properly point to the inside “true diameter” is acute. Of course, we always manage to read, copy, or measure in that way but, in our view, this practice would be questionable in the lab even if it is accepted in the workshop. Most cases such as the type of inner groove do not require high measurement accuracy. Thus, we can tolerate minor variations of uncertainties.



Figure 3.18. Comparator with probe arm for outer measurement (courtesy Mitutoyo)

3.5.2. Sizes of fixed dimensions, or Max–Min

We know that mechanical manufacturing of the parts are often toleranced with regard to the nominal dimension. Every part included within the specified tolerances is normally considered as good, that is to say, it passes the quality control. Then, to control the part, it will suffice to check the means to be used in this control. Among many means of control, we can mention the plug gauges, bore gauges, and pins. For shaft dimensions, we mention the plain bearings and jaw gauges.

3.5.3. Bore gauges

Verification of a bore diameter $\varnothing 25$ H7 using a smooth buffer for control such as “Go, not go” is shown in Figure 3.19.

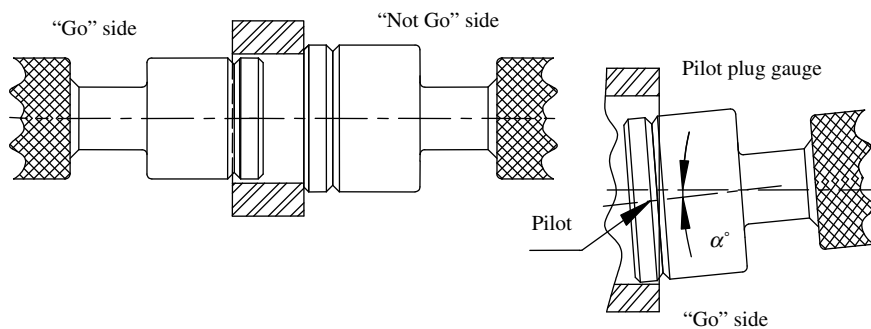


Figure 3.19. “Go, not go” bore gauge

Smooth plug gauges are made of cylinders perfectly smooth at both ends Max–Min of the handle constituting its body. We should avoid the wedging of the plug gauge in case it is incorrectly engaged in the bore. After that, a perfect alignment should be ensured when introducing the plug gauge into the container to be controlled. This is a practical method of engaging a gauge in the bore.

The control approach consists of answering the following yes/no questions, from which the control results are deduced.

3.5.3.1. First scenario (Figure 3.19)

- The “Go” side of the gauge plug enters → yes
- The “No go” side does not enter → yes
- Control result → specification met

3.5.3.2. Schematic illustration

If the hole is too big, here is the appropriate scenario (see Figure 3.20).

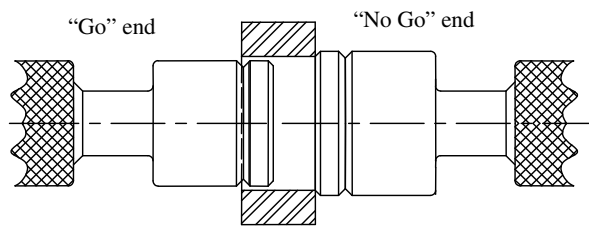


Figure 3.20. Bore gauge (Go; No go), case 1

3.5.3.3. Second scenario

- The “Go” side of the gauge plug enters → yes
- The “No go” side does not enter → no
- Control result → too large hole

3.5.3.4. Schematic illustration

If the hole is too large, here is the case (see Figure 3.21).

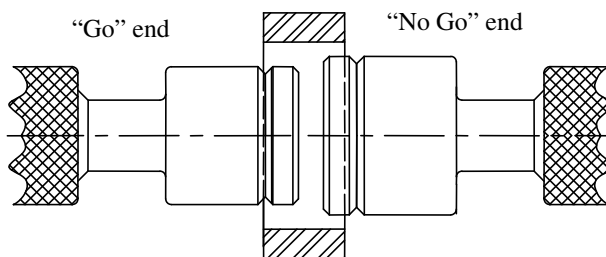


Figure 3.21. Bore gauge (Go; No go), case 2

3.5.3.5. Third scenario (Figure 3.22)

- The “Go” end of the gauge plug does not enter → no
- The “No go” side enters → yes
- Control result → too large hole

3.5.3.6. Schematic illustration

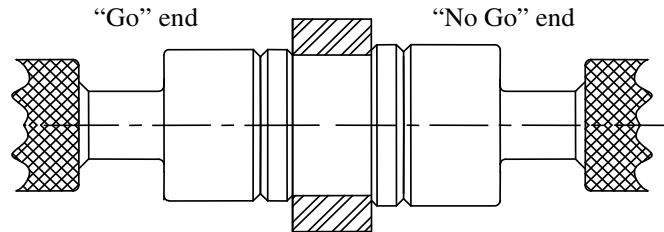


Figure 3.22. Bore gauge (Go; No go), case 3

The same approach exists for the control of threads.

3.5.4. Bore gauges

Bore gauges are used for the same purposes as plug gauges (Figure 3.23). They have the advantage of being less bulky in terms of size, although the contact is limited to the width of the cylindrical surface. The contacts are on both sides of the body of nominal dimension.

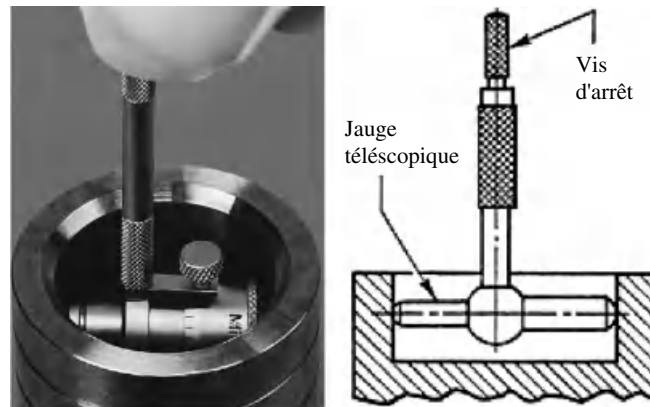


Figure 3.23. Telescopic bore gauge (control by copying)

3.5.5. Plain rings

Plain rings are used for rapid control or serve as shaft gauge. The procedure for their use follows the same recommendations in terms of cleaning and inspection

routine measurement conditions. The plain bearings are also used to calibrate other instruments of control. In many cases, they are used for the calibration of measuring instruments.

3.5.5.1. Shaft gauge – smooth jaws gauges

These gauges are used both in workshop and laboratory for quick control purposes (Figure 3.24). The measure already exists on gauges and it should approve or disapprove the answer “Go” or “No Go”. It is worth taking some important precautions when introducing the piece to control, at the gauge jaws level.

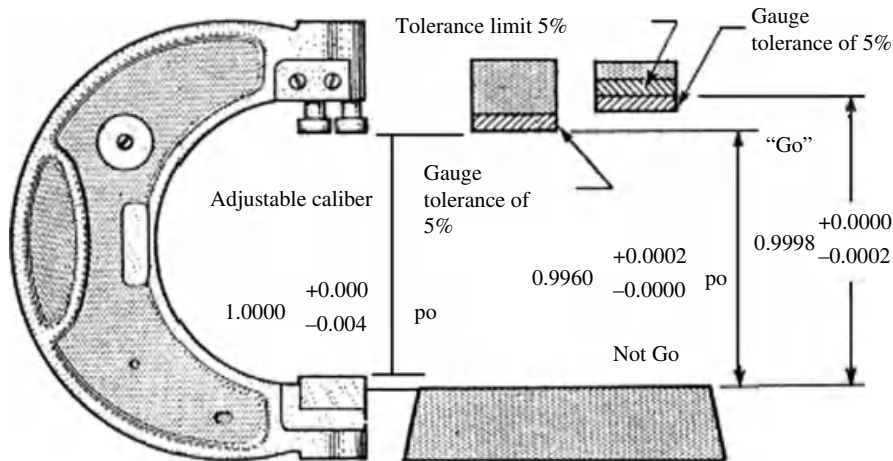


Figure 3.24. Flat gauge double adjustable for nominal dimensions

3.5.6. Spindle bores

Flat gauges are becoming unusable because they are too heavy. They do not tolerate enough penetration to ensure easy control. Thus, they are substituted by spindles with ends intended for this purpose. All spindle bores have the same principle of reading and use. However, they may be differentiated based on several forms of extreme. This can be explained by the shape of the mating part as shown in Figure 3.25.

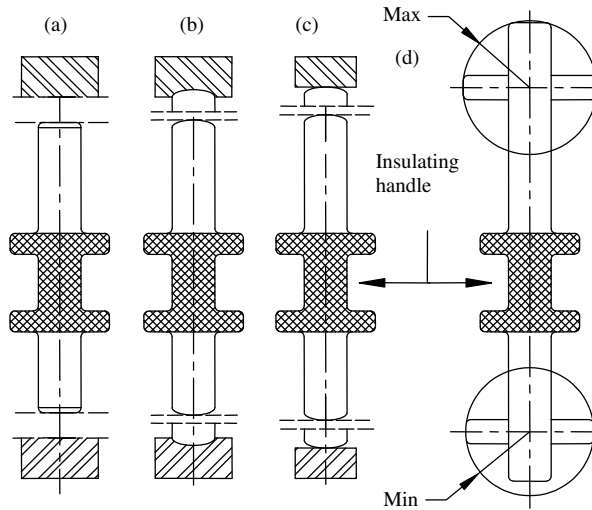


Figure 3.25. Schematic illustrations for bore gauges: (a) plane tipped spindle ends with full contact on plane; (b) spherical tipped spindle with linear contact on cylindrical bore; (c) spherical tipped spindle with punctual contact on plane; (d) double spindle: Max-Min

3.5.7. Inside gauges (micrometer) [MIT 00]

In fact it is a micrometer at 1/100 used for measuring bores (inside/interior dimensions) up to 250 mm when associated with removable rods. The inner gauge allows measurements of grooves or shoulder. The reading principle is identical to that of the micrometer (Figure 3.26).

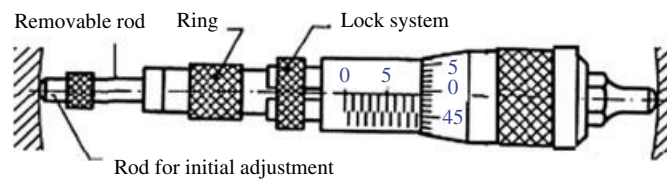


Figure 3.26. Inside micrometric gauge

The inside micrometer (or micrometric gauge) is widely used for measuring diameters of bores which are sometimes too big. Among the difficulties that arise when using interior micrometers, we mention those of errors resulting from inappropriate positioning. If the micrometer is tilted in the axial direction (diametrical) as shown in Figure 3.27(a), this leads to an error in the measurement.

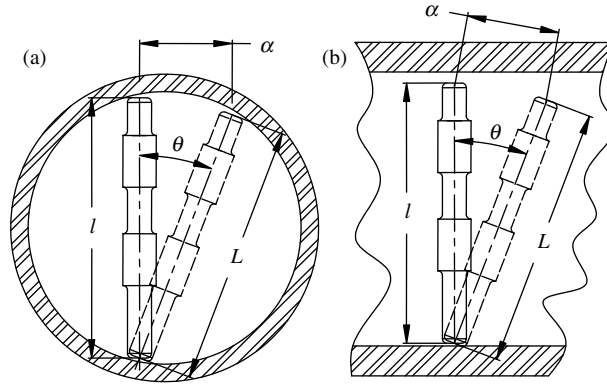


Figure 3.27. Illustration of measurement with positioning defect

If the inside micrometer is tilted in the direction of the lateral direction as shown in Figure 3.27(b), it results in a negative error. In one case as in the other, one should avoid these types of errors in handling. The best guarantee for good measure remains the experience and skill of the operator.

3.5.7.1. Method of calculating the error (Δl) depending on the cases (a) and (b)

The appropriate formula to the above figures is written:

$$\Delta l = (L - l) = \sqrt{(l^2 + \alpha^2 - 1)} \quad [3.17]$$

The following formula is appropriate to Figure 3.27 (b):

$$\Delta l = (L - l) = \sqrt{(l^2 - \alpha^2 - 1)} \quad [3.18]$$

where l : inside diameter of the workpiece; L : measurement length; α : tilt; and Δl : the measurement error.

In terms of laboratory experiments, manufacturer Mitutoyo offers a graphic illustration to show the influence of the error due to the slope of the micrometric gauge along its length. Three lengths – 200 mm, 500 mm, and 1,000 mm – show that the bigger the length, the more likely the error (Δl) is to be high.

APPLICATION EXAMPLE SIMULATED USING MATH CAD SOFTWARE.– Given the example illustrated in Figure 3.27, we simulated and conducted an experiment in the workshop to address the error angle after the length of the micrometer gauge. Here are the results of the simulation based on the data of Figure 3.28; $l_0 = 2$; $l_1 = 5$; $L_2 = 10$; and $\alpha = 0.15$ to 10.

$$f(\alpha) = \sqrt{(l_0^2 - \alpha^2 - 1)}; \quad G(\alpha) = \sqrt{(l_1^2 - \alpha^2 - 1)}; \quad H(\alpha) = \sqrt{(l_2^2 - \alpha^2 - 1)}$$

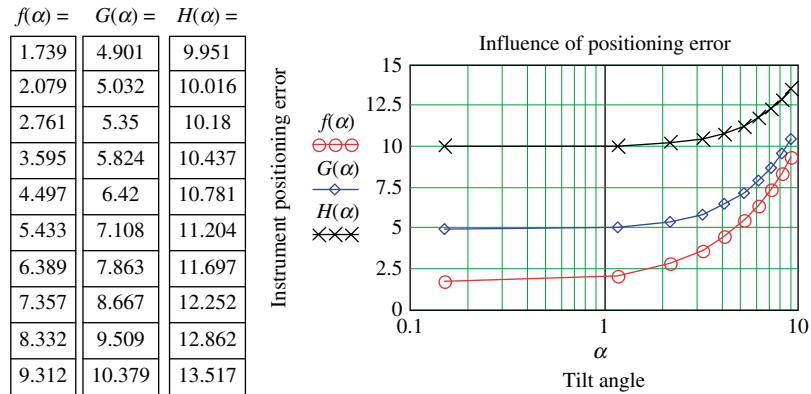


Figure 3.28. Influence of positioning error

3.5.8. Depth gauges [MIT 00]

These allow measurements of hollow or shoulder (Figure 3.29). The reading principle is identical to that of micrometer or caliper (depth). However, if there is allowance in the manufacturing quality, the micrometers are provided with a vernier scale of 1/10th division allowing reading at 1/1000th of a millimeter.

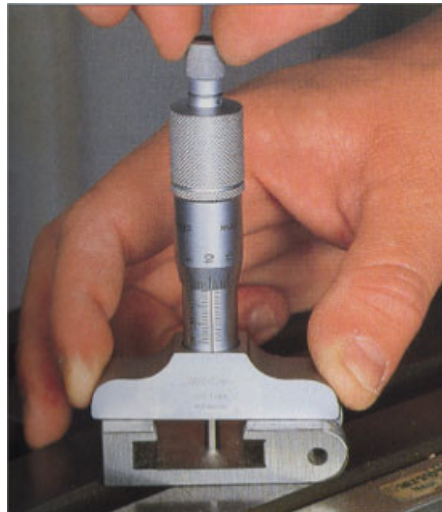


Figure 3.29. Depth gauges for measuring narrow grooves

3.5.9. Telescopic bore gauges

The use of telescopic gauges is common in mechanical manufacturing workshops which do not require high accuracy. This control provides excellent results in workshop.

3.5.9.1. Thickness gauges

They are also known as the set of gauge blocks. They are used to measure distances between 10th and 100th of a millimeter. These gauges are made of thin calibrated steel. The measurement is performed by assessment, by sliding the blades along the distance to be controlled and finding the one that allows a minimal clearance (friction at the edge of tightening). The set of thickness gauge exists in metric and imperial units. Also, we must remove dust and the chips which are likely to penetrate and thus distort the assessment of the control. The principle of use of gauges is similar to that of “Go” and “Not go” gauges. As such, we introduce the gauge in the housing or in an appropriate location and deduce the value of this space based on the value given by the gauge. It is an apparatus dimension, in terms of applied metrology.

3.5.9.2. Telescopic bore gauges

Bore comparators are often used in metrology for verification. The aim is to measure one dimension using another means of precision and then copy this measurement on a telescopic gauge called “bore gauge.”

As shown in Figures 3.30 and 3.31, we insert the nozzle (d) and the spindle play (c) corresponding to the dimension of the bore to be copied and we use the key (e). To ensure that we have properly copied the right dimension of the bore, there are tables provided with the *kit* shown above; before starting copying, measure it and then transcribe it. Prior to carrying out the measurement, we propose two configurations of a tip attached to the rod bore checker.

The verification principle is simple. It suffices to identify a given data and then transcribe it through bush and nipples that are spread out to arrive at dimensions copied on the associated comparator according to the mounting presented in Figure 3.31. The spindle play (c) also called the compensation ring is inserted inside the device, in addition to the nozzle. The name “compensation rings” is appropriate since this ring acts as a supplement to the dimension of the tip which is also chosen based on the index table that accompanies the set of bore gauges.



Figure 3.30. Mounting kit for telescopic bore gauge (see Table A2.1)

Capacity in millimeter	Measurement depth in millimeter	Number of measurement contacts	Number of rings of compensation
6–10	47	9	—
10–18.5	100	9	1
18–35	100	9	2
35–60	150	6	4
50–100	150	11	4
50–150	150	$11 + I^*$	4
100–160	150	13	4
160–250	250	6	7
250–400	250	$5 + I^{**}$	7

(*) contact 50 mm and (**) contact 75 mm

Table 3.2. Bore gauge (source: catalogue Mitutoyo [MIT 00])

Here are the characteristics of two photographic illustrations to show the application of bore gauges:

- wide range of contacts with curvature radius measuring the bore with significant repeatability;
- contacts made of hardened steel and possibly spherical carbide contacts.

The bore gauges in inches, for measuring ranges of the series No. 511 (Mitutoyo), are listed in specific tables (see Table A2.1 in Appendix 2). Having already measured the dimension of the bore with a telescopic gauge, we should still copy it on an outside micrometer, as shown in Figure 3.31.

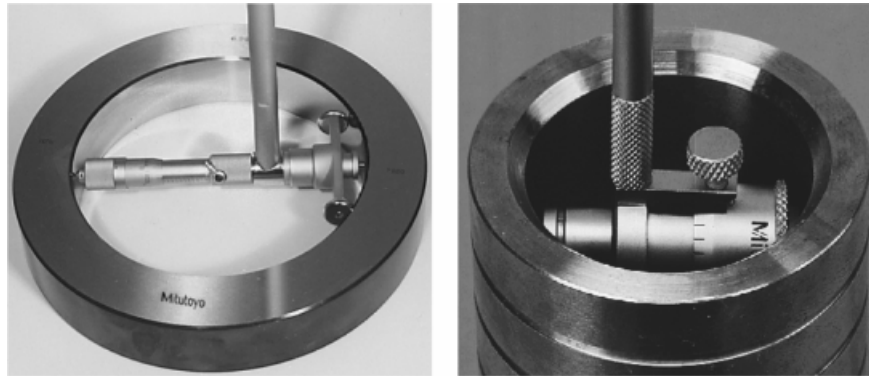


Figure 3.31. Telescopic bore gauges (courtesy of Mitutoyo Canada)

In order to better understand the process of a workshop or laboratory where it is necessary to use telescopic bore gauges, we should follow the approach suggested below.

3.5.9.2.1. Instructions for using telescopic bore gauges

Clean the main machine, parts, and the environment.

Identify the bore to be measured (read on a drawing or measured on a piece etalon).

Choosing the diameter of the bore and the size gives it the appropriate interval.

Clean the probe and the bore.

Choose the rods and the tip (set appropriate intervals; see Appendix 2).

Mount the dial gauge carefully without fully depressing the stem.

Calibrate the device to specific references (micrometer and gauge block).

Set to “zero” (reference) the big needle of the scale on the dial indicator.

Find the farthest point (high is to say before cusp), indicating the “true size” (see Figures 3.33 and 3.34).

Ensure that the measure has actually been transferred by copying the comparator.

Take note of the number of laps (race) covered by the indicator of the hour hand.

Fasten screws and remove the thickness gauge.

Proceed with the measurement of inherent parts of the series and the size of the hole already copied.

3.6. Example of a laboratory model

There are many examples for the use of a telescopic rod for measuring the bore of a cylinder by copying and comparison. This practice is convenient if the pieces mounted on machine tools are numerous and do not require disassembly to maintain the accuracy and isostatism of the assembly.

It may happen that some holes are singularly controlled by this (tool) in laboratory. To do this, we should raise, by copying the target size and then the system plays the role of a caliber “Go” and “No Go”. Sometimes, it happens that boring is not measured by a gauge bore (in micrometer) because of the depth of the bore.

In what follows, we propose to “mount” a laboratory on the basis of the component (workpiece) shown below in Figure 3.34. The goal is to make a set up of bore gauges and explain the process to complete the missing dimensions. Here is a proposed model as an example.

3.6.1. Table of experimental measurements

Based on Figure 3.31, use the technique of bore gauges, and increase its height to report the accurate dimensions on the drawing. The part being machined could be used in other laboratories for roughness tests and for other conventional techniques such as controlling (flatness, straightness).

1. Treatment of experimental data.
2. Mathematical models derived from experimental results.
3. Plotting drawings, GPS, and interpretation(s).
4. Discussion of results and future expansion, possibly.
5. Conclusion.
6. Bibliographic references and/or other documentary sources.
7. Futures developments.

Table 3.3 is a simple spreadsheet template; it is up to the user to customize it.

Measurements (millimeter or inches)	Ø Bore	End tip	Rings	Observations
01				
02				
03				
n_i				
Mean: μ				
Standard deviation: σ				
Uncertainty U				$K = 2$ at probability level 95% (GUM)

Table 3.3. Guide table of experiments

Sometimes, contact problems distort the control and the related measurement error. This illustrates a practical workshop example; we can see clearly that a lack of checking would have a definite impact on the accuracy of the reading. Under the current method of bore gauges (see Figure 3.31), always ensure that you have taken the conceivable contact and properly copied the dimension to the micrometer telescope used for the purpose of reading the final dimension. This dual role of monitoring and reading on the comparator and then copying on to another micrometric assembly creates uncertainties that can be easily avoided by using either a direct micrometric tool or a gauge bore with direct digital reading.

3.7. Precision height

The control of relative positions as that of holes is done in various ways, depending on the purpose, advocated uncertainty, and sometimes the means at our disposal. Among the various methods proposed in dimensional metrology, we retain the method of the heights of precision. This method is sometimes called the method of the booster or the vertical micrometer (*Height Master*), because the micrometer is involved in the reading of the measure. Typically, the approach of dimensional control is simple. It consists of probing two positions ideally extreme, in the case of bores, to deduce the optimum distance. In the case of relative positions, the method is just as simple. We probe two surfaces distant from each other by $\varnothing$ that we read, after telescoping, on a digital micrometer or on a vernier.

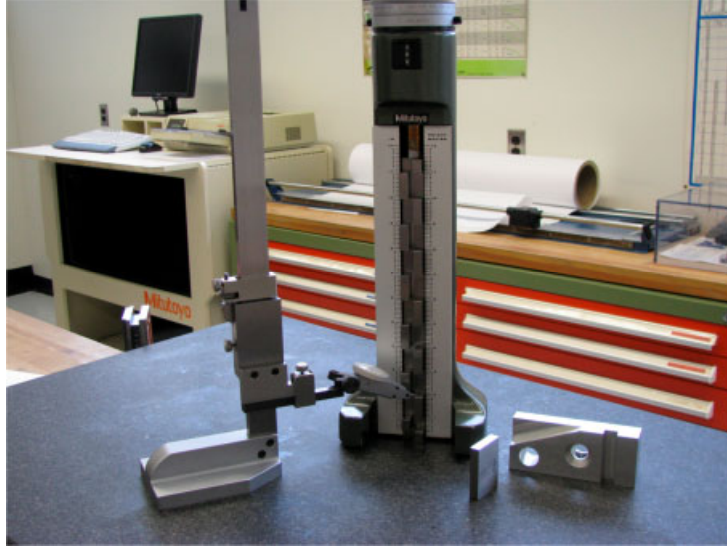


Figure 3.32. *Vertical micrometer*

We note that the micrometer, although included in mechanical metrology, is almost ubiquitous. It suffices to note the position of its mounting on devices, in keeping with the convenience of the type of control or measurement. In summary, we have only seen that mechanical metrology, the comparator, and the micrometer cannot be ignored.

3.7.1. *Directions for use of height masters (or height gauges)*

In this section, we present an example of an approach aiming at checking bores with relative positions of mechanical parts. We use what is called height masters. For this, we should choose and clean a gauge block. After cleaning of the test part, the marble and height masters are cleaned.

It is important to respect the two previous points. Ordinarily when purchasing gauge blocks, cleaning equipment is offered by the manufacturer. The kit shown in Figure 3.30 complies with *ISO 3550* for *Class 1*.

3.7.2. *Adjustable parallel gauge blocks and holding accessories*

These gauge-blocks and accessories show the importance given to the calibration function, in dimensional metrology, for precision. The gauge block remains the

ultimate element of reference, to avoid, at best, influencing the uncertainty ensuing from measurements. The gauge blocks are also subject to adequate and periodic maintenance, otherwise any error will be amplified on the measuring instrument calibrated by such an inadequate gauge. In addition, measurements carried out with this unit will be doubly flawed. The clearance of blocks is intended, among other things, for control and adjustment of micrometers. This clearance is deliberately left with a plain glass in order to check the flatness of the stops of the micrometers. The gauge blocks also allow checking the micrometrical screw to positions totaling a multiple of the nominal pitch and intermediate positions.

1. Orient, preferably, the needle of the comparator to about 15° to the surface to be measured (see Figure 3.34).
2. After considering a gauge block of your choice, using a vernier marking gauge to position a reference (zero).
3. Ensure the rigidity of the control assembly by locking the screws.
4. Copy and transfer the measurement resulting from the gauge block to the appropriate measuring device that is to say, the booster, vertical micrometer.
5. Read correctly the transferred measure and calibrate the device accordingly.

The “ideal surface” is inspected by verifying the needle of the comparator; this surface is likely to reflect the *actual diameter of the bore*. This is done by what has been commonly termed high and low position of the needle of the comparator (see Figure 3.33).

1. Consider the test piece from the reference by sliding against a square.
2. Search “high” and “low” points which would indicate the “real axis” D of the bore (up to the setting back of the comparator needle).
3. Take the reference on the comparator (zero is often recommended for memory).

The measurements will be read on the vertical micrometer as per the below procedure:

1. Transfer the measure already referenced on the appropriate comparator to the appropriate block of the micrometer.
2. Read properly and accurately the measure already copied.
3. Replicate the same manipulation to find the point diametrically opposite on the same axis of the bore.
4. Repeat step 3 as often as necessary; the ideal position for the measurement of the bore diameter is found by searching the most distant points by the automatic

setting-back of the needle of the tilted comparator as shown in Figures 3.32, 3.33, and 3.34.

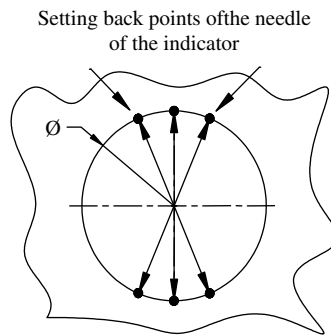


Figure 3.33. Inspection of heights of precision with the cusp of the needle

3.7.3. Example of a laboratory model

There are many examples to show the use of the telescopic rod for measuring the bore of a cylinder by copying and comparison. This method is convenient in cases where the parts mounted on machine tools do not require disassembly to maintain accuracy and isostatism assembly. Sometimes boring is not measured by a gauge bore because of the depth of the bore.

3.7.4. Table of experimental measurements

Measurements (millimeter or inches)	(a) Ø Bore or relative position	(b) Ø Bore or relative position	Relative distance (in millimeter or inches)	Observations
01				
02				
03				
Ni				
Mean: μ				
SD: σ				
Uncertainty U				$K = 2$ at probability level 95% (GUM)

Table 3.4. Table of experiments

The table of experiments (Table 3.4) is proposed as a model used in our workshops and laboratories. It can be varied according to the use and goals of the laboratory in question.

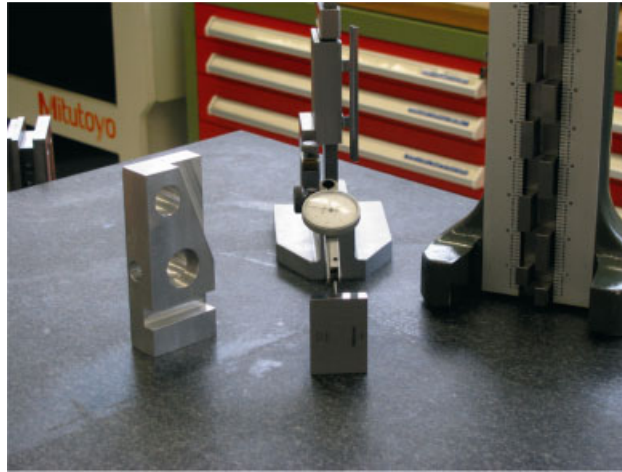


Figure 3.34. *Copying the dimension (to transmit it to the vertical micrometer) in order to measure the relative positions of the two bores of the (standing) workpiece*

Precision heights are in turn controlled by other means of metrology. We mean, by this control, the spacing between the gauge blocks which constitutes the “booster”. Among the various means of control, we quote, the CMM, micrometers, and even the caliper. The advantage of the assembly shown in Figure 3.34 is that it is removable. We can use it in dimensional metrology rooms as well as in machining workshops for the control of machined parts.

3.7.5. Precision height gauge check master

It is not surprising to find that after the use of the precision height gauge, the gauge blocks will not remain at equal distances due to the respective weights of blocks arranged vertically, the fatigue, the frequent use without calibration routine, dust, and lack of maintenance. The means of calibration and verification are also varied. Furthermore, we also use the heights of precision as a means of calibration of some instruments and gauges such as calipers or micrometers, etc. Obviously, the heights of precision serve also for the measurement and calibration.

3.7.6. Caliper gauge control

In the same vein, the heights of precision are also used to calibrate calipers even digital ones. Figure 3.35b shows an illustration of the adjustment of a caliper with digital reading. They are designed for the calibration of different types of calipers.

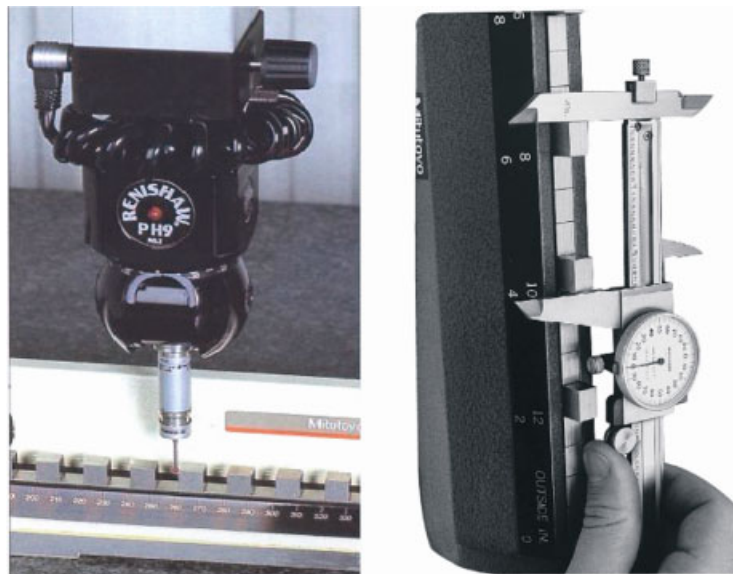


Figure 3.35. (a) *Inspecting the spacing between gauge blocks on CMM;*
(b) *gauge control for calipers (courtesy of Mitutoyo Canada)*

3.8. The universal protractor vernier [MIT 00]

Mariners have often used the astrolabe as an instrument of measurement. It featured as a screening of the sky map which can be rotated over a stereographic projection of the terrestrial globe. In 1597, Philip Danfrie (Britrany 1535–Paris 1606) proposed the graphometer. The latter is a simple instrument consisting of a graded semi-circle whose diameter had a fixed alidade and mobile alidade pivoting around the center. During the 17th century, a circle of land also known as the Holland circle was used.

Nowadays, universal protractors (Figure 3.36) with verniers are frequently used in the fitting workshops and other laboratories of metrology. Their basic principle is much the same; however, there are several models of protractor. The combination square is a versatile set (four graduations) consisting of three elements mounted on a hardened stainless steel ruler.

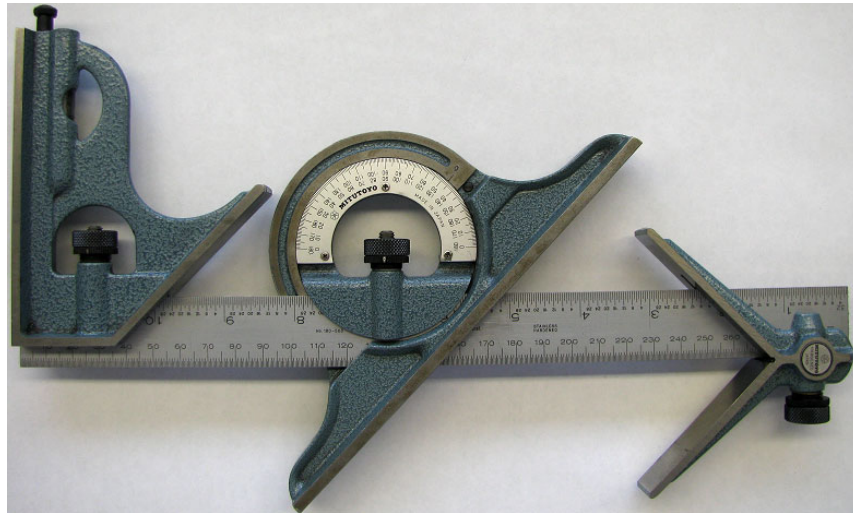


Figure 3.36. *Universal bracket*

Checks and measurements of angles are often performed on dihedral angles. The dihedron is formed by two planar surfaces which intersect. It is characterized by a plane angle. The checks and measurements are also applied at vertexes of conical surfaces (the vertex is formed by two diametrically opposite generatrices).

Given that $\pi/2$ and π are respectively the sums of two supplementary angles and two complementary angles, therefore, an angle can be checked either directly or indirectly. Generally, there are three types of inspections:

- checkers in fixed dimension where the surfaces of reference (SR) are limiting the angle of plane surfaces and of invariable relative positions. We mention, for example, the sizes of angles and prismatic rulers;
- checkers with variable dimension and without reading, also known as false-squares. In this case the position of the flat SR is set up on a prototype or a standard etalon;
- checkers with variable dimension and with reading. In this case, the size of the angle formed by the SR is read on the checker known as a protractor with an approximation varying between (30') and (5') depending on calipers that are used.

To check a right angle, for example, there are many means. We can take as examples a square, a cylinder-square, a gauge-block, or a V-block. We emphasize in this section the protractor as a work objective. It is an adjustable angle gauge, with direct reading.

It comprises two articulated arms whose reference surfaces form a variable angle. A quadrant in degrees connected to one of the arms may turn in front of an index fixed to the other arm, thus indicating the value of the angle measured with an accuracy of about 30'. The small rule forming one of the arms is adjustable in the longitudinal direction. Sometimes, depending on the type of vernier, the index may be replaced by a circular vernier whose principle is similar to that of a caliper, but with an accuracy of about 5'.

The reading principle is simple. After releasing the two arms of the protractor, we apply a surface reference (SR) on one of the surfaces of the dihedral angle to be measured. Then, the second SR is directed following the SR of the dihedral angle. After blocking, we can read the whole number (integer) in degrees between the zero graduation of the quadrant and the zero graduation of the vernier. We add to this the number of minutes determined by the position of the vernier which coincides with a pitch line of the quadrant. This reading can be taken with the help of a magnifying glass because of the closeness of the divisions.

3.8.1. Direct angle measurement

The precise angular measurements can be determined by various means. Mechanical devices of high sensitivity are often used, based on the measurement of the sine of the angle considered. This approach is discussed in another chapter on sine-plate. Moreover, direct measurement can be made with a universal vernier protractor (Figure 3.37). It directly reads out the outgoing angle (α) or the ingoing angle (β).

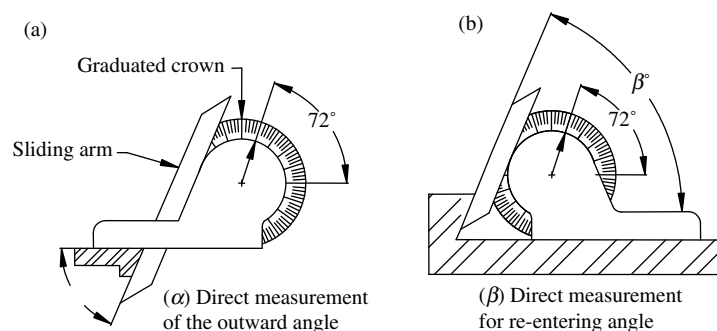


Figure 3.37. Direct measurement of angles

3.8.2. Indirect angular measurement

Indirect measurement is as shown in Figure 3.38.

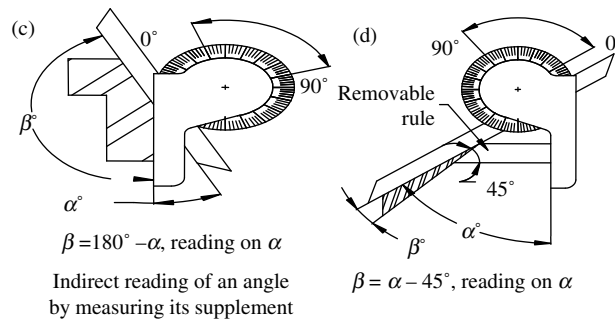


Figure 3.38. Indirect reading of angles

The angle β is deduced indirectly after reading the angle α by the supplement, that is to say $\beta = (180^\circ - \alpha) = (\pi - \alpha)$. Of course, there are other means of measurement that are simple and precise to check and measure angles. We quote the optical metrology (profile projector) or electronic CMM. These two issues will be addressed a little later in the respective chapters (see Chapters 5 and 7). In conventional dimensional metrology, the most common means is the universal angle protractor. It has the advantage of being removable so it can be used both in workshops and in the laboratory without removing the part being measured from the machine tool. This obvious fact cannot be achieved through the CMM or profile projector because of the size of these devices, unless a removable CMM would exist.

3.8.3. Vernier height gauge

The vernier height gauge is an instrument that consists of a fixed part and another moving part. The fixed part consists of a hardened and grounded ruler. The movable part consists of the cursor on the vernier which slides the ruler. The cursor is set to a thousandth of an inch using the scroll wheel fine adjustment of the vernier plate. The scribe is made of hardened steel and is mounted on the height gauge. It is fixed on the vernier cursor. The vernier height gauge is used on a flat surface or tray machine to draw vertical distances and determine the centers. By replacing the scribes by an indicator, the vernier height gauge becomes a gauge of verification or comparison of the heights of precision. What distinguishes the use of vernier height gauge from heights of precision is that it includes accessories that increase its usefulness. The depth device, scribes made of tungsten carbide, out-of-track scribes, and the buttons of the tool maker help in the precise identification of a bore.



Figure 3.39. Vernier height gauge

3.8.4. Gear tooth vernier caliper

The gear tooth vernier caliper is used to measure the big pitch of the teeth. For this, it is necessary to know the chord t . The chordal thickness (T) of gear teeth based on 1 diameter pitch is usually smaller than the regular thickness AB (see Figure 3.39(a)) measured on the pitch circle of a gear tooth in thousandths of an inch.

Before measurement, the gross diameter of the gear must be first determined. As it can be a little higher or lower than the exact diameter, it must be taken into account by adjusting the vertical cursor. This tolerance should be equal to half the difference between the exact diameter and the diameter of the gear. Figures determining the depth adjustment on the vertical cursor and compass readings on the horizontal cursor are presented in tables which may easily be found in the literature (see [STA 97], pp. 33–35). The adjustment is performed by noting that the arch height H was added to the addendum S because the figures to be used are on the S column (the column S). For any further pitch, we must divide the figures in the annexed table (Table A2.2 in Appendix 2) by that required pitch.

M (see Table A2.2 in Appendix 2) can also be measured using a gear tooth caliper in this way (see Figure 3.39(b)).

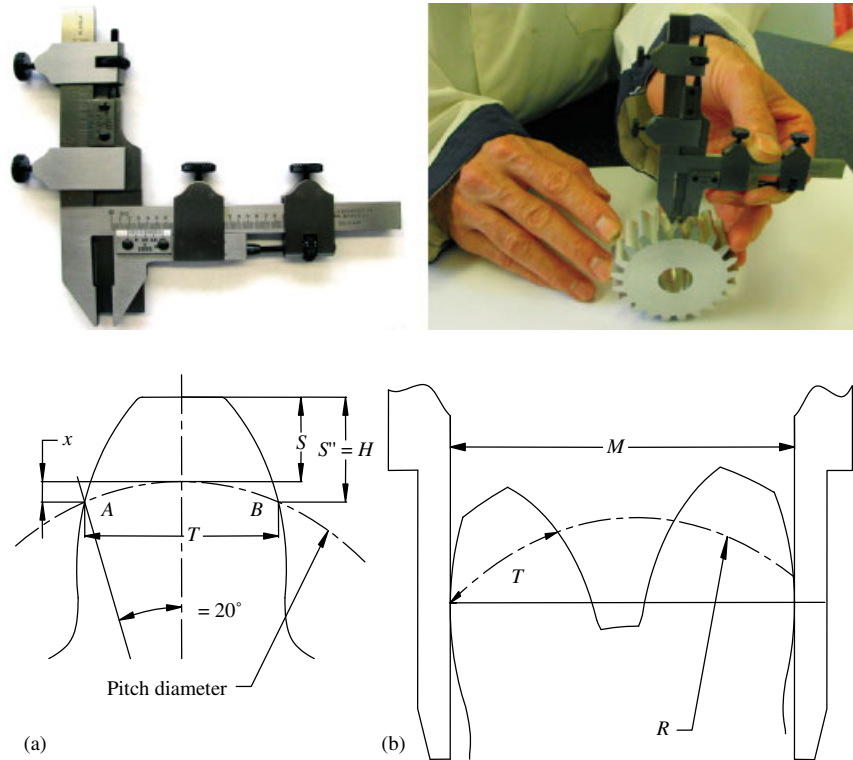


Figure 3.40. Thickness of the base chord of a gear tooth of a drive pitch: S is the modulus or distance from the top of the pitch circle of the tooth; T is the thickness of the chord of tooth; H is the height of the arc $S = S_{corrected} = (x + S)$ (see Table A2.2) Thickness of the base chord of a gear tooth of a drive pitch (source: the AGMA Standards: 112.05, 115.01, and 116.01)

APPLICATION EXAMPLE.— As shown in Figure 3.40 (a) and (b), M is the actual diameter, R is the pitch radius, α is the pressure angle = $14\frac{1}{2}^\circ$ (USA and English Canada), T is the thickness along the pitch circle, $S = 2$ is the number of teeth in engagement between the stops of the vernier caliper, Z is the number of teeth (at least ≥ 13 to avoid interferences), and f is a correction factor under the pressure angle chosen ($f = 0.04303$ for $\alpha = 20^\circ$ and $f = 0.01973$ for $\alpha = 20^\circ = \text{ISO}$, etc.). In the literature, we find complete tables to read the value of f as a function of the angle of pressure [OBE 95]. Teeth considered are then expressed by M :

$$M = d_m = R \times \cos(\alpha) \times \left(\frac{T}{R} + \frac{6.28320 \times S}{Z} + f \right) \quad [3.19]$$

NUMERICAL APPLICATION.— For $R = 2.5$ in., $Z = 13$ to 25, $S = 2$ teeth, $T = 0.2618$ in., and $f = 0.02980$ because the pressure angle (considered ISO) is $\alpha = 20^\circ$.

NOTE AND DISCUSSION.— Under the same conditions, we notice that for $Z = 15$ teeth, $M = 2.284$ in. and for $Z = 20$, $M = 1.792$ in (Figure 3.41). It should be noted also the consideration of the interference factor $f = 0.02980$. In basic conventional metrology, we sometimes tend to measure dimensions without worrying about the physical process and operating conditions of the part being under control. In the previous example, we simulated a change of pinion case (15–25 teeth) to notice that under the same conditions of calculation, the measurement value of the chord M was changed. The phenomenon of interference (*backlash*) has its influence and of course, the pitch radius should change with increased number of teeth.

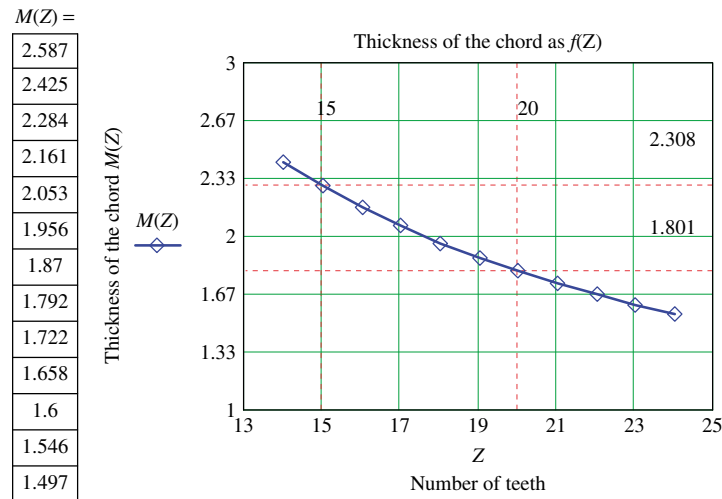


Figure 3.41. Thickness of the chord of two gear teeth Z varying (from 15 to 25 teeth)

3.9. Vernier calipers

Invented by mathematician Pedro Nuñez (1492–1577), the caliper is used to measure the thickness or depth of objects of small dimensions as well as inner and outer diameters (Figure 3.42).

Used by Pierre Vernier (1580–1637) for a long period, this instrument was originally used by surveyors; calipers can make measurements of the internal diameter (bore), external diameter, and length or depth with a precision ranging from a 10th to a 50th of a millimeter. It is constituted by a ruler whose one end terminates in a beak shape wing, over which the beak-shaped cursor wings (vernier)

are present. The ruler is graduated in millimeters and inches. The vernier (named after the French mathematician P. Vernier) is graduated in a 10th of a millimeter. For example, to make a correct reading, the beaks and the workpiece must be clean. Slightly pinch the element to be measured while ensuring that the beaks are well upright on the surface of the workpiece. Slightly tighten the vernier screw and disengage the instrument carefully and then make a reading. The caliper consists of a stable point (head fixed) and a slider with a screw sliding on a ruler that is stable.

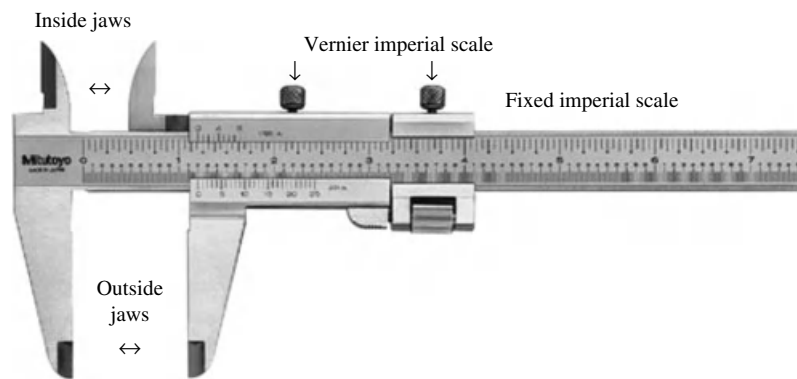


Figure 3.42. Universal vernier caliper (courtesy of Mitutoyo Canada)

Using the vernier we can determine a fraction of the smallest division on the scale of the ruler. The vernier's 10th gradation (1/10th) is obtained by dividing into ten equal parts a given length, on the ruler, equal to nine divisions of the ruler. If the latter is graduated in millimeters, each division of the vernier equals 0.9 mm. The position *zero* of the vernier, which serves as an index reading, can be observed at 0.1 mm, with the naked eye. For this, we should identify the number of the division which coincides with a pitch line under the ruler. In short, the reading of *P* to *C* is done in two stages. Firstly, a simple ruler can take a reading in millimeter. Then, the vernier indicates the reading of tenths of a millimeter by identifying the closest vernier pitch line which coincides with the line of the ruler.

The *absolute uncertainty* of *P* to *C* is generally equal to the smallest division of the fixed ruler divided by the number of divisions of the vernier. If the *P* to *C* that is used is divided into millimeters and the vernier consists of ten divisions, this implies an uncertainty of about ± 0.1 mm. In Chapter 1 of this volume, we have presented the properties of basic uncertainties (absolute and relative uncertainty included). They should be applied to the appropriate cases in the laboratory.

Reminder on the basic calculations of uncertainty – the central value and relative and absolute uncertainty

Consider a measurement value of 1 inch resulting from *P* to *C* ($X = 25.400 \text{ mm} \pm 5/10\text{ths mm}$). “Central value” stands for the value 25.400 mm because this value is *centered* in the interval of uncertainty. “Absolute uncertainty” stands for the value of 5/10ths millimeter because it represents the amount to be added or subtracted to obtain the interval of uncertainty. Relative uncertainty of measurement is the ratio of the absolute uncertainty to the central value and is, therefore, expressed by $\Delta X/x$. In our example, we denote:

$$\frac{\Delta X}{x} = \frac{0.5}{25.400} = 0.019685 \approx 0.020 \text{ or } 2\%$$

Adding the case method of extremes: Consider a quantity U as a function of a quantity μ . $U = F(\mu)$. In doubtful cases, we may calculate the minimum and maximum values in order to obtain the uncertainty. Here is the method:

$$U_{\min} = F(\mu_{\min} \text{ or } \mu_{\max}); U_{\max} = F(\mu_{\max} \text{ or } \mu_{\min}) \rightarrow \Delta U \approx U - U_{\min} \approx U_{\max} - U$$

Unlike the marking gauge and vernier depth gauge, the vernier *P* to *C* is often graded on both sides in order to ensure both inside and outside measurements. The outside readings are done left to right and those on the right are from right to left. When the jaws touch each other (without much pressure), the zero of the outside vernier coincides with the zero of the ruler. As with any precision instrument in dimensional metrology, the caliper should not be forced.

3.9.1. Various measurements of a dimension using a caliper

A caliper measures dimensions both in length and depth. The photographic illustrations below show various ways of measurement with a caliper (Figures 3.43 and 3.44).



Figure 3.43. *Depth reading*



Figure 3.44. Reading inner and outer diameters

Sometimes, it happens that the dimension requires specialized jaws for measurement because of the form of the workpiece. In this case, we should use a caliper, whose jaws are adapted to the form. Of course if this is likely to create problems, then it would be wise to use another measuring instrument that best fits this measurement. We will address this issue in the study of universal micrometers.

3.9.2. Possible errors when using a caliper

Errors are varied and of different sources when taking measurements. In this section, we will mention and show schematically some of them [OBE 95, STA 97]. Note first that the scale on bevel eliminates the risk of a reading error (for examples, see Figure 3.45).

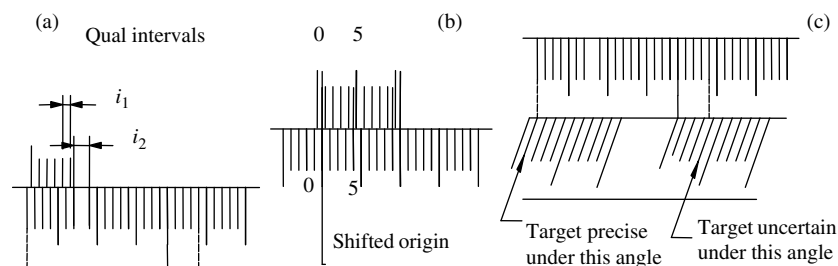


Figure 3.45. Probable errors when using a caliper; error due to unequal intervals: misalignment piece-device (a); reading error due to the shift of origin: measurement error due to possible unequal intervals (b)

As we notice from Figure 3.45(b), that the graduation on the bevel is necessary and it eliminates the risk that may occur while reading. The important thing is to

respect the value of graduation on the bevel to associate it with that of the vernier. When reading the measurement value, we should take the right position to read it, opposite to the instrument and slanting.

As shown in the schematization (Figure 3.45(b)), we notice that if the origin is shifted (b), it distorts the accuracy of reading. The measuring instruments are sometimes tainted by abnormalities inducing reading errors. Among these errors, we include the error of the device (physical) itself.

A caliper after a fall is likely to deform (see Figure 3.46). Although this distortion is often not detectable with the naked eye, we should conduct periodic inspections.



Figure 3.46. Photograph of damaged vernier with offset beaks

Analysis elements of a caliper: Let a and b be two contact points of the beaks with the disc whose diameter is intended to be calculated. C is the contact point of the disc with wings; d is the point of the disc that is diametrically opposed to C ; and h the intersection of $[ab]$ and $[cd]$ (Figures 3.47 and 3.48).

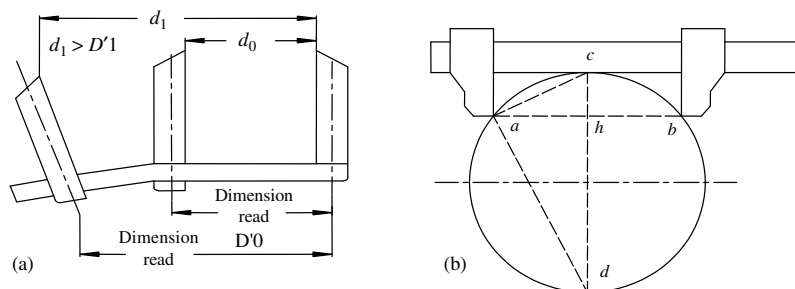


Figure 3.47. Errors (a) and (b) due to apparatus

In fact, C is the length $[ab]$ of the chord enclosed tightly by the vernier. Let f be the given height (ch) = the arrow and let us measure the diameter (cd); let D be its length. The triangle (acd) is a right triangle on a since it traces a half-circle. As shown in Figure 3.36 (a) and (b), we write

$$(a \cdot h)^2 = (hc) \times (hd) \text{ or } (ah) = \frac{C}{2}; \text{ this gives } C^2 = 4 \cdot f \text{ and } C = \sqrt{f \times (D - f)^2} \text{ for } d > 2 \cdot f$$

$$D = \left(\frac{C^2}{4 \cdot f} \right) + f \quad [3.20]$$

By choosing an arrow of length 1 (unit), the vernier will be graded using these results:

D	0	1	2	3	4	5	6	7	8	9	10	...
C	0	1.00	2.00	2.83	3.46	4.00	4.47	4.90	5.29	5.66	6.00	

Between 0 and 2, the scale is linear like on a normal caliper.

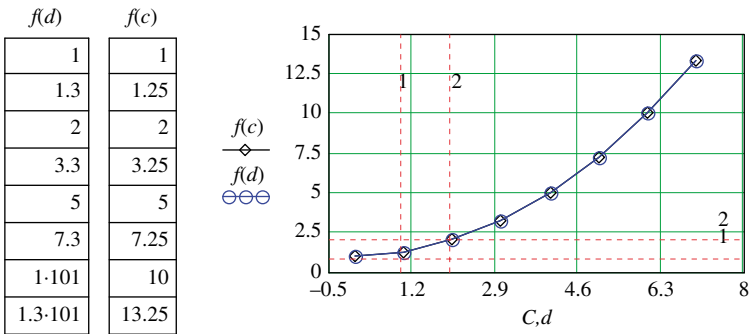


Figure 3.48. Analysis element of caliper graduation

3.10. Micrometer or Palmer [OBE 95, STA 97]

The Palmer micrometer was invented in 1848 by French engineer Jean-Louis Palmer. This precision instrument is used to measure thicknesses and outside diameters. It consists essentially of a micrometer, a bumper, and a barrel (screw head) divided into fifty or a hundred and several parts of the screw, a friction system, and a body of diverse and variable form.

After this brief presentation on the micrometer we will develop the principle of the micrometric screw as it is mainly on such a screw that the micrometer is based.

3.10.1. *Principle of micrometric screw*

The pitch of the screw rod is of the order of $1/40$ in. or 40 threads per inch for inch-micrometers. One complete turn of the handle moves forward or moves away from the face of the rod exactly by $1/40$ or (0.025 in.) from the face of the stop. The longitudinal line on the spindle play is divided into 40 equal parts by vertical lines that match the number of threads on the rod. Also, each vertical line is equal to $1/40$ or (0.25 in.) and every fourth line, longer than the others, amounts to hundreds of thousandths. Example: the line marked 1 is 0.100 in., the 2 is 0.200 in., and 3 is 0.300 in., etc.

The beveled edge of the handle which is divided into 25 equal parts, each representing 0.001 in., is numbered consecutively. By turning the handle by one line to the other, we advance the longitudinal stem by $1/25$ or 0.25 in. or even 0.001 in.. A rotation of two divisions is 0.002 in. and a rotation of four divisions is 0.004 in., etc. So 25 divisions represent a complete turn, that is to say 0.025 or $1/40$ in.

To read the micrometer in thousandths, we multiply the number of vertical divisions visible on the spindle play by 0.025 in. and add to that the number of thousandths indicated by the line on the handle, which coincides with the longitudinal line on the sleeve. When the screw is in contact with the target, the barrel completely covers the millimeter scale and the zero on the barrel coincides with the zero of the millimeter scale. When taking a measure, the spacing of the sleeve gives the thickness in millimeters and, on the circular graduation of the barrel, opposite to the mark, the circular barrel back-up is read in a 50th or 100th of a millimeter.

The use of a micrometer confers several advantages: it is more accurate than the ruler and the vernier calipers, there is no parallax error, and reading is easier than the ruler. Nevertheless, its measurement range is relatively small (25 mm) and it serves only one type of measurement.

The accuracy of the micrometer is typically about ± 0.01 mm (when the screw pitch is 1 mm and the circular graduation is subdivided into 100 parts or when the screw pitch is 0.5 mm and the scale of barrel is subdivided into 50 parts), but there are micrometers with vernier scale with a accuracy of ± 0.002 mm and electronic micrometers with digital display and an accuracy of ± 2 μ .

3.10.2. *Manipulations to perform a measurement with a Palmer*

The graduations above the horizontal line drawn along the barrel correspond to millimeters. The divisions below the line indicate the half-millimeters. The sleeve has 50 divisions. Since it takes two rotations for the sleeve to move 1 mm along the

barrel, each division of the sleeve corresponds to one hundredth of a millimeter. Uncertainty of a micrometer reading can occur generally to its smallest division. The micrometer presented here has an accuracy of one hundredth of millimeter, which means an uncertainty of ± 0.01 mm.

Before manipulating any instrument of this kind, we should first ensure the cleaning of the components and the surrounding environment using appropriate products. Then:

1. fasten the object between the stopper and the screw until the spindle comes into contact with it;
2. if the Palmer is equipped with a stopping device, use it until it runs into space to avoid exerting excessive pressure that would distort the reading;
3. read the measurement starting with the whole number (integer) of millimeters on the millimeter scale, and then the circular graduation.

Graduated micrometer length is generally 25 mm. The instruments have capacities scaled by 0–25 mm, starting from zero up to 300, that is to say, from 0 to 25, from 25 to 50, ..., from 275 to 300. The anvil is adjustable to allow calibration to zero or the length of minimum capacity. A plug gauge is always delivered with the Palmer (Figure 3.49).

3.10.2.1. Reading a metric micrometer

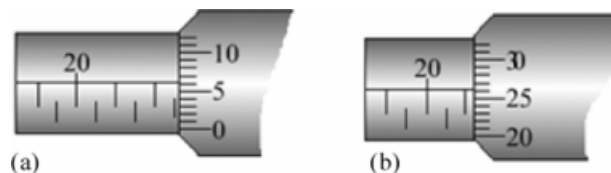


Figure 3.49. Metric graduation reading (millimeter). (a) The fixed rule indicates 22.5 mm and the drum indicates 0.06 mm. The measurement value is: $22.5 \text{ mm} + 0.06 \text{ mm} = 22.56 \text{ mm}$. (b) The fixed rule is graduated in half-millimeter, a drum turn equals 0.5 mm and 50 graduations of the drum. The fixed rule indicates 21 mm and the drum indicates 0.26 mm. Hence, the measurement value is: $21 \text{ mm} + 0.26 \text{ mm} = 21.26 \text{ mm}$

3.10.2.2. Detailed versus concise reading by graduation metric

- The apparent reading (display) on the spindle play is 5 mm.
- Also, 0.5 mm is visible on the sleeve.
- Line 28 on the handle is visible hence: $25 \times (0.01) = 0.28 \text{ mm}$.
- The final reading thus gives: 5.78 mm.

3.10.2.3. *Reading a Palmer graduated in hundredths of a millimeter (0.01) and in two thousandths of a millimeter (0.002)*

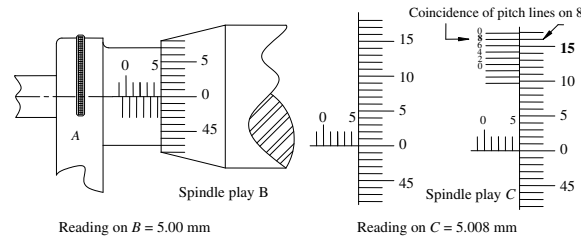


Figure 3.50. Example on metric readings of micrometers [STA 97]

Details of *A* and *B* readings (Figure 3.50):

- the 5 mm is visible on the graduation of the spindle play: 5.000 mm;
- there is no additional pitch line visible on the spindle play so: 0.000 mm;
- line 0 of the handle coincides with that of the spindle play: 0.000 mm;
- lines 0 of vernier coincide with the pitch lines of the handle: 0.000 mm;
- the total sum of four readings gives rise to: 5.000 mm.

Details of *C* reading:

- the 5 mm is visible on the graduation of the spindle play: 5.000 mm;
- there is no additional line visible on the spindle play so: 0.000 mm;
- lines 8 of the vernier coincide with the pitch line of the handle: 0.008 mm;
- the total sum of three readings gives rise to: 5.008 mm.

3.10.2.4. *Reading a micrometer in ten-thousandths of an inch*

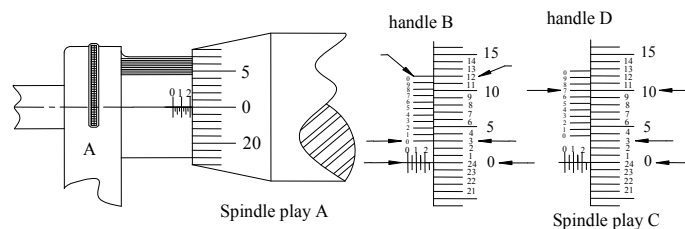


Figure 3.51. Example on reading a micrometer in inches [STA 97]

Details of *A* and *B* readings (Figure 3.51):

- line 2 is visible on the scale of the spindle play: 0.20 in.;
- there are two additional lines visible on the spindle play thus $2 \times 0.025 = 0.05$ in.;
- line 0 of the handle coincides with the longitudinal line of the spindle play: 0.00 in.;
- lines 0 of the vernier coincide with that of the handle 0.00 in.;
- the sum of four readings equals to: 0.250 in.

Details of the (*C*) reading:

- line 2 is visible on the scale of the spindle play *t*: 0.200 in.;
- there are two additional lines visible on the spindle play: 2×0.025 in.: 0.050 in.

The longitudinal line on the spindle play, between the line 0 and 1 on the handle, indicates the ten-thousandths of an inch to be added as read on the vernier. Thus:

- line 7 of the vernier coincides with that of the handle 7×0.0001 in. = 0.0007 in.;
- the sum of four readings equals: 0.25070 in.

3.10.3. Adjusting micrometers [MIT 00, STA 97, FRI 78]

Each micrometer is usually equipped with a gauge block. This allows the user to calibrate the micrometer before each use. This is essential because it allows calibrating the micrometer to the dimension of the gauge block which is considered as the reference dimension. If the reading given is equivalent to that of the gauge block, we can then confirm that the instrument is calibrated and is ready for use. This approach is explained in Figure 3.52. Calibration of measuring instruments is an inevitable process as we just explained above. This applies to all metrology instruments which are associated with a gauge block. Whether it is a caliper or micrometric gauge, this operation should be performed. In some cases, although an instrument is not defective, it requires an adjustment at the reference level, i.e. setting the “zero” of reading with that of the spindle play. It would be wrong to believe that micrometers and other instruments of dimensional with digital display do not require calibration. In the case of a digital display, only the technique of adjustment would be different, while the calibration by a gauge block remains indispensable.

Hand-grip micrometers (friction) allow a uniform contact pressure to get the proper readings. The contact pressure can be kept uniform using a pressure gauge contact. This is particularly important because it allows a controlled longevity of the instrument. When properly controlled, it helps in avoiding errors and controls the overall uncertainty affecting the accuracy of reading. If this issue remains unresolved, it would also affect the fidelity of the instrument.



Figure 3.52. Calibration of a micrometer to the dimension “C” of the gauge-block

3.10.4. Control of parallelism and flatness of the micrometer’s measuring surfaces using optical glass

In addition to the considerations previously addressed, a micrometer may contain flatness defects at the contact stops level. And similarly for parallelism. These defects, when they are not under control, may induce errors on the accuracy of the instrument. One way to control these defects is by the flat glass shown in Figure 3.53.



Figure 3.53. Inspecting the flatness of micrometer stops (courtesy of Mitutoyo)

Placed between the two stops, it allows the creation of an “air wedge” to detect defects of flatness and/or parallelism through visual reading of the interference fringes that may appear as shown in this figure. Any micrometer controlled under this method would show defects of parallelism or flatness. The more the interference fringes are accentuated, the more likely the defects of parallelism or flatness will be. This is reflected by the degree of accuracy of the instrument. For example, the manufacturer Mitutoyo tolerates a deviation of $0.1\text{ }\mu\text{m}$ on flatness and 0.2 on parallelism. Of course there are other ways to control these two types of defect. However, in workshops and laboratories of metrology, all these means of verification (laser, interferometric metrology) are not likely to be always accessible. The plane glass remains the fastest and most efficient tool in the workshop (polished glass, Figure 3.53).

3.10.4.1. *Micrometer with interchangeable contact stops*

This micrometer model offers significant measurement capabilities by simply changing the contact stops (removable stops, Anvil). In fact, most mechanical micrometers and those with a digital display provide the same functions, namely those to read a dimension under contact. Key characteristics are:

- large measuring capacity with large LCD display;
- interchangeable contact stops and initialization of origin;
- zeroing at any pin position the spindles;
- function “hold,” that is to say, maintaining the measured values;
- measurement of the face of the spindle (hardened and grinded) in tungsten carbide.

Nevertheless, it is important to note that the shape of the contact stops and the shape of the frame may differ from one instrument to another. We often adapt these contacts to the form of the target piece as shown in Figure 3.54.

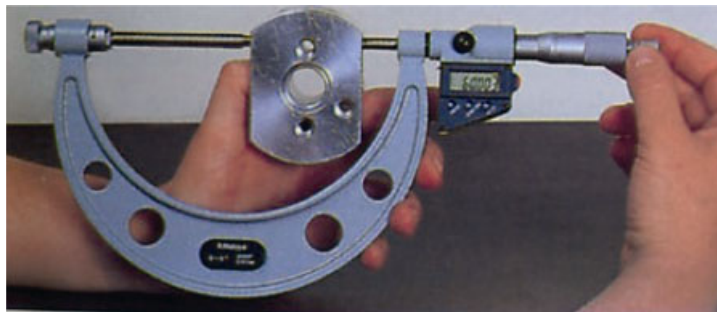


Figure 3.54. *Micrometer with interchangeable contact ends. Resolution 0.001 mm (series 340) (courtesy of Mitutoyo Canada)*

(a) *Precision*: High capacity measurement $\pm(4 + L/75) \mu\text{m}$, with L = maximum length measured $\rightarrow$ conformity to ISO 3611 (up to 500 mm).

(b) *Faces of measures*: Flatness $0.6 \mu\text{m}$ up to 300 mm and $1 \mu\text{m}$ beyond 300 mm; parallelism: $(2 + L/100) \mu\text{m}$, $L = 1$ the maximum measured; force of measurement: 5–10 N.

3.10.4.2. Outside micrometer with fine stepped contact ends

The shape of the keys is adapted to the shape of the measure but the essential part of the device is changed. This is illustrated in Figure 3.55.

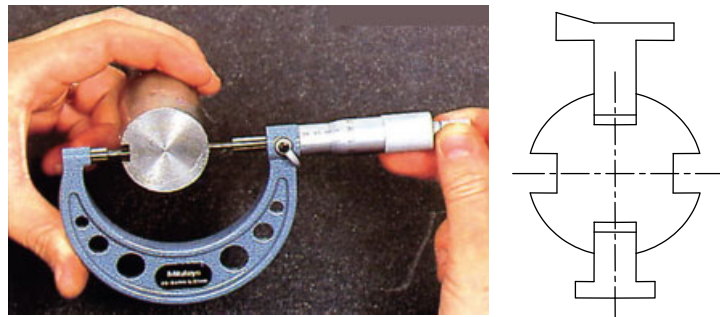


Figure 3.55. Micrometers with fine stepped contact ends. Schematization of the measurement. This model is used to measure grooves, flutings, and other notches. Depending on its compliance with ISO 3611 the precision for models up to 75 mm = $\pm 3 \mu$. Faces of measurement: flatness: $0.3 \mu\text{m}$ and parallelism $(2 + L/100) \mu$. Force of measurement 5–10 N

3.10.4.3. Micrometer to measure cylinder walls with interchangeable stops



Figure 3.56. Standard type micrometer (courtesy of Mitutoyo Canada)

These micrometers are intended for various fields of application. Their interchangeable “universal type” contact stops allow us to measure cylinder walls and other pipes, flanges, heads of rivets, and generally places that are hard to reach (Figure 3.56).

Of course, the stops *fit* the shape of the piece to be measured. We offer practical examples using this kind of micrometers. In Figures 3.57 and 3.58 we can see two illustrations from the catalog of the manufacturer Mitutoyo.

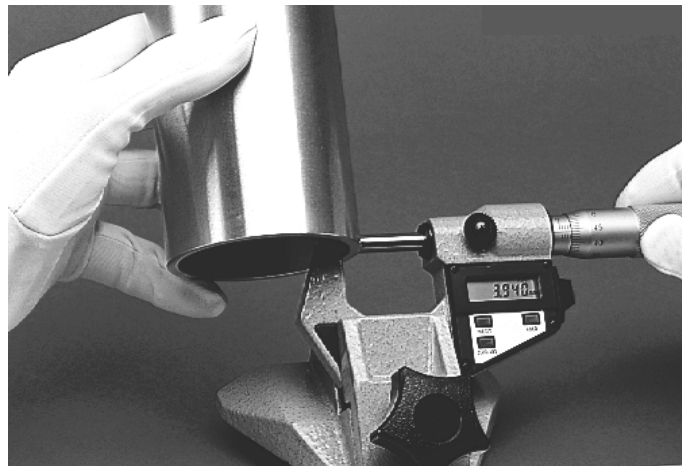


Figure 3.57. Example 1. Use of universal micrometer with interchangeable stop (courtesy of Mitutoyo Canada)

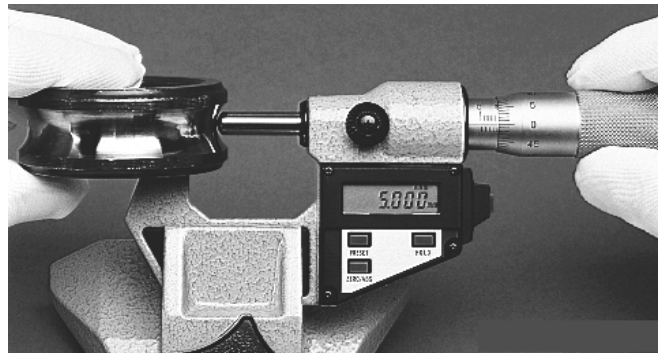


Figure 3.58. Example 2. Use of universal micrometer with interchangeable stop (courtesy of Mitutoyo Canada)

We notice that the stops are concave on one side and convex on the other side. Obviously, there are other ways to control the thickness of the tube shown in Figure 3.57. However, this practice is simpler and faster. It avoids measuring the outer diameter, the inner one, and then deducting the thickness, with all the likely induced uncertainty.

In the case of this figure, we notice that one side (interior) is cylindrical but the other side is concave. Hence we should choose the appropriate stops.

3.10.4.4. *Screw thread micrometers [MIT 00, STA 97]*

Threads are an important component in the mechanics field. They are obtained in various ways, depending on the standard and the means used for this purpose. We will discuss the technique of control by micrometry. We will mention several additional cases. In mechanical metrology of precision, measurements can be carried out using a screw thread micrometer (Figure 3.59) and the best-wire method, in addition, obviously, to gauges, the CMM, profile projector, etc.



Figure 3.59. *Measurement of pitch diameter by screw thread micrometer*

We see clearly that the contact holds are adapted to peaks and valleys.

Characteristics of ISO threading discussed in the literature [OBE 95] are:

$$\left\{ \begin{array}{l} D = d = \text{nominal diameter} \\ H = \left(\frac{\sqrt{3}}{2} \right) \times P = (0.866) \times P, \text{ } p \text{ is the pitch} \\ D_2 = d - 1.0825P \end{array} \right\} \quad [3.21]$$

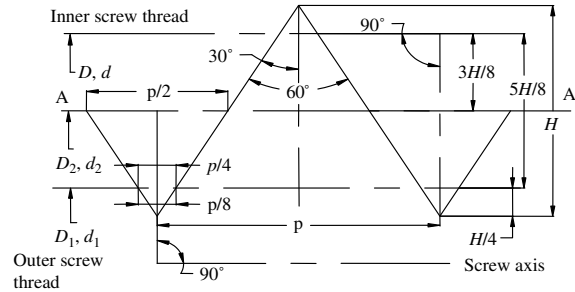


Figure 3.60. Measurement with screw thread micrometer schematization of an ISO thread

The pitch diameter on the screw thread is directly read on the micrometer as shown in Figure 3.60. Line AA corresponds to the reading of the pitch diameter of thread. We will discuss this theme in detail in Chapter 8.

3.10.5. Measurement of screw threads by three-wire method

The illustration in Figure 3.61 is taken from the catalog of Mitutoyo [MIT 00]. It shows the practical approach to be followed to monitor and measure a thread by this method. This figure is, in our view, explanatory by itself and thus further comments are not required. The approach for deductible calculation of the measurement is given as follows.

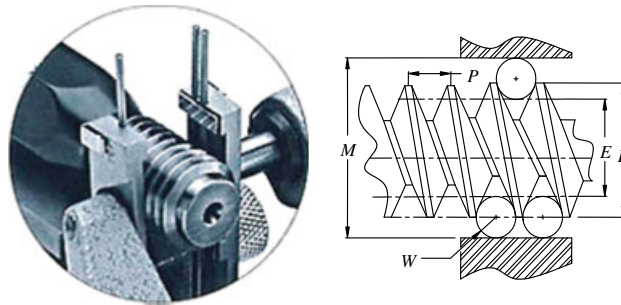


Figure 3.60. Three-wire method (see also Chapter 1, Figure 1.28)

3.10.5.1. Three-wire method of checking the pitch diameter of threads at 60°

$M = d_{ow}$ measurement over wires.

$E = d_p$ pitch diameter of thread.

D = nominal diameter (outer or inner).

$W = d_w$ wire diameter.

P = thread pitch.

C = constant.

$$\left\{ \begin{array}{l} C = 3W - (0.86603) \times \text{Pitch (inch)} \\ PD = M - C \\ W = 0.57735 \times P \end{array} \right\}$$

From equation [3.21], we consider: constant = $3 \cdot W - (0.86603 \cdot P)$ (see Table 3.5 for imperial measurements).

Let: $M = E = \text{Constant}$ or $E = M - \text{Constant}$

3.10.5.2. Table of conversion for metric threads at 60°

Pitch $\varnothing$	Wire size $\varnothing$		ADD		Constant	
mm	mm	in.	mm	in.	mm	in.
0.5	0.4572	0.018	0.6138	0.02417	0.9386	0.03695
0.6	0.4572	0.018	0.4623	0.01820	0.8520	0.03354
0.7	0.4572	0.018	0.3107	0.01223	0.7654	0.03013
0.75	0.4572	0.018	0.2349	0.00925	0.7221	0.02843
0.8	0.6096	0.024	0.6164	0.02427	1.1360	0.04472
1.0	0.6096	0.024	0.3133	0.01233	0.9628	0.03790
1.25	0.7365	0.029	0.3154	0.01242	1.1273	0.04438
1.5	1.0160	0.040	0.7747	0.03050	1.7490	0.06886
1.75	1.0160	0.040	0.3958	0.01558	1.5324	0.06033
2.0	1.1430	0.045	0.3979	0.01567	1.6969	0.06681
2.5	1.3970	0.055	0.4021	0.01583	2.0259	0.07976
3.0	1.6002	0.063	0.2540	0.01000	2.2025	0.08671
3.5	2.0574	0.081	0.8678	0.03416	3.1411	0.12367
4.0	2.3368	0.092	0.9482	0.03733	3.5463	0.13962
4.5	2.7432	0.108	1.4096	0.05550	4.3325	0.17057
5.0	2.7432	0.108	0.6519	0.02566	3.8995	0.15352
5.5	3.0480	0.120	0.8085	0.03183	4.3808	0.17247
6.0	3.2258	0.127	0.5841	0.02300	4.4812	0.17643

Table 3.5(a). Three-wire method (source: Starrett instrument)

3.10.5.3. Table of conversion for all US threads at 60° (imperial)

This table is intended for free use. It is provided by the manufacturers (Starrett, Mitutoyo, etc.) of wires. We reproduce it here to help users easily read the measurement values when using wires in manufacturing workshops and laboratories of dimensional metrology.

We will select the proper size of the wire both for the metric threads and imperial threads (the United States, Canada, and the United Kingdom). Note that the $P.D = E = \text{Pitch diameter } (d_f)$ is a function of the outer diameter *plus* the factor “ADD” *minus* the decimal *constant*.

Screw thread (in.)	Size of the wire	AdditionADD	Constant	Screw thread (in.)	Size of the wire	AdditionADD	Constant
48	0.018	0.02243	0.03596	11½	0.055	0.03321	0.08969
44	0.018	0.01956	0.03432	11	0.055	0.02722	0.08627
40	0.018	0.01611	0.03235	10	0.055	0.01345	0.07840
36	0.018	0.01190	0.02994	9	0.063	0.02061	0.09277
32	0.024	0.02464	0.04494	8	0.072	0.02656	0.10775
28	0.024	0.01781	0.04107	7½	0.081	0.04093	0.12753
27	0.024	0.01587	0.03993	76½	0.081	0.02649	0.11928
24	0.029	0.02385	0.05092	6	0.092	0.02341	0.13166
20	0.029	0.01122	0.04370	5½	0.108	0.04845	0.16654
18	0.032	0.01180	0.04789	5	0.120	0.05689	0.18679
16	0.040	0.02528	0.06587	4½	0.127	0.04421	0.18855
14	0.040	0.01175	0.05814	4	0.143	0.05011	0.21249
13	0.045	0.01842	0.06838	3½	0.185	0.12199	0.30756
12	0.055	0.03870	0.09283	3	0.185	0.04982	0.26632

Table 3.5(b). Three-wire method (source: Starrett, USA)

3.10.6. Ruler and gauges for the control of screw threads

We may also use a ruler or gauges to measure threads. To this end, we should measure the number of threads per inch (for imperial threads) and then deduce the type of thread by means of appropriate forms. We should bear in mind that this method is still in operation in machine shops. It is very efficient and gives good results in addition to its simplicity. For a schematic illustration, see Figure 3.62.



Figure 3.62. *Ruler and gauges for the inspection of threads*

3.10.7. *Micrometer with fine point*

The principle of reading and calibration with micrometers is substantially identical to the set of instruments. The contact holds make a difference because they are chosen depending on the entity to be measured. For example, these micrometers are chosen for areas difficult to access, cotters, and small grooves approximately 0.012 in. size (Figure 3.63).

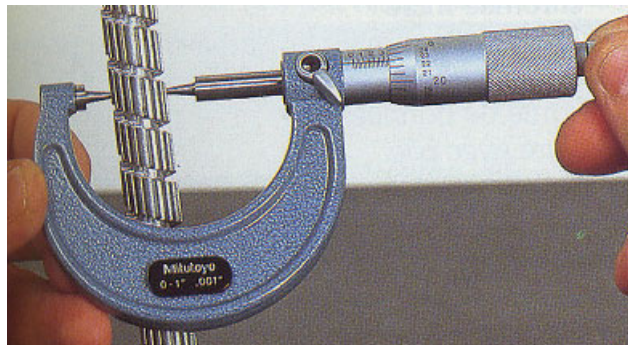


Figure 3.63. *Measuring diameter at the bottom of the throat by a micrometer with fine points*

- Scope of use: contacts and spindle of measurement have tipped ends allowing measurement of the thickness of center blade of drills, fine grooves, notches and in general all dimensions that are difficult to access.
- Depending on its compliance with ISO3611: precision ranges between 0 and 50 mm $\pm$ 3 μ m.

3.10.8. Disc micrometers to measure shoulder distances

Again, the principle of micrometry is similar to other cases, however, the contact stops are chosen as in the form of discs to measure shoulders or similar protrusions. The goal is to choose the appropriate discs and to calibrate them adequately (Figure 3.64).

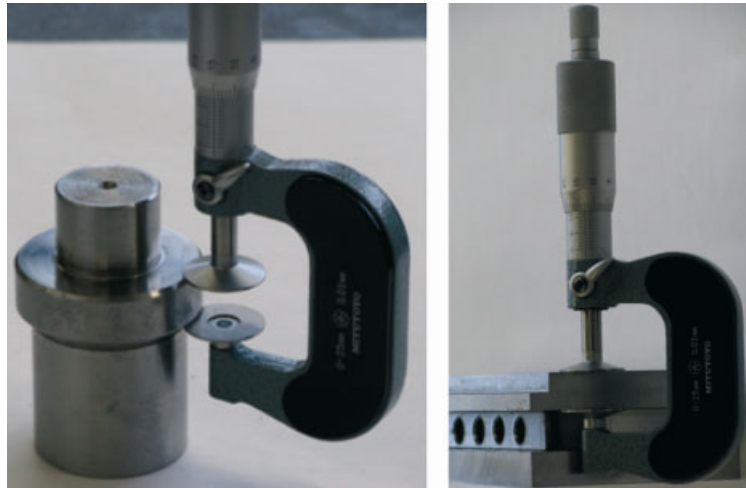


Figure 3.64. Disc micrometer (plate)

This micrometer model is sometimes used to measure the outer gear teeth (Figure 3.65).

For even-numbered gear teeth, [OBE 95] proposes the following formula:

$$d_m = d_p + d_g = d_p \times \left(\frac{Z \times m \times \cos(\alpha)}{\cos(\phi)} \right) \quad [3.22]$$

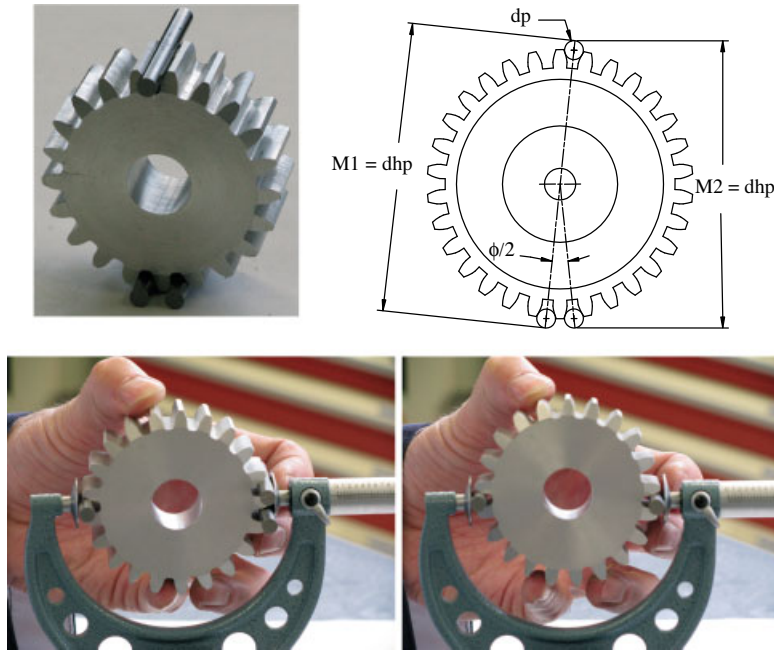


Figure 3.65. Measurement of pitch diameter of a cylindrical gear

For odd-numbered gear teeth, the literature suggests the following formula:

$$d_m = d_p + \left(\frac{d_g}{\cos(\phi)} \right) \times \cos\left(\frac{\pi/2}{Z}\right) = d_p + \left(\frac{Z \times m \times \cos(\alpha)}{\cos(\phi)} \right) \times \cos\left(\frac{\pi/2}{Z}\right) \quad [3.23]$$

where Z is the number of teeth; α is the angle of pressure (20° for spur gears); α is the involute of the circle (m is the gear modulus, M or d_{hp} is the diameter over wire; d_p is the diameter of the wire).

Note that this will be addressed in more detail in Chapter 8.

3.10.9 Outside micrometer caliper type

These micrometers are designed specifically for external measurements which are difficult to access. As shown in Figure 3.66, we find that the stops are deliberately adapted for measuring small grooves which are hardly accessible. With specific mounting stops micrometer, we can access the measurement of the thickness indicated below.

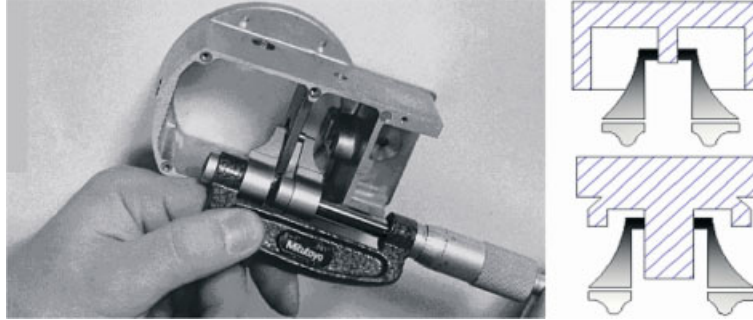


Figure 3.66. Illustration of a measure with outside micrometer caliper type

The technique of reading the measurement is similar to that in other micrometers. We recall that holding the piece between the stops is difficult. This requires a lot of skill. Of course, this control can be achieved with individual parts or in small numbers. The ideal measurement, in the case of serial parts, is obviously carried out by the CMM or profile projector.

3.10.9.1. Inside micrometer caliper type

There are several ways to measure bores – as we shall see more in the laws explained in the chapters dedicated to gauges and measuring rods in micrometry – a bore can also be measured as shown in Figure 3.67.

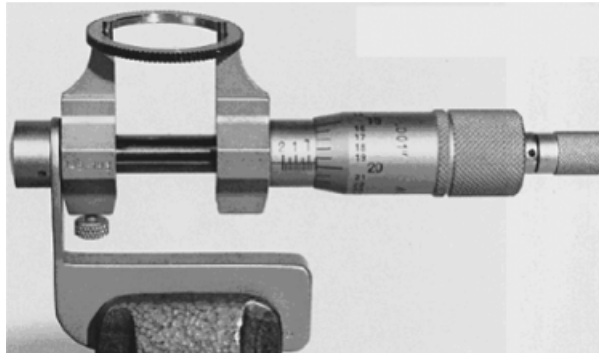


Figure 3.67. Inside micrometer nozzle (courtesy of Mitutoyo)

3.10.9.2. V-shaped outside micrometer for tools with three and five edges (flutes)

These micrometers are very useful in workshop as well as in laboratory measurements related to cutting tools. Sometimes, in the workshop, tools may not be

sharp enough or are deformed and therefore the accurate dimension (CO) is not obtained.

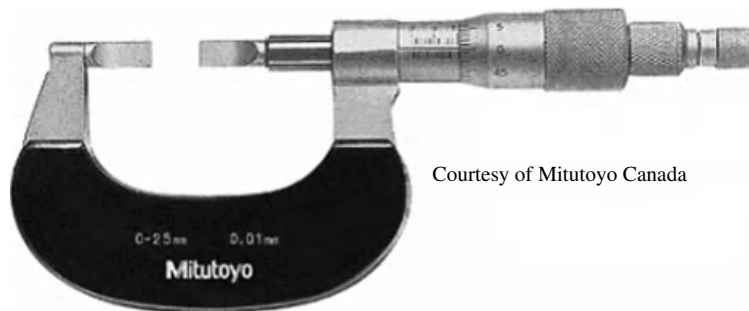


Figure 3.68. *V-shaped outside micrometer for angle inspection (courtesy of Mitutoyo)*

According to the manufacturer Mitutoyo, these models allow the measurement of the outer diameter of cutting tools such as taps, reamers, and cutters, with an odd number of lips. They conform to ISO 3611 with a force of 5–10 N. They have sides measuring flatness = $3\text{ }\mu$ and $(3 + L \div 75)$ micrometers in parallel. Hence, it is beneficial to use this kind of micrometers to check the cutting tool as in Figure 3.68.

3.10.9.3. Blade micrometer

These micrometers are less often found in laboratories, although they remain very useful in machine shops. They allow the control of grooves, cotters, and other hard points which are difficult to access (Figure 3.69).



Courtesy of Mitutoyo Canada

Figure 3.69. Blade micrometer: mechanical keys and non-rotating knives (courtesy of Mitutoyo Canada)

3.10.9.4. *Swan-neck mechanical outside micrometer*

These are identical to other micrometers with simple stop. The Swan-neck shape allows special measurements on parts which are difficult to access such as deep areas where the classic micrometer fails to reach (Figure 3.70).



Figure 3.70. *Swan-neck outside micrometer (Mitutoyo)*

3.10.9.5. *Statistical process control (SPC)*

Some measuring instruments are equipped with an outlet, called a measuring transmitter. This is not a measurement device, but as the name suggests, it serves to transmit values ensuing from the measure “read” to a data processor. This device is used in quality control to analyze the statistical characteristics, basic mathematics such as mean, standard deviation, capability ratio C_p , and capability coefficient C_{pk} of the process. In Chapter 2, we have already addressed formulas of capabilities.

3.11. Summary

In dimensional metrology, the gauge block remains the reference standard, both in workshops and laboratories. The wedges are parallel faces whose nominal dimensions are guaranteed with high accuracy: not less than $2/10\ \mu\text{m}$ for a medium quality. Stacked by adherence to each other, gauge blocks are exemplary reference blocks. Dial indicator shows the difference between the rating measure and that of a known

standard. Comparators allow both the measurement (addition and subtraction) and control by eliminating gaps. Comparators have mechanical amplification systems: pneumatic, optical, electrical, or electronic. Like the gauge block, the indicator is the most commonly used instrument in dimensional metrology.

Max–Min gauges are actually control etalons that do not measure but which allow the control by comparing the answers “yes” or “no” (“Go” and “No Go”). The control of bores, for example, requires, among other things, the use of plug gauges, gauges bore, spherical tipped pins (telescoping rods), etc. For shafts, we use plain bearings or gauges jaws. Nominal dimension is engraved on each of these conventional instruments.

Finally, measurement instruments such as calipers or micrometers allow direct reading with accuracy and without any dimensioning delay. The beaks of calipers, as the micrometers heads, are increasingly interchangeable. The jaws are adapted to the shape of the test piece. The measurement is performed both inside and outside the item subjected to control. The measuring instruments are increasingly varied, depending on their use. We recognize this. In certain cases, we put aside our modesty and claim to know how to measure. This is not a good attitude for a metrologist. We tend to forget that a qualified metrologist is firstly an individual who knows how to accept a measurement result. He does not add “makeup” to it. He only attempts to explain it.

3.12. Bibliography

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Chapter 4

Surface Control

4.1. Control and measurement of angles

This chapter deals with the dimensional control of main surfaces, particularly *inclined, grooved, and threaded* forms. Also, we have deliberately included some details of calculations of machine components. Our primary goal in this chapter focuses on conventional control because in Chapters 5 and 7 we will introduce controls involving electronic and opto-mechanic metrology. Measurement of angles is not a question of choice of appropriate tools. In fact, nothing induces us, *a priori*, to prefer the coordinate measuring machine (CMM) to the mechanical comparison. The choice of the machine depends on whether the control is carried out in the workshop or in the laboratory. The choice of the method is sometimes imposed by the cost of machines and equipments of metrology. Sometimes the operator, in a workshop, has nothing but the common means of mechanical comparison. Otherwise, he would find a laboratory near to the workshops where he will be able to expand his choice of means of control to sophisticated machines such as the CMM, the profile projector, or the interferometer. Thus, the control, by itself, does not raise the issue of skills. Mastering the basic concepts of trigonometry and geometry is essential for the control of surfaces [ACN 84, CAS 78, MIT 00, OBE 96].

Angles in dimensional metrology. The Babylonians counted with base 60, that is, sexagesimal, because they divided the degree into minutes and seconds, which are sixtieths. Similarly, Arab mathematicians have also measured the celestial and terrestrial angles. The measurement of time is, in this way, directly ensuing from astronomical angles.

The degree of arc ($^{\circ}$) is a convenient unit of the angle. A straight angle is measured as 180° . So a degree is $\pi/180$ radians, $10/9$ grades, thus $1/360$ of one complete turn. The SI units are rarely used for the degree of arc and its subdivisions (only to the second of arc). It is the only symbol that does not separate the digit with a space: we should write $12^{\circ}30'$ and not $12^{\circ} 30'$.

Subunits of degree. A degree is divided into 60 minutes of arc (symbol $'$), each of which is divided into 60 seconds of arc (symbol $"$). For example, $1' = 0.0166^{\circ}$ and $1" = 0.000277^{\circ}$. Sometimes, we also use the decimal notation. For example, 1.1° is written as $1^{\circ}6'$. However, we should pay attention to the fact that the minute means $1/60$ degree and the second means $1/60$ arc minute; thus, there is no link in the definition with the minutes and seconds, which are common time indicators for a watch.

Arc minute is a submultiple of a degree, equal to $1/60$ a degree. Similarly, the second of arc is equal to $1/60$ of a minute of arc, that is, $1/3600$ of a degree.

Measurement of angles. In practice, angles can be measured with a goniometer. The angles can be calculated based on the lengths of sides of polygons (triangles) using trigonometry. Sometimes, the angles are designated by their tangent. For example, a slope is expressed in percentage (%); this is the number of meters crossed relative to the horizontal. If α is the angle between the straight line with the highest slope and the horizontal, then the slope is equal to $100 \times \tan(\alpha)$ %. The (angle) protractor is used to measure angles. Some instruments have a dual scale graduation (in degrees and radians); some consist of a full disc, whereas others are simply half-disc.

Geometric angle. An angle is a *mathematical object* that can be represented by an angular sector (Figure 4.1). "Measurement of the angle" is often confused with the term "angle." For example, "straight" angle is improperly called an angle "equal" to 180° .

Definition of radian, unit of measurement of the angle. The international unit of angle is the radian. It is defined as the ratio of the circumference of the delimited circle to the radius of this circle. We commonly use the degree (rarely the grades) because the numbers used can be handled more easily.

Oriented angles. When the plane is oriented, the angles can be positive or negative depending on the direction of rotation. By convention, the plane is oriented in a direction called trigonometric, which is counterclockwise. If we consider two half-straight lines or vectors, then the order in which we state the half-lines or vectors defines the direction of the angle and its sign is written as:

$$\widehat{BAC} = -\widehat{CAB} \text{ or vectorially : } (\overrightarrow{u}, \overrightarrow{v}) = -(\overrightarrow{v}, \overrightarrow{u})$$

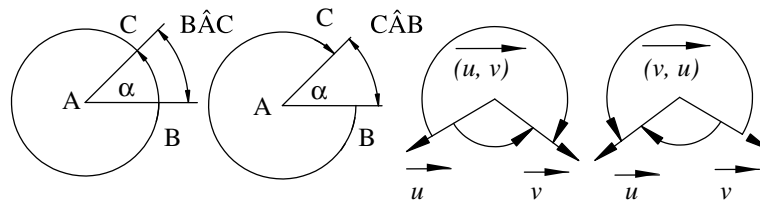


Figure 4.1. Angular sectors (vectors)

The orientation of the plane determines the sign on the angle. The angles are defined in a whole number (integer) of turns. In radians, the angles are set nearly to 2π . For example, assuming a right angle α with direct sense (direction), it will be denoted as

$\alpha = (\pi/2) + 2K\pi$, where $k \in \mathbb{Z}$ or $\alpha \equiv (\pi/2) [2\pi]$; thus, α is congruent to π on two modulo 2π .

4.1.1. Angles defects

An angle is created by the dihedron formed by two intersecting planes or by the position of the generatrix of a surface of revolution, relative to another plane, or the position of two generatrices of a surface of revolution. Each case requires a specific mode of control. In dimensional metrology, there are two main types of angular defects, namely the defects of straightness of the surface, including the flatness or, in some cases, the shape, and defects resulting from a positional deviation of the arms of the angle. Both types of defects are illustrated in Figure 4.2 [CAS 78, MIT 00].

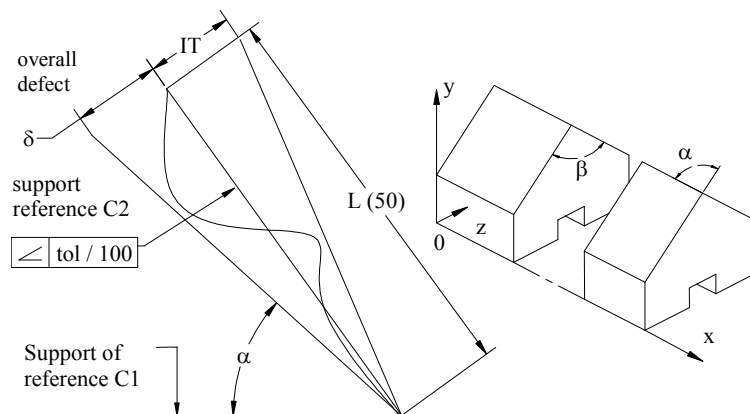


Figure 4.2. Schematic illustration of the types of angular errors

Angular defects are often small. For example, a defect in a single minute of angle ($291 \mu\text{rad}$) results in a deviation of about 0.03 mm for a length of 100 mm. Note that in Figure 4.2(b), a defect of 0.02 mm over a length L of 50 mm is equivalent to a defect of 0.04 mm. For $L = 100$ mm, we get $0.02(100/50) = 0.04$ mm. If we transpose the linear value into an angle, for a length of 40 m, we find

$$L = 100 \text{ mm} \rightarrow 0.02(100/40) = 0.05 \text{ mm}; \text{ thus, } \beta = (0.02/40) = (0.05/100) = 0.0005 \text{ rad} = 500 \mu\text{rad}. \text{ It is as if we write } \tan(\beta) = (0.02/40) = 0.0005.$$

EXAMPLE 4.1. $1' = 1^\circ/60$ or $1/60$ of a degree. There are 90 degrees in a right angle which is written as $1D = 90^\circ$. The following are the commonly used basic notations.

– There are 60 arc minutes in one degree. Thus, $1^\circ = 60'$ (note that the prime on the top of 60 designates the minutes of angle).

– There are 60 seconds of angles in one minute of angle. Thus, $1' = 60''$, that is, $1^\circ = 60 \times 60 = 3600''$ (60' each of which equals 60'') and $1'' = 1^\circ/3600$ or $1'/60$.

– For example, let us convert $45^\circ 20' 50''$ and we get $45 \times 3,600 + 20 \times 60 + 50 = 163250''$.

– In general, we do not use submultiples of a degree (sexagesimal writing). We prefer to use a decimal notation. For example, 30.5° does not mean 30 degrees and 5 minutes of angle, but 30° and $0.5^\circ = 0.5 \times 60 = 30'$; hence, $30.5^\circ = 30^\circ 30'$.

– For example, let us convert 45.8° (the symbol $^\circ$ is the notation on the right of the number and not after 45) into sexagesimal notation. $45.8^\circ = 45^\circ \pm 0.8^\circ$, where $0.8^\circ = 0.8 \times 60' = 48'$; thus, $45.8^\circ = 45^\circ 48'$.

– To convert $25^\circ 50'$ in decimal notation, we should, in fact, convert $50'$ into degrees. As $1' = 1/60$, then $50' = 50 \times 1^\circ/60$ or $50' = 50^\circ/60$ or $50' = 0.833333\dots^\circ$ rounded to 0.83° (after divided 50 by 60); we get, $25^\circ 50' = 25.83^\circ$.

– From degrees to radians, we divide by 180 and we multiply by π . For example, 120° is equal to $2\pi/3$ rad.

– From radians to degrees, we divide by π and we multiply by 180. For example, $3\pi/4$ rad is equal to 135° .

In metrology, there is a control without measurement and a control with measurement. The control alone is done to confirm or refute the veracity of the value of the angle transcribed on a drawing. The measurement of the angle requires careful and appropriate means for measuring and reading. The value of an angle can be simply materialized by size of angle as the square.

Table of Proportional Conversions							
Functions	Arguments						
Degrees	0	30	45	60	90	180	360
Grades	0	33.3333	50	60.6666	100	200	400
Tours	0	1/12	1/8	1/6	1/4	1/2	1/1
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	2π

Table 4.1. *Table of conversions*

4.2. Surfaces of revolution

Many human-made objects have surfaces of revolution. The reason is that the symmetry of revolution facilitates their manufacturing or their use. Surfaces of revolution are parameterized and oriented surfaces, which include toroids, spheres, cylinders, spheroids, hyperboloids, paraboloids, etc. In this chapter, we present some means to control them. The functional surfaces of assembly or adjustment belong to simple elements such as the *plane*, the *cylinder*, the *cone*, or the *sphere*. Each element is defined by its *shape*, *dimension*, and *position*. The duality encountered in dimensional metrology of cones lies in the fact that we need to go, beforehand, through a measurement by mechanical comparison; that is, to involve *gauges*, *balls*, and *blocks*. This fact implies additional geometric and trigonometric calculations.

To get the *plane*, we generate surfaces containing points of a straight line of an arbitrary orientation. For example, control of flatness may be carried out by using a simple beveled miter gauge or a CMM, by probing at least three points on a plane, which is illustrated in Figure 4.3.

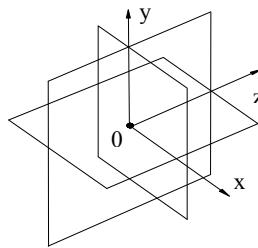


Figure 4.3. *Schematic illustration of a plane*

The cylindrical surfaces (Figure 4.4) consists of a surface with all points being equidistant from a straight line (axis). In Figure 4.4, the generation of the circumference is realized by turning around the straight line (γ). Conventional

control of cylindricity is performed through a dial indicator on the three generatrices (1, 2, and 3) compared with the three directrices (4, 5, and 6).

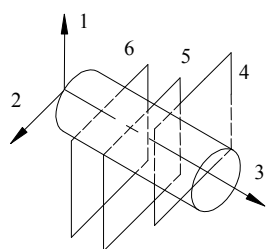


Figure 4.4. *Illustration of a cylinder*

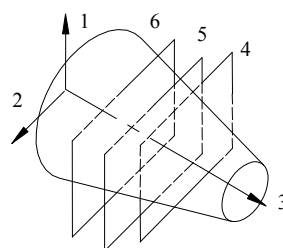


Figure 4.5. *Illustration of a cone*

Surfaces of revolution with linear generatrix rotating around another straight line non-parallel to the axis of its plane give rise to what is commonly called a cone. Figure 4.5 provides an illustration of the generation and control of the cone. We note that the cone is the result of a straight line rotating around another secant straight line called the axis. The control of such a conical form can be carried out in several ways. We mention the comparator where the control will be done, for example, on three generatrices (1, 2, and 3) according to the three directrices (4, 5, and 6).

On each side of the dimension to be controlled, direct contacts like piece/tester are integrated, with a flat contact tip on a flat surface and simply linear or even punctual in other cases. However, to measure parts called “dipping,” direct contact is likely to be impossible. It is then necessary to interpolate between the workpiece and the tester reference elements, thus allowing to

- establish aligned contacts between contact tips on the tester (the reference element and the piece intended to be controlled);
- define the extremities of the dimension to be found, although they are not directly accessible to the contact tips of the tester. We discuss, for example, the case of “dipping” parts with inward or outward angles, but whose apexes are truncated.

The dimensional definition is given relative to another value taken on reference features. The dimension specific to the workpiece being measured is obtained by subtracting from the *read dimension*, the values of these extra elements. This means

$$\text{Actual dimension} = \text{dimension observed} \pm \text{dimension of extra elements} \quad [4.1]$$

According to International Organization for Standardization (ISO), tolerances of cones are classified into four categories (see Appendix 2: Tables A2.1 and A2.2).

Next, we present work on conventional measurement, control, and calculation of the conicity. Then, we discuss other issues related to other surfaces of revolution such as the *cylinders*, the *spheres*, and the *plane*.

Spherical surfaces are represented by points equidistant from a central point r (Figure 4.6). This is the case of a semi-circular surface rotating on its diameter. The means of control of sphericity are numerous and also vary, but for simplicity, we include the comparator.

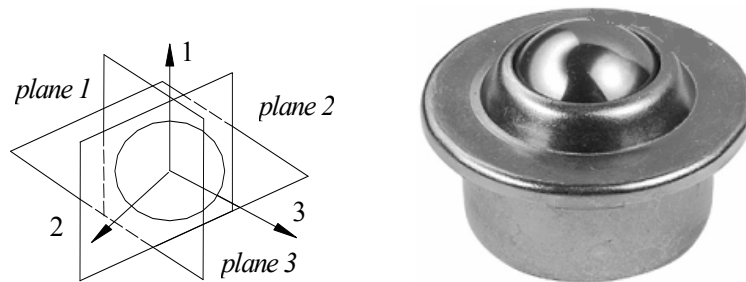


Figure 4.6. Schematic illustrations of a sphere

4.2.1. Fundamentals of the analysis of conical surfaces control

A conical surface is generated by a straight line called generatrix. It passes through a fixed point, called the vertex, and moves constantly on a fixed curve called the directrix. In fact, a conical surface has a directrix, a circle and for the vertex a point, located on a straight line perpendicular to the plane of the circle passing through the center of the latter. It is called “surface of revolution around an axis”. The straight line is the *axis* of revolution. Other defects [CAS 78, OBE 96] of the conical surface such as *concavity*, *convexity*, and even rough waviness, etc. may occur. Concave generatrices come from the fact that the cutting edge, and even its trajectory, is not in a plane containing the axis. For example, these straight lines and axes are neither concurrent nor coplanar even for the edge of the generatrix tool that is placed well below the axis of points of turn. This is a frequent case in terms of conventional rolling. Convex or wavy generatrices initialize from distortions on the workpiece by flexion. This happens sometimes in machining on machine tools. A conicity out of specifications is due to incorrect setting of the tool of form or the trajectory of envelop tool (tool-slide, slide of reproduction, i.e. rule to be followed, etc.).

Control process for conical assembly. The main concern in the case of conical assembly consists of the alignment of the workpieces without clearance adjustment. To achieve this, the conicity must be the same for two cones in contact. The control of cones focuses particularly on the accuracy of the conicity and on the straightness

of the generatrix. The diameter of reference base of the cone and the circular shape of the cross-section will be verified accurately only exceptionally. Therefore, the control of the conicity remains a difficult issue in mechanical metrology. Conicity is the ratio of the difference in diameters of the cone to the axial length between them as shown in Figure 4.7.

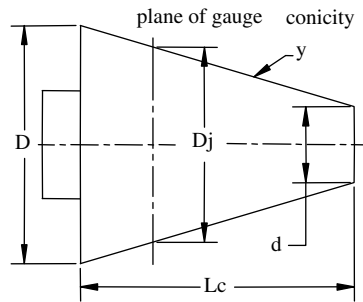


Figure 4.7. Dimensioning a cone to calculate conicity

$$C = \left(\frac{D-d}{L_c} \right) \times 100 \quad [4.2]$$

where C is the conicity in percentage; D is the largest diameter (gauge) in millimeters or inches; d is the smallest diameter in millimeter or inches, L_c is the conical length in millimeter or inches. Basically, the conicity reflects the slope. By transposing the vertex (α) of the cone, conicity can be written as follows:

$$\tan\left(\frac{\alpha}{2}\right) = \left(\frac{D-d}{L_c} \right) \text{ with } \alpha = 2 \times \arctan\left(\frac{D-d}{L_c}\right) \quad [4.3]$$

There exists a principal series of normalized conicities, for example,

$$1/3, 1/5, 1/10, 1/20, 1/50, 1/100, 1/200, 1/500$$

The diameter on which the control is focused is called gauge diameter D_g . It is totally independent of the nominal diameter D_n of the conicity. *Toleranced* dimensioning according to CSA [ACN 84] is important and relates to the diameter of the gauge defined as a cylindrical body. It is independent of the angular tolerance.

EXAMPLE 4.2. Consider the characteristics of a conical assembly: $C = 1/5$, $\varnothing_{\text{gauge}} = 31.2$ mm, $D_n = 32$ mm, span length(L) = 36 mm.

QUESTION.— If the tolerance on the gauge diameter is $\varnothing 31.2 \text{ H}8/\text{h}8$, find the deviation of recess of the buffer corresponding to the tolerance $\text{H}8$ and limit values of the distance L for tolerance $\text{h}8$ of the shaft.

SOLUTION.— Based on the formulae of conicity, we deduce

$$\text{If } C = \left(\frac{D-d}{L_c} \right), \text{ hence } L_c = \left(\frac{D-d}{C} \right) \quad [4.4]$$

Basically, the conicity C reflects the slope along the span length L . In our case, C and L are known by assumption. Standardized adjustments according to CSA and ISO enable reading the toleranced dimensions depending on the required qualities. After calculating, it follows:

$$\varnothing 31.2\text{H}8 = 31.2_{-0}^{+0.039}, \text{ which can be written as } \{31.239 \text{ and } 31.200\}$$

The exact reading of the tolerances of the bore allows us to find the deviation of the recess of the buffer and based on equation [4.4].

NUMERICAL APPLICATION.—

$$\text{If } D = 31.239, d = 31.200, \text{ and } C = \frac{1}{5}; \text{ Then, } L_c = \left(\frac{D-d}{C} \right) = 0.195 \text{ mm}$$

Following the approach made for a bore, we now calculate the parameters that characterize the shaft and deduce the corresponding limits of distances. The limits of

$$\varnothing 31.2\text{h}8 = 31.2_{-0.039}^0, \text{ which can be written as } \{31.200 \text{ and } 31.161\}$$

By analogy with the previous calculation, it follows from equation [4.4] that $L_1 = 0.195 \text{ mm}$, $L_2 = 0.195 \text{ mm}$, $L = L_1 - L_2 = 0 \text{ mm}$, so there is perfect coincidence:

– We see that the tolerance of recess is in the order of 0.195 mm . This is related to the nominal diameter whose tolerance is high, compared with this low conicity of $1/5$.

Control with a ring and tapered buffer do not tolerate wedging during the fitting of parts. It is plausible to see this case of wedging in the case of conicity less than or equal to $1/50$.

4.2.2. Control by comparison to a standard

Control by comparison to a standard is carried out using a buffer and a ring or pneumatic differential comparator (not discussed here). Conventional control by measurement and calculation is performed using one of the following four methods:

- using a buffer and a cone-shaped ring,
- using cylindrical gauges or calibrated balls,
- using the sine-bar, and
- using the sine-table.

4.2.3. Using the buffer and the cone-shaped ring

For the bore of the female workpiece (Figure 4.8), the conical buffer is coated with a thin layer of dye, at least at three of its generatrices with approximately 120 degrees spacing.

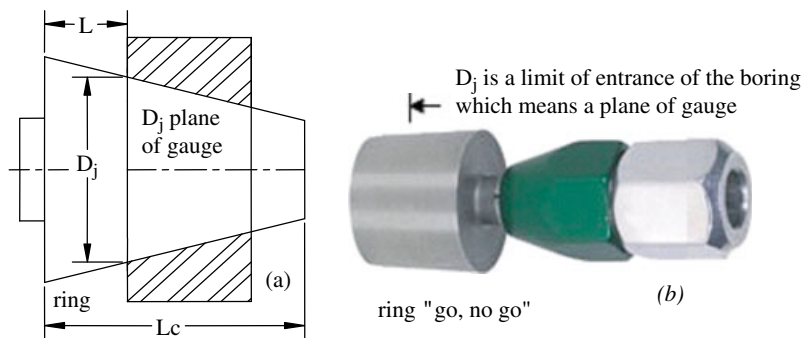


Figure 4.8. Conventional conicity control with conical buffer and ring

It is calculated in the same manner as the male workpiece (Figure 4.8(b)) in which we control the conicity with the conical ring. We recall that the control of the conicity must be followed by the control of the gauge diameter D_g . For this, the buffer could have a circular line at that diameter, and thus marking the boundary in the bore. As for the conical ring, we do consider that the dimension calculated earlier limits the recess of the piece.

4.2.4. Measuring angles with gauges and balls

This classical method is widely used in the laboratory. Many books from technical literature worldwide use it under this schematized form. The cone intended to be controlled stands on the marble plate, and the balls are then inserted inside the bore, one after another, and to measure the dimensions A and B carefully as shown in Figure 4.9. The expressions that we present throughout this section are classical. Most textbooks [CAS 78, FRI 78, OBE 96] on dimensional metrology develop

similar approaches. The mathematical expressions that support the corresponding diagrams are the result of analytic geometry and trigonometry. Figure 4.9(a) represents the classical issue of control of conicity using gauges and balls.

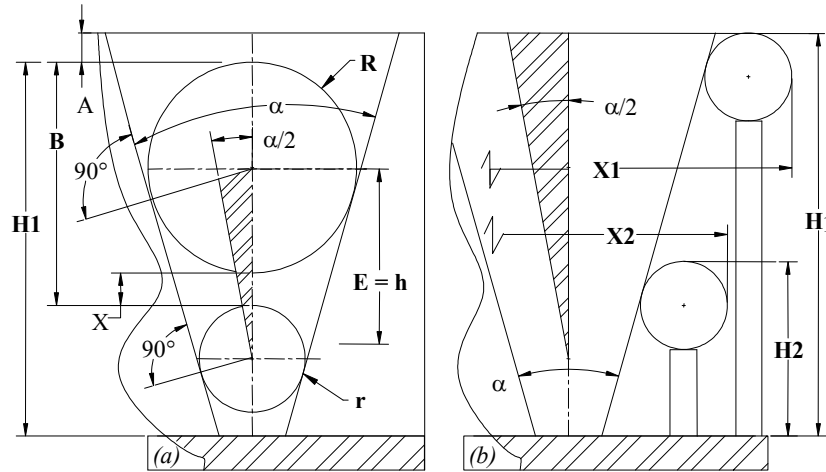


Figure 4.9. Conventional control of conicity (male and female) using balls and gauges; (a) and (b) are perfectly symmetrical relative to the respective axes

4.2.4.1 Female cone

We performed many assemblies in our own workshops to verify the measurement results obtained from our metrology and machining laboratories. To measure the angle or $\alpha/2$ shown in Figure 4.9(a), we use the following formula:

$$\sin\left(\frac{\alpha}{2}\right) = \left(\frac{R-r}{E}\right) = \left(\frac{R-r}{(R+r)+X}\right) \quad [4.5]$$

where R is the radius of the large ball (known value), r is the radius of the small ball (known value), E is the distance between the centers of balls, and X is the residual diametrical difference between the two balls (all the values are expressed in millimeters or inches); and $\alpha/2$ is the half-angle resulting from the geometric construction (Figure 4.9(a)).

From equation [4.5], we deduce the value of X as follows:

$$X = \left(\frac{R-r}{\sin(\alpha/2)}\right) - (R+r) \quad [4.6]$$

If equation [4.6] is applied for the expression of E (or h),

$$E = \left(\frac{R-r}{\sin(\alpha/2)} \right), \text{ then [4.6] becomes } X = E - (R+r) \quad [4.7]$$

4.2.4.2. Male cone

To find $\Delta X = (X_1 - X_2)$ with respect to Figure 4.9(b), we consider the following:

$$2 \tan\left(\frac{\alpha}{2}\right) = \left(\frac{\Delta X}{\Delta H} \right) = \left(\frac{X_1 - X_2}{H_1 - H_2} \right) \quad [4.8]$$

From [4.8], we deduce $\Delta X = (X_1 - X_2)$ as follows:

$$\Delta X = (X_1 - X_2) = 2 \tan\left(\frac{\alpha}{2}\right) \times (H_1 - H_2) \quad [4.9]$$

NUMERICAL APPLICATION 1.–

For $\alpha = 20^\circ$; $H_1 = 60$ and $H_2 = 10$; from equation [4.9], $\Delta = 17.633$ mm

NUMERICAL APPLICATION 2.– Measuring a gauge angle in Figure 4.9(b). Let us determine, based on measure X_1 on disc, a committed angular error. X_1 is the actual distance between gauges:

- assuming $\alpha = 32$, $r = 10$ and $R = 20$, $X_{1 \text{ real}} = 5.21$ mm .
- $E = h = (R + r + X) = 20 + 10 + 5.21 = 35.21$ and $(R - r) = 20 - 10 = 10$ mm.

Figure 4.10(b) illustrates the issue of the female conicity control (Figure 4.10) using gauges and balls. This representation is substantially similar to that of Figure 4.9. Depending on the assemblies that we have in workshop, we formalize our equations accordingly.

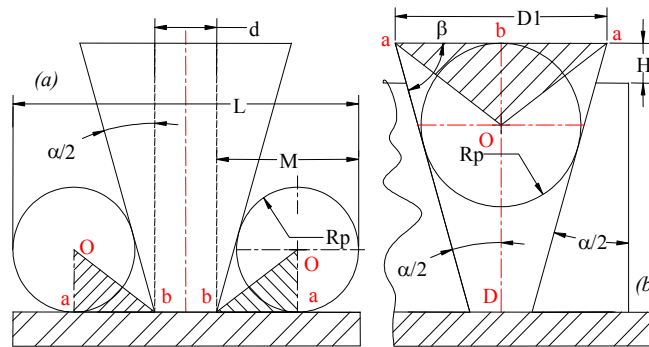


Figure 4.10. Classical representation of female conicity control

For a male cone, the calculation of the small base diameter results from the study of Figure 4.10(a). With the known gauge diameter d_g , we measure the dimension M as follows:

$$d = (L - 2M) \quad [4.10]$$

$$\text{with } M = (r - a \times b) \text{ and } ab = \frac{r}{\cotan(\beta / 2)} \quad [4.11]$$

$$2M = (d_g + 2ab) = d_g + d_g \times \cotan\left(\frac{\beta}{2}\right) = d_g + \left(1 + \cotan\left(\frac{\beta}{2}\right)\right) \quad [4.12]$$

$$\text{equation [4.10] becomes } d = M - d_g \times \left(1 + \cotan\left(\frac{\beta}{2}\right)\right) \text{ with } \beta = \left(\frac{\pi}{2} - \frac{\alpha}{2}\right) \quad [4.13]$$

The measurement of the gauge diameter D for a female cone (Figure 4.10(b)) is done by a calculation that is similar to the previous calculation. The same calculation, when applied to the same figure gives the value of the gauge diameter D , namely

$$D = D_1 - 2 \times H \times \tan\left(\frac{\alpha}{2}\right) \quad [4.14]$$

$$D_1 = d_g \times \cotan\left(\frac{\beta}{2}\right) \text{ considering } D = d_g \times \cotan\left(\frac{\beta}{2}\right) - 2 \times H \times \tan\left(\frac{\alpha}{2}\right) \quad [4.15]$$

The methods previously developed are not restrictive, and the means of control vary from one laboratory to another. The basics of calculations are the same because they obey fundamental concepts of analytic geometry. It is recommended for this purpose, to comply with the methods and tools available to assess and judge the adequacy to the ultimate goal: measurement and control of the male and/or female conicity.

4.2.4.3. Application on the control of an angle using cylindrical gauges

The example shown in Figure 4.11 (see also Figure 4.13) is taken from a laboratory's work on the control of an angle by means of cylindrical gauges. We know that the gauges are cylinders whose diameter should be known, with precision, in advance. We propose a calculation example on control of the angle α using this method. We place, inside a dihedron, a gauge P_1 of diameter D_1 . It is then convenient to measure the length l_1 , that is, the farthest generatrix up to the side (BB') of the piece. Then, we place another gauge P_2 of diameter D_2 different from the first one (smaller), and thus we measure the length l_2 (Figure 4.11).

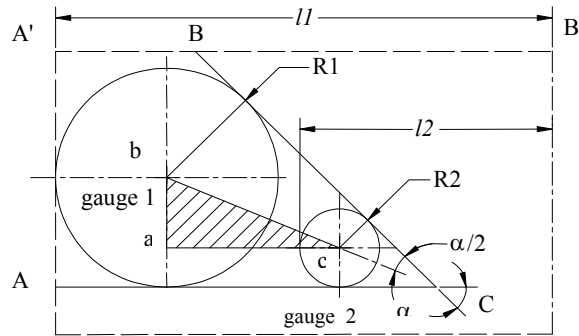


Figure 4.11. *Classic example of measure (control) of an angle by cylindrical rods*

It is easily understood that the calculation (or the control) of the value of half-angle $\alpha/2$ formed by the dihedron is based on the difference between the values (D_1-D_2) and (l_1-l_2) . The measurement means of l_1 and l_2 are ordinarily a caliper correctly calibrated. Here, it could be both a measurement and a control, and both cases are explained as follows.

4.2.4.3.1. First case of measuring the angle α : here, we calculate the angle α .

The angle (acb) measures the half-angle opposed to the apex $\alpha/2$. In a scalar, the expression of (ab) is as follows (Figure 4.11):

$$(ba) = \left(\frac{D_1 - D_2}{2} \right) \quad [4.16]$$

$$(ca) = \left(l_1 - \frac{D_1}{2}\right) - \left(l_2 - \frac{D_2}{2}\right) = (l_1 - l_2) - \left(\frac{D_1 - D_2}{2}\right) \quad [4.17]$$

From the triangle (abc) , we consider the following expression using equations [4.16] and [4.17]:

$$\tan\left(\frac{\alpha}{2}\right) = \left(\frac{ba}{ca}\right) = \frac{\left(\frac{D_1 - D_2}{2}\right)}{[(l_1 - l_2) - ((D_1 - D_2)/2)]} \quad [4.18]$$

NUMERICAL APPLICATION.— The trigonometric table easily allows us to measure the angle α and, thus, calculates the half-angle $\alpha/2$, as will be shown as follows. Assume that $l_1=72.35$ mm and $l_2=52.00$ mm, having used gauges with $D_1 = 22$ mm and $D_2 = 8$ mm. Let us calculate the half-angle $\alpha/2$ using formula [4.18] and then deduce the angle α :

$$\left(\frac{ba}{ca}\right) = \frac{((22-8)/2)}{[(73.35-52)-((22-8)/2)]} = 0.4880; \text{ knowing } \tan\left(\frac{\alpha}{2}\right) = \left(\frac{ba}{ca}\right)$$

$$\text{Hence, } \frac{\alpha}{2} = \arctan(0.488) = 27.949^\circ \Rightarrow \alpha = 55.898^\circ$$

4.2.4.3.2. Second case of control of the angle α

In this case we control the angle α .

In this case, the value of the angle α is known with precision. We try to demonstrate that for gauges of known diameters, respectively, D_1 and D_2 , the difference of the lengths (l_1-l_2) corresponds to the value assigned to the angle α . Formula [4.18] can also be written as follows:

$$\tan\left(\frac{\alpha}{2}\right) \times \left[(l_1-l_2) - \left(\frac{D_1-D_2}{2}\right)\right] = \left(\frac{D_1-D_2}{2}\right) \quad [4.19]$$

From equation [4.18], we can derive equation [4.20].

$$\begin{aligned} (l_1-l_2) &= \left(\frac{D_1-D_2}{2 \times \tan(\alpha/2)}\right) = \left(\frac{D_1-D_2}{2}\right) \times \left(\frac{D_1-D_2}{2 \times \tan(\alpha/2)}\right) \\ &= \left(\frac{D_1-D_2}{2}\right) \times \left(1 + \cotan\left(\frac{\alpha}{2}\right)\right) \end{aligned} \quad [4.20]$$

Numerical application of values in the problem allows ΔL to be calculated for:

$$\begin{aligned} D_1 &= 22 \text{ mm}, D_2 = 8 \text{ mm}, \alpha = 60^\circ \\ \Rightarrow (l_1-l_2) &= \Delta L = \left(\frac{D_1-D_2}{2}\right) \times \left(1 + \cotan\left(\frac{\alpha}{2}\right)\right) = 19.124 \text{ mm} \end{aligned}$$

Obviously, we should first make sure that the segments (AA') and (BB') are *perfectly perpendicular*, then:

$$\begin{aligned} \text{From } \Delta L &= \left(\frac{D_1-D_2}{2}\right) \times \left(1 + \cotan\left(\frac{\alpha}{2}\right)\right) \\ &= 19.124 \text{ mm} \quad l_1 = 22 \text{ mm}; \quad l_2 = l_1 - \Delta L = 45.156 \text{ mm} \end{aligned}$$

PRACTICAL EXERCISES BY NUMERICAL APPLICATION.— We propose two significant examples encountered both in dimensional metrology and in machining. We performed them both in workshop and in laboratory. Here are classical problems sets from conventional metrology. In Figures 4.12 and 4.13, we consider:

$$AC = (l_1 - R) - (l_2 - R) = (l_1 - l_2) \quad \text{and} \quad \cotan(\alpha) = \left(\frac{l_1 - l_2}{h}\right) \quad [4.21]$$

NUMERICAL APPLICATION.— If $R=10$, $l_1 = 52.57$; $l_2 = 42,10$ and $H = 20.15$ mm; $AC = 10.47$, $\cotan(\alpha) = 0.5769$, and thus $\alpha = 60.015$.

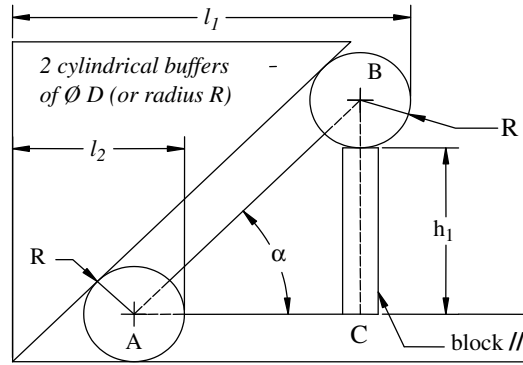


Figure 4.12. Applications on the calculation of the value of an angle α using a block and cylindrical buffers. Classical case of a fixed slide of a milling machine (machine tool)

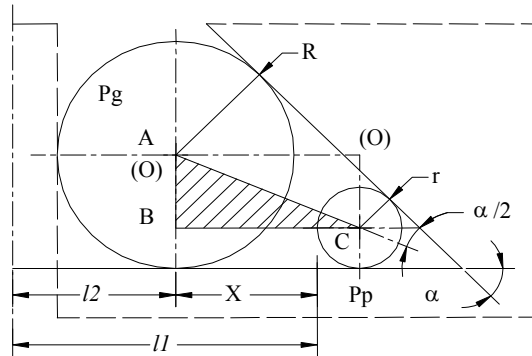


Figure 4.13. Application on the calculation of the value of an angle (α) with two cylindrical buffers of different diameters (see also Figure 4.11). Case of a fixed slide of milling machine

$$\left\{ \begin{array}{l} BC = (l_1 + r) - (l_2 + R) ; AB = (R - r) \text{ and } \tan(\alpha) = \left(\frac{R - r}{(l_1 + r) - (l_2 + r)} \right) \\ \text{or } \cotan(\alpha) = \left(\frac{(l_1 + r) - (l_2 + r)}{(R - r)} \right) \end{array} \right. [4.22]$$

For $R = 25.4$, $r = 6$, $l_1 = 59.95$, $l_2 = 25.4$

then $\cotan\left(\frac{\alpha}{2}\right) = 1.731 \Rightarrow \alpha = 60.03^\circ$

4.2.4.4. *Advantages and disadvantages of measuring the conicity by comparison*

It is obvious that the measurement of conicity based on CMM is far more advantageous than measurements based on gauges. There are many advantages of the CMM, such as the quasi-absence of mechanical frictions. The device is insensitive to temperature variations, within current limits in workshops. Response time is in the order of fractions of a second. However, it is undeniable that the drawbacks of measuring by mechanical metrology are obvious, witness the additional mounting of cylinders and gauges for the control of the conicity, which inevitably adds uncertainty to the measurement. The choice of balls for a tapered bore is probably tainted with uncertainties and the manipulations are often subjected to the skill of the operator, cleaning of contact surfaces, and hygrometric conditions, affect the accuracy of the measurement. The advantages for measuring by mechanical metrology are undeniable in a workshop.

In the case of mounting on machine tool, the control using CMMs seems to be compromised because of the dimension. In the case of concave, convex, or undulating generatrices, due to a possible deregulation of cutting, it is not obvious for the CMM to identify the cone. In the field of angular measurements, the extra elements must be of “durable” precision (steel that is hardened and ground) to prevent introducing further errors. We use gauge blocks, balls, cylinders, and sometimes half-cylinders. It is worth recalling that mastering the fundamentals of trigonometry and analytic geometry is essential.

The principle of sine measurement [OBE 96] is based on the calculation of the “sine” function that allows the angle α to be deduced. For accurate angular measurements, we often resort to mechanical apparatus of high sensitivity, based on the measurement of the sine of the angle and that is why they are called as sine devices. In this case, a rule of reference is rigidly linked to two identical ground cylindrical buttons, with a fixed and known center distance L , as shown in Figure 4.14.

4.2.5 *Principle of measurement called “sine”*

4.2.5.1. *Using a sine-rule-measurement known as “sine”*

The sine rule schematized in Figure 4.14(b) bears two buttons with identical diameters separated by 100 mm and placed parallel to the edge of the reference. The support of the device placed on a marble plate receives the articulation axis based on the rule, whose inclination is adapted to the cone, is obtained by interposing a block, and holds with which the buttons are in contact. The vertex of the cone α is written as:

$$\sin(\alpha) = \frac{H}{L} \quad [4.23]$$

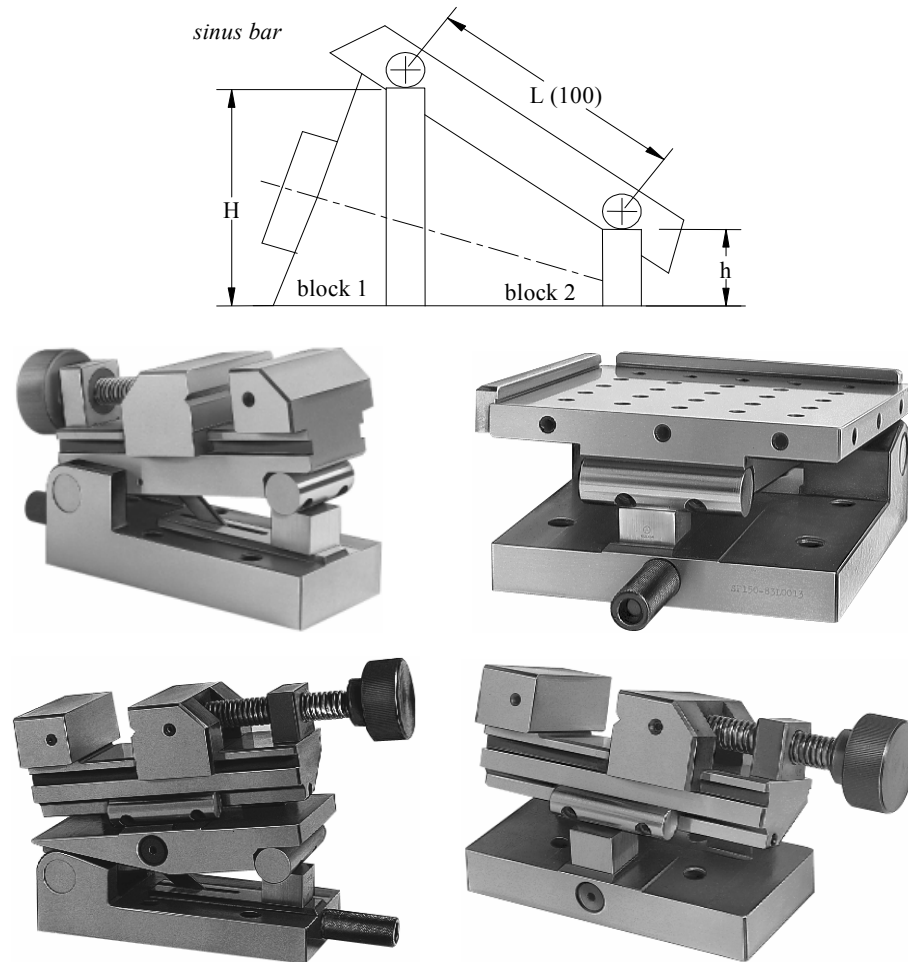


Figure 4.14. Trigonometric measurement, different uses of the bar sine (sine angle)
(courtesy of Mitutoyo, Canada)

The adjustment is also set through the coincidence of the rule on the reference surface (RS). Dimensions H and h are linked to the value of the angle α . Let us consider:

$$L \times \sin(\alpha) = (H - h) \quad [4.24]$$

NUMERICAL APPLICATION.— Assume $L = 100$, the measurement result of $H = 46.475$ and $h = 12$; thus using [4.24], $\sin(\alpha) = 0.34475$ rad = 19.753 deg (in degrees, minutes, and seconds):

$$\alpha = 0.34475 \text{ rad} = \begin{pmatrix} 19 \text{ deg} \\ 45 \text{ min} \\ 10 \text{ sec} \end{pmatrix}, \alpha = \arctan(0.34475) = 19^\circ 45' 10'' \text{ with } \alpha \approx 20^\circ$$

The measurement of H and h is ordinarily carried out using gauge blocks. The direct sensitivity which reaches $\approx 5 \mu$ may be improved with the use of an auxiliary cylindrical plug gauge of diameter 2.5.

4.2.5.2. Using the sine-table

The sine-table [MIT 00, EBO 96] is generally used for handed cones in opening. The reading of the deviation, on the comparator, marks the incorrectness of the conicity or the straightness of the generatrix. Here, the length L is an intrinsic characteristic of the device. The adjustment is made depending on α and is obtained by interposing a block of height H calculated using equation [4.23]. The control of large parts, which may be clamped justifies the use of a sine-table derived from the sine rule, but with larger dimension and whose RS has V-grooves to allow the use of fixation bolts. Under the same table, we have two cylinders, one serving as a pivot and the other guides for the inclination. They are parallel to the table and the distance between their respective axes ranges between 200 and 500 mm and parallel to the table. The latter is supported directly on a marble or against a T-square, thus enabling the observation.

4.2.5.3. Example application of measuring distances on gauges

In fact, rods are cylinders whose diameter is known with high accuracy. The example we present later has become a classic case both on the practical aspect and on the manipulation of geometric and trigonometric expressions accordingly. We present the following two cases: case 1, angle $\alpha > 90^\circ$ and case 2, angle $\alpha < 90^\circ$. The dovetail guides that should be machined at a precise width are generally verified using cylindrical gauges by measuring the dimensions x and y shown in Figure 4.15(a).

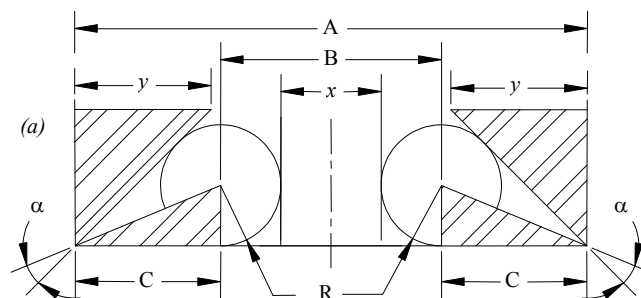


Figure 4.15a. Angular measurements on female dovetail (slides)

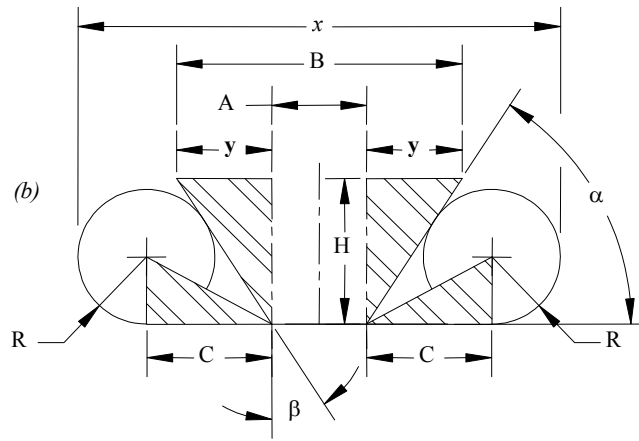


Figure 4.15b. Angular measurements on male dovetail (slides)

To obtain the dimension x for a male dovetail, we use the following formula:

$$X = A + D \times \left(1 + \cotan\left(\frac{\alpha}{2}\right) \right), \text{ where } D = 2r \quad [4.25]$$

To obtain the dimension y for a female dovetail, we use the following formula:

$$y = B - D \times \left(1 + \cotan\left(\frac{\alpha}{2}\right) \right), \text{ where } D = 2 \cdot r \quad [4.26]$$

Dimension C is equal to r multiplied by the cotangent of the angle α :

$$C = r \times \cotan(\alpha) \quad [4.27]$$

The cylindrical gauge used must be thin enough so that the contact point (e) is located at some distance below the edge of the dovetail:

$$\text{Single male: } A = X + (Z + r) \text{ with } B = A - y \quad [4.28]$$

$$\text{Double female: } A = X + 2 \times (Z + r) \text{ with } B = A - 2 \times y \quad [4.29]$$

$$C = \frac{r}{\tan(\alpha / 2)} \quad [4.30]$$

4.2.5.3.1. Numerical application of an example related to Figure 4.15(a)

Measuring a dovetail: Determine the value of X , an actual distance to be measured on gauges. Consider the data $\alpha = 60^\circ$, $r = 10$, and $A = 80$, $X = ?$

SOLUTION.— We should first properly ensure that the elements constituting the measurement correspond to Figure 4.15(a), hence the formula for the calculation of X .

NUMERICAL APPLICATION.—

$$A = 80; r = 10; \alpha = 60^\circ \text{ and from [4.30], } C = \frac{r}{\tan(\alpha/2)} = 17.321 \text{ mm}$$

$$\begin{cases} \text{Single male: } A = X - (C + r), \text{ where } B = A + Y \\ \text{Double male: } A = X - 2x(C + r), \text{ where } B = A + 2xy \\ \text{For a single male dovetail: } X = A - (C + r) \end{cases} \quad [4.31]$$

NUMERICAL APPLICATION.— For a single male dovetail:

$$X = A - (C + r) = 52.679 \text{ mm}$$

4.2.5.3.2. Numerical application of an example related to Figure 4.15(b)

Control of a dovetail. Determine an error in the width occurring on a measurement X' with gauge, on a dovetail similar to that shown in Figure 4.15(b), that is, “double male.”

SOLUTION.— We should first properly ensure that the angles relative to the measurement of distance conform with the specifications given on the drawing. After that, we choose gauges, preferably having their contact at mid-height of the flanks.

Consider $\alpha = 60^\circ$, $r = 12$, $B = 60$, and $h = 25$; $X'_{\text{real}} = 96.820 \text{ mm}$.

Thus, the error on X'_{real} will be $\delta = X' - X$?

Let us first calculate the value of X and then $\delta = (X' - X)$. The formula relative to a double male, presented previously, is written as follows:

NUMERICAL APPLICATION.—

$$X_{\text{theoretical}} = A + 2 \times (C + r) \text{ and from } B = A + 2 \times y, \text{ we get } A = B - 2 \times y$$

$$\text{Using } \tan(\beta) = \left(\frac{y}{h}\right) - (\alpha), \text{ we calculate } \beta = \arctan\left(\frac{y}{h}\right) - (\alpha) = 90^\circ - 60^\circ = 30^\circ$$

$$\text{knowing } C = \frac{r}{\tan(\alpha/2)}, \text{ then } y = 25 \times \tan\left(\frac{\alpha}{2}\right) = 14.435 \text{ mm}$$

$$\text{From } C = \frac{r}{\tan(\alpha/2)} = \frac{12}{\tan(60/2)} = \frac{12}{0.577350} = 20.785 \text{ mm}$$

we calculate the value of $A \rightarrow A = 60 - (14.4330 \times 2) = 31.1340$; thus,

$X = 31.340 + 2 \cdot (20.860 + 12) = 96.060 \text{ mm}$. The expression of the error by excess will therefore be: $\delta = X' - X = 96.20 - 96.06 = 0.14$ (114/1,000th).

4.2. Metric thread (M) measurement on gauge

The metric thread is generated by an equilateral triangle (angle = 60°) with one side parallel to the axis. The nominal diameter d is the diameter of the male thread measured on the truncation measured $H/8$ as shown in Figure 4.16(a). It is equal to the diameter on *sharp angles* ($H/4$). From the literature [FAN 94, KAL 06], we consider:

$$\text{Case (a): } d_2 = d_{\text{pitch diameter (medium)}} = d - \left(\frac{3}{4}\right) \times H \quad [4.32]$$

Diameter of the basic cylinder: $A = (d_2 - H)$

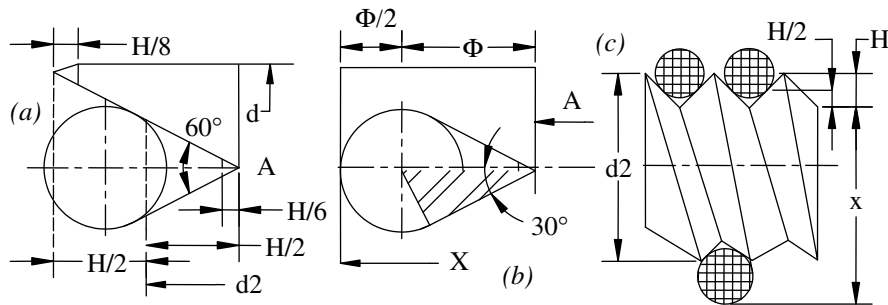


Figure 4.16. Metric thread (M) measurement on gauge

Case (b): The diameter on gauge X = diameter measurable on the gauges Φ :

$$X = A + 3 \cdot \Phi \text{ with } A = (d_2 - H) \quad [4.33]$$

EXAMPLE 1.— Calculation of X , knowing the nominal diameter d and the diameter of the gauges Φ :

$$\begin{aligned} X &= A + 3\Phi; \text{ with } d_2 - H = d_2 - H + 3\Phi = (d) - \left(\frac{3}{4}\right) \cdot H = d - \left(\frac{3}{4}\right) \cdot H - H + 3\Phi \\ &= d - \left(\frac{7}{4}\right) \cdot H + 3\Phi \end{aligned}$$

EXAMPLE 2.— Determine an error of the medium diameter d_2 , occurring on a measurement X' , on a gauge. We have $d'_2 = X' - 3\Phi + H$ and $d_2 = (d) - (\frac{3}{4}) \cdot H$, hence the difference: $\delta = (d'_2 - d_2)$ indicates the error of execution measured diametrically.

4.3.1. *Laboratory control of the conicity with balls and gauges*

4.3.1.1. *Objectives and goals of the laboratory*

- To manipulate geometric concepts of prismatic surfaces.
- To control the straightness and the conicity with the diameter of the gage.
- To note the accuracy of measurement resulting from the mechanical metrology and to compare it with the same measurement resulting from three-dimensional (3D) metrology.
- To understand the principle of generating a cone on machine-tool.

The goal is to control the conicity by means of a mechanical comparison and, then proceed similarly using a profile projector to finally make a realistic comparison of the result accordingly. Then, it is useful to make a comparative analysis and make the argumentative scales showing how the overall uncertainty is different from one method of control to another. Some dimensions of this piece are deliberately omitted.

4.3.1.2. *Conicity by mechanical metrology (conventional)*

- Explain, briefly, the fundamental principle of control and measurement of a male cone and a female cone, using cylindrical gauges and calibrated balls. For the male cone, we use the machined workpiece. For the female cone, the student will receive a laboratory specimen as well as a set of balls and blocks, during the experimentation (Figures 4.17 and 4.18).
- Draw the female cone to be used as part of this laboratory.
- Carry out an appropriate and detailed mounting for the two cones proposed above.
- Perform the same experiment using the CMM or the profile projector.
- Based on provided parts, perform the missing measurements.
- Dimension properly the parts with all the required tolerances by correct drawing (GPS).
- Make a correctly dimensioned drawing including the geometric and the trigonometric relations.
- Present and discuss the numerical calculations.
- Conclude with discussion of results and personal comments.



Figure 4.17. Control of the male conicity of a chuck manufactured locally (engineering drawing)

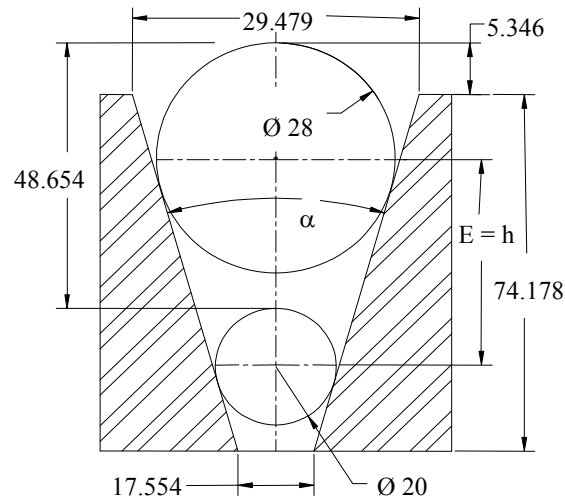


Figure 4.18. Control of female and male conicity (interior and exterior)

NOTE.— The tolerances accepted are in the order of ± 0.01 mm or 0.0004 in.

4.4. Controls of cones on machine-tools

Cones (tapers) as cylindro-conical surface forms are often used in machine tools [POI 66]. We cite, drills and live centers, standardized taper-shaped tails. The cylindro-conical form is deliberately chosen for its strong capacity for adhesion to the machine and it ensures accurate alignment (tool with the axis of the bore), smooth and free of any traces of oil. There are three basic methods for turning conical forms, namely:

- using the swivel slide,
- using lateral displacement of the tailstock, and
- using telescopic device (rule to follow of which some lathes are equipped).

4.4.1. Method of swivel slide

Swivel slide method is common in cases of chamfering. Of course, there is full autonomy on the angle but not the length because of the size of the machine-tool. If the angle α is neither specified on the manufacturing drawing nor on the machining range, then we should calculate it as shown schematically in Figure 4.19.

As shown in Figure 4.19, we should draw, based on the small diameter d , two parallels to the axis of the piece, this gives rise to two equal right triangles (Δabc) of which the side (bc) is opposite to the angle α and can be written as:

$$\tan(\alpha) = \left(\frac{bc}{ab} \right) \text{ knowing } (bc)$$

then:

$$\tan(\alpha) = \frac{(D-d)/2}{l} \quad [4.34]$$

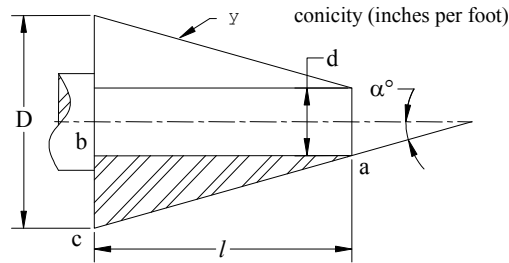


Figure 4.19. Internal conical turning

We calculate the value of the angle α using formula [4.34]. Let us do a simple explanatory exercise. The taper ring gauge must be machined on a conventional lathe, whose dimensions are shown on the drawing (Figure 4.19). It will be supported on a jaw-turntable. Then, we adjust the swivel slide relative to the axis of the parallel lathe to bore this cone. Using formula [4.34] and considering $D = 1$, $d = 0.625$, and $l = 1.5$, let us find the angle α .

NUMERICAL APPLICATION.—

$$\tan(\alpha) = \frac{(D-d)/2}{l} = 0.125 \text{ rad} \Rightarrow \alpha = \begin{pmatrix} 7 & \text{deg} \\ 9 & \text{min} \\ 43 & \text{sec} \end{pmatrix} = 7^\circ \text{ nearly}$$

Final result: $\alpha = 7$ degrees, 9 minutes, and 43 seconds. When the drawing suggests the conicity in inches per foot, then we should divide [4.34] by 24:

$$\tan(\alpha) = \frac{\text{conicity in inches/foot}}{12 \times 2} = \frac{\text{conicity in inches/foot}}{24} \quad [4.35]$$

4.4.2. Method of lateral displacement of the tailstock of a lathe

The lateral displacement method (setting over the tailstock) is still used in machining workshops. It is used particularly for non-pronounced cones. Components mounted between centers can be turned in conical shape (in both lateral senses of displacement, depending on the chosen direction of the cone). We assume that the piece is conically turned over a length that is exclusively singular to its length (Figure 4.20). To ensure that the generatrix of the cone is parallel to the axis of the lathe, and also parallel to the path of the tool, the displacement δ of the tailstock must be equal to one-half of the difference between D and d .

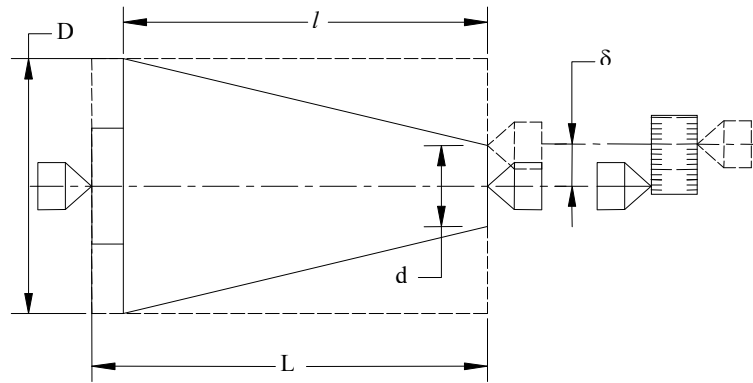


Figure 4.20. Schematization of the piece after the lateral displacement in conical turning

There are various methods for measuring the displacement effects over the tailstock of a lathe. We can measure this movement either by placing a ruler between the points (Figure 4.20) or by using a comparator with a non-inclinable probing needle, that is, a transversal needle in contact with a perfectly smooth surface.

From [4.34], let $\delta = \left(\frac{D-d}{2} \right)$; conicity on imperial foot $\rightarrow CO_{\text{inch}} = \left(\frac{D-d}{l_{\text{conical}}} \right)$

If the cone had been extended to a certain length L , the expression of conicity in inches would be affected by the multiplier and would therefore be:

$$CO_{\text{inch}} = \left(\frac{D-d}{l} \right) \times L \quad [4.36]$$

where D is the largest diameter of the cone at some point (given); d is the smallest diameter; l is the length of the conical portion; and L is the total length of the piece.

Note that all these dimensions are in inches. Since the tailstock should be moved by only one-half the difference of diameters ($D - d$), the formula of displacement (δ) of the tailstock is then written as:

$$CO_{\text{inch}} = \left(\frac{D - d}{l} \right) \times \left(\frac{L}{2} \right) \quad [4.37]$$

Exercises:

1. Calculate the conicity CO, in inches/foot, of a male cone assuming that the large diameter $D = 3/8$ in., $L = 7\frac{1}{2}$ in., conical $l = 1\frac{1}{4}$ in., and the smallest diameter $d = 7/8$ in.
2. Calculate the necessary displacement for the tailstock of a conventional parallel lathe (in inches) to machine a male cone. Deduce the large diameter D assuming: the conicity CO, in inches/feet $C = 1\frac{1}{2}$, $L = 4\frac{1}{2}$, conical $l = 2\frac{3}{4}$ in. and the smallest diameter $d = 9/16$ in.

Solution of calculations on a tapered reamer: let us calculate δ , the displacement of the tailstock:

with $D = 1.375$; $d = 0.875$; $l = 4.25$; $L = 7.5$ and using [4.37], then $CO_{\text{inch}} = 0.4410$ in.

Assuming the case where it is the conicity to the foot that would be given as in Exercise 2, then the displacement calculated above will take the following expression:

$$\text{If } L = 4.5; CO = 1.5 \text{ then, } \delta = \left(\frac{\text{conicity per foot}}{12} \right) \times \left(\frac{L}{2} \right) = 0.28125 = \frac{9}{32} \text{ in.}$$

The diameter (D) will be deduced from expression [4.36].

We do not intend to explain here the third method (telescopic device of the rule to follow) of the realization of a cone on conventional machine tool. The reason is that this method of calculation is similar to that previously explained, that is, calculating the angle or conicity at the foot. However, the practical method is more interesting from the standpoint of the rigidity of the assembly and stability. For more details, we can refer to the machine tools manuals.

4.5. Control of flat surfaces

Flat surfaces are realized with precision because of the contact of parts to each other. In machining, for example, the goal of the precision of flat surfaces is to provide joint tightness, an accurate relative positioning (marble), and a guidance

of motion (tables and slides guides). A flat surface is geometrically delimited by at least three points.

The literature [CAS 78, KAL 06] defines the flat surface as “a surface such that, given any two points on the surface, the surface also contains the unique straight passing through these points.” This definition is a bit subject to question, when we know the control of the material plane, when assessing defects of straightness and flatness. A flat surface is, in fact, an unlimited surface that contains any straight line joining two of its points A and B . The plane is two-dimensional. The generalization of the plane to higher dimensions is called a hyperplane. The angle that intercepts the various planes is called dihedral (polyhedral in the case of a multiplane). To find the equation of a plane, we need a point on the plane to locate it in space and then a vector orthogonal to the plane to determine its direction (Figure 4.21).

Let r be an arbitrary vector starting from the origin. The vector $(r - P_0)$ is in the plane orthogonal to a . This is expressed mathematically as $a(r - P_0) = 0$.

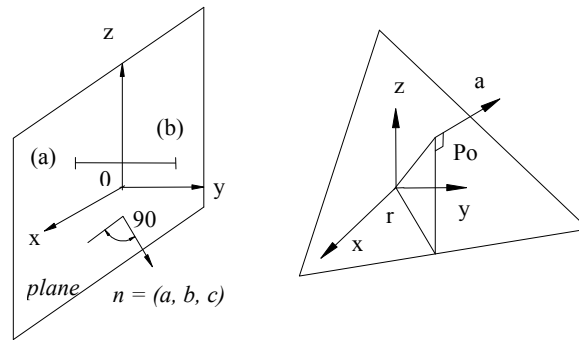


Figure 4.21. A plane containing an arbitrary straight line (ab)

This equation is never studied under this form. Solving this equation gives rise to the mathematical expression of the plane represented by its straight line d :

$$d = a_1x + a_2y + a_3z \quad [4.38]$$

A dihedral angle is formed by two half-planes, resulting from the same straight line (ab).

4.5.1. Properties of a dihedron

The perpendiculars (Ox) and (Oy) traced in each plane from a point O belonging to ab , define the straightness of the dihedron: two dihedra are equal if their

rectilinears are equal. A plane may drag on itself. Two planes are superposable (Figure 4.22).

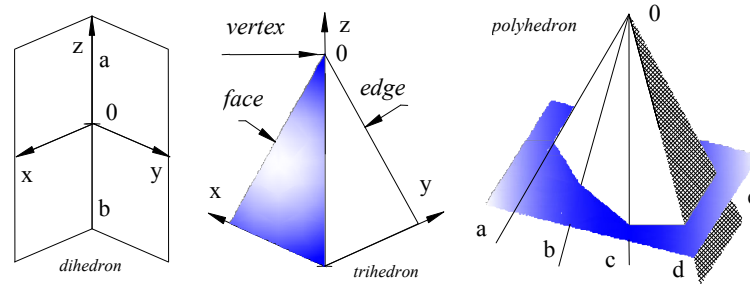


Figure 4.22. Dihedral angle → trihedral angle → polyhedral angle

After this brief overview of the plane, we address the control of flat surfaces.

4.5.2. Control of large flat surfaces

4.5.2.1. Verification of a plane with a spirit level (bubble level) [MIT 00]

We use the bubble level to indicate whether a surface corresponds to the angle of reference. The device contains – in a small window – a transparent tube partially filled with ethanol because of its low freezing point (-114°C), in which an air bubble is enclosed. Two lines indicate the position where the bubble should be positioned to match the level. The toroidal curvature of the tube allows the adjustment of the bubble between its bearings.

To use the spirit level, it suffices to simply place it on a flat surface. If the bubble is opposite to the mark, the surface is considered horizontal. The bubble occupies the median part between the markers r_1 and r_2 . The curved flask of radius R is incompletely filled with ether (alcohol or carbon disulfide). L is perfectly the flat length of total frame of the base plate.

From Figure 4.23, we can clearly see that the bubble C has a center of curvature O and the segment AB represents the horizontal position. When the base plate bends, it moves to $A'B'$ but the bubble remains at C while the initial position was at C' . The ensemble will be rotated at a small angle α called amplification factor:

$$\alpha = \left(\frac{d}{h} \right) = \left(\frac{R}{L} \right) \quad [4.39]$$

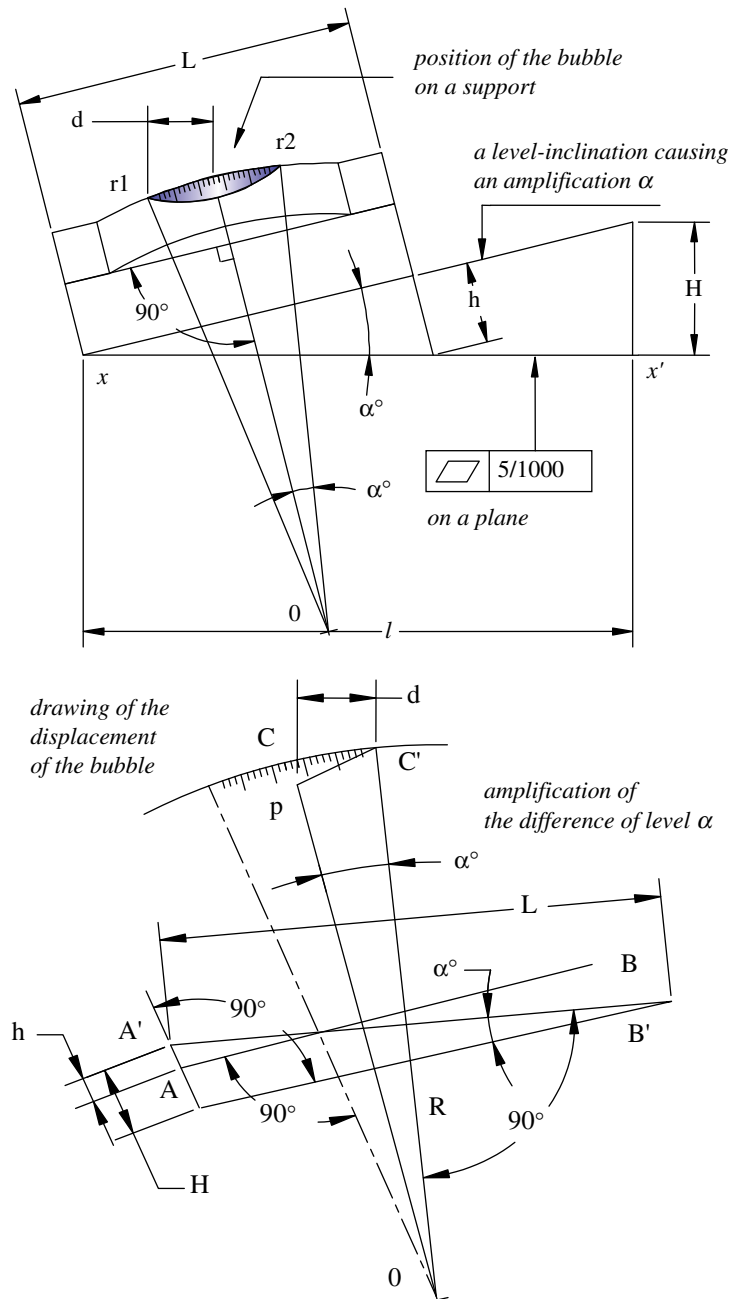


Figure 4.23. Classic displacement of the bubble and amplification α of the difference of level (experiment simulated in a workshop)

The bubble level has a function to control the horizontality of a plane. It is therefore used to measure the defects of a plane by elements. Manufacturers [MIT 00] of measuring devices, such as the spirit level, suggest an R between 10 and 60 m and L between 150 and 300, as well as a normal division (1 division = 0.1 mm/m) ordinarily corresponding to a difference of level of 0.05 mm/m. *Slope levels* present two grooved base plates, often with fine adjustment (vernier with 1 division = 10 mm) and for the flask (1 division = 1 mm). The methods described on the control by level of precision, in this section, are not complete. Temperature may change the length of the bubble, sometimes distorts the flask and the support. For this reason, it is often suggested to perform measurements in isotherm environment. Even under normal conditions of measurement, a level can sometimes show difference in the position of the bubble. This difference is due to *hysteresis* consistent with rheological phenomena (adhesion and viscosity).

Manufacturers of measuring instruments [MIT 00] recommend identifying, by turnaround, the exact position of “zero” on the graduation of the level of precision. we note, however, that the degree of development of today’s levels allows many uses.

Example: $L = 200$ mm (length of level), $R = 50$ m (radius of the flask). The amplification factor α will be

$$\alpha = \left(\frac{R}{L} \right) = \left(\frac{50,000}{200} \right) = 250 \text{ times } \alpha = \left(\frac{d}{h} \right) = \left(\frac{R}{L} \right) \Rightarrow d = h \times \left(\frac{R}{L} \right) \leftarrow \text{see sketch.}$$

The distance between two divisions for $h = 0.05$ is

$$d = 0.05 \times \left(\frac{50 \text{ m}}{1 \text{ m}} \right) = 2.50 \text{ mm}$$

Assuming that this interval contains five divisions spaced 0.05 mm but visible (with magnifying glass), thus each spacing would be equivalent to one variation of level $(0.05/5) = (0.01 \text{ mm/m})$ or 0.002 per 200 mm length of the level in Figure 4.23. Based on the foregoing, we can note that the level is a precision instrument. The measurement uncertainty is around $\pm 10 \mu\text{m/m}$. The angle of inclination is often very low (about 3'). This result is not negligible in view of the classic metrology based on marble and prototype.

4.5.2.2. Sensitivity through an example

The sensitivity of a level of precision with a bubble is represented by the function $\tan(\alpha)$ which gives the displacement of the bubble equal to a division (Figure 4.23). Let us reconsider equation [4.39] under this form: $\tan(\alpha) = (d / R)$.

When we assign to d the unit value (1) of a division unit and assuming additionally that $d = 2$ mm and $R = 50$ m, we then find:

$$\text{For } d = 2; R = 50,000; \text{ from [4.24], } \alpha = \left(\frac{d}{R} \right) = 4 \times 10^{-5} \text{ rad} = \begin{pmatrix} 0 \text{ deg} \\ 0 \text{ min} \\ 8 \text{ sec} \end{pmatrix}$$

The slope m represented by this value has a height H when $L = 1$ m (10^3 mm). As shown in Figure 4.23, for $d = 2$; $R = 50,000$ mm, we find:

$$\text{from [4.24], } \alpha = \left(\frac{d}{R} \right) = 4 \times 10^{-5} \text{ rad, and for } L = 1,000 \text{ mm, we calculate } H$$

$$\text{Method 1: } H = \left(\frac{d}{R} \right) \times L = 0.040 \text{ mm; Method 2: } H = \tan(\alpha) \times L = 0.040 \text{ mm}$$

We note that the sensitivity and the slope per *meter* correspond to the displacement of the bubble equal to one division and are inversely proportional to the radius of curvature R of the flask. The more this radius increases, the more the level is sensitive and therefore likely to detect low slopes. With the level values, we see that at a displacement of the bubble of one division (i.e. 2 mm) corresponds to a slope of 0.04 mm/m. If the level had $l = 250$ mm, the height h would have, in terms of divisions, the following [4.25]:

$$H = l \cdot \tan(\alpha) = 250 \times 4 \times 10^{-5} = 0.01 \text{ (1/10 division)}$$

We can detect defects (Δ) in the order of $(0.01) \times (0.1) = 0.001$ or 1μ .

HYPOTHESIS. Considering the hypothesis where we would be forced to consider half the radius of curvature, that is, $R = 25$ m instead of 50 m, all values would have been doubled (Figure 4.24). As such $\tan(\alpha) = 8 \times 10^{-5}$, and accordingly $H = 0.08$. The verification of the accuracy of H allows the following:

$$\text{for } L = 1; d = 2, \text{ and } R = 25 \Rightarrow H = \left(\frac{d}{R} \right) \times L = 0.08 \text{ mm}$$

We simulated an example to confirm or refute the accuracy of levels modeled using formula [4.39]. We conclude that this relationship is not only perfect but also it confirms the above hypothesis.

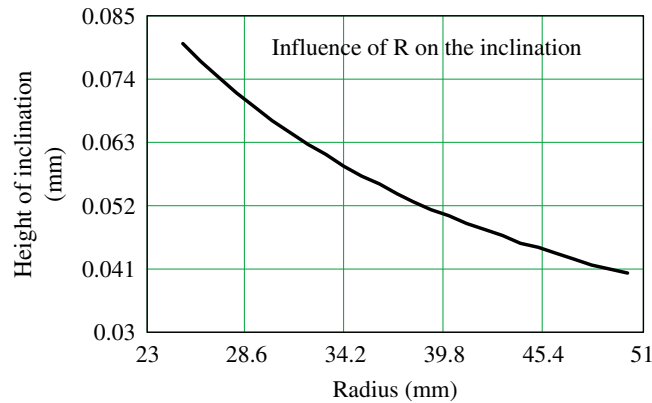


Figure 4.24. Curve based on the calculation of difference of level (“inclination”)

4.6. Control of cylindrical surfaces (of revolution)

We present here some methods for inspecting cylindrical surfaces of revolution (with circular cross-section). We will not address the appropriate apparatus. We deliberately deal with the measurement and the localization of possible defects on the cylindrical surface. In fact, the plane is also a special case of different classes of surfaces called *cylinders*. When the word cylinder is used, we automatically think of a form of a tube (full or empty), which we call a right cylinder. However, cylinders may actually have any form ensuing from a section and it is not absolutely necessary for such a section to be closed (bounded).

4.6.1. Cylindrical surface

A cylindrical surface is generated by a straight line called generatrix, which moves parallel to a fixed direction, by constantly leaning on a fixed curve called the directrix (Figure 4.25).

4.6.2. Associated definitions

A cylindrical surface, having a circle as a directrix and a straight line perpendicular to the plane as a generatrix, is called “surface of revolution around an axis” ($z'z$) since the straight line remains at constant distance R from the axis. If we were to increase indefinitely the number of faces of a prismatic surface placed

in a cylindrical surface, the prismatic surface tends to the cylindrical surface. This property is used in some CAD software programs by modeling the cylindrical surfaces (and cylindrical volumes) with prisms of many facets. In fact, we obtain a true “faux 3D”. A cylinder of revolution (or right circular cylinder) is the volume generated by the rotation of a rectangle $ABCD$ around one of its sides (e.g. AD), which becomes the axis ($z'z$) of the cylinder. With CAD, we use either the function extrude or revolve.

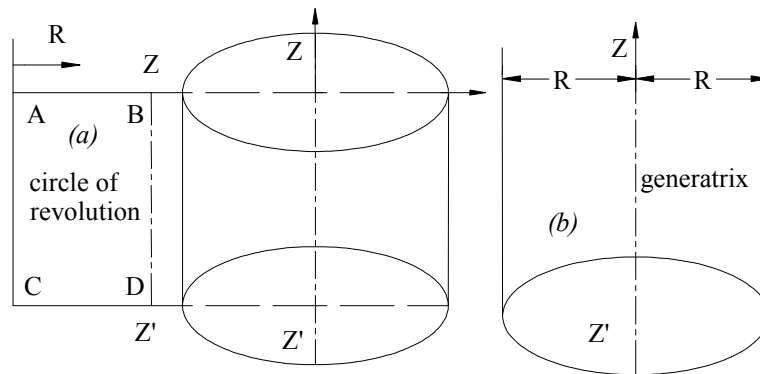


Figure 4.25. (a) Cylindrical surface of revolution and (b) cylindrical surface by extrusion

In summary, we mention the defects of cylindrical surfaces as follows:

- Non-rectilinear generatrices are due to a defect of parallelism between the cutting tool and the axis, or due to distortions.
- Non-circular section is due to incorrect mounting of the axis or the tool. In this case, the cross-section becomes elliptical, oval, or polygonal.

The detection of cylindricity defects is carried out in various ways. Mounted on a CMM, the defect of a piece is easily detected by probing eight points over the whole cylinder. We may also achieve detection by control the diameter by probing the upper generatrix of the cylinder by means of a comparator of precision sliding on a reference plane parallel to the axis of the points.

4.6.3. Cylindricity defects

Cylindricity defects [MIT 00, KAL 06] are easily detected by a micrometer even if this requires measuring several diameters on each of the concerned sections (Figure 4.26). There are four major defects.

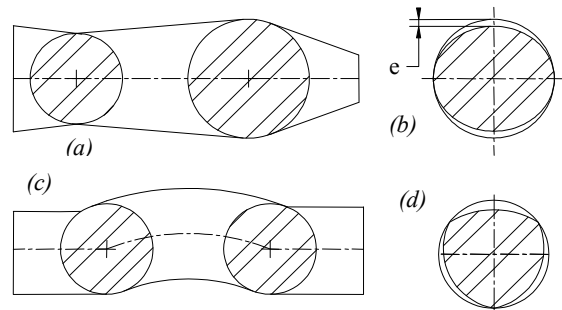


Figure 4.26. Measuring actual deviation (e) in diameter on simple support: (a) section variation, (b) defect of straightness for the axis, (c) profile defect: elliptical section; and (d) profile defect: polygonal section

To inspect these defects, we use various choices, such as the CMM, the profile projector and the mechanical comparison. We prefer the last alternative because it requires a skill from the user and also it is used both in machining shops and in metrology laboratory. Using a comparator, gauge blocks and a V-block, we carry out such inspections by mechanical comparison. We opt for this procedure to inspect spherical-shaped surfaces on flat support. This approach is simple and still used in daily practice.

4.6.4. Control of a cylinder on three contact tips on a V-block

For the overall height H and M is measurement value at the center of D of radius R , if we pass through α , then we consider the following (Figure 4.27):

$$M = (H + R + a) \text{ with } (H + a) = \frac{R}{\tan(\alpha/2)} \quad [4.40]$$

We note the influence of the angle α on the actual deviation E is as follows:

α	$E/(D-d)$
30°	1.5
45°	1.2
60°	1.07

There is also a deviation by defect on ellipse (Figure 4.27(b)) and a deviation by excess on the polygon (Figure 4.27(c)). The measurement of the diameter is at the center of cylindricity defects. Thus, the measurement of the diameter would be sufficient to assess the defect.

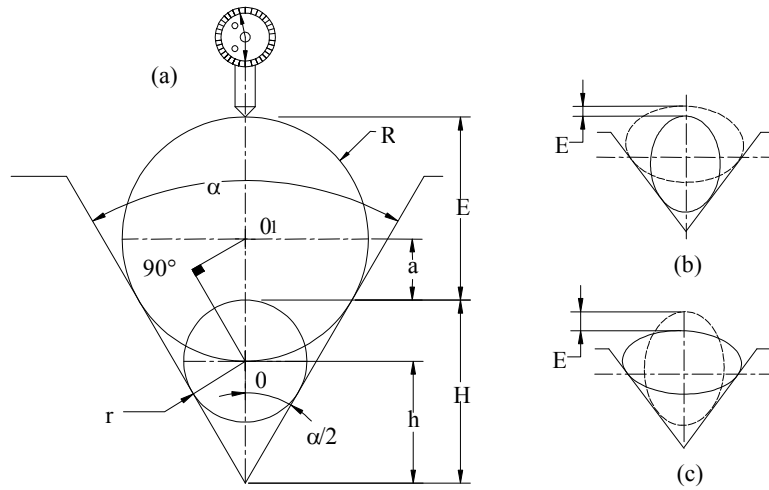


Figure 4.27. Classic measurement of the diameter deviation by amplification on V-block

We limit ourselves to expose the extent of the inner diameter by direct measurement where we may use a gauge caliper or a micrometer. Indirect measurement is performed by comparison to a standard (prototype), where the obtained deviation provides information about the degree of geometric incorrectness. We use the dial comparator to derive the deviation δ as shown in Figure 4.27(a).

We can also amplify the reading of the deviation by placing the part materialized by O_1 on a V-block. The calibration will be based on a cylinder of reference or gauge of verification materialized by O as shown in Figure 4.27(b).

We now present the mathematical approach that allows the assessment of the actual deviation in diameter δ .

Assumptions of calculations. Let E be a deviation in reading and compare the actual deviation δ in diameter in millimeter. From Figure 4.27(a), we find that:

$$E = (a + R) \text{ with } a = (O_1O - r) \text{ then } E = (O_1O - r) + R \quad [4.41]$$

Consider the triangle (O_1RO) and put (in scalar):

$$O_1O = \frac{R - r}{\sin(\alpha / 2)} \quad [4.42]$$

Substituting equation [4.42] in [4.41], we obtain [4.43]:

$$E = \frac{R-r}{\sin(\alpha/2)} + (R-r) = (R-r) \times \left(\frac{1 + \sin(\alpha/2)}{\sin(\alpha/2)} \right) \quad [4.43]$$

From equation [4.43], we deduce $(R-r)$,

$$(R-r) = E \times \left(\frac{\sin(\alpha/2)}{1 + \sin(\alpha/2)} \right) \quad [4.44]$$

Knowing that the actual deviation (δ) is expressed by the equation:

$$\delta = (D-r) = 2 \times (R-r) \quad [4.45]$$

we ultimately get the formula of δ :

$$\delta = 2 \times E \times \left(\frac{\sin(\alpha/2)}{1 + \sin(\alpha/2)} \right) \quad [4.46]$$

Typically $\alpha = 60^\circ$, hence $\alpha/2 = 30^\circ$, consequently $\sin(\alpha/2) = 1/2$. Substituting these numerical conditions in [4.46], we find the value of δ :

$$\text{For } \left(\frac{\alpha}{2} \right) = 30^\circ; \sin\left(\frac{\alpha}{2}\right) = \left(\frac{1}{2}\right); \text{coefficient of correction} = \left(\frac{2}{3}\right)$$

$$\text{and } E = \left(\frac{3}{100}\right)$$

$$\delta = 2 \times E \times \left(\frac{\sin(\alpha/2)}{1 + \sin(\alpha/2)} \right) = 0.02 \text{ mm and if } \alpha = \left(\frac{\pi}{2}\right) \Rightarrow \delta = 0.025 \text{ mm}$$

$$\text{or } \delta = \left(\frac{E}{1.2}\right) \text{ mm}$$

It should be noted that the actual deviation δ has the same sign ($\pm$) as the reading (Figures 4.28 and 4.30).

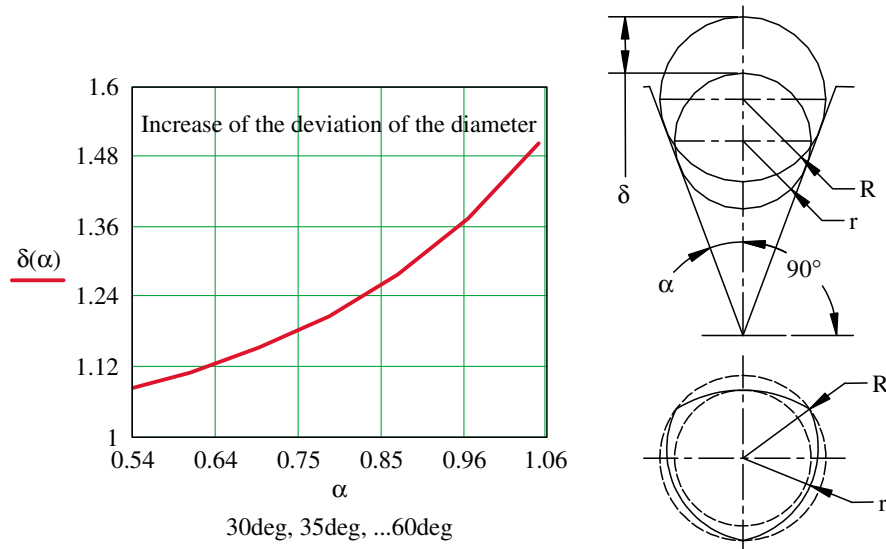


Figure 4.28. Measurement and calculation of $\delta(\alpha)$ and the diameter deviation by amplification on a V-block

EXERCISE OF APPLICATION 1.– From the literature [OBE 96], consider:

$$\delta(\alpha) = (R - r) \times \left(1 + \frac{1}{\cos(\alpha)} \right) \quad [4.47]$$

EXERCISE OF APPLICATION 2.– From the technical literature [OBE 96], consider:

$$R = \left(\frac{(1 - d^2)}{8 \times (H - d)} \right) + \frac{H}{2} \quad [4.48]$$

Problem related to a concave form for the following experimental data:

$$l = 25 \text{ mm}, d = 3 \dots 10 \text{ mm } H = 12 \text{ mm } R(d) = \left[\frac{l - d^2}{8(H - d)} \right] + \frac{H}{2}$$

Position of the second experimental problem (convex form, Figure 4.29) in millimeter:

$$R = \left(\frac{1 - d^2}{8 \times d} \right)$$

[4.49]

$$l = 25 \text{ mm}, \quad d = 3 \text{ to } 10 \text{ mm}, \quad H = 12 \text{ mm}, \quad R(d) = \left(\frac{1 - d^2}{8d} \right)$$

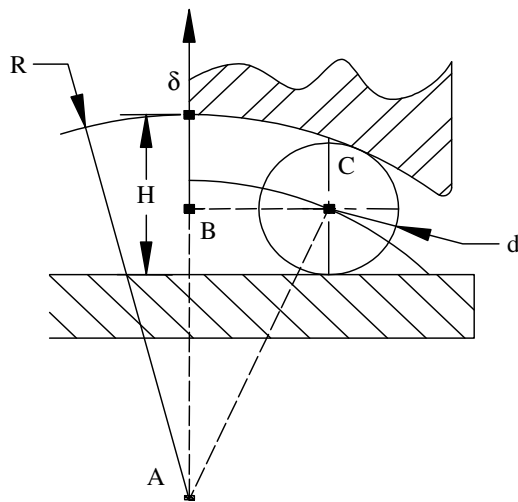
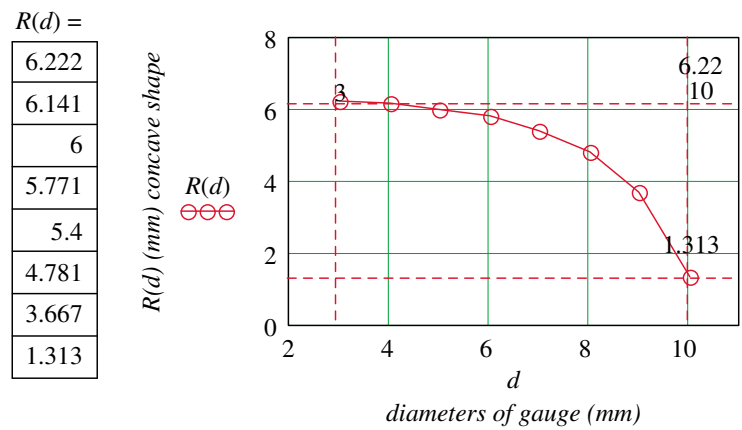


Figure 4.29. Method of concave gauges sets to find R

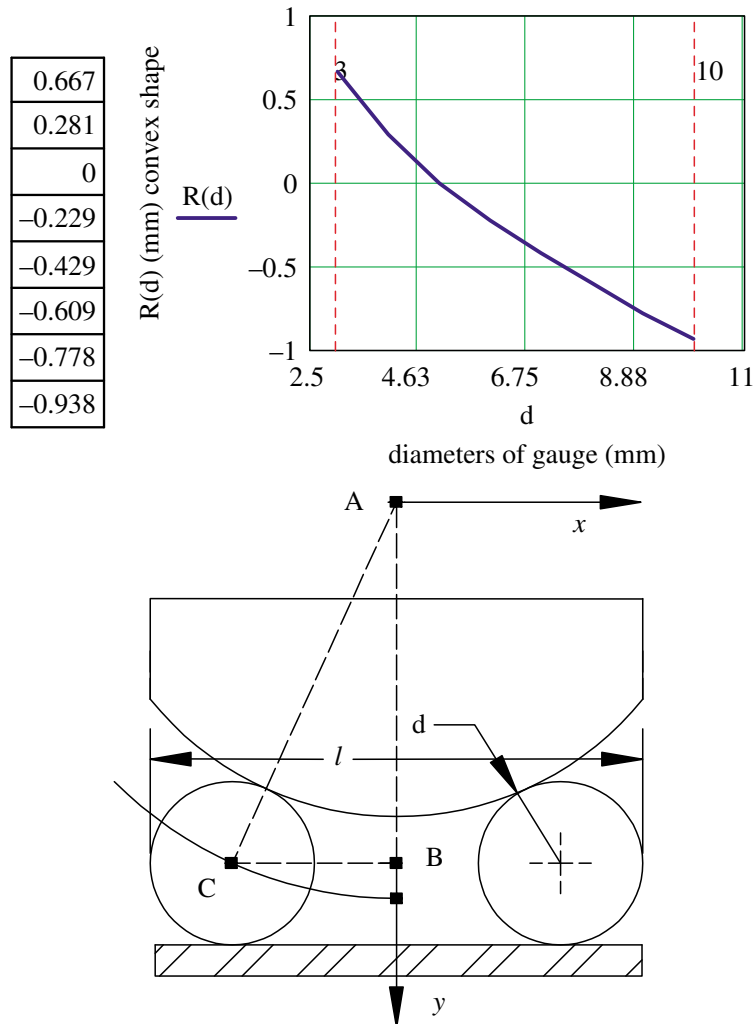
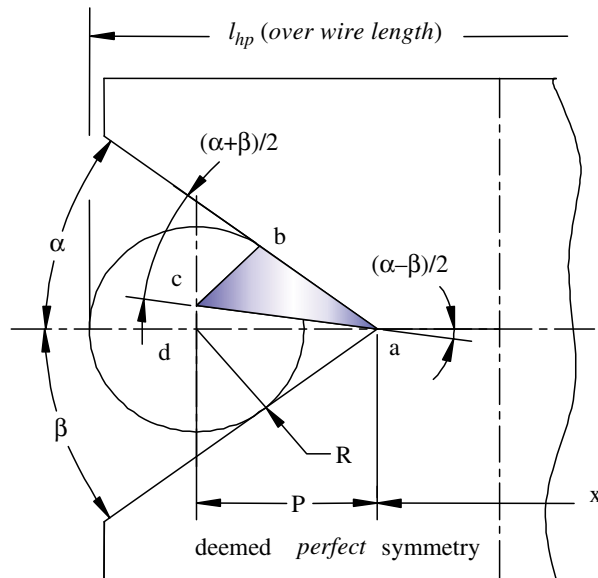


Figure 4.30. Method of convex gauges sets to find R

EXERCISE OF APPLICATION 3.— Consider: $l_{ol} = f(R)$ and write the equation expressing the distance p as a function of the radius (in inches) of gauges used in the workshop. l_{ol} is the overall length easily measurable since it is accessible (Figure 4.31).

$$f(R) = x + \left(2 \cdot R \times \cos\left(\frac{\alpha + \beta}{2}\right)^{-1} \times \cos\left(\frac{\alpha - \beta}{2}\right) + 1 \right) \quad [4.50]$$



For $x = 1.825$ in., $\alpha = 45^\circ$, $\beta = 37^\circ$, $R = 0.5, 0.7, \dots, 2$ in.

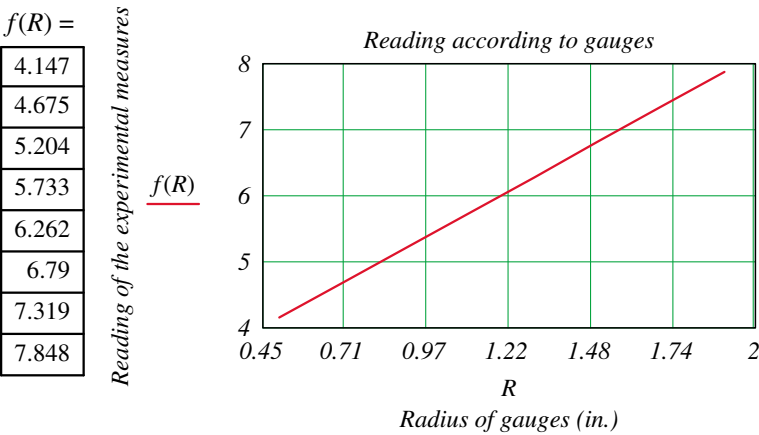


Figure 4.31. Set of gauges used to inspect an angular form (on V-block) $l_{ol} = f(R)$

4.6.4.1. Direct measurement of the form

Here, the direct measurement is performed by using the following:

- Sliding gauge: It cannot reach the ends of the bore.
- Inside micrometer: It cannot reach deep bores.

- Gauges of three contact tips (120°): The micrometric screw that controls a conical core deviates the contact ends. Extensions would be mounted to reach deep bores.
- CMM for non-deep diameters.

4.6.4.2. Indirect measurement

In terms of indirect measurement, this may be used only after applying a calibration, which requires a plug gauge with a jet connected to a pneumatic micrometer or a comparator with two opposing sensors with a centering skid (Figure 4.32).

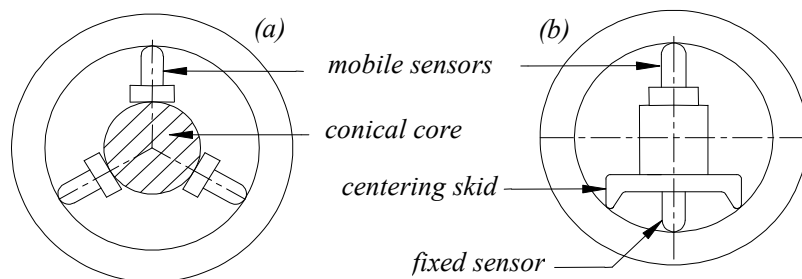


Figure 4.32. (a) Three-sensor gauge 120° (star-shaped for boring); (b) gauge with two opposed sensors and a centering skid (boring) (see Figure 3.31 and Tables A2.1 in Appendix 2)

The measurement of form defects is also carried out over a flat support. We also use this procedure in the control of surfaces of spherical shapes. In the case of the flat support, we place a cylinder on two flat supports (identical) and rotate the piece manually under the probe against the vertical faces. The flat contact surface leans on the upper generatrix of the cylinder and the comparator, and then we “measure” (control) the radial runout or the defect of profile depending on the variation of the reading, imposed on the needle. For an actual deviation (e) or a deviation on actual magnitude (elliptical cross-section) and (e') for an uncertain deviation (polygonal cross-section).

Ovalization defects generate a characterized variation but the polygonal section does generate a slight motion of the comparator’s needle. Moreover, it would be wise to take *measurements on V-support*: the cylinder will lean on one of two identical V-blocks and the probe is positioned on top of the generatrix of the cylinder. Deviations due to a possible ovalization or even an unusual polygonal shape would be different from the actual deviation because of the displacement of the support points in the V-block (see Figure 3.14).

4.6.5. Practical control of the straightness of the generatrix of a cylinder

The most traditional conventional control is to place a ruler against the generatrix and observe whether a light could be filtered. This practice does not require any particular skill in metrology, but it remains effective in the first place. In the remaining cases called simple, we should move a probe along the generatrix and see either the eccentricity or the runout, as indicated in Figure 4.33.

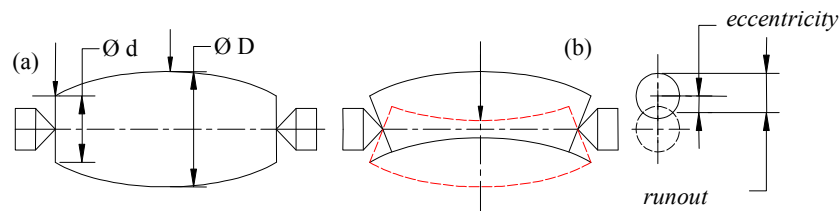


Figure 4.33. Conventional control of the straightness of the generatrix

4.6.6. Control of the perpendicularity of the generatrix and the drive circle

It is recognized that defects straightness of the axis cause an inevitable form defect. The most common defect is the net curtain (lateral runout). In dimensional metrology, by mechanical comparison, we can perform a “truing” simply by placing the probe of a comparator against the flat surfaces of the cylinder as shown in Figure 4.34(a).

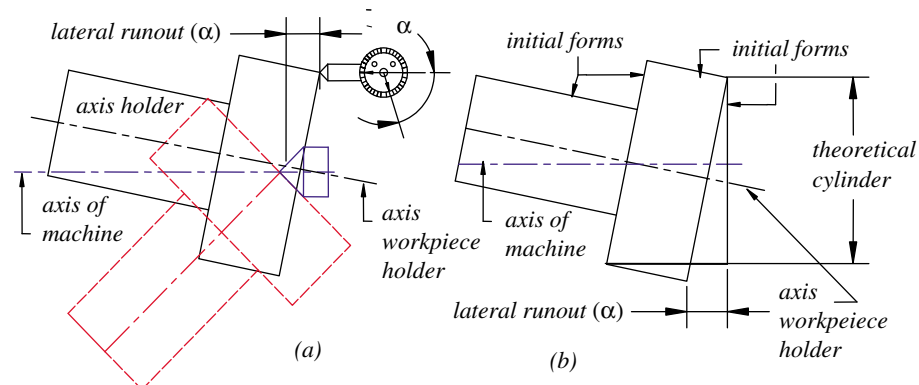


Figure 4.34. Conventional inspection of radial runout

A defect in the positioning within the chuck causes a misalignment of the axis and thereby a defectiveness in turning. Even by using a four-jaw-chuck on such a piece, it would cause a defect of form as shown in Figure 4.34(b). The radial runout of the faces is due to a straightness defect.

NOTE: Note the position of the axis of machine, the position of the workpiece axis, and the workpiece holder axis relative to the theoretical cylinder. There are three axes. In dimensional metrology applied to mechanical manufacturing, it is strongly advised not to refer to the axis during geometrical tolerancing. We should rather refer (direct the arrow indicating the reference) to a generatrix (the case of rotating cylinders).

4.7. Control of surfaces of revolution with spherical forms

In this chapter, we present some methods of inspection of spherical surfaces of revolution. We discuss the spherical surface generation, the most common surface defects, and some control processes by spherometry. Before that, let us discuss some basic definitions of a spherical surface. A spherical surface is a surface generated by a semi-circumference that moves 360° around its diameter (Figure 4.35).

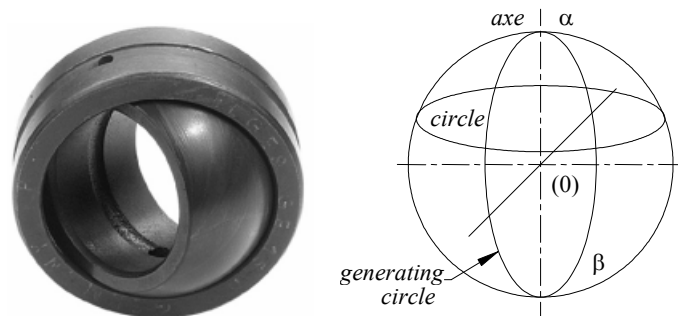


Figure 4.35. *Geometric representation of a spherical surface*

ASSOCIATED DEFINITIONS.— A sphere is defined as a volume generated by rotating a semi-circumference around its diameter. As for the control of previous surfaces of revolution, spherical surfaces are widely used in mechanical engineering (knob, cap, ball joints, etc.). Certainly, spherical forms are not as much exploited as cylinders (axis and parallel generatrices); however, they remain important. These surfaces are named after the Greek (*sphèra*). Commonly used apparatus for this purpose are the spherometers.

4.7.1. Description and functioning of a spherometer

4.7.1.1. First approach

Spherometer is used to measure small thicknesses, radii of curvature of spherical surface (concave and convex), such as a mirror or a face of a lens, etc. The spherometer also allows the measurement of thicknesses and calibration of gauges. A rigid support leans on three measurement pins (Figure 4.36). The friction button initiates the graduated drum called as “limb.” A graduated rule (index) fixed on the support allows the rotation of the screw to be measured as well as a direct measurement of the translation. For example, assuming the pitch of the screw is 0.5 mm and the graduated drum has 500 divisions angularly equidistant, it would then be possible to measure the translation to one micron nearly. We place the spherometer on a solid and flat surface and we raise the pin into contact with the plane. Then, we note the position of the graduated drum as shown schematically in Figure 4.36.

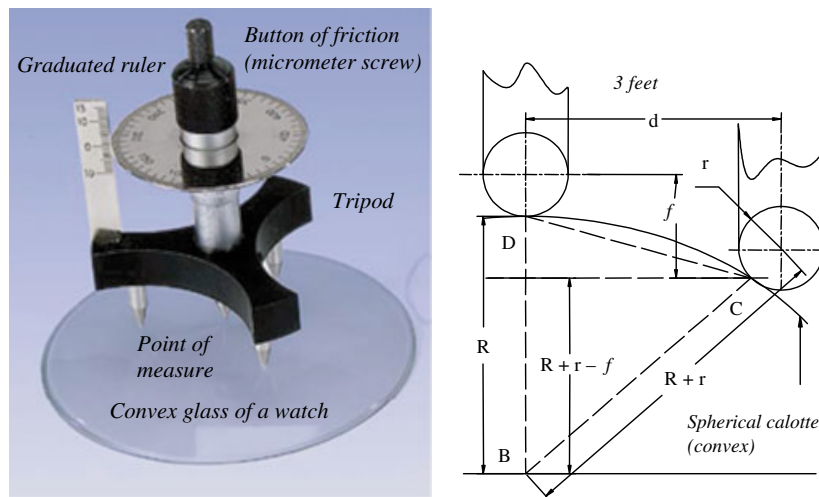


Figure 4.36. Schematization of the measurement of a crown with a spherometer

4.7.1.2. Measurement of the radius of curvature

To measure the curvature of radius ρ , we place the instrument on the spherical surface (convex in our case). The contact points of the tripod form an equilateral triangle (BCD) and define the basis of a spherical cap (Figure 4.36) whose screw tip will reach the summit. A micrometric screw, whose motion is controlled by a button of friction, passes through the center of the tripod. We measure the arrow f as thickness. The ruler (vertical index) in millimeters is used to count the turns and

serves at the same time as an index to the graduated drum. The radius r of the spherometer (a one-pitch distance to the center) being known data, we measure the radius of curvature R . To determine the radius of curvature of a spherical cap, we place the spherometer so that the feet and the pin simultaneously touch the surface.

For a convex surface, we use the sign $(-)$ and for a concave surface, we use $(+)$. The accuracy is restricted to the assessment of the contact of the feet and the measuring pin with the spherical surface. The reproducibility of the button of friction is of $(\mu \pm 3)$.

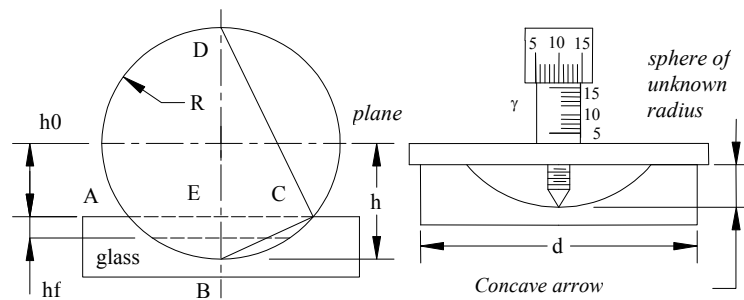


Figure 4.37. Schematic representation of the measure (concave) with a spherometer

4.7.1.3. Manipulation for measuring a curvature radius

Using a caliper, we measure the distances d separating two feet of spherometer. Based on its mean value, we deduce the radius $(\rho \pm \Delta\rho)$ of the circle passing through the three feet. The radius of curvature $(r \pm \Delta r)$ is common to the ends of the feet and the measuring pins.

As the micrometric screw is raised off, we place the spherometer on the convex side of the glass cap (Figure 4.36). We thrust the screw by turning carefully the button of friction (and not the graduated drum) until the measuring pin touches the cap (we will hear a clicking sound). We examine the division of the graduated drum. Then, we place the spherometer on a flat glass surface, and continue turning the screw in the same direction until being in contact with the measuring pin. We count the number of screw-turns and observe the position of the graduated drum. By subtraction, we deduce the outer arrow f_{out} of the spherical cap. We repeat these operations in different points, and calculate the mean value of f_{out} and its uncertainty Δf_{out} .

Starting by placing the spherometer on the flat surface, and then on the concave face (Figure 4.37) of the glass cap, we determine, similarly, various inner arrows f_{in} . We calculate the mean value of f_{in} and its uncertainty Δf_{in} .

We calculate the outer curvature radius ($R_{\text{out}} \pm \Delta R_{\text{out}}$) and the inner curvature radius ($R_{\text{in}} \pm \Delta R_{\text{in}}$) of the spherical cap. The difference between these two values gives the thickness of the cap ($e \pm \Delta e$). We compare this thickness with the thickness measured directly, using a caliper.

The core of a spherometer is a screw with a small pitch. It ends with a pin whose base of support is a radius r . In Figure 4.37, the vertical traverse of the pin is of ± 15 mm. The radius r of the circle of the support base is of 20 mm. When we turn the central screw, the pin moves vertically. The horizontal circle O , divided, moves against the fixed vertical scale represented by γ . The fractions of turns are read on O , in front of γ . By analogy to equation [4.50] dedicated to a convex spherical form, the radius of the concave spherical surface, in this case, is expressed mathematically as:

$$\text{Since } R^2 = r^2 + (R-h)^2, \text{ then } R = \pm \left(\frac{r^2 + h^2}{2h} \right) \quad [4.51]$$

where $d = 2r$ is the radius of the support base of the spherometer. This is the distance between the foot and the tip of the central screw; h is the vertical displacement of the pin; and R is the radius of the spherical surface (radius of curvature; all values are expressed in millimeter or inches).

To further substantiate the foregoing, we propose a workshop on spherometer.

4.7.2. Laboratory (workshop) simulated on the appropriate use of spherometer

The aim is to properly use the spherometer. To do this, using equation [4.51] as a function of h , we will plot the representative chart. The spherometer consists of a tripod with a micrometer at the center. The ends of the three fixed feet determine a flat surface. The micrometric screw can be either lowered under the plane of the tips of the feet to measure the concave surfaces or raised to measure convex surfaces. This small device is used to find the radius of curvature of mirrors or lenses.

All the accuracy of the spherometer is focused on the screw and it should never be forced during its use. Here are the sequences of manipulations to follow:

- Put the spherometer on a glass plate.
- Adjust the central rod so that the pins of the four feet is on the same plane. The latter is obtained when the tip of the rod and its reflected image coincide, that is, when there is a space between the rod and its image.
- Record the reading shown on the screw. This reading constitutes the “zero” of the apparatus h_0 .

- Make the dress pattern of the four pins, with carbon paper on a white sheet. With the calipers, measure d , the distance between the tip of a fixed foot and the tip of the central screw. Measure the distance d for the two other pins.
- To measure the curve arrow h , you must either raise or lower the screw using the same method as for the reading of h_0 . Note the reading h_f on the screw.
- Calculate the mean value d and estimate its uncertainty (see Chapter 1).
- Calculate the value of the arrow using the formula, $h = |h_f - h_0|$.
- The spherical cavity in the lens is viewed from the side.
- Knowing d and h , find the radius of curvature R of theoretical curvature of the spherical cavity. Triangles $\triangle CBE$ and $\triangle DCE$ are similar.
- Find and calculate the theoretical formula of uncertainty on $R(h)$ (see Chapter 1).

Figure 4.38 represents the results of simulated experiments

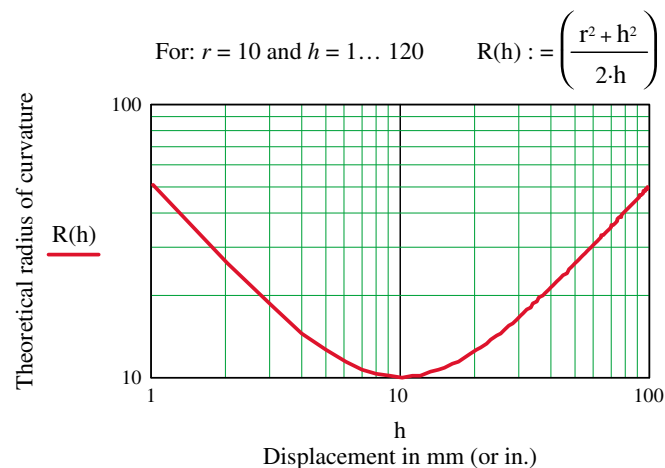


Figure 4.38. Measurement results of a (simulated) spherical surface using a spherometer

4.7.3. Control and measurement with spherometer (second approach)

For large radii of curvature, we should use a spherometer as shown in Figure 4.39. This measurement is indirect and allows deducing the radius r of the sphere. For large radii, we use the spherometer to carry out controls and derive the calculations necessary to the final measurement of the sphere diameter.

First of all, the contact of the pins of the spherometer with the workpiece allows easy reading of the diameter of the circle of circumference C . We initially assume that C is a known value. The comparator, whose probe is located at the center of the three contact tips, will help us reading directly (or by comparison) the value of the arrow f . We first perform the calibration using a block. The three contact tips are based out on the same plane where the block is located under the probe. Figure 4.39 allows reformulating the expression the diameter D_1 of the spherical surface:

$$D_1 = \left(\frac{C^2}{4 \cdot f} + f \right) \quad [4.52]$$

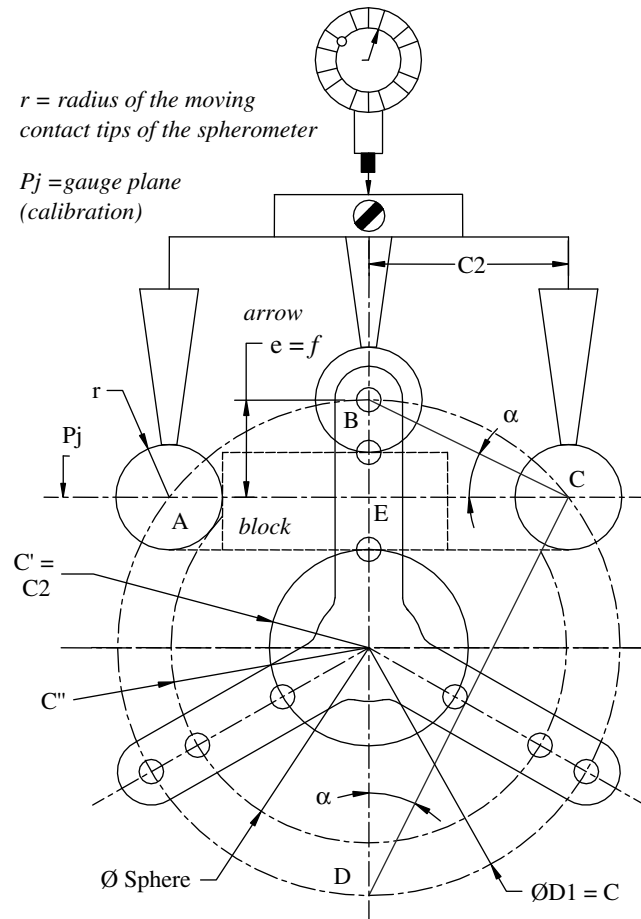


Figure 4.39. Measurement procedure of the spherical surface (concave) with the spherometer

By subtracting $(2r)$ from equation [4.51], then diameter D will become

$$D = D_1 - 2 \cdot r = \left(\frac{C^2}{4 \cdot f} + f \right) - 2 \cdot r \quad [4.53]$$

The diameter may also be calculated through the equation:

$$D = \frac{C/2}{\sin(\alpha) \times \cos(\alpha)} \quad [4.54]$$

This approach complements the mathematical formulae on the calculation the radius of curvature of spherical surfaces.

Why spherometers are less used in metrology? The presence of more and more accessible CMMs and profile projectors stands as a sufficient reason to penalize the use of conventional spherometers. Additionally, spherometers are used to measure small radii of surfaces curvature (both concave and convex).

4.7.4. Generating a spherical surface

The spherical surface is usually processed on machine-tools either by a tool of form depending on circular motion or by an envelope tool relative to the two circular movements [CAS 78, POI 66]. In the case of a tool of form, it is the cutting edge that is circular, that is, the shape of the tool (dimension tool) that imprints a spherical shape to the surface. Only a single rotation of the workpiece in a chuck is necessary. In the case of an envelope tool, the form is obtained by the combination of two movements of rotation around two perpendicular axes.

When the spherical form is generated by a tool of form, a poor adjustment of the tool leads to defects such as flattened or pointed forms, if the penetration of the tool is insufficient or if the edge is not sufficiently positioned at an appropriate height relative to the axis or even if the tool is misaligned. When the generation of the spherical shape is led by an envelope tool (two circular motions), this may result into a hollow torus for non-concurrent axes.

4.7.4.1. Detecting defects on the sphere form

The spherical shapes are specifically used for design purposes, as it requires no formal quality, that is, the dimensional aspect lapses (equilibrium mass or end of crack). Therefore, it becomes less important to care about the fact that the form tool, used for the preparation of a spherical surface, has the appropriate form. In the case of spherical forms for a specific use such as spherical joints and dial gaskets,

the verification requires considerable care to obtain the required form. Hence, the control must also be performed with care. As far as balls and rings are concerned, it becomes imperative for the control of the form to have the dimensional priority over that of the form itself. Note that here the rolling-bearings are often chosen based on their matching hence the importance of measurements.

4.7.4.2. Observation using a bezel and using measure differences in comparator

There are several possibilities for the control [CAS 78, KAL 06] and the measurement, and both function simultaneously. The apparatus of control, as such, differ depending on the degree of accuracy and the diameter of the sphere to be controlled. It may happen that we measure the radius when considering a fraction of a sphere. The form defect can be controlled using a simple tubular apparatus (bezel) at the beveled end that serves as a RS placed directly on the sphere. The linear contact should be complete or under lateral lighting. A light coming from inside the tube shows the defects of sphericity. The method is simple but has the disadvantage of being only visual as the form itself is imprecise. We may also use a V-block of three contact tips (120°) mounted in connection to a very sensitive indicator. This method is used, *inter alia*, for the control of precise balls.

4.7.4.3. Control by visual comparison with a gauge

By analogy to “Go” “No Go” gauges, the gauge used to control the spherical surface has the form of a spherical cap. It is hollow for a male sphere and ball-shaped for a circular groove (concave and convex) (Figure 4.40). Practically, the gauge is coated with a dye. It is rubbed gently against the surface being controlled; hence, the defects of form will be printed on it. It is sometimes possible to use other types of gauges such as (*max* and *min*).

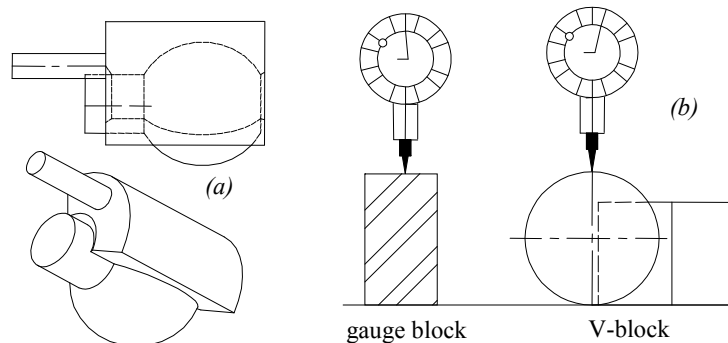


Figure 4.40. Procedure for the control of defects on a spherical surface

The control may also be done on plane support. The sphere is applied in the opening of a V-block laid flat (Figure 4.40(b)) serving as a locator. The piece rests on flat support. A comparator of high sensitivity and calibrated in advance, allows a direct reading of diameter deviations. The accuracy in these cases is significant.

To measure the defects it is necessary to, first, control the diameter of the piece, for example on V-block with three contact tips. Thus, we set the sphere on the V-block while making sure that the contact tips are in contact with the spherical surface to be controlled. Deviations δ that would appear by reading E correspond to the position of calibration on a standard ball (Figure 4.27). By analogy to the various mountings realized for gauges and balls during the control of an inner cone, we propose a mathematical expression through which we calculate the difference in the diameter deviation δ . Thus, the deviation δ is calculated using equation [4.46]. Ordinarily $\alpha/2 = 60^\circ$, thus the actual deviation takes this form:

$$f(\alpha) := 2 \cdot E \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{1 + \sin\left(\frac{\alpha}{2}\right)} \right)$$

$E := 1 \quad \alpha := 0\text{deg} \dots 360\text{deg}$

For example $\quad := 0.667 \cdot E$

$\alpha := 60\text{deg} \quad f(\alpha) := \delta \quad \Rightarrow$

	0
0	0
1	0.648
2	0.914
3	0.999
4	0.952
5	0.749
6	0.247

Regarding inner spherical surfaces, we use inside micrometers, gauges with three contact tips at 120° , or even two sensor-tips comparators (telescopic stems) with center-bottom (Figure 4.32(b)).

4.7.4.4. Summary on surfaces of revolution and angles

It has been briefly explained that the control and measurement of angles are carried out using various means. The control of cylindrical surfaces is characterized by the *straight axis* and the *circular and invariable straight/cross-section, centered on the axis*. We have, throughout this chapter, discussed the control and sometimes the measurement of the defects of straightness of generatrices. It goes similarly for diameter deviations δ . Form defects are measured separately. We have presented the measurements on flat supports or on Vs. It has been demonstrated that radial runout of faces is the result of any defect of straightness of the axis. The control of the *straightness of the axis* is also valid for conical surfaces.

Finally, we discussed the control of spherical surfaces where we addressed classical means of control, namely bezel, comparator, V-block as well as the spherometer.

We remind that the axis is not *physically attainable*. It is a mathematical understanding that helps to situate the metrologist in the “true center” of a physical entity. No apparatus in the world or any instrument or device will be able to point on a center and indicate that it *is the axis*. Clearly, the axis is an absolute necessity that mathematics has facilitated to benefit instrumentalists (metrologists). Although sometimes we attempt to point *theoretically* to the axis, for the purpose of the convenience of reading on a drawing, nevertheless the axis remains physically inaccessible.

4.8. Control of the relative positions of surfaces

The mechanical ensembles are generally positioned to meet the dimensional geometric specifications [CHA 99] underlying their functions. That is, the requirements expected from relative positions of contact surfaces require a rigorous and often a tedious control, in terms of assemblies of control. This is explained in Chapters 5 and 6. One of the best means of control is the CMM, suitable for such cases of relative positions. Unfortunately, this variant is possible only in the laboratory. For an illustrative example of constraints on descriptions of relative positions, see Figure 4.41.

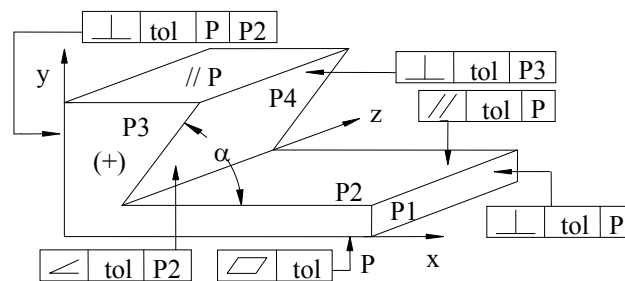


Figure 4.41. Relative positions of planes P_1 , P_2 , P_3 , and P_4 relative to the plane P

In the workshop, the controls require a small size and an ergonomomy that is sometimes hardly accommodative. The purpose of this section, which is complementary to the topics covered in Chapters 5 and 6, is to show that in the case of relative positions, we emphasize on correction of elementary surfaces after their verifications. The advantage of using the CMM in these cases is the ease in probing

surfaces and the edges of dihedral useful for the measurement and the relevant calculation. Figure 4.41 shows some cases of relative positions with tolerances of parallelism, perpendicularity, and angular position between flat surfaces. Next, we present some, non-exhaustive, selected cases on the control of relative positions between flat surfaces and other edges of dihedral. The control of these geometric constraints on CMM is explained in Chapter 8.

4.8.1. Control of parallelism for surfaces or edges

From geometry, we already know that if two planes are parallel, all points on one plane are equidistant from the other. Under the foregoing, we can make direct measurements of the deviation using a caliper or a CMM. The control will be valid only if it is made in several points sufficiently spaced relative to each other. In the case of measurement with CMM, we make sure that probing many spaced points and with the aid of data processing software, we will assess the resulting computations of tolerances [MIT 00]. We can also perform measurements of the differences of deviations by conventional means (comparison).

Also, it is not necessarily required to know the spacing (deviation δ) between two planes to deduce their parallelism. It is possible, by using a comparator, to measure the dimension deviations relative to a reference plane. Marble may be used as a reference plane if desired. In this case, the piece and the comparator are placed directly on the marble. We proceed as explained in Chapter 3.

4.8.2. Control of parallelism for two dihedral edges

The inspection of the facets of a dovetail-shaped sliding is carried out by inspecting the parallelism of the edges of the corresponding dihedral. In fact, this control is done on two straight lines $d_1 = d_2$ depending on the positioning of the gauges of equal diameters and which are supported on the respective facets. The spacing δ between the gauges will be measured between two distant areas. The interposition by a set of gauge blocks and later the control with “Go; No Go” gauges constitute the mounting shown in Figure 4.42.

$$\begin{cases} x = D \left(1 + \cotang \left(\frac{\alpha}{2} \right) + a \right) = h \times \cotang(\alpha) \\ y = b - D \left(1 + \cotang \left(\frac{\alpha}{2} \right) \right) \end{cases}$$

In some cases, the sliding dovetail implies the use a bacon-backlash with a clearance of longitudinal slope (Figure 4.42(c)). The difference in the readings l do

not correspond to the difference in thickness δ of the bacon for this same length. Therefore, we obtain what is shown in Figure 4.42(c). The variation in thickness of bacon is expressed as follows:

$$\delta = l \times \sin(\alpha) \quad [4.56]$$

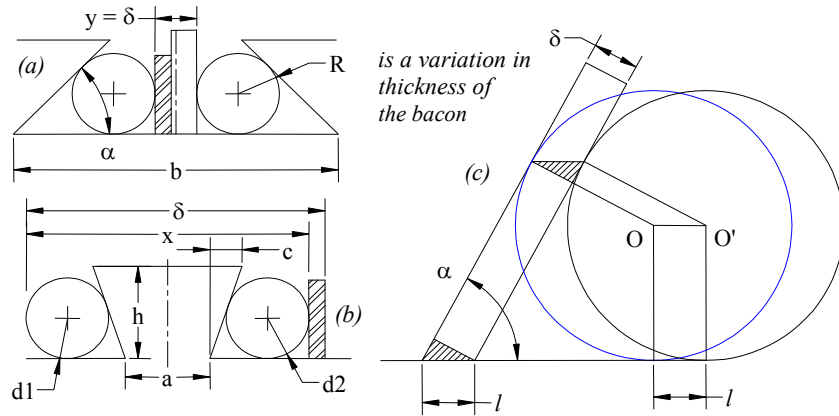


Figure 4.42. Control of the parallelism (a) the edges of a dovetail-shaped sliding and (b) variation in thickness of bacon

4.8.3. Control of the angular position of surfaces, distance between the axis of a bore and the plane

The measurement of this distance is linked to the classical control of parallelism. We know that we can materialize (embody) an axis. So by placing an expandable chuck (without clearance) in the bore, it will be possible to realize this. The axis is an “imaginary entity” and not physical. This is one reason why we use a (expandable) chuck, which is shown in Figure 4.43.

From Figure 4.43(a), we note that it is possible to measure the distance h_1 between the plane and the lower generatrix of the chuck. We can also measure the distance h_2 between the plane and the upper generatrix. As the chuck has a known diameter D , it is easy to find the distance H as follows:

$$H = h_1 + \left(\frac{D}{2}\right) \text{ or } H = h_2 + \left(\frac{D}{2}\right) \quad [4.57]$$

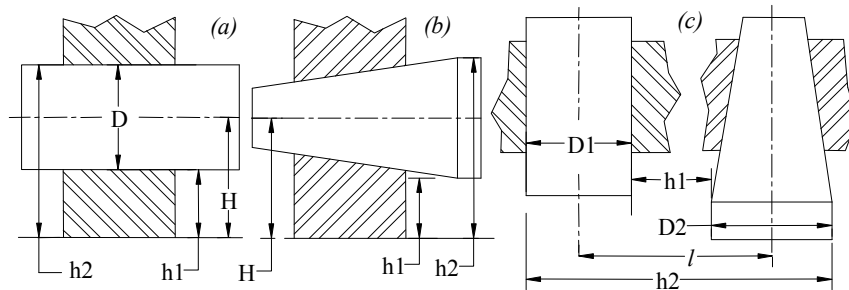


Figure 4.43. (a) Distance of the axis of a cylindrical bore to a plane; (b) distance of the axis of a tapered bore to a plane; and (c) distance between the two axes of a bore

4.8.4. Control of distance between the sphere center and the plane

A male sphere located in a free end is shown in Figure 4.44.

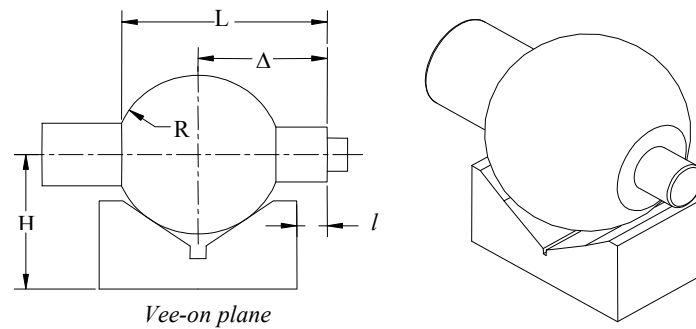


Figure 4.44. Distance between a sphere center and a plane

The distance Δ from the center to the plane surface of reference is calculated using the following formula:

$$\Delta = L - \left(\frac{D}{2}\right) \text{ or } \Delta = l + \left(\frac{D}{2}\right) \quad [4.58]$$

In the remaining cases, we can make use of a sphere mounted on a cylinder gauge block forming a cone-shaped hollow as indicated in Figure 4.44. We can thus calculate the distance l if the parallelism of the axis and the marble is verified (formula 2 [4.58]).

4.8.5. Control of the position of the edge of a dihedron

The edges of dihedra are never physically defined because they are protected during the abatement in chamfer. If these edges are protruding and if we remove them, they will result in hollows. It is therefore concluded that the measurement of their position can be done only indirectly, that is, using for example gauges or other means of control. The control of relative positions is drawing significant attention in dimensional metrology particularly in the case of mechanical comparison of means. These relative positions do not generate the same degree of difficulties in the case of use of an electronic or optical CMM. However, the latter two means are not always portable; therefore, they are best suited for laboratory control, not in the workshop.

4.9. Methods of dimensional measurement

Before developing other cases of relative positions, it would not be vain or useless to recall at least three methods of dimensional measurement:

- direct method (calibration curve),
- indirect method (by comparison or differential), and
- indirect method known under the term “at zero.”

4.9.1. Direct method (calibration curve)

In what follows, we consider a classical workshop case. It involves using a one-inch gauge block, as shown in Figure 4.45. We try to calibrate the caliper with an appropriate metrological accuracy. Although fast, this method does not eliminate the deficiency in terms of accuracy along the measuring-chain. To compensate the result of the value of the accuracy defect, we should plot the calibration curve as already done in Chapter 1. Therefore, it is imperative to know the measurement result value for all measurements.

4.9.2. Indirect method (by comparison or differential)

This method is based on the mechanical comparison. That is, the quantity intended to be measured will be compared with a similar quantity with known value. We measure the difference in magnitude and conclude with the mean dimension as shown in Figure 4.46. Block staking is equal to the mean dimension being under control. The value of the measured quantity is normally equivalent to the value of stacked block (+), and the deviation is indicated by the comparator (Figure 4.46).

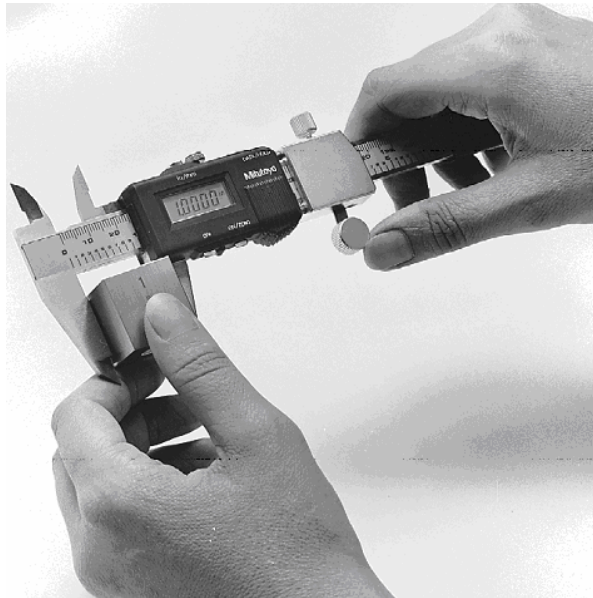


Figure 4.45. Direct method called “calibration curve”

The deviation shown on the indicator denotes a deviation (δ) of the needle. Compared with the height of the blocks stacking, we note the difference hence the appellation differential comparison.

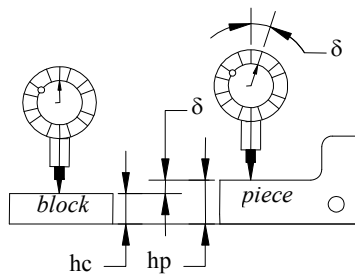


Figure 4.46. Measurement by comparison

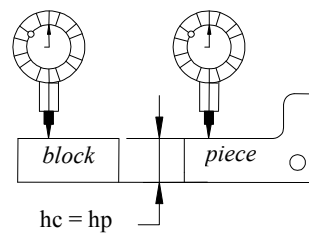


Figure 4.47. Measurement method called “at zero”

4.9.3. Indirect method known under the term “at zero”

The value of the quantity being measured is determined by balancing. We adjust quantities of known values until equilibrium corresponding to a value identical

to the two measures, that is, the value resulting from the measurement of blocks will be equal to that resulting from the measurement of the specimen as shown in Figure 4.47.

In the following sections, we present some cases of controls for relative positions. We choose for this purpose a flatness defect.

4.9.4. Measurement of flatness defect

According to ISO 1101, it is stated that the surface must be between two parallel planes separated by 15/100th mm (IT). The flatness tolerance would be respected if the flatness deviation ($d \leq IT$). It reflects the distance between the envelope of the actual surface and the point on the actual surface, that is, the farthest point from the plane envelope. The *element envelope* is a perfect geometric element (i.e. a straight line, cylinder, plane, etc.) located on the free side of the material. It touches the surface without cutting it. It is important to recall that it is possible to have several element envelopes.

The ISO standard indicates that the orientation of the element envelope must be chosen so that the distance d of the farthest point to this element envelope is minimal. For a schematic illustration of the foregoing see Figure 4.48.

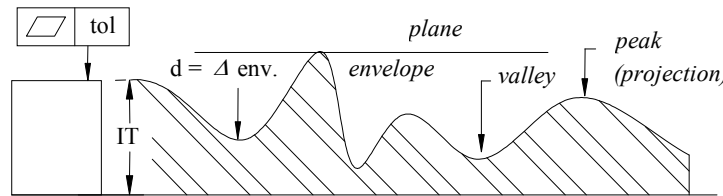


Figure 4.48. Classical control of a surface specification (flatness)

4.9.5. Method for measuring flatness deviation

We have re-performed them both in machining shop and in conventional metrology laboratory. For example, measuring the deviation of form (flatness) consists of measuring a set of distances point/plane. This measurement can be carried out by a conventional means or on a CMM [CHA 99, MIT 00].

We note that the flatness deviation is supposed to be equal to the maximum difference of measured values. The direction of measurement is perpendicular to the model associated with the real image. According to the standard ISO1101, the model associated with the actual surface must be the plane envelope that minimizes the

deviation in the flatness of the surface. This plane is actually very difficult to determine. Three methods of modeling are then particularly considered:

- envelope plane through three extreme points of material;
- plane passing through three points chosen arbitrarily; and
- plane called least squares, that is, Gaussian method.

We present a popularized approach for each of these three methods.

4.9.5.1. Plane envelope passing through three extreme points of the material

The plane envelope (Figure 4.49(b)) is materialized by a marble considered as geometrically perfect. The measured flatness deviation will be the distance between the plane envelope and the lowest point. It should be noted that the piece shall inevitably remain stable during the measurement. This is where we see the advantage of using the CMM, that is, the simplicity and speed with which the result is obtained. The disadvantages are diverse and convenient. For example, the associated envelope plane chosen is not necessarily that which is intended to *minimizes* the deviation. This method could be applied in good conditions for concave surfaces with small dimension compared with the marble. The displacement of the test piece is not always possible. A special marble should be used (which could not be obtained). This fact complicates the task because of the “price” and the “time” induced by this control. We may therefore call for another method, where the plane passes through three arbitrary points.

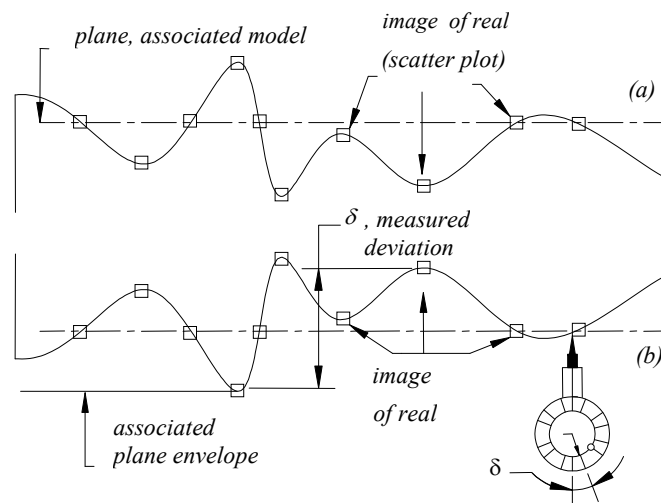


Figure 4.49. Conventional method for measuring the flatness defect (according to ISO 1101)

4.9.5.2. Plane passing through three points chosen randomly

The associated plane is embodied by three points arbitrarily chosen on the surface of the workpiece. The plane is then oriented parallel to the marble plate through successive adjustments of screw jacks (Figures 4.50 and 4.51). The deviations are then measured by the comparator that moves on the marble. The measured deviation expresses the distance between the plane envelope parallel to the arbitrarily defined plane and the lowest point indicated by the comparator. This method has the advantage of being adapted to parts that cannot move and which requires no additional calculations. This advantage is not negligible in mechanical manufacturing using dimensional metrology in machining (workshop). Thus, this substitutes the CMM. The piece is well supported on two screw jacks placed on the marble as shown in Figure 4.50.

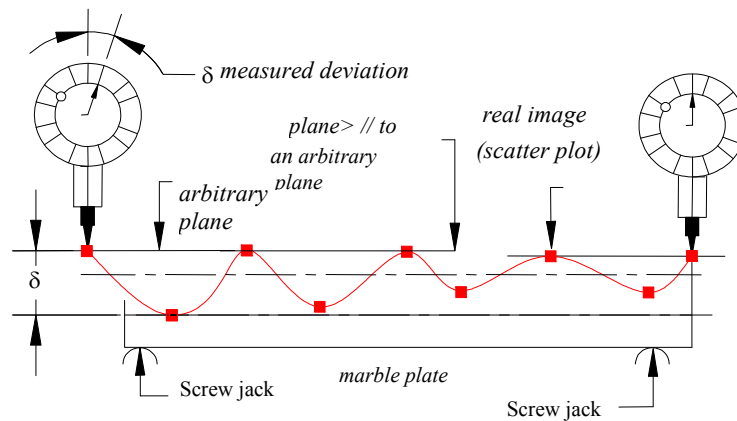


Figure 4.50. Classical schematic illustration of the second method of flatness deviation measurement

The disadvantages are explained by the fact that the associated plane is not the plane envelope. Depending on the chosen arbitrary plane, the results are different and the deviation may be well above the actual deviation. Therefore, there is a risk of producing scrap on parts, which could be good under a different control method. For these reasons, this method would be questionable. It has not often been purchased in machining shops, even less in metrology laboratories.

4.9.5.3. Plane called of least squares method (or Gauss)

Several works [CHA 99] discussed this method both in dimensional metrology and in mathematics. It is therefore appropriate to refer to them for further details. See CMM in Chapter 5.

4.9.6. Operating procedure for flatness deviation measurement

Method 1: The surface of the test piece is placed directly on the marble (Figure 4.51) without necessary meshing. Once the dial is zeroed on the standard plane (can be arbitrarily chosen but supposed to be confused with the plate marble) by moving the dial along the surface of the piece, we read over time the maximum deviation shown on the dial gauge.

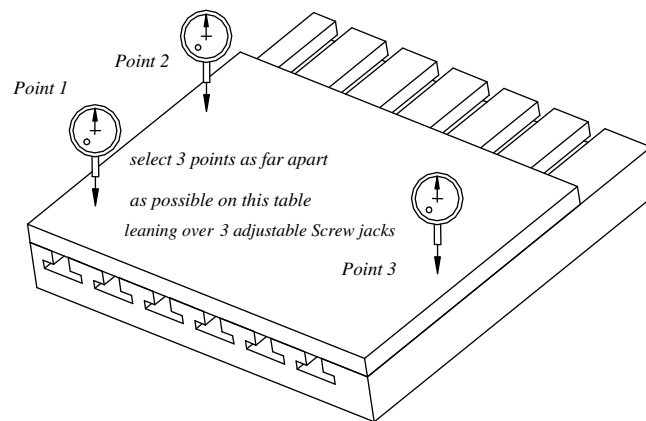


Figure 4.51. Operating procedure for measuring a flatness deviation

Method 2: We perform a meshing (preferably square) on the surface of the test piece. Then, we place the piece as shown in Figure 4.51, over three screw jacks, under the table. We choose three points (Pt_1 , Pt_2 , and Pt_3) as much distant as possible. The three jacks are set such that at each adjustment the comparator is zeroed. We note the deviations of each point compared with the previously defined plane by moving the dial on the surface of the workpiece. The difference relative to the minimum deviation measured allows determining the value of flatness defect.

Method 3: We can use a spreadsheet such as Microsoft Excel. Procedures will be carried out in respect of columns (X -axis) and rows (Y -axis). Using an optimization program in Excel (or Math CAD), it becomes easy to find the minimum point and maximum point. Hence, we find the flatness defect as expressed by [4.59].

$$\text{Flatness defect} = \text{Point of Max deviation} - \text{Point of Min deviation} \quad [4.59]$$

Example of control of parallelism for a surface relative to a reference plane: Parallelism of a given surface to a plane is a classical specification for relative position (Figure 4.52).

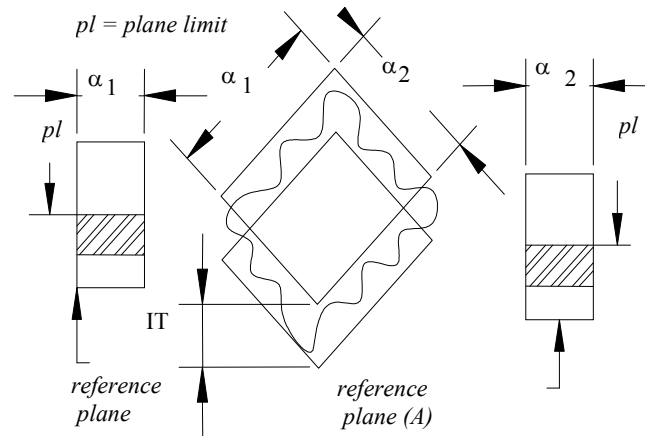


Figure 4.52. Control of parallelism between a surface and a reference plane

Ordinarily, the control is conducted to verify whether the *surface connected* to the frame of tolerance is included in a specified *tolerance zone*. The deviation δ to be measured is the distance between the *plane envelope* to the specified surface (*toleranced*), parallel to the reference plane and the plane passing through the point on a specified surface, and this point is the farthest possible point from the plane envelope parallel to the reference plane (Figure 4.53).

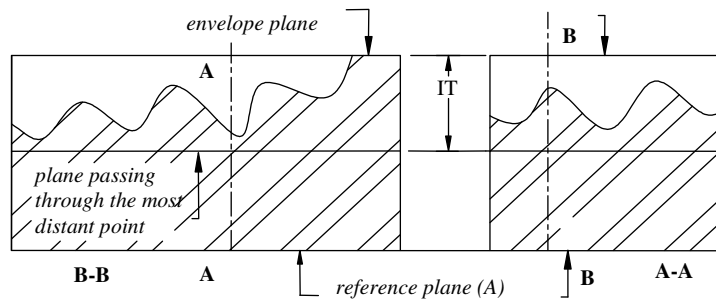


Figure 4.53. Quantity (δ is a deviation) to be measured

In some cases, form defects may be neglected as in Figure 4.54. In this case [CHA 99], parallelism deviations δ_1 and δ_2 represent the distances between the parallel planes and the reference plane. In addition, they are tangent to two segments of straight lines with, respectively, lengths l_1 and l_2 and which intercept the surface under control as well as the respective planes of measurement P_1 and

P_2 . The distances are therefore measured perpendicularly to these planes in the measurement plane P_1 and P_2 . Note that the measurement planes P_1 and P_2 are orthogonal and perpendicular to the reference plane α . In practice, in machining shop, we measure the deviation δ_m^1 , the distance measured perpendicularly between the planes parallel to the reference plane over a length δ_m^1 in P_1 (measurement plane). We deduce the slope $\tan(\alpha)$ and the deviation δ_1 over a length l_1 . Similarly, we measure the deviation δ_m^2 , the distance measured perpendicularly between the planes parallel to the reference plane over a length l_m^2 in P_2 (plane perpendicular to P_1), and we deduce the slope $\tan(\beta)$ and the deviation δ_2 over a length l_2 . In our machining shops (machine tools 2), we have repeatedly tested this approach on the vise base below:

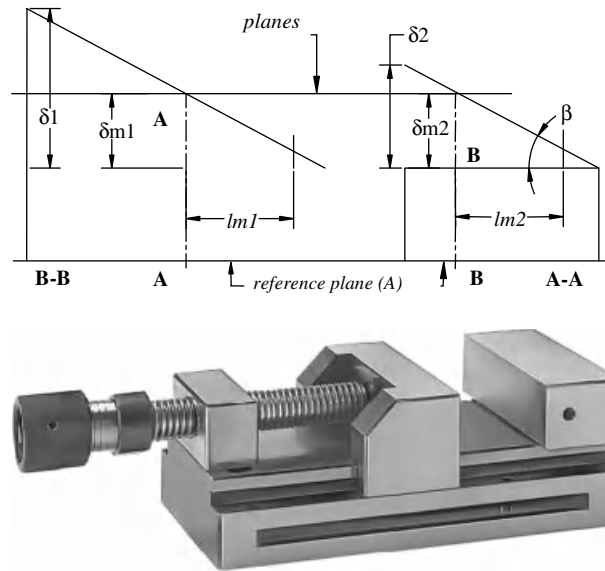


Figure 4.54. Quantity (δ , deviation) intended to be measured (if we neglect the defects of form)

This reasoning allows us to write the following equations:

$$\left[\begin{array}{l} \delta_1 = \tan(\alpha) \times l_1 \text{ with } \tan(\alpha) = \left(\frac{\delta_m^1}{l_m^1} \right); \text{ therefore, } \delta_1 = \left(\frac{\delta_m^1}{l_m^1} \right) \times l_1 \\ \delta_2 = \tan(\beta) \times l_2 \text{ with } \tan(\beta) = \left(\frac{\delta_m^2}{l_m^2} \right); \text{ therefore, } \delta_2 = \left(\frac{\delta_m^2}{l_m^2} \right) \times l_2 \end{array} \right] \quad [4.60]$$

From the condition for minima, the formula of δ is:

$$\delta_{\min} \leq \sqrt{(\delta_1)^2 + (\delta_2)^2} \quad \text{with} \quad \delta_{\min} \leq \text{TI} \quad [4.61]$$

We conclude this section by recalling the principle of measurement on geometric models. Measuring procedures proposed here are focused on the dimensional control of parts during machining. For several years, we use this example in the frame of surfacing parts for vise base. In the case of machining, the positioning of the piece seems to be important. If the entire surface is scanned, the direction and the stop will not be necessary. The positioning of the piece is shown in Figure 4.55.

4.9.7. Relative position of measuring instruments and the workpiece

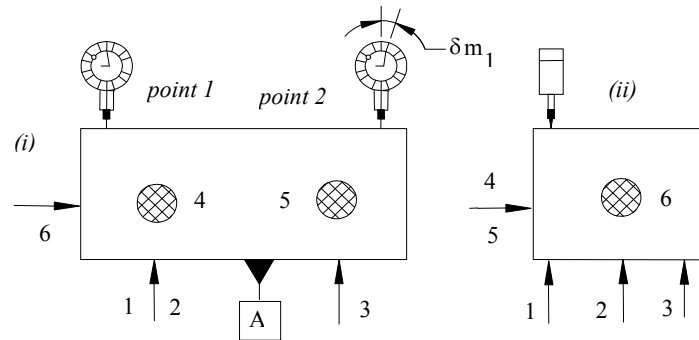


Figure 4.55. Example of control during the surfacing process with the conventional milling machine

The axes of the probes of the comparators should be perpendicular to the reference plane. During the measurement, the relative displacements instrument/workpiece should be parallel to the reference plane. We relocate preferably the instrument or piece-holder rather than the piece itself. The instrument is calibrated to zero on a point of the surface being under control. In the case of using multiple comparators, they must be calibrated to zero with respect to the reference plane using gauge blocks. We conclude this theme by presenting two examples of measuring procedures.

The workpiece should be placed as shown in Figure 4.55, and the comparator installed in position (i). Then, we set the comparator to zero. The latter will be moved in translation in the measurement plane from the position (ii). Thus, we read the indication δ_m^1 . Mathematical expressions [4.60] and [4.61] are applicable to these cases. We perform the same operations as those previously presented,

in a measurement plane perpendicular to the previous one and we deduce δ_m^2 . For example, we place the piece in position on the reference plane. Then, the comparator is installed and zeroed on any point. We scan the whole surface. We read the deviation δ corresponding to the maximum amplitude shown by the comparator.

4.9.8. Control of the perpendicularity of a line to a plane

Perpendicularity, as parallelism explained earlier, is also a specification of a *relative position*. We mean by line, the axis of a cylinder. By convention, we materialize the axis by one (or many) generatrix of the surface of revolution and then perform the appropriate measurements. The *direction* is on the *fictitious element*, which is the *axis* of the cylinder relative to a *reference element* which is the plane.

4.9.8.1. Tolerance zone

The tolerance zone projected onto a plane is bounded by two parallel straight lines distant from each other by IT and perpendicular to the reference plane. Its length h is that of the specified element which represents the length of the surface of revolution (here, it is a projecting cylinder). The projection plane P of the tolerance zone corresponds to the plane of the drawing in which tolerance is prescribed.

The orientation of this plane with respect to other elements of the part is represented in an implicit way. Extreme positions of the axis of the probed cylinder are considered depending on its generatrix, both with a CMM or conventional control using comparator probing (Figure 4.56).

Perpendicularity deviation δ represents the distance between the projections on P of two straight lines envelope of a generatrix with a length h embodying the cylinder axis. The schematic illustration supports our reasoning (Figure 4.57).

$$\text{with } \delta = h \times \tan(\alpha) \text{ and } \tan(\alpha) = \left(\frac{\delta_m}{l_m} \right) \Rightarrow \delta = h \times \left(\frac{\delta_m}{l_m} \right) \quad [4.62]$$

In this case, the defects of form are neglected. The straight-line envelope is perpendicular to the reference plane B . In the practice of metrology, we measure the deviation δ_m over a length l_m , in the measurement plane P . Finally, we deduce the slope $\tan(\alpha)$ and the deviation (δ) over a length h . Cases of perpendicularity measuring are diverse similar to the means that we may apply to this end.

Up to now, we presented cases dealing with engineering schematization, applied to metrology in accordance with our practice of measurement by workshop means, that is, cases which involve conventional comparators.

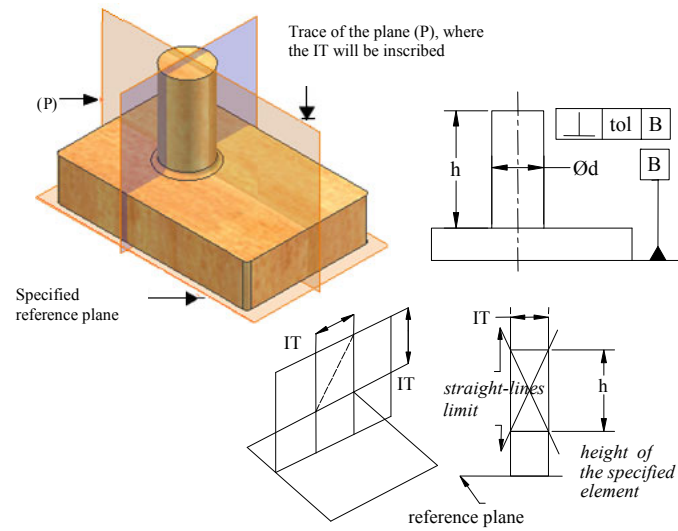


Figure 4.56. Control of the perpendicularity of a line to a reference plane

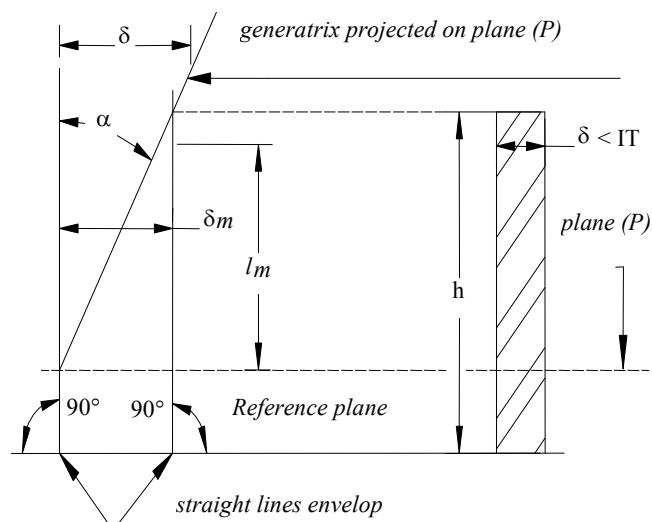


Figure 4.57. Quantity intended to be measured during the control of perpendicularity of the line relative to the reference plane

4.9.9. *Relative position of measuring instruments and the workpiece*

Our objective is to position the conventional measuring instruments, namely comparators, relative to the plane of reference and the plane of measurement, which is commonly termed as “rough planning operation.” The axes of the probes of comparators should be parallel to the plane of reference and contained in the measurement plane. We often resorted to the use of two comparators to slide by a relative translation movement (part/comparator).

The measurement is performed by mechanical comparison. The instrument should be zeroed. To achieve this, we use a gauge block; however, we assumed that the generatrix is perfectly perpendicular to the plane of reference.

4.9.10. *Other controls of dimensions in relative positions*

There are various methods for the measurement of a dimension. We present some brief cases as a summary of this chapter. For each of the surfaces S_1 and S_2 , the image of the real is defined by the point of contact between the surface and the contact tips of the instrument.

4.9.10.1. *Direct measurement on conventional caliper*

The measuring instrument generates distance on a point-to-point basis (Figure 4.58). The control the dimension to be inspected is the distance measured by point-to-point is included within the dimensions called the “limit”.

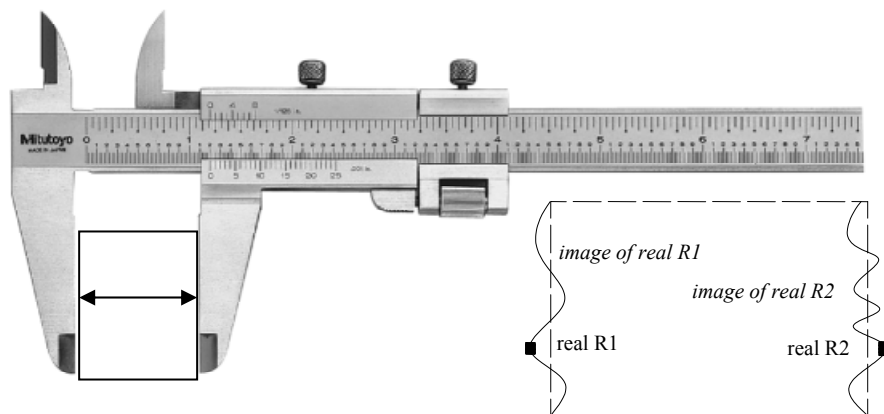


Figure 4.58. *A classic example of real measurement with a caliper*

4.9.10.2. Other indirect measurement

The *real image for surface 1* is defined by one or more points probed on the surface. There is no associated model. The *real image for the surface 2* is defined by the points of contact between the marble and the machined surface (Figure 4.59).

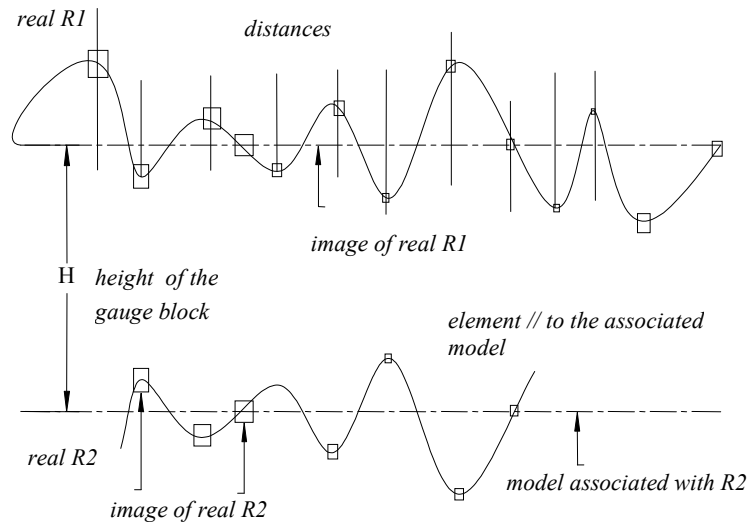


Figure 4.59. Indirect relative dimension with marble and comparator

The geometric model [CHA 99] associated with the real image is a geometric envelope element to the surface passing through the contact points (straight line or plane). After calibration, the comparator measures the deviations between the different points of S_1 and the geometric element (straight line or plane) parallel to the model associated with S_2 and distant by H (dimension of the gauge block). The measurement direction is perpendicular to the marble plate (and to the associated model). The measurement realized after calibration corresponds to a set of distances point/straight line or point/plane. The control of the dimension consists of verifying whether the measured distance lies between the dimension limits.

4.9.11. Direct measurement of an intrinsic dimension using micrometer

We consider that the instrument is geometrically perfect. The measuring method is based on the *real image* that is defined by the points of contact between the surface and the micrometer contact tips. The model associated with the real image is a circle or a cylinder passing through the points of contacts, with a diameter equal to the distance between the two contact tips. The measuring instrument is in this case

a *point-to-point* distance (Figure 4.60). The control of the dimension consists of verifying whether the distance measured via this method lies within the limit dimensions $(\varnothing 17 \pm 0.15)/2$.

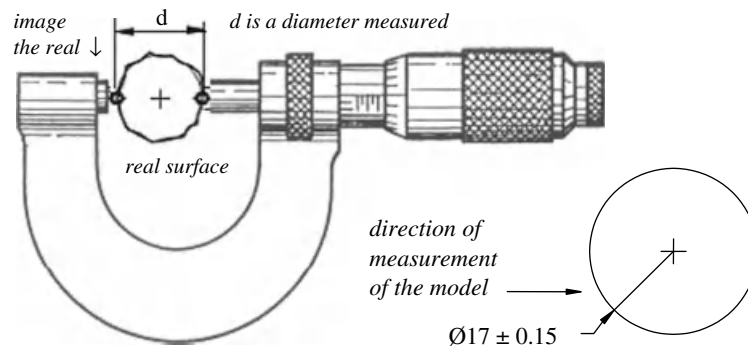


Figure 4.60. Classic schematization of direct measurement by micrometer

This method is consistent with the definition of the principle of independence stipulated by the Standard. It is wise to take several measurements at different locations to tend towards the principle of the standard.

4.9.12. Summary on relative positions

We note that the real image and the associated model are different based on the instrument used and thus the results are different. Therefore, we should consider that compliance with the standard of independence is related to the choice of the method of measurement and the means to use to this end. Control of the relative position of surfaces constitutes the necessary complement of the control of the state of shaped surfaces. It would be hardly credible, or even subject to question, to believe that the measurement of dimensions in relative situations is identical to the classical measurement of a piece using a given conventional instrument. Relative positions require first and foremost an unambiguous understanding of the definitions: actual surface, real image, actual model, actual axis, imaginary axis, sense (or direction) of the measurement, etc. Then, the choice of instrument will condition the *accuracy* and the *precision* of measurement. It is same for the *fidelity* of measurement. Choosing a device or a measuring instrument is not problematic by itself. However, using such a device is problematic because the dimension of the part to be measured and the environment are largely dependent on the choice of measuring tool.

In the workshop, for example, it would be traditionally hardly credible to measure a small vise base for measuring a small vise to the same table of a milling machine, using a CMM or a profile projector. Often, in our current machining, we found ourselves faced with these problems. Certainly, modern means such as optical or laser instruments are increasingly used in workshops; however, the cost induced by the measurement is significant and it is always a choice to be decided when we know the advantage and the disadvantage of portable instruments. A serious work on quality control would doubtlessly facilitate the task of choosing an instrument in dimensional measurements of relative positions.

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Chapter 5

Opto-Mechanical Metrology

5.1. Introduction to measurement by optical methods

5.1.1. *Description of profile projector (type Mitutoyo PH-350H)*

This chapter deals with measurements that are taken without contact between the test piece and the measuring instrument. We discuss the profile projector, its principle, and its various applications. Then, we propose an introduction to interferometry to control defects in parallelism and flatness for micrometers and gauge blocks. Optical means measurement is essential to magnify without deforming or altering the measurement accuracy. Therefore, this provides a different approach to quality control. Non-contact measurement allows viewing and controlling, as a standard (shadow), parts with forms that are hardly visually accessible to measurement when using a conventional instrument such as a micrometer. This method relies on the accuracy of optical elements, excellent lighting, and the ability to measure extremely precise worktable. These devices consist of a steel frame building with protective cap and digital display with a resolution of up to 0.001 mm. The objectives are interchangeable (10 to 20-50-100X). Projection may be diascopic, that is, to say by halogen lamp and incident light with fiber-optics in two directions. The new versions of devices are characterized by high brightness and ease of use when good precision is added. The optical system consists of interchangeable lenses and condenser optics with mirrors.

Profile projectors are generally equipped with a digital display for angular dimensions. This apparatus is also available with horizontal and vertical projection

systems. Many industries, especially the medical equipment industry, use them in radiography for various dimensional reading or magnified images. Developments in the field of profile projectors have greatly contributed to their widespread use. The non-contact between parts and the measuring instrument has certainly promoted the use of this technology since it is recognized that profile projectors occupy reduced space in a metrology laboratory.

The optical comparator projects an enlarged profile on a screen to visualize and classify it with ease. The demarcation of lines and contours is amplified via appropriate lenses. The physical principle of the device is based on the projection of light coming from a light-bulb and traversing a *converging lens* which is, itself, projected on the workpiece. The resulting shadow passes through a *diverging lens* that magnifies and projects it onto a mirror. The reflected shadow, magnified once again, is then projected onto the screen. The size of the shadow is generated by the lens inducing the amplification, e.g. 5x, 10x, 31.25x, 50x, 62.5x, 90x, 100x, and 125x. Some optical comparators feature a variety of fixtures reclining the seat of a micrometer allowing high celerity and definite accuracy of dimensions (in both directions). We expose the profile projector PH-350H, for demonstration (Figure 5.1), with the courtesy of Mitutoyo, Canada.

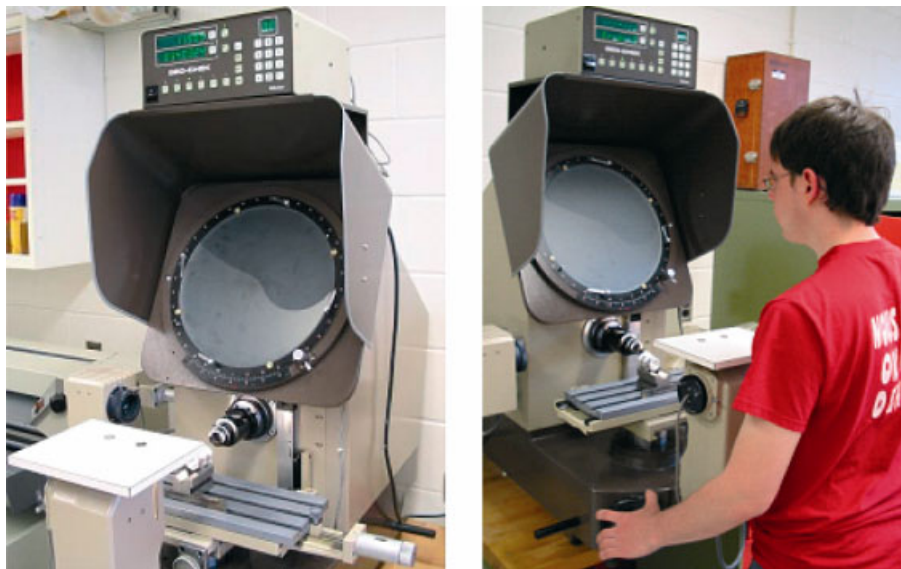


Figure 5.1. Overview of profile projector Mitutoyo PH-350H

The rays of a profile projection from the source to the screen are shown as in Figure 5.2.

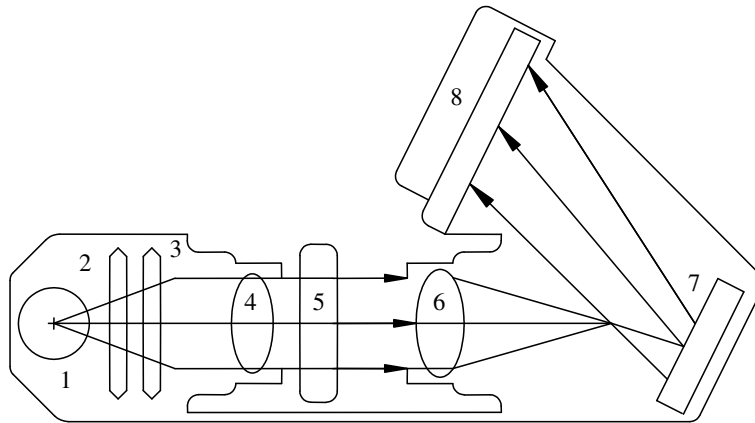


Figure 5.2. Principle of profile projection; 1, bulb; 2, thermal radiation filter; 3, color filter; 4, condenser lens; 5, part to be measured; 6, projection lens; 7, mirror of reflection; 8, Screen

Without going into detail on optics, we can say that the principle is simple and is based on projecting an image of the test piece illuminated by halogen lamp. The projection lens projects the image onto a mirror which in turn reflects it on to a screen. The magnification is based on the type of the lens. Therefore, if the objective is to control a thread, one should choose the appropriate lens to magnify the desired profile. The projection can be *diascopic* (transparent or tracing paper) or *episcopic* (profile projector with mirrors whose dark side corresponds to weight of the piece intercepting light rays). Figure 5.3(a) and (b) illustrates this principle:

– *Diascopic projection.* The case of diascopic lighting (Figure 5.3(a)) or backlight allows projecting a highly magnified track made on a transparent support.

– *Episcopic projection.* (Figure 5.3(b) shows screen (a), test piece (b), lamps (c), condenser (s), objective (e), and mirrors (f)). In the case of episcopic projection where the lighting is frontal, the profile of the piece (b) to be observed is illuminated by the source (c) and magnified by the optical system, and later projected by means of two mirrors on the ground glass screen. The dark part corresponds to the mass of the piece intercepting light rays.

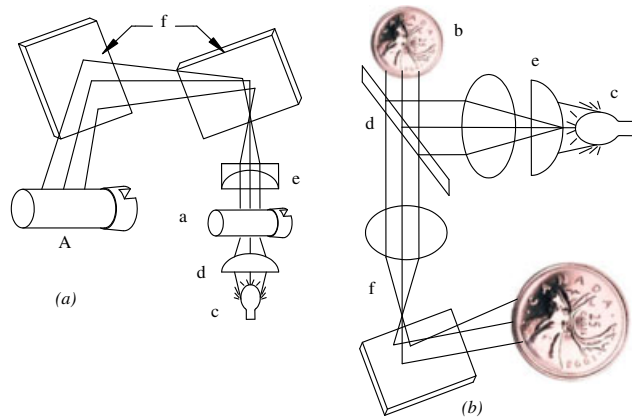


Figure 5.3. (a) Diascopic projection and (b) episcopic projection

We propose a summary of the main functions of profile projector PH-350H, which is used in the context of our laboratory work.

5.1.2. Presentation of the main operating functions of GEOCHECK

To run the Mitutoyo GEOCHECK console, first of all, the main power button should be activated, and then the light source is selected to project contours of the piece to be measured. In a second step, the measurement process starts by activating the main power to the digital displayer: to measure a test piece whose size is too small to be measured by conventional classical means (micrometer, caliper), or even by a coordinate measurement machine, we use the profile projector.

A	- 00.0000	X	Q	INC	O	R	0 0
B	- 00.0000	Y	P	ABS	N		
	ON	C					
D	OFF	E	F	G	H	I	S
M	ENTER	78		9			
L	STORE	45		6			
K	RECAL	12		3			
J	PRINT	●	0	+/-			

A : X-axis	F : polar coordinates	K : recallmemory	P : validateY-axis
B : Y-axis	G : complementary angles	L : storememory	Q :validate X-axis
C : current	H : relative position	M : enter	R : memory
D : radius	I : relative position	N : units(mm /inch)	S : spreadsheet
E : angle	J : printing	O : modes (INC/ABS)	PH- 350H

Figure 5.4. Display Counter for Profile Projector- Mitutoyo model -PH-350H

The development of display counters has seen many advances. Profile projectors are increasingly accompanied by software, and statistical process controls are there to make computations simpler; such as a distance or angle to the geometric constraint of perpendicularity or parallelism. An additional advantage is the possibility to create and store databases useful in more than one way.

We propose, in this section, some examples of measurement using the profile projector connected to a display counter, known under the name of GEOCHEK (Figure 5.4). The reader easily gets inspired from the steps presented here to practice the measurement in the laboratory even if it does not possess the same profile projector as in our case.

5.1.3. *Selecting the point of origin (preset operation, zero reset)*

We must choose a reference point on the part that allows us to measure different dimensions from this point. Once we have used this reference by placing a cross of sight in the chosen location, we realize the following manipulation:

X	Y	INC/ABS	ENTER

1. Press “X” (the X -axis becomes worthless);
2. Press “Y” (the Y -axis becomes worthless);
3. Press the “INC/ABS” key;
4. Press “ENTER.”

This results in resetting the point of origin within the axes X and Y . Coordinates 0.0000 (0.000 mm) are displayed on digital display screens. We can now take the required measurements.

To save zero, in incremental, we will follow this:

X	Y	ENTER

1. Press “X;”
2. Press “Y;”
3. Press “ENTER.” This will set the zero (0.0) both on X and Y .

NOTE.— The warning light switches for incremental, without a proof to absolute.

5.1.3.1. *Measurement in incremental mode based on a measured point*

X	Y	INC/ABS	1	•	2	ENTER
Y	X	INC/ABS	1	•	2	ENTER

1. Make sure there is a warning light;
2. Press the “X” key (the *X*-axis becomes worthless) if necessary;
3. Press the “Y” key (the *Y*-axis becomes worthless) if necessary;
4. Press “ENTER.”

Note that the term “if *necessary*” here, means that it is not necessary to press the key “Y” if the transfer takes place in “X” only.

The following measurement will be given in incremental mode.

By pressing the “INC/ABS” key, the absolute mode comes back; the value is then obtained based on the initial reference.

X	INC/ABS	3	ENTER

By resetting a new origin either INC or ABS, GEOCHEK will signify that it will be programmed starting from the new origin distant from the location at the time of this measurement. Here is a practical example which resets the new origin to 3. We will therefore see 3.0000 inches shown on the *X*-axis. This new dimension is automatically assigned to measures that would follow our dimensioning. Here is another example combined with *X* and *Y*, whose dimensions to the origin are, respectively, affected by 1.2500 and 1.5600 inches. To perform the foregoing, the procedure on GEOCHEK is simple and consists of typing the following:

X	1	•	2	5	+/-	ENTER
Y	1	•	5	6	+/-	ENTER

In general, when we proceed with such measurements, we are tempted to remove the deviation between the actual size from the incremental one, that is, to say, the displayed measurement: {1.2552; 1.5623} and not {1.2500; 1.5600}. When measuring, it is desirable to see {0.0052; 0.0023}. As such, the screen displays {− 1.2500; − 1.5600}.

5.1.3.2. *Choosing a point of reference different from the usual “0”*

1. Press the “X” key;
2. Press “INC/ABS” key to select the operational mode;
3. Press the digital keys corresponding to the value you want to display on indicator of the relevant axis;
4. Press “ENTER” to finalize the operation;
5. Repeat the same procedure for the “Y”-axis.

5.1.4. *The main functions of optical comparator*

5.1.4.1. *Alignment function*



- Press the button (see symbol). In the upper right wedge will appear the indication “P2” to indicate that we must take two points.
- Align the cross sights on the edge of the workpiece at one point and press “ENTER.” The indication “P1” will appear.
- Align the cross sights on a second point of the edge and press “ENTER.” Both axes will have the value “0”.

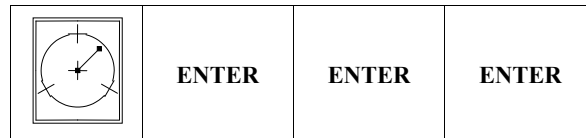
NOTE.– The angular misalignment must be less than 45° . The two points where the measurement has been taken should be as far distant as possible.

Example of Application: write the process for aligning and finding (0, 0):

1. Target a point on the horizontal edge of the workpiece;
2. Press the “Y” key,
3. Press the “ABS” key;
4. Press “ENTER;”
5. Target a point on the vertical edge of the piece;
6. Press “X;”
7. Press the “ABS;”
8. Press “ENTER” and confirmation of the result of (0.0).

5.1.4.2. Radius measurement function

As such, we measure the radius by targeting three points, the farthest possible from each other:

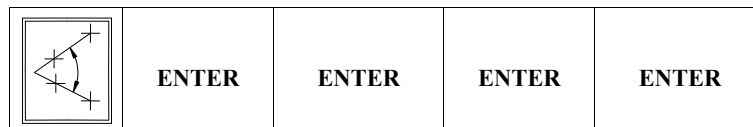


5.1.4.3. Procedure for radius measurement

1. Press the touch “Radius” → the warning light switches on and P_3 appears;
2. Press “ENTER” after targeting the first point;
3. Press “ENTER” after the targeting the second point;
4. Press “ENTER” after targeting the third point;
5. Press “2” to get the diameter ($\varnothing$) = $F(0X)$.

5.1.4.4. Angle measurement function

Be sure that the key “polar” is disabled!



1. Press the key “ANGLE” → P_4 appears;
2. Target a 1st point * → press “ENTER;”
3. Target a 2nd point → press “ENTER;”
4. Target a 3rd point * → press “ENTER;”
5. Target a 4th point → press “ENTER.”

NOTE.— (*) As far as possible from the apex of the angle to be measured. This explains why GEOCHEK is released automatically from the previous mode.

The angle appears at the Y-axis. The value of the segment “appears” on X.

See Figure 5.5, for some angles positions selected by the angle function.

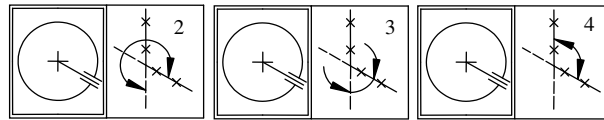


Figure 5.5. Example of measurement and display of an angle and its supplements

5.1.4.5. Measurement function on polar mode

The “Polar Mode” key is schematized as:

The GEOCHEK polar function simultaneously records a distance (direct line) from one point to another in addition to the angle which composes them. The following are the steps:

1. Press the target *Polar* until light goes off *;
2. Target a point P_1 at bottom wedge from the reference;
3. Target a point P_2 ;
4. Press the “Polar.”

The screen displays, in mm or inches, the direct line between P_1 and P_2 on the location of X on Y , it will display the angle measured. The order followed by the signs (+) or (–) is the conventional trigonometric order. Figure 5.6 shows eight possibilities of GEOCHEK display.

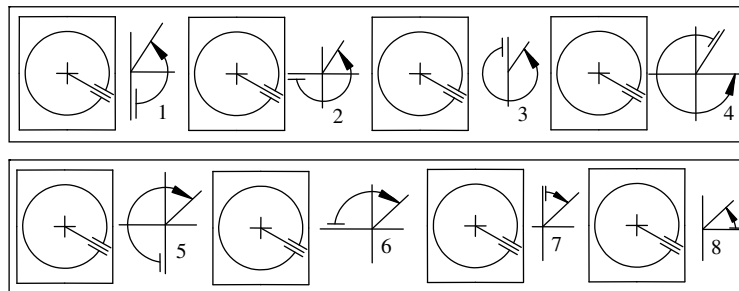


Figure 5.6. Example of measurement and display of an angle and its supplements

5.1.4.6. Procedure of measurement for a relative function

The key “relative distance” is schematized as:



This function allows complex dimensioning. This can, for example, be used to measure the relative distance between the centers of two circles. Here is the approach of this function:

RECALL	0	1	ENTER
	0	2	ENTER

1. Apply the function “Radius”, of which three points will be taken on the first entity (circle1);
2. STORE 01: press “ENTER”;
3. Apply the function “Radius” of which three points will be taken on the second entity (circle 2);
4. STORE 02: press “ENTER”;
5. RCL (to recall entity 1) press “ENTER”;
6. Activate the button “Relative Position” press “ENTER”.

Read the result of the relative distance between two entities (C_1 and C_2)/Frame of references.

5.1.4.7. Procedure of Measurement for a Center Line

We may also search the central position of a line via this procedure:

INC/ABS	X	Y	ENTER	1	ENTER

1. Make sure you are on incremental mode. This means that the first button, schematized earlier, is (ON);
2. Position the cross laid on P_1 and activate the buttons schematized earlier;
3. Position the cross laid on P_2 and activate the buttons schematized earlier.

5.1.5. Metrology laboratories on profile projector

Laboratory 1. Briefly explain the basic principle of optical profile projector and highlight the advantages and disadvantages of optical comparison against other methods presented in this handbook. According to the following drawing of a workpiece, you are assigned to:

1. Machine, using molybdenum steel (SAE 4010), the part shown in of Figure 5.7;
2. Measure, using the projector, the required dimensions and to report them on the drawing;
3. Write up the report in accordance with the expressed requirements.

NOTE.— The permissible tolerances are in the order of: ± 0.005 mm or $\pm 0.5'$.

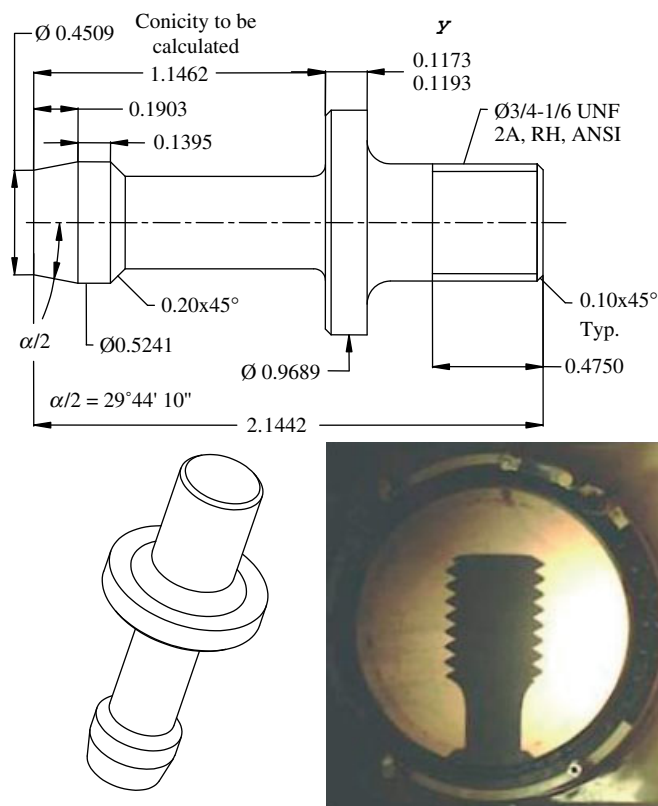


Figure 5.7. Technical drawing of a screw (very fine thread)

Laboratory 2 (non-contact). Optical measurement on profile projector. According to the drawing of the following part, perform the work already assigned in Laboratory #1. The part in Figure 5.8 is an alloy of copper, by machining of your choice. Tolerances are ± 0.005 mm or $\pm 0.5'$. It is recommended to adequately re-measure (after processing/machining) all the dimensions in inches.

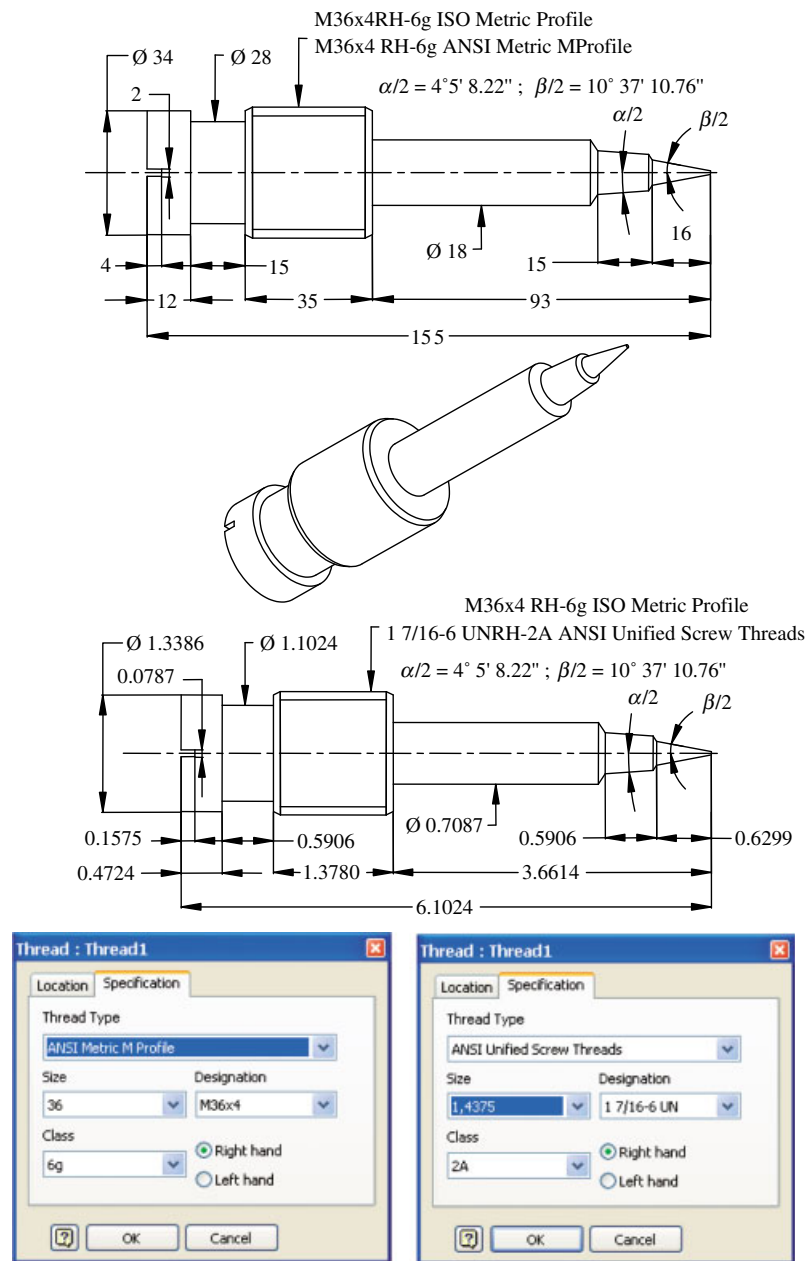


Figure 5.8. Drawing of a special screw thread (very fine for a conventional micrometer)

DISCUSSION.— We have locally produced this screw (Figure 5.8). During control, we deliberately proceeded with metric and imperial units. The result in metric is unambiguous: this is a screw M36x4- Class 6g, both under the international ISO standard and ANSI Metric M Profile. On the right-hand side, we show the result of the control of the screw. This time, according to ANSI Unified Screw Threads, the characteristics are 1 7/16-6UN Class 2A.

We have processed in this way to emphasize the standard to be applied during manufacturing. Interchangeability is not always automatically provided even if we add the ISO registration.

5.1.6. Plates measurement standards for profile projector

It is possible to measure the threads or grooves using measuring standard. These standards are provided with graduations appropriate to the nature of the entity to be measured (pitch diameter of threads or transition radius, etc.). The operating principle of these standards is simple. It suffices to mount them on the screen of the profile projector, to project the slide or workpiece to be compared with the standard, and to superimpose the images (part/standard). If there is a coincidence, we read the displayed measurement value. To monitor and measure the dimensions of a part by projecting its profile, it is necessary to follow a methodical approach. We propose an approach that has yielded good results in the laboratory. For example, to control a very fine thread, 60°, it is important to:

1. Mount the appropriate lens in the comparator;
2. Place the adjustable assembly on the cross slide of the micrometric support;
3. Adjust the reclining assembly/installation to the helical angle of thread;
4. Put the workpiece between the tips of the reclining assembly;
5. Mount the graphic protractor and center it horizontally on the screen;
6. Switch on the light bulb;
7. Adjust the lenses to obtain the required sharpness;
8. Move the micrometer's support with cross slide to center the image;
9. Rotate the graph protractor until reading 30°;
10. Adjust the cross slides for the coincidence image/protractor line;
11. Control the other side of the thread in similar way.

In view of the foregoing, here is a summary:

- Optical metrology remains an important tool to measure complex shapes and profiles, even if difficult and of inconvenient access.
- It should be used wisely because of the effective advantages offered by metrology by mechanical comparison.

5.2. Principle of interferential metrology (example: prism spectroscope)

We address principle because it presents a remarkable interest in the calibration of measuring instruments such as micrometers or the control of straightness for gauge blocks. Interferometry is a domain of physics that has its ramifications in dimensional metrology, and therefore, we consider this last point. The principle of magnification involves the interferometric properties of non-uniformity of light. In a straight line, light moves at a celerity C (3×10^8 m/s) depending on a vibratory move with short waves λ .

We know from the literature [GUI 66, SER 85, DEC 97, DEC 99] that white light consists of radiations with different colors, each of which having specific wavelength: the classic phenomenon of a prism. Often the naked eye is unable to assess (quantify) this phenomenon directly. We see that at the output of the prism, the rays are deflected to go then spread on a screen, thus forming a spectrum (rainbow). One can see distinctly separate networks in many different colors, namely: *purple, indigo, blue, yellow, orange, and red*, as shown schematically in Figure 5.9. We note that the wavelengths variation, from purple at 0.4μ to red at approximately 0.7μ . In principle, these wavelengths are given to eight digits after the decimal point. These radiations are stable under certain ambient conditions, that is to say, temperature, pressure, and humidity of the air. We say that there is *interference* between two vibratory movements when they overlap on the same point. This happens once these movements are driven by the same source, that is to say, within the same period of time, crossing the same trajectory also.

This simple reminder on interferometry is proposed to situate the reader within the scope of our work in metrology. In what follows, we will present the theme of the control of defects appearing on certain measuring instruments (micrometers stops) and how to control them by interferometry.

The superposition of an infinite number of monochromatic light radiations gives a white light. Each monochromatic radiation is characterized by a frequency ν in Hertz (Hz), by a wavelength λ . The refractive index n of transparent milieu varies depending on the wavelength of the light radiation passing through it.

The deflection, via a prism, of a light beam in monochromatic light depends on the index n of the prism, thus λ the wavelength of light irradiation. If the prism is illuminated by white light, the deflection is greater for violet radiation ($\lambda = 400 \text{ nm}$) than for the red radiation ($\lambda > 700 \text{ nm}$). The prism allows separating the different radiations constituting white light as shown schematically in Figure 5.9.

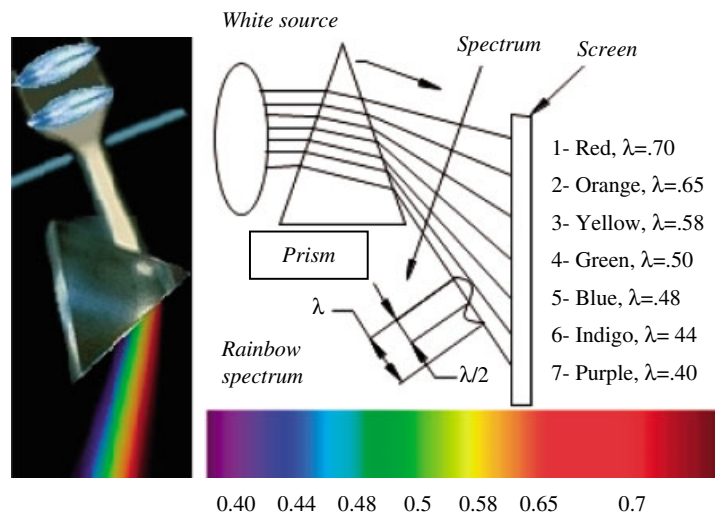


Figure 5.9. Classical schematic diagram of radiation spectrum

Some sources such as incandescent lamps emit a white light whose spectrum contains all the colors (continuous spectrum). We simulate, in what follows, a program that addresses numerically the phenomenon of interference of two sinusoidal waves of the formula:

$$f(\lambda) = \frac{1}{2} \cdot \left(1 + \cos \left(\frac{2\pi}{\lambda} \times \delta \right) \right) \quad [5.1]$$

where λ is the wavelength.

5.2.1. Function of two sine-waves interference

– First number in the series: start = 0.400 and last number in the series: end = 0.700.

– Increment: incr = 0.005. Function of creation of the vector $\delta = 1.500$ (Figure 5.10) [CHE 64, GUI 66].

Vecteur V résultant : $\text{Range2Vec}(\text{sec}, e, i) :=$

$\lambda := \text{Range2Vec}(\text{start}, \text{end}, \text{incr})$ De (5.1) $f(\lambda) := \frac{1}{2} \left(1 + \cos\left(\frac{2 \cdot \pi}{\lambda} \cdot \delta\right) \right)$	$\begin{array}{l} \text{count} \leftarrow 0 \\ \text{for } i \in \text{sec}, \text{sec} + 1..e \\ \quad v_{\text{count}} \leftarrow i \\ \quad \text{count} \leftarrow \text{count} + 1 \end{array}$

$\downarrow$
 V

Vecteur V Résultant = Resulting V -Vector

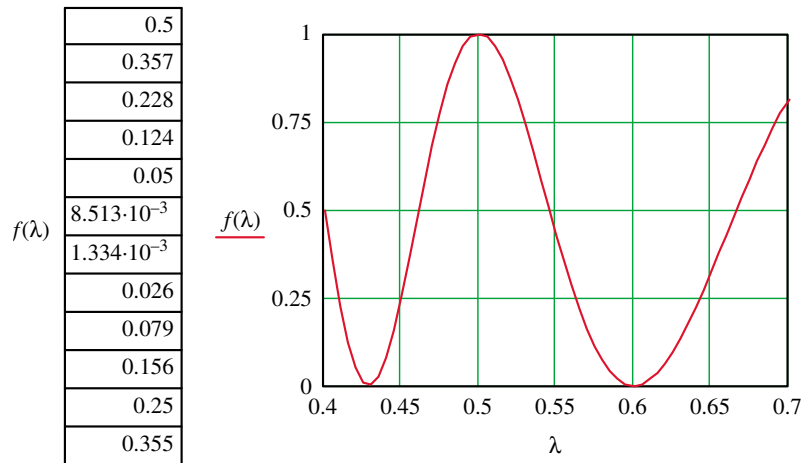


Figure 5.10. Function of two sine-waves interference

5.2.2. Statistical description

The statistical description of this function is expressed by the probability density. It is actually a quantity that has a probability density formulated by $S(\lambda)$. We split $S(\lambda)/5$ for convenience of graphic plotting (Figure 5.11):

Data $:= \lambda$ and $\lambda := \text{length}(\text{data})$; $\text{SD}(\lambda) = \text{Standard deviation}(\lambda) \times \sqrt{\frac{\lambda}{\lambda - 1}}$
 let $\bar{x} := \text{mean}(\text{data}) = .55$ Standard deviation $\rightarrow \text{SD}(\text{data}) = .089$

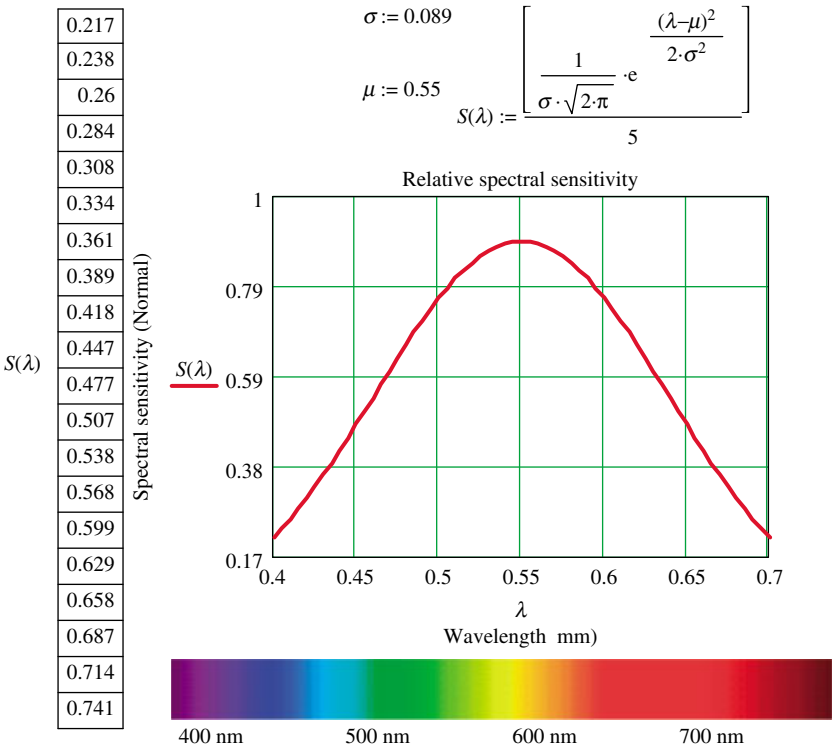


Figure 5.11. Relative spectral sensitivity simulated under Math CAD

5.3. Flatness measurement by optical planes

In the photographs of Figure 5.12, we visibly observe the ends of a micrometer [CAS 78, MIT 00].

A perfect plane disc of polished glass is placed between the spindle and anvil of a micrometer (1 inch). We clearly distinguish the “fringes” on these photographs: three fair strips and three dark stripes. To each visible band width corresponds a parallelism defect of $\lambda/2$. As such, this may be well illustrated by the interference fringes created by the *air wedge* and the plane of the standard which is actually a polished glass as shown in Figure 5.12.

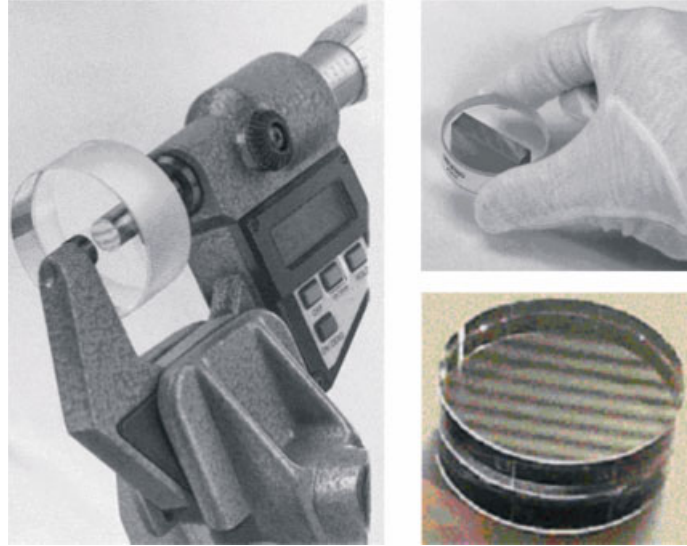


Figure 5.12. *Interference fringes observed with the naked eye (source: Mitutoyo Canada)*

5.4. Principle of interferoscope

The light beam driven from a monochromatic punctual light source a passes through the A -side of the standard-plane of a lens to be then reflected on O across the plane d forming the mirror and is then directed to the bezel to meet again the standard-plane in M . Another ray r_2 , from the same source is reflected directly on the standard-plane in M to follow the same reflection path as r_1 . SR is a reference surface [CAS 78, CHE 64].

Interference occurs in M and the path difference between r_1 and r_2 is the path AOM . If it is an even multiple of $\lambda/2$ (hence a multiple of λ also), there will be, on M , “a bright fringe” since M is a node (see Figure 5.13(b)). However, if AOM is equal to an odd number of $\lambda/2$, there will be, on M , a “dark fringe” and M would then be on what is called a “belly.” In fact, the angle AOM (Figure 5.13(a)) is very small. Everything happens as if the air wedge below the standard-plane was formed in equal and parallel amounts alternately light and dark of $\lambda/2$. Thus on any polished piece, placed under the standard plane and lighted under the same conditions, there will arise a network of fringes forming contours lines of the same surface each of which showing an unevenness degree of $\lambda/2$ or about $3/10 \mu$ per fringe.

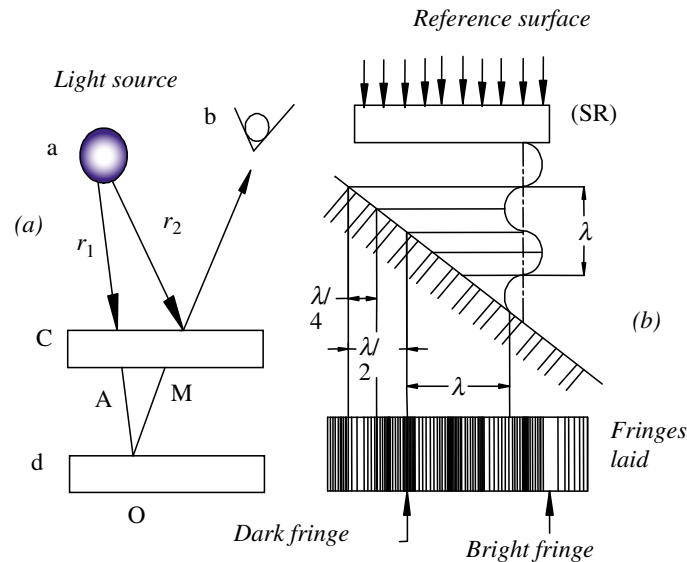


Figure 5.13. Principle of producing interference fringes by an optical disc: r_1 and r_2 are two rays forming an air wedge, b a spotting scope, C a plane disc (polished glass), and d the surface of the workpiece.

The principle of the interferometer is simple and the apparatus is a compact unit. It provides a high accuracy ($1/10 \mu$), especially in comparative measurements of up to 100 mm and up to approximately 20mm in terms of indirect measurement. This kind of laboratory apparatus requires the use of, at least, seven radiations ensuring, as such, measurements at 0.025μ nearly and very efficient thermal stability-protection, at less than $1/10^\circ\text{C}$. This condition is very difficult to achieve and it is probably for this reason that often dimensional metrology laboratories (Cégep) do not own them. Of course, professional calibration laboratories are often equipped with such an apparatus.

In the second experiment between a disk-plane made of glass (1) and a part (2), we send a light beam (wavelength network). Depending on the angle α , the sprawl will be huge for $\alpha = 0$ or low for $\alpha = \text{few seconds of arc}$.

In summary, we can say that wherever the surface of a piece intercepts either a glass or a node of wave, the fringe appears. If there was no parallelism defect between the two contacts no fringe could exist. The equation explaining this phenomenon is hence deduced $\rightarrow f = \text{distance between two successive fringes} \leftrightarrow \lambda/2$.

By placing a wedge between two parallel plane discs (P_1 and P_2) made of polished glass and perfectly straight, this creates an “air wedge” of the angle α and a monochromatic light (blue-green spectrum is emitted by a mercury lamp). Interference fringes appear parallel to the line formed between the two polished glasses. This phenomenon is called “equal thickness interference.”

By reading Figure 5.14(b), it appears clearly that the monochromatic light of wavelength λ passes through the disk P_1 along the direction (ab) and then reflects on the surface of polished glass, passes through P_2 , in the sense of direction (cd). The light is then reflected on P_1 along the path $e \rightarrow f \rightarrow g$ as shown in Figure 5.14.

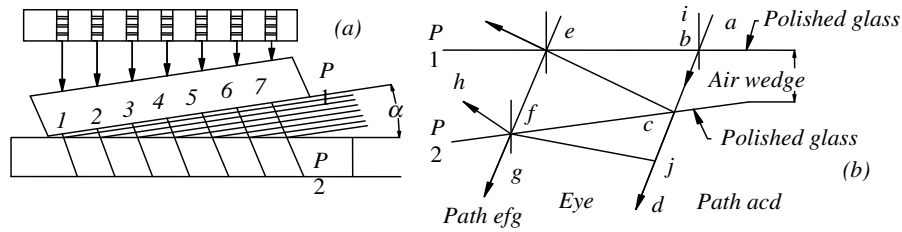


Figure 5.14. Experiments on two polished glass discs

The surfaces of the discs P_1 and P_2 are slightly tilted relative to one another. Hence (cd) and (fg) are not exactly parallel. If we used a lens to converge them, there will be interference phenomenon caused by the phase difference. If the “air wedge” equals a space t and the angle of incidence i , the path difference δ , displacement between (cd) and (fg) is then written as:

$$\delta = 2 \cdot t \times \cos(i) \quad [5.2]$$

$$\text{if the path difference } \delta = (2n+1) \times \frac{\lambda}{2} \quad (n \text{ is an integer } > 0) \quad [5.3]$$

$$\text{so that equation [5.3] gives } \rightarrow (2n+1) \times \frac{\lambda}{2} = 2 \cdot t \times \cos(i)$$

The two waves of light reduce their intensities with each other and a shadow line appears. Otherwise, if the thickness t satisfies the expression [5.4], the equation of δ would be as follows:

$$\delta = 2 \cdot t \times \cos(i) = 2n \times \left(\frac{\lambda}{2} \right) = n \times \lambda \quad [5.4]$$

The two light waves increase their intensity, with each other and a line of bright light (fair) appears. As indicated in the previous equations, the appearance of interference fringes depends on the angle of incidence i . Moreover, the parallel waves of light (that is to say, with the same incidence angles) must be used to obtain a clear image of interference fringes. If $i = 0$, such that waves of light would have perpendicular incidence to the disk P_1 , a line of bright light appears during the period t as expressed hereinafter:

$$t = 2n \times \left(\frac{\lambda}{4} \right) \quad [5.5]$$

A line of dark light appears during the next period:

$$\delta = (2n + 1) \times \left(\frac{\lambda}{4} \right) \quad [5.6]$$

If the surfaces of polished glasses are *completely straight*, the “air wedge” shows a line parallel with the line of intersection of the two surfaces. Thus the interference fringes are also parallel to each other. The difference between the air wedge and the bright light (dark) adjacent to the positions will be expressed by $\lambda/2$:

$$(2n + 1) \times \left(\frac{\lambda}{4} \right) - (2n - 1) \times \left(\frac{\lambda}{4} \right) = \left(\frac{\lambda}{2} \right) \quad [5.7]$$

To each difference of air space corresponding to $\lambda/2$, there is an interference fringe. The smallest angle α between the two surfaces of polished glass, slightly tilted relative to one another gives rise to a larger interval of fringes. Therefore, the wavelength λ of the *monochromatic light* will be in wide intervals. The interval S between adjacent lines of bright light (Figure 5.14(b)) takes the following form:

$$S = \left(\frac{\lambda/2}{\tan(\alpha)} \right) = \left(\frac{\lambda}{\tan(\alpha)} \right) = \frac{\lambda}{2\alpha} = \frac{1}{2} \quad [5.8]$$

If the argument α is very small, it would be identified by the function itself, that is to say: $\tan(\alpha) \approx (\alpha)$. If the surface of one of the two polished glasses is not straight enough, the shape of interference fringes resembles to a contour line of a map. This means that the contour indicates lines too close to each other. If white light were used, many interference fringes with several different colors (rainbow) appear only at the period ($t = 0$). The path differences are zero and there seems no positioning

distance. This is explained by the fact that the difference of interference wavelengths appears at the same time. The principle of equal thickness interference is widely used in the measurement of parallel steel gauge blocks where straightness, parallelism, and good surface condition of gauge blocks are inspected.

Partial summary. An interferometer (interferoscope) is an instrument that uses the phenomenon resulting from interactions of two rays of light [GUI 66, SER 85] in which a single ray is split into measure small lengths, to measure the light wavelengths, and to analyze a narrow region of the spectrum. Among the many devices used in interferential metrology, we quote:

- Disks (standard-plane) of polished glass for direct geometric verification of small and sufficiently polished surfaces analysed in the context of metrology;
- Micro-geometric control apparatus for surfaces whose fringes define the relief;
- Measuring devices, both direct (in absolute value) and indirect, called interferometer.

In dimensional metrology, we use interference fringes to control the flatness of gauge blocks and the micrometer's stops (see Figure 5.13). The basic principle of these techniques is the coherent superposition (interference) of beams of light representing different states of the test object. An interesting property and common phenomenon lies in the usable signal output: the result is always materialized by networks of periodic interference fringes. This important concept of periodicity is determined by the wavelength λ of interfering beams. Of course, analysis of the observed image can be defined as the conversion of a fringe figure into a continuous network which depicts the quasi-sinusoidal intensity distribution.

The importance of the introduction of new computer technologies is justified by a considerable gain in processing time and especially the suppression of manipulation errors. Nowadays, we are witnessing the emergence of specific systems for processing images, especially images of interferometry.

5.5. Control of parallelism (case of parallel gauge-blocks)

In the frame of our dimensional metrology laboratories, we conducted an experiment to control, using a polished glass, parallel gauge blocks. The control focuses on the parallelism and flatness of gauge blocks. The experimental procedure is diagrammed as in Figure 5.15 [DEC 97, DEC 00].

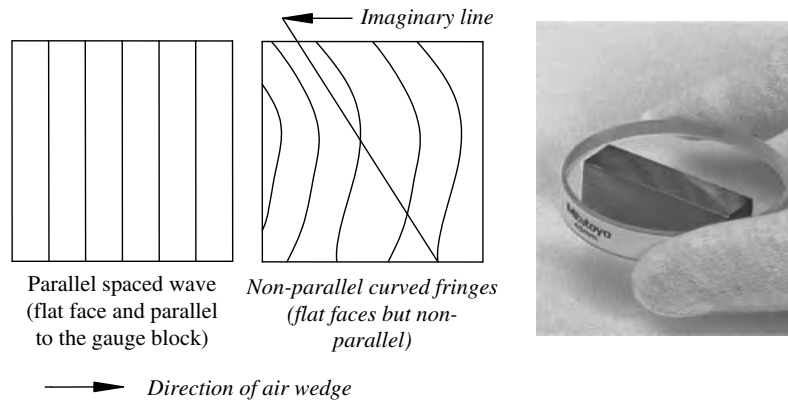


Figure 5.15. Interference fringes observed during the experiment on a parallel gauge blocks

We leaned a stack of three parallel blocks next to a single gauge blocks, having the same size as the sum of other three gauge blocks. To control the flatness, we have placed a gauge blocks under a monochromatic light source ($\lambda_{\text{Sodium}} = 589.10^{-9}\text{m}$). Then, we installed an optical plane, while creating a small air wedge to ensure the presence of interference fringes. The fringes may appear as shown in Figure 5.15. We distinguish three cases:

- We have previously explained that the presence of “perfectly parallel” fringes indicates that the surface is flat and thus there is no significant flatness defect.
- In the case of the Figure 5.15 in the middle, the surface may be concave or convex. To control the flatness, we turn the optical polished glass until we get *a* an air wedge in one of the chosen directions. Figure 5.16 shows that the surface may also be convex or concave.
- In the case of the third figure, we clearly observe interference fringes indicating that the cylindrical surface is convex. The fringes are directed along the axis parallel to the direction of the air wedge created for the circumstance. We can draw these fringes of $\frac{1}{2}\lambda$ and deduce the amplitude *h* (to read height *h*, see application example later). Obviously, the third figure shows a flatness defect which should be quantified with a certified uncertainty.

To control the parallelism, by optical means (interference fringes), we performed this assembly on a perfectly clean marble. We use a gauge blocks parallel (high-precision gauge block) and next, we stack, for shoulder, a series of three parallel blocks, to control the parallelism. It is advisable to use at least two, for this kind of shoulder/conjunction. We must ensure that the height of the gauge block and height

resulting from the stack of three blocks, on the right, are exactly the same. We also measured the thickness (example $t = 9$ mm). This would simplify the calculation when the interference fringes are parallel as shown in Figure 5.17.

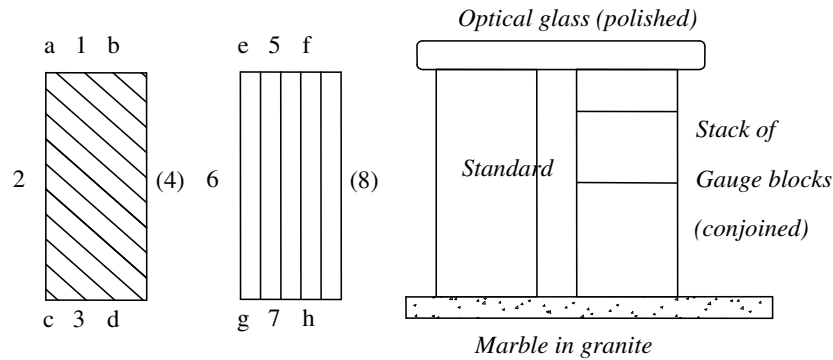


Figure 5.16. Schematic assembly observation of interference fringes

Let us apply for the second phase of the experiment, a small pressure localized at the center of the optical polished glass. The wedges (a , b , c , or d) are there to show where would the contact be situated between the polished glass and gauge block. It is, in other words, a reference. This reference is taken, for example on the wedge a , we then find the height h of the other three wedges (b , c , d) in good adequacy with the wedge a . The height of the wedge b relative to the wedge a will indicate the number of fringes on the face E – the number of fringes on the face A multiplied by a half-wavelength λ , etc.

After designating the six sides of the gauge block, here is the summary of the experience:

- $h_{(b)}$ relative to wedge (a) = Nb. fringes side (E) – Nb. fringes side (A) $\times (\frac{1}{2}\lambda)$
- $h_{(c)}$ relative to wedge (a) = Nb. fringes side (F) – Nb. fringes side (B) $\times (\frac{1}{2}\lambda)$
- $h_{(d)}$ relative to wedge (a) = Nb. fringes side (G) – Nb. fringes side (C) $\times (\frac{1}{2}\lambda)$ +
Nb. fringes side (F) – Nb. fringes side (B) $\times (\frac{1}{2}\lambda)$
→ etc.

Parallelism is thus defined as the separation between two planes containing the surface of the profile. These planes must be parallel to other reference surfaces. In this case, we subtract the value of the height of the smallest wedge of the value

corresponding to the wedge with the highest height value. We then compare the results with the specification indicated on the gauge block to confirm or refute the parallelism.

Let us recall what is commonly called the contour lines of a surface. We show again in Figure 5.18 how the light source S emits light rays corresponding to a known radiation [CAS 78, CHE 64]. Upon reading this figure, we see that the point M appears dark and the point N (between P and P') will be much lighter. The flat surface L is at the origin of the appearance of a network of parallel and equidistant interference fringes. We see that the lines would cut L based on heights regularly measured at $\lambda/2$. Note again that if the surface L was not flat, the interference fringes would therefore appear as a non-uniform relief as in Figure 5.17.

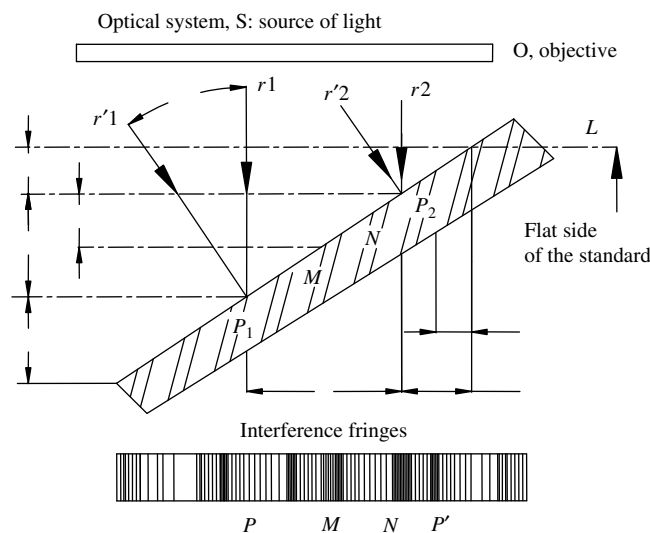


Figure 5.17. Schematic illustration of contour lines and flat versus non-flat surfaces

In the case of a surface deemed perfectly flat, the interference fringes appear in the form shown in Figure 5.17. We will now compare surfaces and interpret six plausible observations on them. We will proceed in a manner similar to that used in the context of the previous experiment. We place ourselves on the side referenced L and examine the face of the workpiece resulting from surface flat grinding. By repeating the previous experiment, we observe the following (in Figure 5.18):

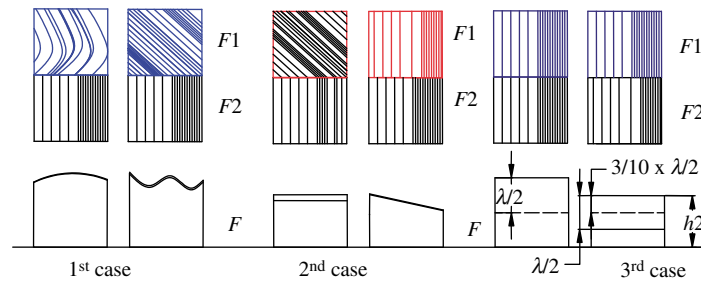


Figure 5.18. Schematization of the dispositions relative to interference fringes

NOTE.— The schematization in Figure 5.18 is approximate. It is inspired from the technical literature [CAS 78, CHE 64], simply for pedagogy purposes. As part of our tutorials, we were able to verify only the good adequacy for the proper procedure presented in the literature, we present later a summary of three cases of observations of interference fringes in respect of the authors content [CAS 78]. We use for this purpose, a program designed with MathCAD software, which will be presented in section 5.5.1.

From the foregoing, we observe three different cases, the readings of which are:

– First case:

- f_1 : not straight fringes on the face F ; this area does not contain any straight line right,

- f_2 : regularly spaced straight fringes on the face F ; this surface is not flat but has lines deemed straight;

– Second case:

- f_3 : regularly spaced straight fringes on the face F ; this surface is flat but shows two sub cases:

(i) if the direction of the fringes is not the same as on the piece F and on the plane L , the face F is not parallel to the support plane L ;

(ii) if the fringes on F and the support L are parallel, but if their spacing is not the same on both, the face F is not parallel to the support L ;

– Third case:

- f_1 : parallel fringes, parallel and with same spacing: the face F is parallel to the face of the support L . In the latter case, if the fringes of F and those of L coincide exactly with f_1 , the height of the piece is a multiple of $\lambda/2$, that is to say:

$$h_1 = x_1 \times \left(\frac{\lambda}{2} \right) \quad [5.9]$$

- f_2 : if the fringes of F and those of L are shifted, the height of the piece is not an exact multiple of $(\lambda/2)$. It will be different by a fraction of $(\lambda/2)$. We estimate the shift between the two fringes networks. The shift, in our case is of 3/10 interval between two fringes. This corresponds to an excess of $(0.3 \times \lambda/2)$. The height of the piece will be calculated as follows:

$$h_2 = x_2 \times \left(\frac{\lambda_2}{2} \right) + \left(\frac{3}{10} \right) \times \left(\frac{\lambda_2}{2} \right) \quad [5.10]$$

For example, for the *red of the helium* (#4), this height is written:

$$h_4 = \pm x_4 \times \left(\frac{\lambda_4}{2} \right) + (\delta_4) \times \left(\frac{\lambda_4}{2} \right) \quad [5.11]$$

Measurement using interferometer is carried out through two distinct ways. The first is called measurement by comparison and the second called direct calculation. We present a complete example (including a numerical application) and prefer explain the second one without numerical application. In the case of measurement by comparison, we consider a piece of unknown height H_1 . We stack (by conjunction) the piece and the standard side by side on the reference L as in Figure 5.19.

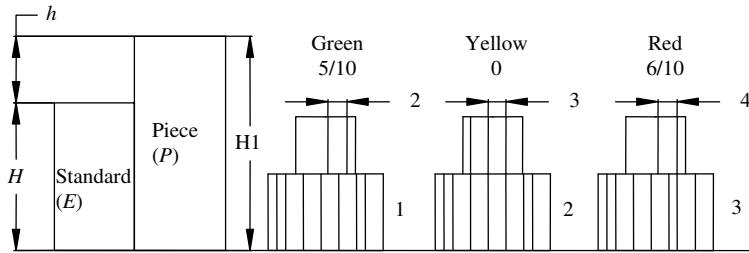


Figure 5.19. Schematic illustration of an example by comparing the fringe

We expose the standard and the test piece to a beam of monochromatic light and we observe the interference fringes. We later assess the respective shift of these interference fringes. H is a height (in micrometer or microinch) either of positive or negative value. For each radiation used, h is then a multiple ($\pm$) of $\lambda/2$, increased by

the excess observed (but still positive). This height should therefore correspond to all the relations formulated:

$$h_4 = x_i \times \left(\frac{\lambda_i}{2} \right) + (\delta_i) \times \left(\frac{\lambda_i}{2} \right) \quad [5.12]$$

and so on for each color observed. Here is a schematic illustration for this purpose (Figure 5.20).

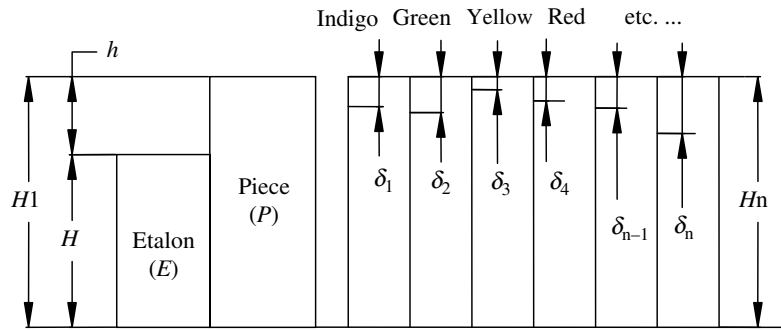


Figure 5.20. Schematic illustration of the measurement by comparison of interference fringes

We now present the application example as discussed earlier.

5.5.1. Numerical example of laboratory

Positioning of the problem of indirect measurement: let the height $H_1 = 10.955 \mu\text{m}$ of a piece grinded in conventional workshop. After grinding, this piece was measured using a micrometer at 1/10th with a tolerance of $\pm 1 \mu\text{m}$. We used the MathCAD software to model this problem. This application example is inspired from the literature [CASE 78]. We deliberately used the three common radiation and had noted observed excesses (Figure 5.20):

- $e_1 = 2/10^\circ$ with $\delta_1 = x_1 = 3/10$ for the green of the helium;
- $e_2 = 2/10^\circ$ with $\delta_2 = x_2 = 3/10$ for the yellow of the helium;
- $e_3 = 2/10^\circ$ with $\delta_3 = x_3 = 3/10$ for the red of the helium.

We conducted 11 experiments that gave 11 h_1 . For this, we used the MathCAD software for programming the function of h_1 relative to each of the radiations. We have observed the three cases at the end, when the results were grouped, the value (−0.72) was common to all of three cases, at the eighth observation.

5.5.1.1. Green helium

This (simulated) workshop creates a vector of figures on an interval defined as follows: for the green helium, we obtain the following (Figure 5.21):

$$\text{For the green helium, } \lambda_{\text{GreenHelium}} := 0.2507852 \quad \frac{\lambda_{\text{GreenHelium}}}{2} = 0.251$$

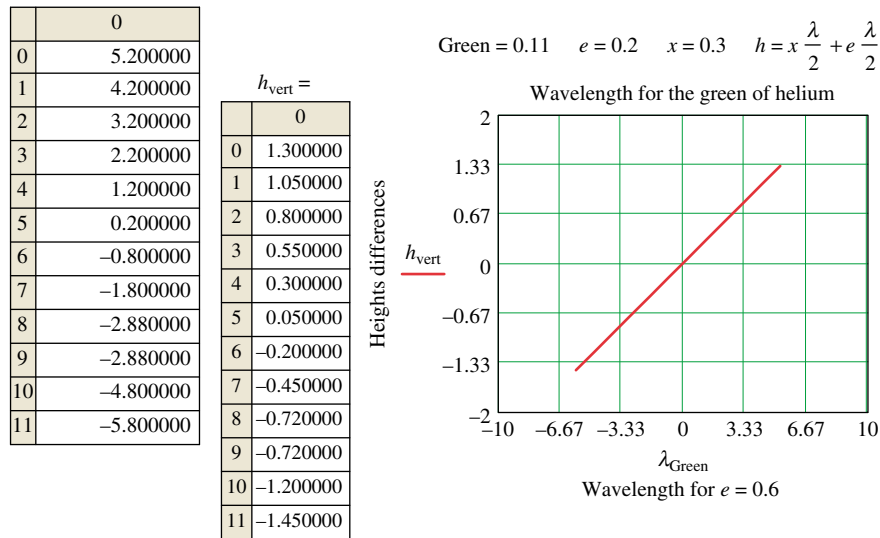


Figure 5.21. Results of measurement by comparison of interference fringes

5.5.1.2. Yellow helium (Figure 5.22)

$$\text{For the yellow helium, } \lambda_{\text{YellowHelium}} := 0.293782702 \quad \frac{\lambda_{\text{YellowHelium}}}{2} = 0.293783$$

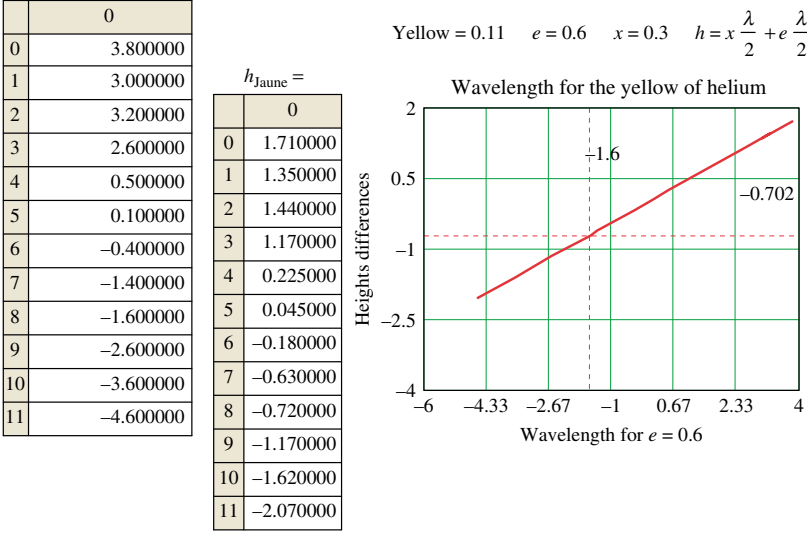


Figure 5.22. Results of measurement by comparison of interference fringes

5.5.1.3. Red helium (Figure 5.23)

For the red helium, $\lambda_{\text{RedHelium}} := 0.333909192$ $\frac{\lambda_{\text{RedHelium}}}{2} = 0.333909$

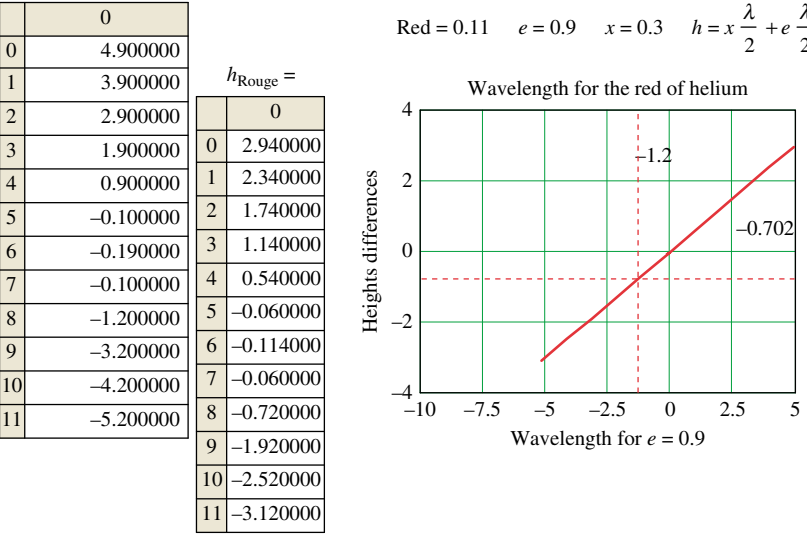


Figure 5.23. Results of measurement by comparison of interference fringes

5.5.1.4. Graphical summary of the three wavelengths simulated

We clearly notice that at the eighth simulation for the three colors of the considered helium, the value of the height ($h_1 = 0.7200,000,000$) appears. This approach is so successful and offers great accuracy (Figure 5.24).

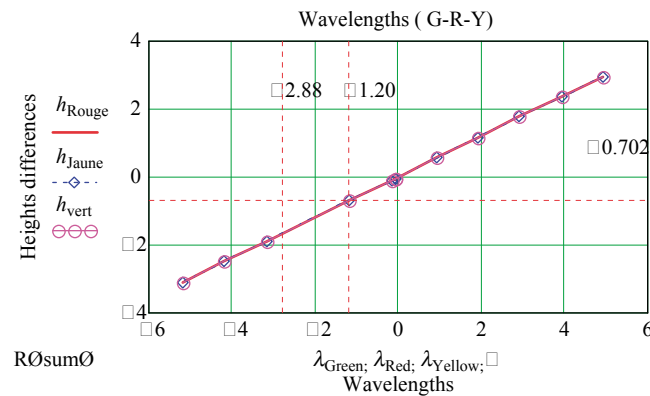


Figure 5.24. Summary of results of measurement by comparison of interference fringes

Our goal is to eventually calculate the height H_1 of the piece by the following equation:

$$H_1 = H + h_1 = 10.955 + (-0.00072) = 10.95428 \mu\text{m}$$

with $H = 10.955$ mm, the height of the standard of comparison and $h_1 = 0.72 \mu\text{m} = (-0.00072)$ the difference of heights, calculated for each radiation.

We do not evoke here direct measurement since we have not tested, using the interferometer, parts consistent with gauge blocks. However, this technique is simple and straightforward. The piece will be placed on the base plate of the device and its height will be measured without any intermediary element. The accuracy is largely significant (at minimum, the order of 1/10th of micrometer). The interferometer should be used in the laboratory and requires stabilization, in severe environmental conditions, to ensure accuracy.

5.6. Conclusion

First, we have seen that the profile projectors are devices that are increasingly important; not only in dimensional metrology, but particularly in image magnification techniques where the mechanical contact between the object and the

instrument is inefficient or impossible. The illustrative example is that of the screw studied in the laboratory setting number 2. It was impossible for us to know the characteristics of this thread, even though we built it ourselves with deliberately erroneous data. Identifying it required that we use a profile projector and the dimensions listed on it are still approximate. Measurement or laboratory testing devices using interferometry are suitable for metrology in workshops, which will accommodate normal usage conditions. For high precision (test) measurements, we often use light interference. This phenomenon means that we can eliminate virtually all the errors which are attributed to contact, to amplifying deviations and assessing readings.

Ultimately, the interferential observations would not be justified in dimensional metrology (and are not possible) except when the surfaces to be inspected are perfectly polished with a high-quality finish. Yet, it is for this reason that we use the principle presented here in this subsection for controlling the *flatness* or *parallelism* of micrometers. It is not recommended to use the measurement by direct calculation. The expected result is certainly *correct* and error free. However, it is important to meet conditions of temperature, pressure, and total stability during measurements.

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Chapter 6

Control of Surface States

6.1. Introduction to surface states control for solid materials

The study of surface states, also known as surface roughness, deals with the remarkable irregularity of surfaces by measuring altitude variations. Their descriptions are realized using tools of statistics, signal processing, image processing and, in some cases, fractals theory. Rugosimetry also enables a predictive approach to properties of materials and structures in use, in addition to a “desired” final outcome of the object in study.

In manufacturing engineering, surface of a material (even after an appropriate desired exposure using an appropriate machine) is never perfectly smooth, not only in terms of the type of machining method and tools used, but also according to the material’s nature itself. The machined surface has many micro- or macrogeometric irregularities. In a geometric reference, these irregularities are identified by comparison with a “mean” line, and classified as asperities or “peaks,” and cavities or “valleys.” All of these surface defects constitute the roughness. The functional role of a surface depends on a number of factors, including the surface state which is already well explained in tribology. The lower the roughness index of a material is, the harder the estimation of its value will be. This constraint necessarily increases the manufacture cost of a material. According to ISO 4287, dealing with surface states, knowledge of topography of surfaces is fundamental in order to:

- minimize the wear of parts in contact;
- optimize the ability of materials to adhesion or assembly, machining, etc.;
- qualify the stretched paintings, the function of materials, their brightness, etc.

- describe temporary aging by natural degradation; and
- increase resistance to corrosion of surfaces subjected to aggressive environments.

Optimization of surface roughness allows a better mechanical anchorage, especially in devices where frictional forces play a functional role (Cone Morse). In order to define surfaces' state for classification purposes, it seems useful to:

- develop systems for acquiring and processing data; and
- prepare tools that would help in the classification of surfaces.

In this handbook, we limit ourselves to an experimental study using Mitutoyo Surftest 211. Our goal is to measure at least one roughness parameter, namely R_a based on a workpiece that we prepared in the workshop: machined using a conventional milling machine. In dimensional metrology, testing, measuring, and analyzing the surface states require a procedural approach and increasingly sophisticated means (instruments and apparatus). In line with ISO 4287, we illustrate the evaluation of surface states with a general flowchart shown in Figure 6.1.

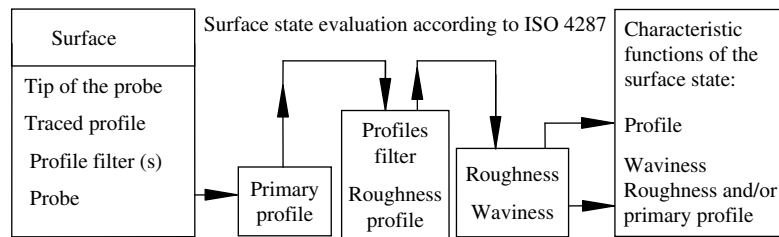


Figure 6.1. Diagram of surface evaluation according to ISO 4287

In this section, we first begin by defining the parameters which reflect the roughness, and then we discuss specific examples that highlight the criteria of surfaces. Surface texture is governed by both national standards [OBE 96] ANSI/ASME B46.1-1995 (Canada and the United States) and international organizations such as ISO 468, 4287, 4288, 5436, 12179. In the field of surface roughness, waves and striations, the ANSI/ASME B46.1-1995 standard covers the geometrical irregularities of surfaces of solid materials. Some stylus devices (ISO 1878, 12179) and other standards or specimens (ISO 5436) are being used to indicate the state of the tested surface. The ANSI/ASME standard defines both the state and characteristics of the surface to be inspected. It also offers the conventional model (ISO 1302) for use in engineering drawings. Roughness calibration is governed by the standard of ISO 12179. Note that neither the US standard nor the international one (ISO) deals with the appearance, color, corrosion, wave resistance,

hardness, and/or microstructure of the surface or any other specific consideration. The units of measurement are either the micrometer or the microinch.

Surfaces are usually very complex to characterize. For this reason, the appropriate standard deals with height, width, and direction of the irregularity of surfaces. This is justified by the fact that these features are useful in many industrial applications. The surface state, more precisely the texture of the surface, is in itself insufficient as an index of information to use the material.

Why knowledge of surface roughness is essential? For materials that are subjected to a process of material removal, the resulting surface state is dependent upon several factors. When visual and tactile comparisons are no longer sufficient to describe the roughness of surfaces, a profilometer is then used. The latter incorporates the operational principles discussed in this chapter and determines reliably and accurately the roughness parameters such as R_a and R_z , etc. A surface is defined here as a physical entity that characterizes the boundary separating an object from the other. Among many industrial applications requiring the use of a profilometer, we are specifically interested in:

- the use of a profilometer that allows control in the quality of definition at the end of the production line in order to detect potential problems that are encountered during the machining process; and
- the measurement of roughness, which allows evaluating the quality of a surface treatment process.

6.1.1. Terminology and definition of surface states criteria

First, we limit ourselves to the usual terminology; then, we detail some of the most commonly used terms [OBE 96]. We list some of these, e.g.:

- the sensitivity to thickness defect is important because it is, somehow, a typical discontinuity of surface topography. This defect is related to the metallurgy of the material, which influences roughness measurements;
- the lay reflects the predominant direction of the surface features. It is directly related to the manufacturing process. Therefore, dithering (number of skin passes, during machining) is an important factor in this phenomenon; and
- roughness characterizes the fineness of irregularities inherent to the applied technological methods (machining, molding, etc.). Irregularities concern an initially limited sampling length. The measured surface is consequent to the type of measuring instruments used. The resulting dimension is then called an “apparatus dimension.” The nominal surface of a material characterizes the contour subject to roughness;

- the surface state is a statistical notion resulting from random repetition of the actual surface concerning the roughness, the wave, the lay, and thickness (flaw); and
- the waviness is a widely spaced periodic component of the surface state. Unless otherwise indicated, the waviness integrates all irregularities on a sampled length. It may be due to various factors such as machine tools, arrows (deformations), thermal treatments, etc. It is, in fact, a superposition of waves from a warped surface.

In roughness, we must clearly distinguish between the above and terminology related to the measurement of surfaces state, such as:

- the profile is, in fact, the contour of the flat surface normally measured (or perpendicular) unless a specific angle is indicated;
- the arithmetic mean or mean of Gaussian, or least-squares line is called the central line (Graphical centerline);
- the mean line is used to calculate deviations. It is parallel to the direction of the profile containing the limits of the sampled length. The centerline ensuing from the filter is established starting from a cutoff line of the material under study;
- the profile measurements along the direction normal or perpendicular to the nominal profile is represented by height z , $Z(x)$;
- measured profile is a representation of the actual profile obtained from the device; and
- modified profile is a profile measured through a filter (apparatus reference included). It is used to minimize certain characteristics of the surface;
- roughness profile is obtained by filtering wavelengths;
- topography of roughness is the modified topography obtained by filtering wavelengths. Topography is a three-dimensional (3D) representation of geometric surface irregularities;
- sampling length is the nominal spacing where surface characteristic is determined;
- measured topography is a 3D representation of geometric surface irregularities, resulting from the measure;
- valley is the point corresponding to the maximum depth, on a portion of the profile with respect to the centerline;
- the evaluation length, L , is the length over which waviness parameters are calculated. The long-wavelength cutoff, l_{cw} , is the wavelength taken from a profile to identify its parameters;

- waviness short-wavelength cut-off, l_{sw} , is the spatial wavelength where roughness parameters are derived from an electric or digital filter; and
- topography of waviness consists of the modified topography obtained by filtering the shortest wavelengths of roughness and the longest wavelengths associated with errors of form.

6.1.2. Surface states (texture) and sampling lengths [MIT 82]

The sampling length is an interval which covers the value of a singular parameter surface. It is like a harmonic recurring at equal intervals. Traversing length is a profile explored by the stylus of Surftest to establish a representative length. It is always greater than the length of evaluation. We should refer to ANSI/ASME B46.1-1995 for the recommended values respectively for each type of measurement. Table A3.1 (see Appendix 3) provides information on ISO standards in this matter. Traversing length is, in fact, an actual length of the profile required to define the parameters of roughness to be controlled [MIT 76, 79, 80, 82, 00, OBE 96]. Note that a traversing length includes one or more basic lengths as shown in Figure 6.2.

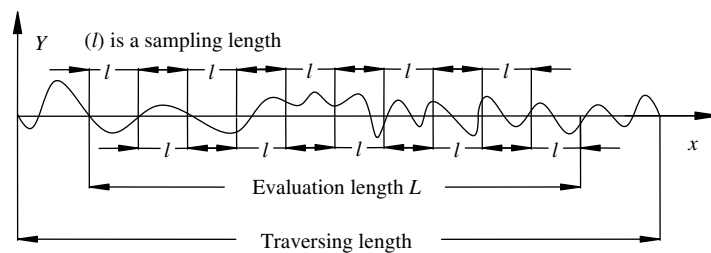


Figure 6.2. Traversing length traveled by the Surftest stylus

The evaluation length (L_e): as its name implies, is the length from which the surface characteristics are evaluated. For statistical reasons, we conventionally consider five sampling lengths (l). If more than five measurements were performed, this should be mentioned. In that case, we consider the evaluation length (L_e) of the roughness.

The cutoff or cutting length millimeter (or inches) is an electrical response characteristic of the measuring instrument. The cutoff is chosen at the limit of spacing of surface irregularities to be included in the assessment of surface condition. The roughness mean is defined as the arithmetic mean ($XX' = AA'$ Arithmetic Average) of absolute values of height deviations measured by the evaluated length, L_e . This is illustrated in Figure 6.3 by the shaded area (from a to u).

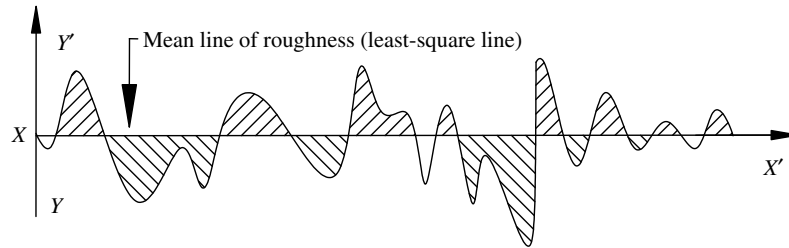


Figure 6.3. *Illustrative diagram of R_a and its mean line*

Typically, either the sampling lengths or the cutoff is included in any case. Graphically, we measure the average roughness (in micrometer) based on the deviations of heights measured normally or perpendicular to the mean line (XX'). Again, the average roughness is expressed in micrometer, i.e. 1/1000000th of a meter. Roughness average value, R_a , is a uniform interpretation of the reading of the stylus of a particular device, on a magnitude as small as possible. This roughness is mathematically indicated by the root mean square (RMS*), which is, in fact, the arithmetic mean.

(*) Root mean square (RMS). In physics (vibrations and sounds) the RMS translates a crest factor, in electricity, it reflects an effective value of a periodic quantity, in mathematics, it expresses the mean square, and in statistics, it reflects a rms of a single-valued function within an interval.

As roughness is a statistical indication of the RMS, an evaluation length over least squares line, we calibrate the meter to read at about 11% above the height of the device since this is expected to minimize an arithmetic average. Rugosimeters are often recalibrated to obtain the arithmetic mean (XX'). Some rugosimeter manufacturers consider that the difference between the (RMS) and the (XX') is small enough so that the RMS coincides with the mean line XX' . Manufacturer Mitutoyo, strongly recommends recalibration of the Surftest after many replicated measurements, even if these measurements were performed following the same process and under unchanged working conditions.

6.1.3. Waviness parameters

In fact, waviness reflects the wide spacing of the components of a surface state. Thus, roughness, under this concept (waviness), could be less easy to define. Therefore, we rather discuss the concept of waviness height, W_t , and waviness

length evaluation. The waviness height, W_t , is a height of peaks from valley to valley of the profile changed with the roughness and also with the formal errors, due to filtering or polishing processes (statistical entities). The obtained measure is considered in relation to normal or perpendicular to the nominal profile containing the waviness of the sampled length.

Waviness length, L_w , is evaluated in order to determine the waviness parameters. Note that for waviness, this is now obsolete. We prefer to use the wavelength of the cutoff (l_w).

As in functional dimensioning, there is a relationship between a rough surface and tolerances (upper and lower deviation) of a work piece obtained during manufacturing. We address this issue in relation to the assessment of surface states. We do so because the surface roughness measurement requires determining deviations from of the mean-line called the least squares or Gauss line (statistics). Thus, we notice that there is a direct relationship between surface roughness and dimensional tolerances. This requires an accurate measurement, taking into account the tolerance limits typically shown in an engineering drawing, prior to manufacturing. If this condition is not met, we practically find ourselves de facto in another important area in metrology i.e. uncertainty. This latter concept, which in itself is the proper definition of dimensional metrology, is illustrated in Figure 6.4. Note that the method of surface roughness measurement implies first determining the deviation from the mean. Generally, we approximate $4 \times$ roughness calculation (peak to valley height) compared to the height of the profile. This approximation is variable depending on the specific character of the surface and the test material. Surface roughness estimated values R_a depend on the manufacturing process, in accordance with ANSI B 461.1.1978 (United States and Canada).

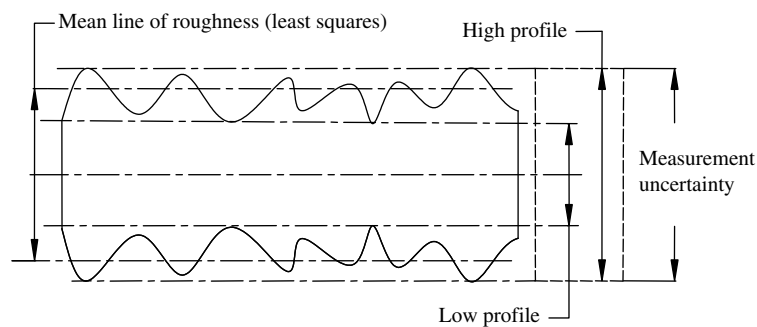


Figure 6.4. *Schematic illustration of measurements uncertainty*

6.2. Instruments for measuring surface state

Roughness measuring instruments are used to measure roughness and waviness. They are classified into six types [FAN 94, MIT 76, KAL 06], as listed below:

- Type 1 instrument: With profilometric contact, these instruments (profiling contact skidless instruments) are applied to control very smooth surfaces to measure waviness and roughness;
- Type 2 instrument: Non-contact profilometry instruments, these rugosimeters measure the total profile and analyze the topography of the surface (profiling non-contact instruments). Profiles may or may not be filtered. Since these devices would present disadvantages regarding inclined surfaces positions, an interféroscope could be included (see Chapter 5);
- Type 3 instrument: This is in itself a scanning probe microscope. This device is similar to the coordinate measurement machine (see Chapter 7) where an electronic probe is used; however, in the case of surface roughness a microscopic probe is more appropriate. With a very high resolution, since measurement is limited, scanning tunneling microscope (STM) is likely to be better suited to scan the surface;
- Type 4 instrument: These are profiling contact skidded instruments. To eliminate the longest wavelengths, they are used with a skid as a reference. Unfortunately, these devices are excluded in cases of waves control or form errors assessment;
- Type 5 instrument: These instruments deal exclusively with only skid (skidded instruments with parameters only) and hence do not generate profiles. The skid in this case allows eliminating the highest wavelengths;
- Type 6 instrument: (area averaging methods) These instruments measure the average parameters on specific surfaces. They do not generate profiles.

6.2.1. *Selecting cutoff for roughness measurements*

Typically, the analyzed surfaces contain irregularities, which cover a large area. Equipment and instruments used for their measurements are intended to measure the irregularities at a distance less than that of the given values. This is called a cutoff value and is chosen for different surfaces requiring measurement that are suitable for control of irregularities. For example, the effect of cutoff variations is shown in Figure 6.5.

We notice that the profile on the top reflecting Surftest stylus movement corresponds to the surface whose rugosities are spaced nearly by 1 mm. Profiles are retained on surfaces with a cutoff of about 0.8 mm, 0.25 mm, and 0.08 mm. We note

that on the surface corresponding to the choice of a cutoff of 0.8 mm, there is a trace of high irregularities. The trace on a cutoff of 0.25 mm excludes high irregularities and processes those called medium or fine. The trace based on a cutoff of 0.8 mm includes fine irregularities. Usually, R_a represents the roughness of the finite (except in specific cases) of the machined surface, in engineering drawings. In the recent past, many types of surface states were in use. Still, confusion added to a vocabulary sometimes overused to describe the surface state, the cutoff, and the sampling length.

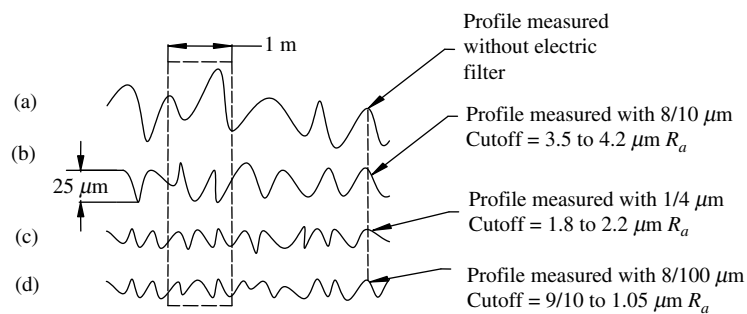
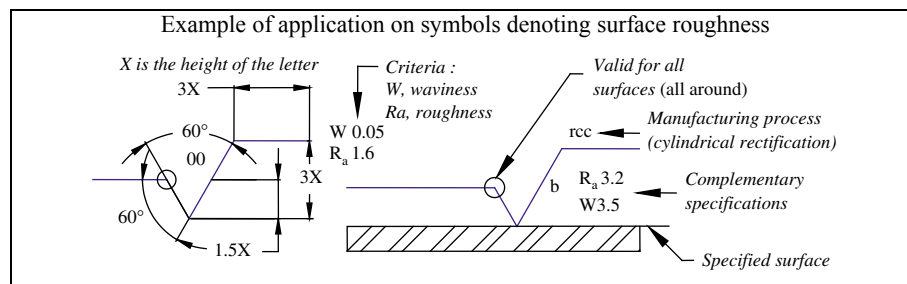


Figure 6.5. Influence effects as function of different values of the cut-off [MIT 82, OBE 96]

6.3. Symbols used in engineering drawings to describe the appropriate surface state according to ANSI/ASME Y14. 36M-1996

The ISO and ANSI (United States and Canada) standards offer a brief method for selecting symbols in a drawing to denote the surface state of a solid material. These symbols also indicate the technologic means of used in obtaining the surface to be controlled. The US standard is equivalent to the international one. It uses the International System (SI) of Units [OBE 96]. Table 6.1 summarizes the commonly used symbols in a drawing.



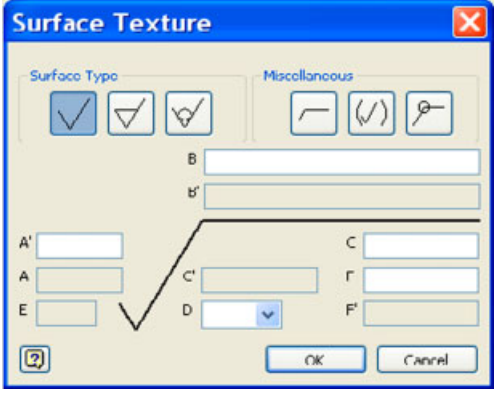
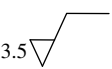
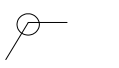
	
Bilingual signification of symbols from left to right →	
	– <i>Basic surface texture symbol</i> (a) Basic surface texture symbol. This surface may be obtained by any process unless the symbol has a circle (see d) or a complementary bar (see b)
	– <i>Material removal is required</i> (b) Material removal required. Horizontal line indicates that material removal by machining is required; material must be provided for that purpose
	– <i>Material removal required</i> (c) Material removal allowance. For example, the number indicated (3.5) in inches (or millimeters) indicates the density of material to be removed by machining. The tolerance may be added to the basic symbol, in many cases
	– <i>Material removal is prohibited</i> (d) Removal of material prohibited. The circle added to the symbol indicates that the surface must be the result of technological process such as forging, casting, sintering (powder metallurgy) or plastic injection, without subsequent removal of material
	– <i>Specifically designed = Force tail</i> (e) is a symbol used to characterize a specific surface condition. Information on the technological process of obtaining the surface state (i.e. grinding, lapping, etc.) is added either to the horizontal bar or next to the symbol information on the technological process. The surface is obtained in this case by any method unless the circle is added inside the triangle to specify the non-material removal
	– <i>Majority identical to the first symbol (a) = Majority for the drawing</i>
	– <i>All-around, values for the surface characteristics</i>

Table 6.1. Designation of roughness symbols

6.3.1. Surface characteristics in a drawing using CAD–CAO software

The surface characteristics in this regard are listed below:

A indicates the minimum roughness value;

A' indicates the maximum roughness value (roughness value R_a maximum);

B shows the method of manufacturing, processing, or cup according to ISO and ANSI;

B' also indicates manufacturing method according to ISO or DIN (Germany);

C indicates the cutoff of the roughness or sampling length for the average roughness according to ISO or DIN. It sometimes specifies the wave height or sampling length. According to the Japanese standard JIS, it specifies the value of the cutoff length and evaluation;

C' indicates the cutoff of the roughness or the sampling length value of additional sampling for roughness. For ISO and DIN, it specifies the length of sampling for the additional roughness. According to the Japanese standard, it is a baseline evaluation length measurement;

D indicates the direction of the striations (specifies the direction of lay). This option is no longer available when the option “prohibition of material removal” is active;

E is not available when the option “prohibition of material removal” is selected;

F indicates the tolerance allowed/permissible xxx specifies the machining allowance; and

F' indicates the waviness surface under the Japanese standard, JIS. This option is not used by ISO, ANSI, and DIN.

W 0.05 is a parameter based on physical criteria and R_a is a statistical criterion. The normalized values of the series are called Renard Series R10 of profile parameters or criteria in micrometer (Table 6.2).

Each value placed in a drawing shows a quantitative assessment of the surface. One reads easily, e.g. that a surface state whose finish is superfine would show a value of $R_a = 0.0125 \mu\text{m}$ and a value $R_a = 2.5 \mu\text{m}$ would correspond to a very rough

surface, etc. Sometimes, we deem it necessary to add the conventional roughness symbol, another symbol (perpendicular, parallel, or crossover) to clearly indicate the direction of the striation compared to the projection plane. Table 6.3 is an illustrative example in this regard.

0.005	0.0125	0.1	0.2	0.4	0.8
High degree of finish	Extra fine finish	Extra fine finish	Very high level	Finished very fine	Very good finish
1.6	3.2	6.3	12.5		
Good finish surface	Average finish	Coarse finish	Rough surface		

Table 6.2. Renard series R10

Upper value: a_1 Lower value: a_2	$\sqrt{\frac{a_1}{a_2}}$	Lay perpendicular to the plane of projection	
Standard length of 2.5 mm	$\sqrt{2.5}$	Lay perpendicular to the plane of projection	
Waviness height of 0.005 mm	$\sqrt{\frac{W_t}{0.5}}$	Lays crossed in two oblique directions	

Table 6.3. Normalized dimensioning of surface state according to ISO

In the field of machining, material removal is a big/wide topic of compromise that continues to raise many questions about the optimization of cutting parameters. In all cases of reading the roughness, it is always beneficial to refer to the recommendations of the ISO standards in this area. Even if sometimes the rule in question is doubtful, it would have the merit of attempting to standardize and promote the work of interchangeable parts that depend on them through functional dimensioning geometrical products specification (GPS). The standardized symbolization is presented in Table 6.4.

Material removal by machining			Observations *
Optional	Obligatory	Prohibited	
Ra 0.8 ✓	Ra 3.2 ✓	Ra 3.2 ⊙	Surface with R_a maxi = 3.2 μm
Ra 6.3 Ra 1.6 ✓	Ra 6.3 Ra 1.6 ✓	Ra 6.3 Ra 1.6 ⊙	Surface with R_a maxi = 6.2 μm and $R_a = 3.2 \mu\text{m}$
Ry 0.4 ✓	Ry 0.4 ✓	Ry 0.4 ⊙	Surface whose roughness is different from R_a , here $R_{y\text{Max}} = 0.4 \mu\text{m}$
✓ Rz 0.8 Rz 0.4	✓ Rz 0.8 Rz 0.4	⊙ Rz 0.8 Rz 0.4	Surface whose roughness is different from R_a , $R_{z\text{Max}} = 0.8 \mu\text{m}$ and $R_{z\text{Min}} = 0.4 \mu\text{m}$
basic surface texture symbol	Material removal is required (MMR)	Material removal prohibited (NMR)	Standardized symbols according to ISO 1302

Table 6.4. Summary of symbols used in rugosimetry, according to ISO 4287

We present in Figure 6.6 a sample application as an example.

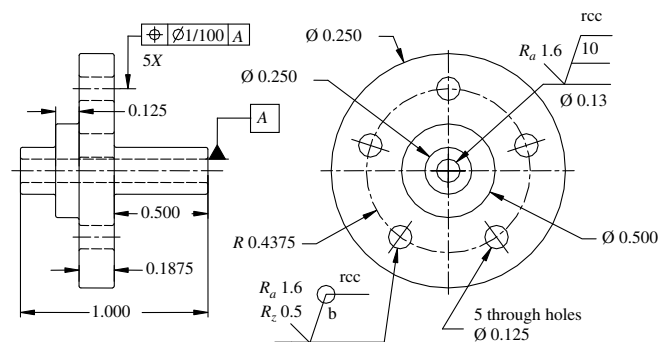


Figure 6.6. Example of surface texture indications on drawings according to ISO 1302. Drawing dimensioned according to ASME Y 14.36M-1996 equivalent to the ISO standard (GPS) 4287

Additionally, we propose, in Table A3.1a, (Appendix 3) summary table of the main ISO standards related to roughness. Of course, a standard alone does not solve the problem of roughness. It is imperative to respect the conditions that govern the problem, from the calibration to the techniques and experimental conditions. To this end, we propose experimental tables recommended by manufacturer Mitutoyo and used in rugosimetry. We provide an example in this regard in Table 6.5.

Basic lengths for measures R_a , R_q and basic lengths for measures R_z , R_v , R_p , R_c and R_t according to ISO 4288 standard			
R_A (micron)	R_z (micron)	Roughness basic (sampling) length l_r (mm)	Evaluation length of roughness l_n (mm)
$(0.006) < R_A < 0,02$	$(0.025) < R_z < 0.1$	0.08	0.4
$0.02 < R_A < 0,1$	$0.1 < R_z < 0.5$	0.25	1.25
$0.1 < R_A < 2$	$0,5 < R_z < 10$	0.8	4
$2 < R_A < 10$	$10 < R_z < 50$	2.5	12.5
$10 < R_A < 80$	$50 < R_z < 200$	8	40

Table 6.5. Standardized assessments of a surface state (source: [MIT 82])

When a roughness symbol is used, it affects the whole surface, unless otherwise specified. Measurements of roughness and waviness apply depending on the direction where the lay allows a maximum reading. The recommended series of cutoff are grouped in Table 6.6.

Mm	mm	Mm
0.08	0.25	0.80

Table 6.6. Series of sampling length (cutoff) (source: [MIT 82])

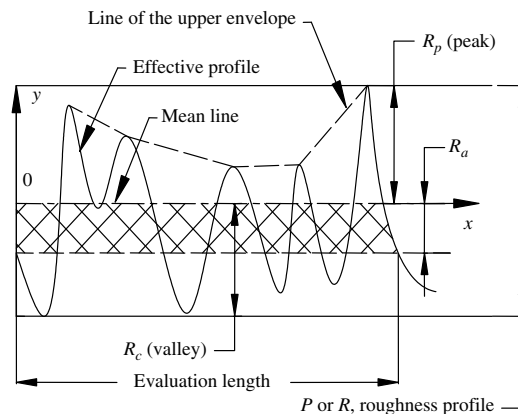
In cases that concern surface roughness, the ISO standards are close to the ANSI/ASME Y14.36M standards. We cite, e.g. ISO 1302:1992, which covers methods of specification of surface conditions and symbols used in engineering drawing (Table 6.7). The ISO 1302 standard defines the criteria of roughness, waviness, grooves, etc. Considerations of inspection and multiple imperfections are addressed by the ISO 8785 standard. The ASME Y14.36 and ISO 1302 standards do not address the aspects of brightness, painting or design, much less corrosion, microhardness, or the analysis of microstructures. When the sample length is specified in the drawing of the test piece, the cutoff of the wavelength (L_c) is equal to the sampled length.

	= Lay parallel to the line representing the surface to which the symbol is applied.
	Lay perpendicular to the line representing the surface to which the symbol is applied.
	X Lay angular in both directions to line representing the surface to which the symbol is applied.
	M Lay multidirectional.
	C Lay circular relative to the center of the surface to which the symbol is applied.
	R Lay radial to the center of the surface to which the symbol is applied.
	P Lay particular, non-directional, or protruding.

Table 6.7. Indication of lay symbols

6.3.2. Expressions of the terms of surface roughness

Many definitions [FAN 94, MIT 82, OBE 96] are available to signify roughness. The definition we use in this handbook is the arithmetic average roughness, R_a . It is equal to the average value of $|y|$ on the length L of analysis, as shown schematically in Figure 6.7.

Figure 6.7. Designation of profiles: R_t , R_p and R_c

On the surface analyzed portion, roughness measurement provides access to:

- R_p , maximum peak height: $R_p = y_{\max}$;
- R_c , maximum valley depth: $R_c = |y_{\min}|$; and
- in this case, the total roughness is then: $R_t = R_p + R_c$.

We distinguish an arithmetic average roughness R_a , which is the most commonly used in textbooks and various laboratories.

6.3.2.1. Arithmetic average roughness value, R_a (according to DIN 4768)

R_a is the arithmetical mean deviation of the roughness profile. It is, therefore, an average value of $|y|$ over the analyzed length. This is the case where the assessment requires R_z (DIN 4768), which is the average value of roughness depth values of five successive evaluation areas or in other words, the quadratic mean deviation of peaks. Figure 6.8 is the graphical representation that would result as a consequence.

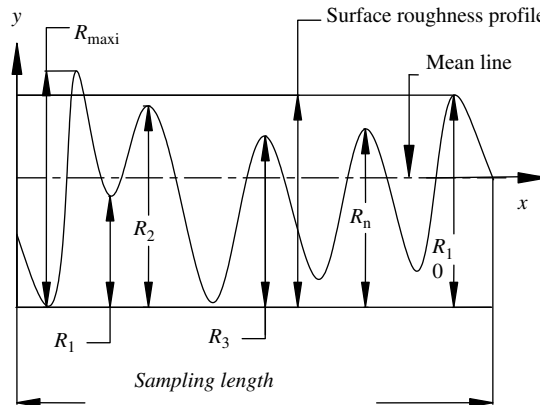


Figure 6.8. Quadratic mean deviation of peaks R

$$R_z = \left(\frac{R_1 + R_3 + R_5 + R_7 + R_9}{5} \right) - \left(\frac{R_2 + R_4 + R_6 + R_8 + R_{10}}{5} \right) \quad [6.1]$$

In the case where we consider the average quadratic mean of peak Z , we provide here a schematic illustration followed by its mathematical expression R_z (Figure 6.9).

$$R_z = \left(\frac{Z_1 + Z_2 + Z_3 + Z_4 + Z_5}{5} \right) \quad [6.2]$$

$$l_m = 5 \times (l_e) \quad \text{with} \quad R_z = Z_i \text{ (maxi)} \quad [6.3]$$

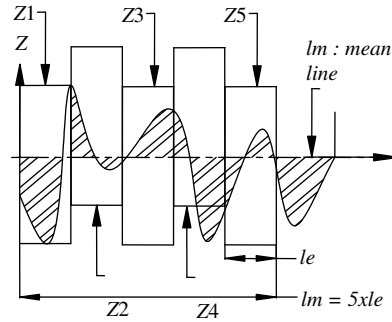


Figure 6.9. Quadratic mean of peaks R_q .

The average value y is zero if the equation of the mean line is $y = 0$. Of course, one may also use the mean quadratic roughness whose value is derived from a simple statistical treatment, represented by R_q which is the square root of the mean value y^2 over the sampling length lm . Maximum peak $\rightarrow R_p = y_{\max}$ and maximum valley $\rightarrow R_c = |y_{\min}|$. The total roughness is the sum of maximum peak and maximum valley observed over the assessment length ($R_t = R_p + R_c$) (Figure 6.7).

The US standard ANSI/ASME B46.1 consists of a clear case of all manufacturing processes and provides information on the roughness (R_a in micrometer). In addition to ISO standards dealing with the roughness, we present the whole process in conjunction with the corresponding ranges of R_a . There are, e.g. tables for information on the association with the technological process for obtaining the surface texture of the material under study as summarized in Table 6.8.

Table of R_a (micrometer or microinch), depending on the technological process

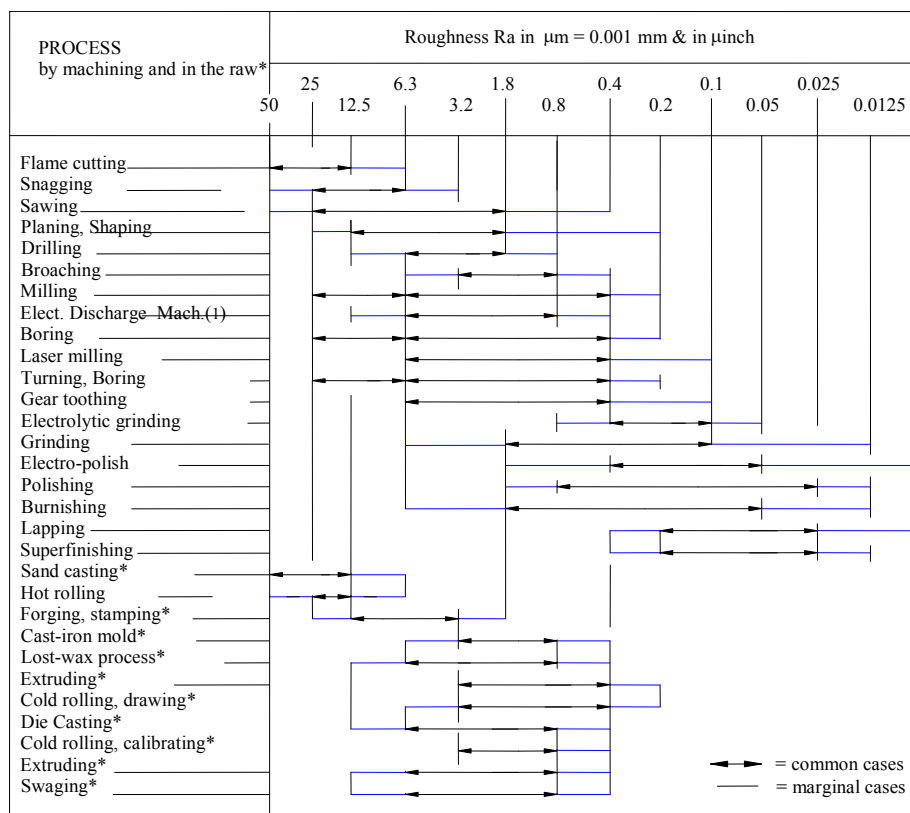
6.3.2.2. Average peak-to-valley height

According to ISO 3274, we consider a sampling length representative of the navigation range (surf cutoff). Of course, the more the testing samples are numerous, the more significant will be the resulting average value of the surface profile. Based on the fact that the surface irregularities reflect a non-compliance with the nominal surface and hence also are reflected by the undulations, flaws, angle of heel, and the profile (hence the term profilometer), it is important to:

1. let the device to rest after it was powered up;
2. check the calibration of the device according to the instructions specified by the standards;
3. calibrate in accordance with the definition of manufacturer and standards;

4. properly clean the test surface and the stylus of reading;
5. travel the probe (1/8 in./s) over the test surface, slowly and evenly; and
6. read and memorize the roughness value R_a , R_z , and $R_{\max}$.

Typically, there are four types of surface.



(1) Electric discharge machining

Table 6.8. Table of R_a in μm and μinches , depending on the technological process

6.3.3. Description of the main surface states

In roughness measurement, there are four major surfaces and one profile [FAN 94]:

– geometric surface:	SG	→ defect of order I
– specific surface:	SS	→ defect of order II
– real surface:	SR	→ defect of order III
– measured surface:	SM	→ defect of order IV
– surface profile:	PS or R_a	→ defect of order V

6.3.3.1. Geometric surface, SG (perfect surface)

This surface shows specific properties in the drawing regarding flatness, straightness, roundness, etc. In the drawing, it is geometrically defined by the design department using nominal dimensions. Table 6.8 is usually included in all handbooks dealing with materials technology. It is not exhaustive but it includes most of the processes used in mechanical manufacturing.

6.3.3.2. Specific surface (SS)

In fact, this surface results from the geometrical surface (GS). It takes into account the tolerance of form, position, and descriptions of surface state. The specific surface area is bounded by the limits laid down in the drawing. Flaws that are usually associated with them are generated by deformations caused by machinery, vibrations, chatter, or by heat treatments. This surface is transformed by the design department which prescribes the limits of the realization of the surface with the aid of symbols and numerical values in addition to the nominal dimensions of the drawing.

6.3.3.3. Real surface (RS)

This is the surface resulting from the manufacturing process. It is inherent in the values from the mechanical manufacturing, such as lay of roughness left by the furrows of tools which may be distinguished as:

- L , which is the basic length of the average profile;
- R_z is the average depth of roughness parameter. The arithmetic mean of the ordinate values y of all points of the profile on the base length L is written as:

$$R_z = \frac{1}{L} \times \int_n^L f(x) dx \quad \text{with } y = f(x) \quad [6.4]$$

R_a is the arithmetic mean deviation of the surface, that is to say, the arithmetic mean of the absolute values of the ordinate OY (between each point of the curve and the axis OX). In view of Figure 6.7, let us consider

$$R_a = \frac{1}{L} \times \int_n^L \left| \frac{\partial}{\partial x} f(x) \right| dx = \frac{1}{L} \times \int_n^L |f(x) - R_z| dx \quad [6.5]$$

6.3.3.4. *Surface being measured (SM)*

This surface is determined, e.g. by the position of the unit of measurement from the real surface. According to the jargon used in manufacturing and mechanical metrology, it is based on the dimensioning device. This surface is not immutable because it is subject to the control of the measurement device. The defects correspond to avulsion or slots sometimes accidental, after machining or even the use of mechanical parts, differences in shape or position may result. It is the same for periodic waviness. One can distinguish roughness located within the faults already listed. This roughness generally appears through striations, furrows, avulsions (metal), tool marks, and pitting (corrosion cracking seats). The actual measured surface is determined via measuring instruments (profilometer or rugosimeter) on real surface. The SM resulting from the exploration of the RS image should be closest to it.

6.3.3.5. *Evaluation (basic) length (L)*

The evaluation length L is divided into sections that provide information on irregularities. The middle line of each section is a straight line. The mean line known under the term “least squares” defines the direction of the profile. For each basic length, the line is assimilated to a straight line and the sum of the areas above the mean line is equal to those below as shown in Figure 6.8. The defects that we measure using a rugosimeter may result from various causes. These types of defects can be macrographic or micrographic.

We have previously presented four surfaces and one profile. We now present the principles, definitions, and criteria of defects:

- First-order defects: These correspond to the geometrical surface defects such as flatness, straightness, roundness, etc. The applicable exploration methods include electronic sensing device (inductive touch sensing), etc.;
- Second-order defects: These are related to waviness, kinds of successive hills and valleys included in the profile generated by the vibrations, machines deformations, chattering, heat treatment, etc. The applicable exploration methods include sensing electronic device (inductive), etc.;
- Third-order defects: These are roughness ridges, furrows traced with regularity of the undulations relief by cutting tools. The applicable exploration methods include electronic device sensor (inductive or piezoelectric) sensor-needle, optical devices, etc.; and
- Fourth-order defects: These are more irregular, sometimes accidental, they correspond to pull-outs, cracks, or fissures in the material. The applicable exploration methods include electronic sensing device (piezoelectric or inductive), optical apparatus, sensor, etc.

The cutoff length is the length of the sample profile for determining the roughness R_a . The recommended lengths in millimeters are $\{0.08 \rightarrow 0.25 \rightarrow 0.8 \rightarrow 2.5 \rightarrow 8.0 \rightarrow 25.0\}$.

6.3.3.6. Designation of roughness abbreviation R_a as a function of the process

Table 6.9 includes all abbreviated technological processes, used to obtain a given surface condition. In French, as in other languages, the abbreviation is sometimes added to properly dimension a design that requires an indication of the roughness during the manufacturing process.

Normalized abbreviation of processing (French)					
English	French	Abbreviation (Fr)	English	French	Abbreviation (Fr)
Boring	Alésage	al	Counterbore	Lamage	lm
Broaching	Brochage	br	Hot-rolling	Laminage à chaud	lac
Cutting	Découpage	de	Cold-rolling	Laminage à froid	laf
Facing	Dressage	dr	Swaging	Matricage	ma
Electrical discharge machining	Electro-érosion	é	Grinding	Meulage	me
Electroforming	Electro-formage	éf	Sand-casting	Moulage sable	mos
Electro-polish	Electro-polissage	ép	Cast-iron mold	Moulage coquille	moc
Stamping	Estampage	es	Drilling	Perçage	pe
Flash welding	Etincelage	ei	Honing	Pierrage	pi
	Etirage	et	Polishing	Polissage	po
Thread	Filetage	fl	Planation, planing	Rabotage	rb
End-Milling	Fraisage en bout	frb	Cylindrical grinding	Rectification cylindrique	rcc
Out-Milling	Fraisage en roulant	frr	Planar grinding	Rectification plane	rcp
Drop-Forging	Forgeage	fo	Lapping	Rodage	rd
Burnishing	Galetage	ga	Wet Sandblasting	Sablage humide	sah
Scrapping	Grattage	gr	Dry Sandblasting	Sablage à sec	sas
Angular shot-blasting	Grenailage angulaire	gna	Sawing	Sciage	sc
Spherical shot-blasting	Grenailage sphérique	gns	Superfinish	Superfinition	sf
			Turning/Spinning	Tournage	to

Table 6.9. Processing normalized abbreviation (English/French)

Classification of surface defects: Surface defects are represented by the form, waviness, and roughness. The specifications are represented by the currently applicable standard (ISO 1302). Contact measurement devices are represented, according to the standard ISO 3274 which includes:

- sensors;
- transducers;
- probing reference; and
- signal processing (ISO 11 562).

Calibration is in line with ISO 5436. Procedures for roughness measurements, according to ISO 4288 are influenced by many considerations, namely the precautions to be taken, the choice of measurement conditions, and operating results.

6.4. Presentation of Mitutoyo Surftest 211

To measure surface roughness, various devices [MIT 00, 79] may be used (Figure 6.10). In this section, we present some useful information about the manipulation of the rugosimeter used in our workshops and laboratories.



Figure 6.10. *Mitutoyo Surftest 211 (courtesy of Mitutoyo Canada)*

6.4.1. Components of rugosimeter 211

Rugotests (visual-tactile comparators) and electronic rugosimeters can assess the surface roughness. We present the portable model whose pad can be fixed below the housing or disconnected:

- several parameters can be measured including R_a , $R_{\max}$, and R_z in accordance with three standards: ISO (International), JIS (Japan), and DIN (Germany);
- detector by inductive system;
- automatic calibration;
- function good/bad; and
- alimentation: battery or alternating current (AC) adapter.

Rugosimeters are usually equipped with:

- detection unit: it can be integrated to the monitor and can also operate outside it. However, it must always be connected to the monitor;
- sensor: while “surfing,” it penetrates the surface valleys under the effect of the pressure of its support and it sends information to be processed by the monitor;
- connecting cable: it serves to connect the sensor unit to the monitor;
- AC adapter: it converts AC 110 V into direct current (DC) 9 V;
- monitor: the principal component of the rugosimeter provides the operator with various measurement values taken by the detection unit;
- standard for calibration: serves as a reference to calibrate the roughness;
- support for calibration: it is the plate that serves to support and properly position the set monitor/detection unit, during calibration.

Later in this chapter, we develop the role of the measuring chain of device with a probe. Now, we present the components of Mitutoyo Surftest 211 (Figure 6.11).

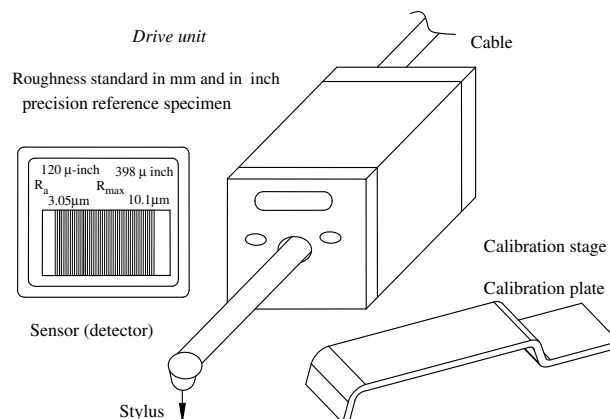


Figure 6.11. Rugosimeter accessories (see also Figure 6.12)

The seven components mentioned above are interconnected as follows: the sensor is placed within the detection unit carefully, taking care to orient the small sapphire down. We already know that the monitor and the sensor unit can be physically separated from each other while being electrically connected by a cable connection. The AC adapter supplies power needed to the monitor. Connections are made at the rear of the unit. The standard and the calibration plate are used together for the calibration of the rugosimeter. The main setting and connection functions are indicated to facilitate the assembly and the various options appropriately.

Before each experiment, it is imperative to calibrate the device. Therefore, it is important to recalibrate the device when the handling duration was prolonged. In addition, the calibration shall be carried out after having selected the different measurement parameters. Small switches are placed behind the rugosimeter (Figure 6.12). They allow an operator to select among the different measurement parameters mentioned above such as: type of the surface roughness being measured, the measuring system, units of measure, and the cutoff length. A small screw allows adjustment of the upper limit of the roughness considered and a second screw is used to calibrate the apparatus in accordance with the roughness standard.

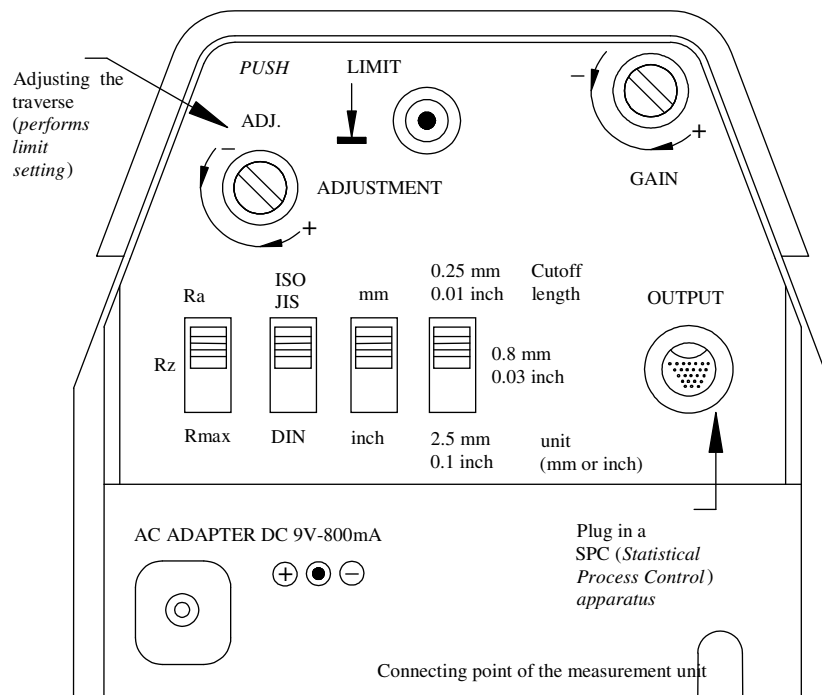


Figure 6.12. Back view of rugosimeter 211 features

6.4.2. Calibration of Mitutoyo rugosimeter 211

First, we should activate the apparatus (by pressing the “ON” button) and then calibrate the rugosimeter. Next, the START button should be activated. The probe moves perpendicularly to the profile. At the limit switch, a reading is provided (R_a , R_z , or even $R_{\max}$ expressed in the unit initially chosen in micrometer or microinches). If necessary, we may turn the adjustment screw while reading, and resume reading until adjusting to the measure indicated by the standard.

6.4.3. Measurement

We should make sure that the upper limit of the chosen parameter is high enough; otherwise, the flashing image would appear on the screen as shown in Figure 6.13 showing that the actual measured value is higher than the upper limit in force (active). While holding the black limit button (Figure 6.12) and turning the adjustment screw located on top left, we change the upper limit to correct the situation. The measurement is launched by pressing “START”. We see at this time a series of dotted lines which gradually disappear over time during the measurement, as shown schematically in Figure 6.13.

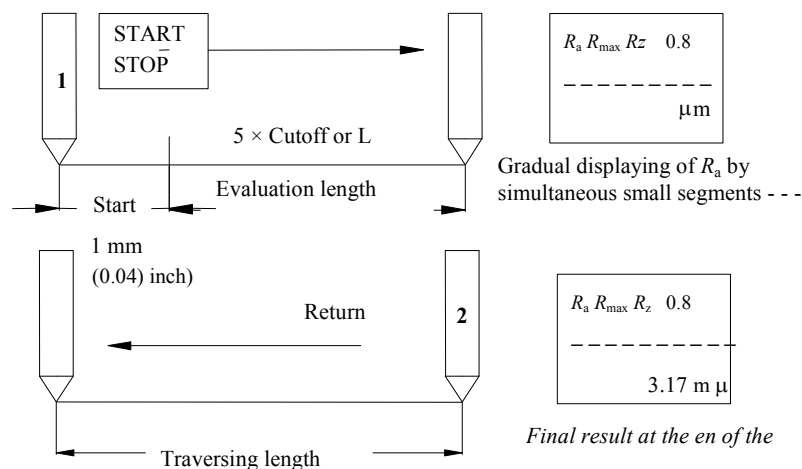


Figure 6.13. Measurement of R_a using a rugosimeter

6.4.4. Practical example on the application of Surftest 211

The assembly that we have prepared illustrates that Mitutoyo Surftest 211 can be mounted as exhibited in Figure 6.14. This device is not equipped with a sliding

block as is the case for the apparatus SJ-401 (Figure 6.15). For reasons of guiding without skid, it is therefore necessary to make sure that the basis of Surftest 211 is coplanar with the front part to be tested. This function is very important to fulfill in order to obtain a correct sampling.

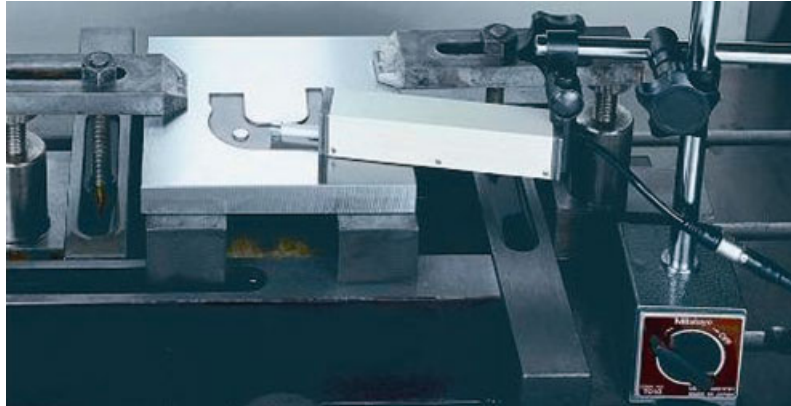


Figure 6.14. Photograph illustrating an example of surface testing using a rugosimeter (211)



Figure 6.15. Portable rugosimeter SJ-400 (courtesy of Mitutoyo Canada)

In the section dedicated to laboratory tests on roughness, we follow the same procedure in terms of the verification of the surface state of the grind workpiece. We deduce, R_a for an average value of 10 consecutive measurements and compare it to the roughness value given in Table 6.10 of ANSI B46.1-1978 standard (Revised 1984).

6.4.5. Portable rugosimeter SJ-400 by Mitutoyo

According to Mitutoyo, the SJ-201 is the smallest SJ model. The measurement scale for R_a , R_y , R_p , R_t , and R_z , increased from 160 to 300 μm . Surfaces, whose roughnesses were previously too large for the (ST) Surftest 211, are perfectly measured with the new LS-201. The selection of the number of measured sections and measurement length are adjustable and optional.

According to the manufacturer Mitutoyo, the ST-211 had always been performing measurement on the basis of $5\times$ the (filtered) section of adjusted measure. With the SJ-400, one can choose to measure either on $1\times$, $3\times$, or $5\times$ the measured section. It can measure in places where we do not have space for measuring greater lengths. This is advantageous when one knows the cramped tables of some fixtures on machine tools. The measurement length L for the unfiltered profile P can be perfectly adjusted to the choice for the SJ-201 (up to 12.5 mm maximum) so as to accurately measure the path that was selected. The difference, in this case, is based particularly on how the surface profile is measured.

For most portable surface rugosimeters, measurement is based on a sensor with a skid because the drive unit can be kept as compact as possible due to the lack of inner guidance. With the skid, the sensor follows the surface of the workpiece automatically. This measure shows, however, drawbacks because we know nothing about the “form” of the surface. To simplify things, the profile elements belonging to the surface within the wavelength that is shorter than the adjusted value (the filter) of the cutoff are perceived as roughness, whereas elements of the surface profile at a greater wavelength are perceived as the shape of the workpiece and are therefore not included in the value of roughness. By using a pad in the measurement, we actually create an additional (mechanical) filter since the pad follows the shape of the piece so that the sensor does not record this form.

For measurements of normal roughness, this has no significant influence. This is due to the fact that surface wavelengths that we “miss” on the workpiece, represent a multiple of the wavelength cutoff of the electronic filter. Regarding non-flat pieces, the skid method can cause slight differences since the point on which the pad rests on the workpiece and points where the stylus records the shape of the workpiece can never be exactly the same. As for the skidless measurement, a drive unit with a built-in guide is needed. The skidless measurement offers a range of additional features such as:

- accurate measurement in the absence of limitations of the method with skating in tightened locations;
- determining the waviness of the workpiece; and
- use of the tracked profile (unfiltered) to “display” the surface.

In order to allow measurement without skid, the drive unit of SJ-400 is not only equipped with accurate control facility, but also an adjustable height and angle for the alignment of the unit compared to the workpiece. At the end of this section, we notice that there are both advantages and disadvantages in using skids. We try our best in what follows to develop an argument inspired from the catalog of Mitutoyo [MIT 00].

6.4.5.1. Measurement with and without skid

Measurement devices are considerably developed. For example Surftest SJ-400 uses a probe allowing measurement with or without skid. Thus, it allows covering the majority of special measurement conditions. If we measure without a pad, we could detect irregularities of the surface with the help of the reference guide of the feed unit. It is, however, possible to measure the waviness and the profile (see Figure 6.16(a)). When measuring with a pad, surface irregularities can be detected because in practice the pad follows the surface waviness (except the waviness with a walk). The type of measurement (Figure 6.16(b)) accommodates the alignment of the test piece from the feed unit.

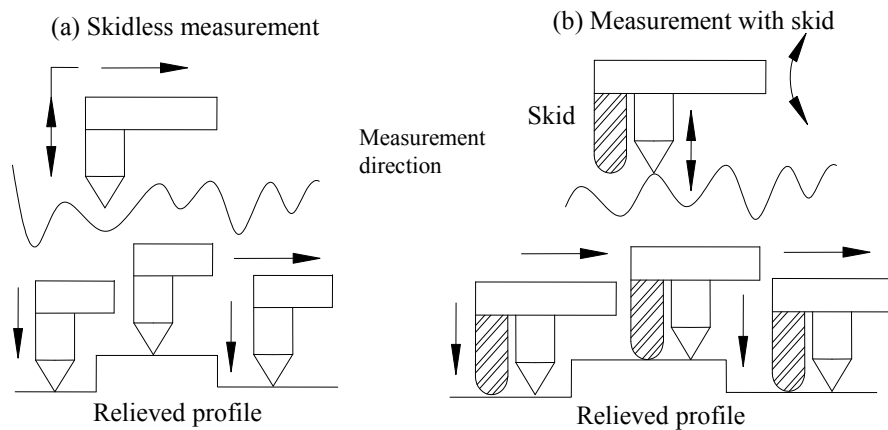


Figure 6.16. Illustration of measurements with and without skid

The unit includes a preleveling device allowing efficient alignment during skidless measurement. This handy feature provides a very high accuracy. We provide here a measurement chain with skid versus skidless measurement pattern (Figure 6.17).

In rugosimetry, surface testing is performed in various ways. The measure, however, is dependent on the shape of the surface to be probed.

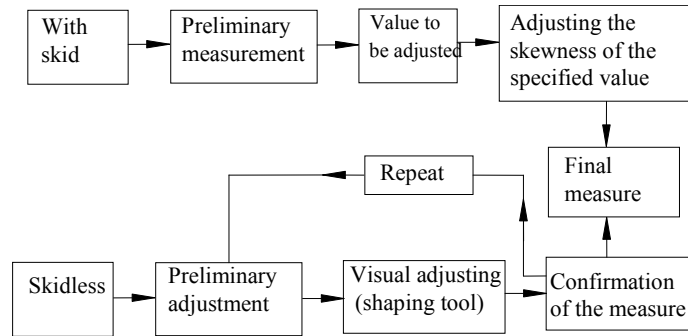


Figure 6.17. Comparison of two types of measurements (with and without skid)

6.4.5.2. Photographic illustrations of surface testing position

Photographs in Figure 6.18 are excerpts from the catalog with the courtesy of Mitutoyo Canada. We use them here in order to emphasize the various uses of Surftest 211 and SJ-400.

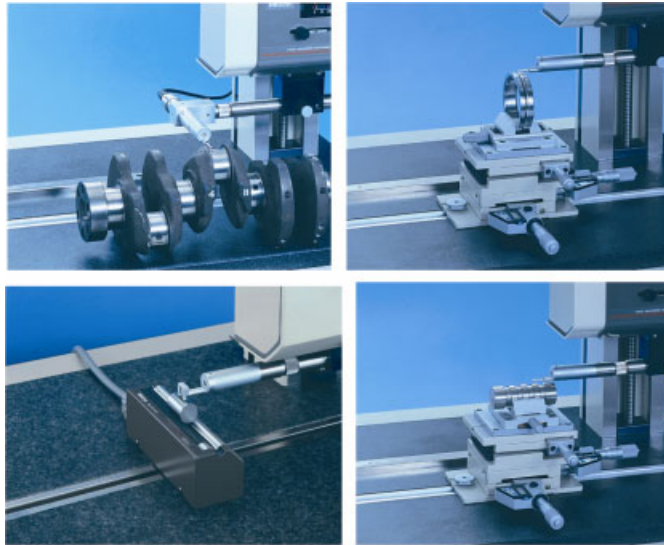


Figure 6.18. Examples of profile measurement based on the position [MIT 00] (courtesy of Mitutoyo Canada)

The development of portable testing devices for surface roughness control is significant. The ability to measure in various positions has become an important

feature to provide a wide range of applications. For example, the Mitutoyo SJ-400 allows direct measurement, either on pieces of machine tool or on large pieces difficult to move, as shown in Figure 6.19.

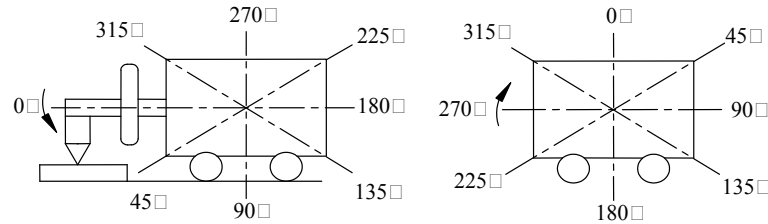


Figure 6.19. Displacements in longitudinal and lateral tilt; $l_p = 12.5\text{ mm}$

A comparison work [MIT 82] was conducted to analyze the rectitude of horizontal displacement in a skidless measurement (probing length = 12.5 mm). This Mitutoyo laboratory work clearly shows the influence of the angle of inclination on the rectitude in lateral and longitudinal positions.

6.5. The main normalized parameters of surface states used in the industry, their formulas, and definitions

In terms of rugosimetry regarding roughness measurement, there are various methods of control that involve appropriate means [GRO 90, MIT 00, SLE 99]. To this, we propose a summary table (Table 6.10) that summarizes the evaluation methods on a given surface and the exploration and devices for measuring a profile. As part of this handbook, we endeavor to confine ourselves to rugosimetry. It is useful to revisit the definitions already given in this chapter. The terminology and the general definitions given in this section are excerpted or taken in part from ISO 3274. The appliances and devices are classified according to whether the assessment is done by exploring a surface portion or exploration of a profile (common case).

Devices and systems for measurement of surface geometric parameters			
Evaluation on a given area	Measurements on a given profile		
Visual–tactile comparison sample	Progressive assessment of a given profile	Overall exploration of a given profile	
Optical metrology (reflection–refraction) Interferometric Metrology	Contact devices	Contactless devices	
Photogrammetry	Profilometer	Profilometer	Device with optical section
Capacimetry			

Table 6.10. Main standardized parameters in surface metrology

DEFINITION OF PROBE APPARATUS.— A measuring device that explores the surface using a probe, records the differences of the surface profile, calculates the parameters, and can protect the profile from measurement-related procedures. Some appliances have a displacement measurement and a digital recording of the profile with filtering criteria. The measuring device of a probe is a closed chain, which includes all mechanical components between the test object and the tip of the probe. We cite, e.g. ways of positioning, clamping, measuring device, the feed unit, and the sensor (pick-up).

DEFINITION OF SENSOR (PICK-UP).— The sensor is a component of the measuring device containing the element probing with the tip of the probe and transducer. The probing element is the element that transmits the displacement of the tip of the probe to the transducer.

DEFINITION OF THE TIP OF PROBE.— It consists of a circular cone having an angle normally defined (60° or 90°) and a spherical end with a defined radius ($2\text{ }\mu\text{m}$, $5\text{ }\mu\text{m}$, and $10\text{ }\mu\text{m}$) resting on the surface to be measured. The device that converts the vertical coordinates of the profile into a signal used by the device is a transducer.

There are four main filters that are defined as follows:

- Profile filters (P): these separate the profile components and long-wavelength components of short wavelength;
- Profile filter λ_s : it defines the separation between the components of roughness and those of shorter wavelength present on the surface to be measured;
- Profile filter λ_c : it defines the separation between the components of roughness and waviness components; and
- Profile filter λ_f : it defines the separation between the components of ripple and wave components even longer present on the surface.

During the measurements, it is necessary to distinguish between the surface profile and the path. This is actually a geometric seat in the center of the probe having a conic geometric shape with a spherical end. Based on this profile layout, other profiles are defined (primary, waviness, roughness, etc.). For example:

- Primary profile: It serves as a basis for evaluating the parameters of the primary profile. This is the profile resulting from the total profile after applying the filter on short wavelength λ_s .
- Roughness profile: It is a profile obtained from the primary profile by suppressing a component of long wavelength by applying the filter profile λ_c . This profile is used to calculate the roughness parameters whose symbol is (R). The current ISO standard on this subject is ISO 3274 (Figure 6.20).

– Waviness profile: It is obtained by applying a filter to the primary profile. The evaluation of waviness parameters is dependent on the waviness profile. The symbol is W .

– Total profile: It is a digital representation of the profile plot.

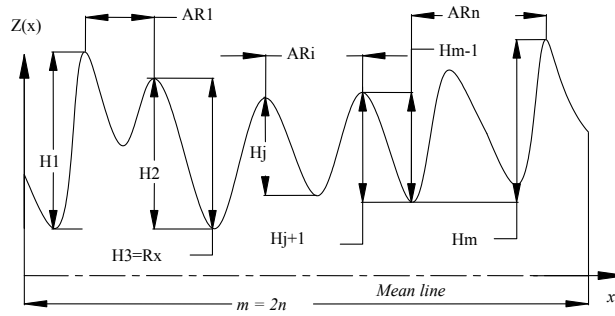


Figure 6.20. Roughness parameters according to ISO 12085

Further, AR is the mean pitch of roughness and R is the depth of roughness. The mathematical expressions of roughness parameters are written in micrometer (microinch) as follows:

– average depth of roughness:

$$R = \frac{1}{m} \times \sum_{j=1}^m H_j \quad [6.6]$$

– mean pitch of roughness:

$$AR = \frac{1}{n} \times \sum_{j=1}^m AR_j \quad [6.7]$$

This represents the arithmetic mean of lengths, AR_i , which helps us to understand that roughness is located within the length of evaluation.

– average height of roughness:

$$R_x = H_3, \text{ with } m = 2n \quad [6.8]$$

6.5.1. Waviness parameters

In what follows, we continue to define separately the parameters of waviness and roughness. We should not confuse the roughness profile R_a with the waviness profile

W . The latter is derived from the primary profile by successive application of filters λ_f on the profile (removing the components of long wavelength) and λ_c (removing the components of short wavelength). The graph in Figure 6.21 clearly shows a waviness pattern.

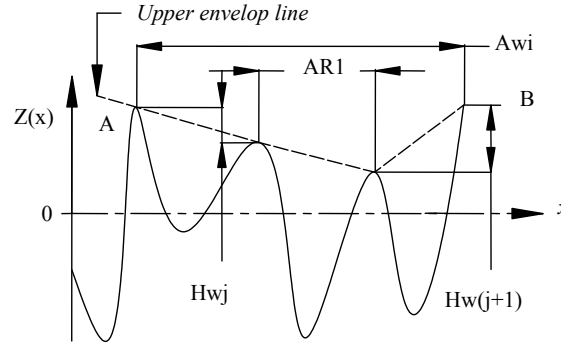


Figure 6.21. Waviness profile according ISO 12085

Waviness parameters (W) are calculated, by analogy to roughness parameters and are expressed as follows:

– average depth of the waviness pattern:

$$W = \frac{1}{m} \times \sum_{j=1}^m HW_j \quad [6.9]$$

W represents the arithmetic average depths, HWJ , of waviness patterns found within the length of evaluation.

– mean pitch of the waviness pattern:

$$AW = \frac{1}{n} \times \sum_{j=1}^m HW_j \quad [6.10]$$

AW represents the arithmetic average value of the lengths of waviness patterns found within the evaluation length.

– mean height of the waviness motifs:

$$W_x = HW_{j_{\max i}}, \text{ with } m = 2n \quad [6.11]$$

By comparison to roughness motifs, waviness motifs represent an entire wave of period T , that is to say between A and B (AW_i , where: $A \leq \text{pitch} \leq B$), as shown

in Figure 6.21. Roughness motifs reflect the graphical expression from peak to peak, that is to say between 0 and B (AW_i , with $0 \leq \text{pitch} \leq B$). The vertical axis $Z(x)$ expresses the height of the profile (primary, of waviness, of roughness), measured in any position x . It is negative if the intercept is located under the X axis (mean line), and positive otherwise. For illustration, we propose the scheme shown in Figure 6.22.

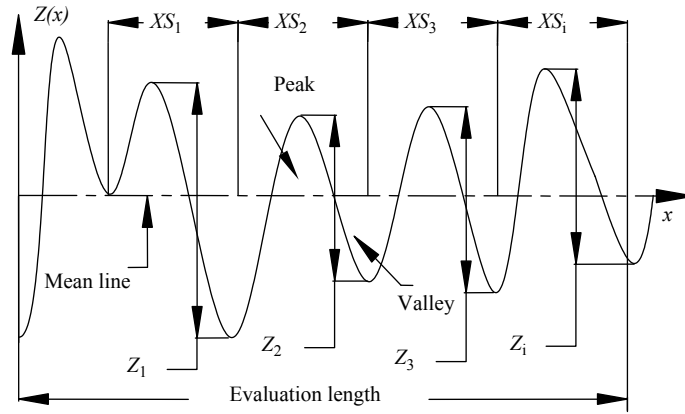


Figure 6.22. Average height and width of profile features (W_p and WSM): WC represents the average height profile features; WSM is the average width of profile features

Mathematical formulae for waviness pattern parameters are:

– average height of the waviness pattern (W_c for the valley):

$$W_c = \frac{1}{m} \times \sum_{i=1}^m Z_i \quad [6.12]$$

– average width of the waviness pattern (WSm for the average ridge)

$$WS_c = \frac{1}{m} \times \sum_{i=1}^m XS_i \quad [6.13]$$

For the primary profile, the mean line is determined by calculating a least-squares line. By reference to the mean line, we should calculate this:

$$\sum_{i=1}^m (\text{area of surface above center line}) = \sum_{i=1}^m (\text{area of surface below center line}) \quad [6.14]$$

The basic length (lp , lr , and lw) constitutes the length being in the direction of the X -axis, used to identify irregularities that characterize the profile to be evaluated:

- the basic length of the primary profile, lp , is equal to the evaluation length;
- the basic length of the waviness profile, lw , is equal, in numerical value, to the wavelength characteristic of the filter profile λf ;
- the basic length of the roughness profile, lr , is equal, in numerical value, to the wavelength characteristic of the filter profile λc . Examples of basic lengths of common roughness include: $0.08 \rightarrow 0.25 \rightarrow 0.8 \rightarrow 2.5 \rightarrow 8.0$ mm.

Evaluation length (ln) or cutoff length is the length in the direction of the X -axis, used to establish the surface profile to be evaluated. This length can include one or more basic lengths. The evaluation length should be adjusted at each measurement. Here are some recommendations:

- evaluation lengths recommended by ISO: $0.64 \rightarrow 3.2 \rightarrow 16$ and 80 mm; and
- usual evaluation length procedure for roughness: $0.4 \rightarrow 1.25 \rightarrow 4 \rightarrow 12.5$ and 40 mm.

According to laboratory experiments conducted by Mitutoyo, evaluation must be performed along a line supposed to be the edge of the crosssection, and the probing length (lp) must imperatively be taken from a section which does not contain very irregular peaks and valleys resulting from scuffs (Table 6.11). The recommended probing lengths [MIT 82] are:

Range of maximum heights		Probing length lp (mm)
Upper	Lower	
–	$0.8 \mu\text{m } R_{\text{max}}$	0.25
$0.8 \mu\text{m } R_{\text{max}}$	$6.3 \mu\text{m } R_{\text{max}}$	0.8
$6.3 \mu\text{m } R_{\text{max}}$	$25 \mu\text{m } R_{\text{max}}$	2.5
$25 \mu\text{m } R_{\text{max}}$	$100 \mu\text{m } R_{\text{max}}$	8
$100 \mu\text{m } R_{\text{max}}$	$400 \mu\text{m } R_{\text{max}}$	25

Table 6.11. Maximum height as a function of the probing length (source: Mitutoyo)

In the past, controllers of surface roughness were used to record only the amplified profile traced by the stylus. Irregularities showing large amplitudes were sometimes excluded because of scratches that could be considered and included as “correct data.” Currently, the calculation of R_{max} is done using a rugosimeter. The operator determines the surface to be measured as representing the entire machined surface without scratches or furrows. Scratches are often

beyond control and the determination of this parameter depends, again, on the operator. The roughness on an arithmetic average deviation of the waviness pattern (W_c , valley) is written as:

$$P_c \text{ or } R_c \text{ or } W_c \quad \text{with} \quad W_a = \left(\frac{1}{L} \right) \times \int_0^L |Z(x)| dx \quad [6.15]$$

Roughness parameters “ R ” constitute the set of parameters calculated or determined from the roughness profile such as R_p , R_v , R_z , R_b , R_a , R_q , and RS_m . To determine R_z , the evaluation is performed (“ Z ” comes from the German Zehn which means 10) for the probing length at the interior of the profile curve. The five highest peaks and the five other deepest valleys are measured in the direction of the horizontal amplification from a line parallel to the mean line of the surface profile, thus they do not intersect. The average difference between the sums of the heights and depths is then calculated to give R_z (in micrometer or microinch). Table 6.12 shows the probing lengths to measure R_z . It should be noted that some compressed surfaces may not contain the number of peaks and valleys required for experimentation. In this case, a probing length of 6–10 times the stroke of the probe must be considered.

Range of R_z		Probing length l_p (mm)
Upper	Lower	
—	$0.8 \mu\text{m } R_{\text{max}}$	0.25
$0.8 \mu\text{m } R_{\text{max}}$	$6.3 \mu\text{m } R_{\text{max}}$	0.8
$6.3 \mu\text{m } R_{\text{max}}$	$25 \mu\text{m } R_{\text{max}}$	2.5
$25 \mu\text{m } R_{\text{max}}$	$100 \mu\text{m } R_{\text{max}}$	8
$100 \mu\text{m } R_{\text{max}}$	$400 \mu\text{m } R_{\text{max}}$	25

Table 6.12. R_z as a function to the probing length [MIT 82] (source: Mitutoyo)

The ISO 1302 standard specifies a method for determining surface roughness by focusing on R_a . Although the cutoff values (0.08; 0.35; 0.8; 2.5; 8; and 25 mm) can be chosen, the values shown in Table 6.13 are used except, of course, if otherwise specified.

Range of R_z		Cutoff length (mm)
Upper	Lower	
—	$12.5 \mu\text{m } R_a$	0.8
$12.5 \mu\text{m } R_a$	$100 \mu\text{m } R_a$	2.5

Table 6.13. Standard cutoff values for determining R_a [MIT 82]

For regular patterns, cutoff values must be chosen (Table 6.14).

Sideways movement of the machine		λ_c (mm)	l_e (mm)	L_m (mm)
Upper	Lower			
0.01	0.032	0.08	0.08	0.4
0.032	0.1	0.25	0.25	1.25
0.1	0.32	0.8	0.8	4
0.32	1	2.5	2.5	12.5
1	3.2	8	8	40

Table 6.14. Adjustment depending on the distance between valleys (regular profiles) [MIT 82]

For irregular profiles such as satin-finished profiles, the cutoff values are chosen in correlation with the R_z of the DIN standard indicated as follows (Table 6.15).

R_z (μm)		λ_c (mm)	L_m (mm)
Upper	Lower		
—	0.1	0.25	1.25
0.1	2	0.8	4
2	10	2.5	12.5
10		8	40

Table 6.15. Standard values for measuring R_a (irregular profiles) [MIT 82]

R_q is a value equal to the square root of the integral of squared deviations from the mean of the roughness curve $f(x)$ over the probing length l .

$$R_q = \sqrt{\left(\frac{1}{l}\right) \times \int_0^L f(x)^2 dx} \quad [6.16]$$

R_q is equal to the deviation of the distribution of distances to the mean line of the roughness curve, following a Gaussian distribution. Since the square of R_q is representative of the shape of the curve, it is often used in theoretical analysis. This approach is well known in the United States and to some extent in Canada, even if the standards do not include it. Sometimes one is led to calculate R_z (DIN), the mean roughness depth. This is clearly shown in Figure 6.23. A length intended to be probed equaling five times the cutoff value is extracted from the roughness curve. The formula for the calculation of R_z is also written in the form below followed by “DIN” (for the German standard, see equation [6.2]).

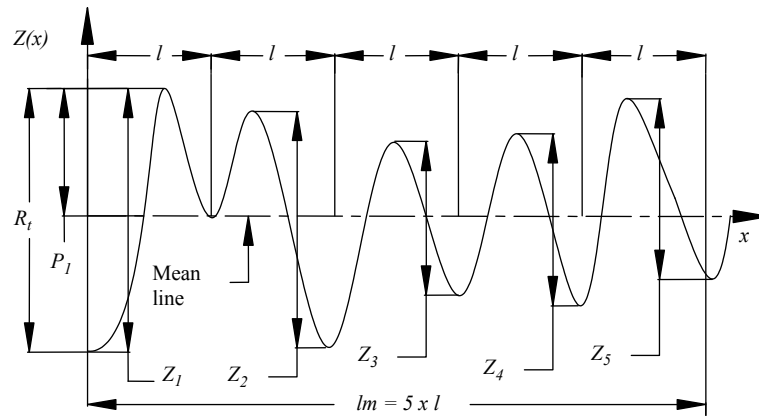


Figure 6.23. Illustration of R_z according the German DIN standard

The cutoff values shown in Table 6.13 apply to regular profiles, whereas those of Table 6.16 apply to irregular profiles depending on the surface unevenness.

R_z en μm		λ_c (mm)	l_e (mm)	L_m (mm)
Upper	Lower			
—	0.5	0.25	0.25	1.25
0.5	10	0.8	0.8	4
10	50	2.5	2.5	12.5
50		8	8	40

Table 6.16. Standard cutoff values for measuring R_z (irregular profiles)

A length of probing equaling three times the cutoff may be taken into consideration if one cannot obtain a probing length equaling five times the cutoff value.

R_{max} DIN is the maximum roughness depth which is also expressed by the deepest valley between Z_1 and Z_5 determined to R_z DIN (Figure 6.23). In Figure 6.23, (R_z DIN), Z_1 represents R_{max} DIN. The cutoff values are the same that are used for R_z DIN.

R_t is the total roughness depth. R_t is the maximum peak-to-valley height point on the probing length equaling to five times the cutoff. This allows us to write:

$$R_t \geq R_{\text{max}} \text{ DIN} \quad [6.17]$$

R_p is the depth of flattening of roughness. On mode R (roughness curve), R_p represents the mean peak height (average value of P_1 to P_5), representing the distances between the highest points of the profile and the mean line specific to each of the five specified probing lengths shown in Figure 6.23. R_p is then expressed as follows:

$$R_p (\text{RMODE}) = \left(\frac{P_1 + P_2 + P_3 + P_4 + P_5}{5} \right) \quad [6.18]$$

On mode P (meaning profile curve), R_p represents the distance between the highest point of the surface profile and the mean line along the probing length. These profiles are similar to those shown in Figure 6.24.

P_c is the number of peaks per unit of length on both sides of a band C_1 which is the neutral zone centered along the mean line (least squares). The band C_1 is used in order to exclude very small peaks and peak noises from the total of peaks. This is illustrated in Figure 6.24.

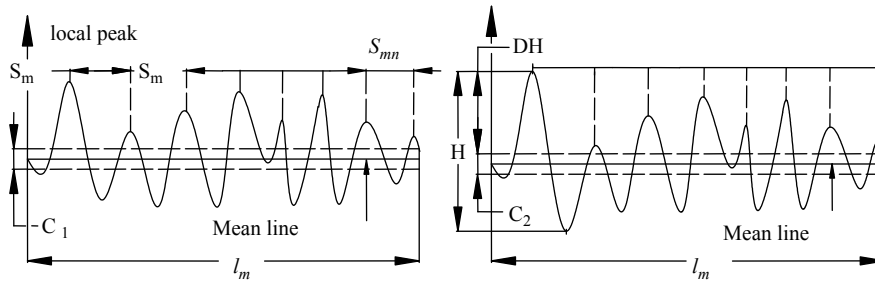


Figure 6.24. P_c and S_m : number of peaks (source: [MIT 82])

The number of peaks is also the number of ridges per unit length located above a central line (mean line), at a depth DH to the highest peak (Figure 6.25). To remove various noises, a neutral zone C_2 is adjusted below the line and its width is 1/100th the size of each range. The ridges are specifically used for the control of sheets subject to coating, crankshaft bearings, and surfaces of machine tools guidance. In the following cases, S represents the average spacing between local peaks of the profile. As depicted in Figure 6.25, S is the average spacing between adjacent local peaks over the probing length. This difference is expressed by the following equation.

$$S = \left(\frac{S_1 + S_2 + S_3 + \dots + S_n}{n} \right) \quad [6.19]$$

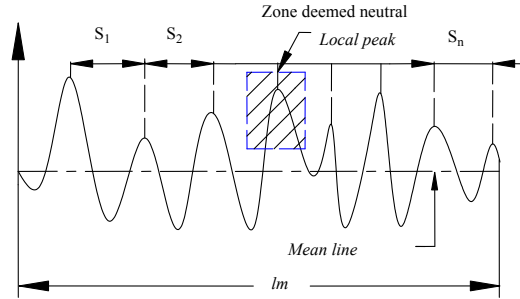


Figure 6.25. Is an average deviation of local peaks of the assessed profile

A peak, as its name suggests, is the highest point on the segment comprised between two adjacent hollows of the profile. According to ISO 468, a peak which is distant from the adjacent local peak of the profile by less than 1% of the value of the cutoff, is not considered a local peak. This applies to a peak whose height is less than 10% of the value of $R_{\max}$ in height. Further, S_m is a average spacing between the profile irregularities. In other words, it is the average spacing between profile irregularities at the mean line of a roughness curve or a profile over the probing length:

$$S = \left(\frac{S_{m1} + S_{m2} + S_{m3} + \dots + S_{mn}}{n} \right) \quad [6.20]$$

In addition, ϱ_m is the arithmetic average of the slope profile θ_m is the arithmetic average of the absolute slopes of a profile curve along the probing length [MIT 82] and is expressed by the following equation (Figure 6.26):

$$\theta_m = \frac{1}{n} \times \sum_{i=1}^n \left| \frac{\Delta y_j}{\Delta X_j} \right| \quad [6.21]$$

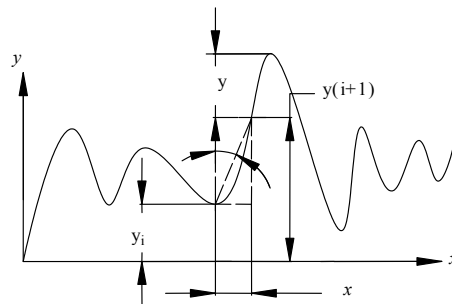


Figure 6.26. Local slope of the profile θ

This parameter is used to control the surface of the workpiece to be joined by adhesion. The ratio of bearing length and total length t_p , occurs when the profile is intersected by a line parallel to the center line or middle line along the length of probing (Figure 6.27). The graph of this parameter is also called “curve of reach” (CP) and is frequently used to evaluate resistance level to abrasion.

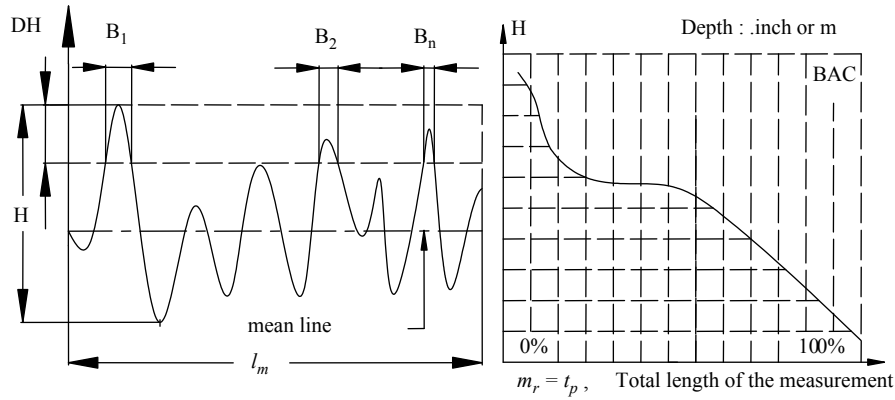


Figure 6.27. Relationship between the bearing length and total length of the measurement (t_p)

t_p is the ratio of the bearing length to the total length of measurement. It is expressed in percent as follows:

$$t_p = \frac{\sum b_i}{l_m} \times 100\% \quad [6.22]$$

LABORATORY PROBLEM.— In our laboratories, related to the study of surface states of machined parts, we often use Surftest 211 to evaluate the surface roughness parameters. We tested a grinded steel sheet (SAE 4041). By exploiting the standard JIS 1994, we took five tests (on 12/05/2002). Measurements conducted on a profile (considered to be standard) R, are filtered by a Gaussian filter. The scope (range) of measures is left freely automatic, i.e. chosen automatically by the apparatus. The speed attained in this experiment is 0.02 in./s. (The results of these tests are shown in Figure 6.28.).

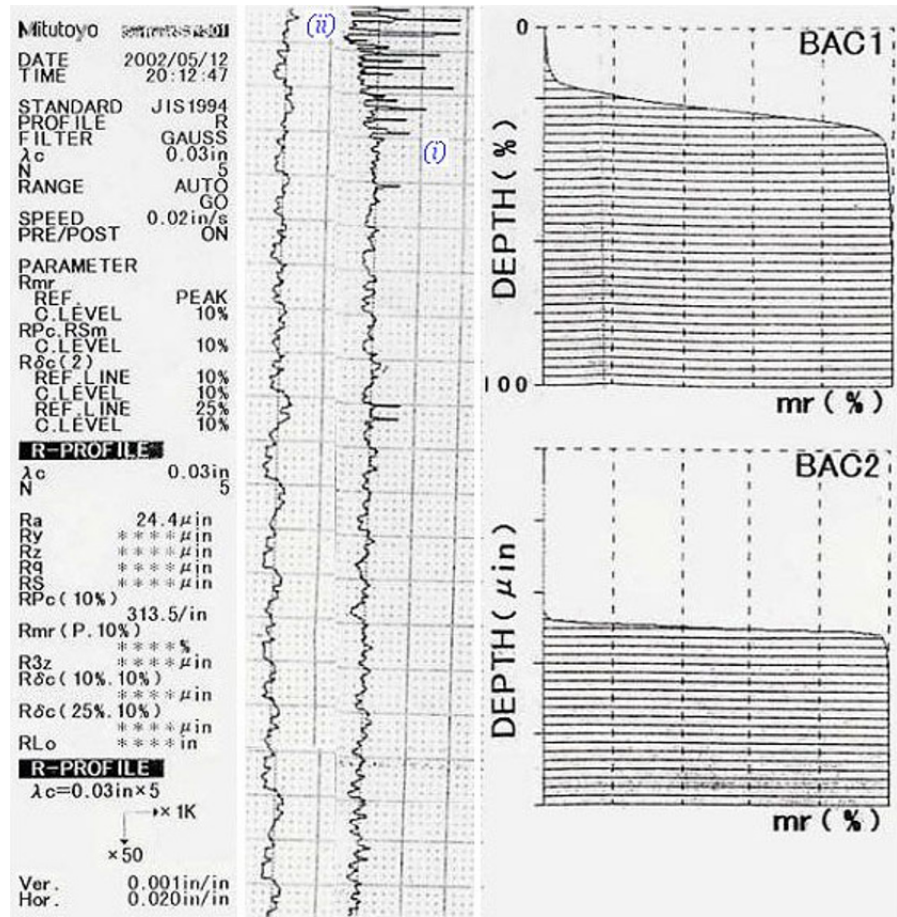


Figure 6.28. Criteria of roughness, R_a and profile λ , that is to say via a Gaussian filter

We have outlined the parameters of roughness and those of waviness (by wavelength). However, we are limited to the exclusive experience of roughness R_a (universal), as in mechanical manufacturing, we content ourselves with this parameter to make a sufficient assessment of the surface state of the object being analyzed (Table 6.17). Of course, (R_a) is measured under specific conditions that were previously explained. We wanted to expose all other criteria because we believe that the analysis of a surface state is not confined only to the analysis of R_a . Yet, the ISO standards related to surface states are grouped in the same way. The user has to refer to the appropriate standard depending on his/her case for better understanding of the conditions for use of his/her apparatus and to know the limitations of these specific criteria.

Chain of standards on surface states						
Link number	1	2	3	4	5	6
Roughness → profile	ISO 1302	ISO 468 ISO 4287 ISO 12085 ISO 13565 ISO 11562	ISO 4288 ISO 12085 ISO 13565-2 ISO 11562	ISO 4288 ISO 12085 ISO 2632	ISO 1878 ISO 1879 ISO 1880 ISO 3274 ISO 2632 ISO 11562	ISO 2632 ISO 5436 ISO 12179
Waviness → profile	ISO 1302	ISO 4287 ISO 12085 ISO 11562	ISO 12085 ISO 11562	ISO 4288 ISO 12085	ISO 1880 ISO 3274 ISO 11562	ISO 5436 ISO 12179
Primary → profile	ISO 1302	ISO 4287 ISO 11562	ISO 4288			
Surface → imperfections	ISO 8785	ISO 8785				

Table 6.17. Chain of standards relative to surface states (source: [MIT 82])

In the last section, we refer to a laboratory that highlights the importance of roughness parameters/criteria in the assessment of a surface state of a grinded workpiece.

6.6. Example on the control of the roughness of a plate grade 6061

We propose to measure, in micrometers and microinches the three main roughness parameters: R_a , R_z , and $R_{\max}$. For this purpose, the ISO standard is selected for the Mitutoyo 211 device to:

1. Describe the major surface imperfections.
2. Define criteria for evaluating the roughness, its normalization, and its dimensioning.
3. Using a rugosimeter, control the surface roughness: R_a , R_z , and $R_{\max}$ in both micrometer and microinch.
4. Draw and discuss the graphs resulting from point 3.

The fixation piece, made of aluminum 6061—thickness 0.375 in. (Figure 6.29), was planed in production shops using conventional mechanical milling.

The overall dimension of the workpiece is not important. It is essential to subdivide (by fine tracing) this piece into 10 equal parts as proposed in Figure 6.30.

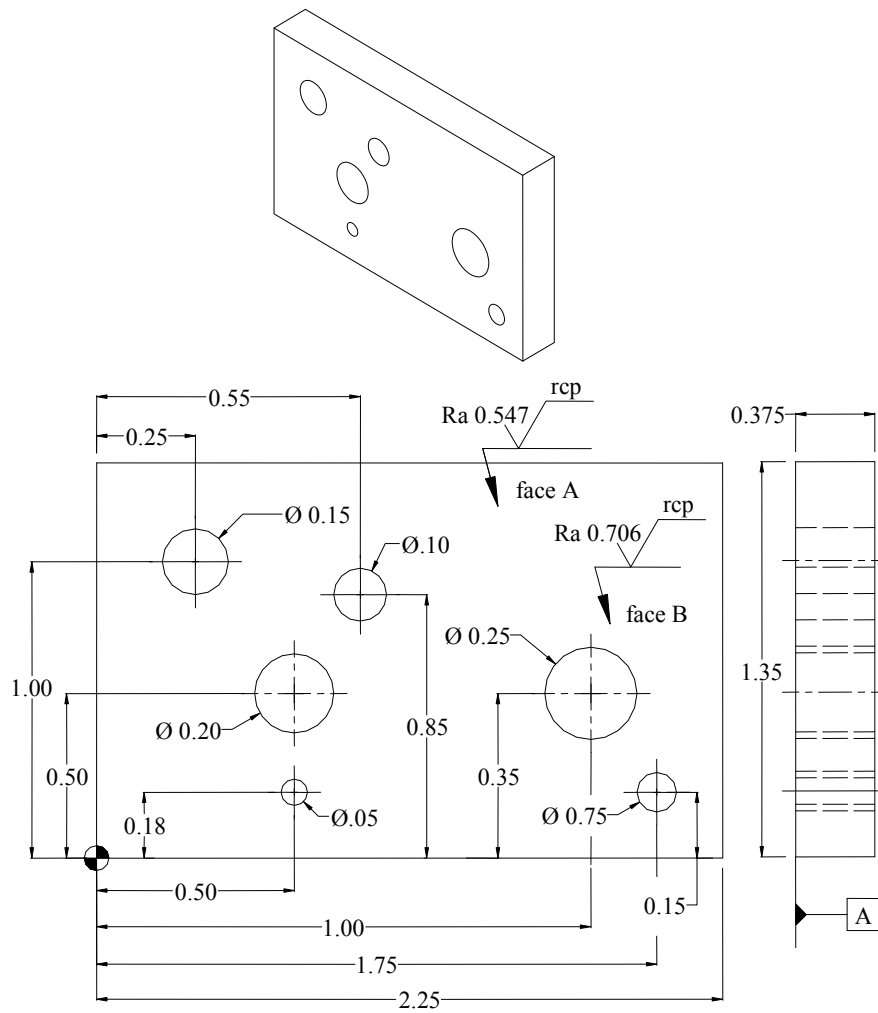


Figure 6.29. A workpiece in aluminum (6061), thickness 0.375 in.

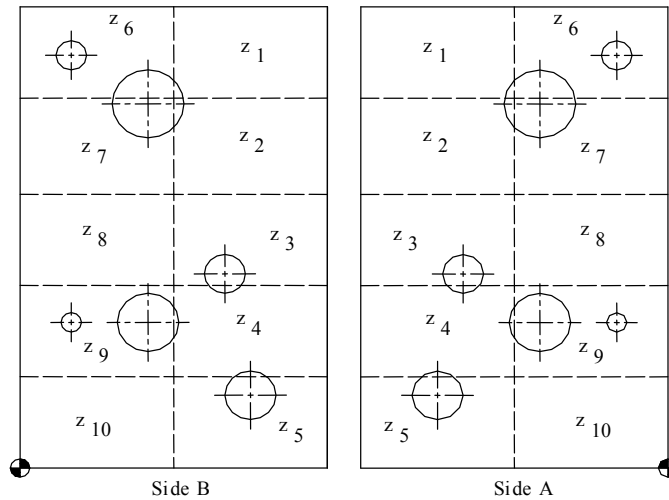


Figure 6.30. Aluminum sheet 6061, thickness 0.375 in.

6.6.1. Questionnaire and laboratory approach

This includes the following points:

- Carry out the calibration of the apparatus and explain the procedure.
- Fill in the tables as shown in Tables 6.18a and 6.18b with your own experimental results, R_a [micrometer]; [microinch].
- Using graph paper (use Excel or MathCAD), plot the spectrum of R_a of your choice in micrometer and/or microinch.
- Find the peaks and assign to them a physical and mathematical context as appropriate.
- Calculate the mean of the measured R_a (only one is parallel to $R_{a\text{mean}}$ at 0X).
- Proceed to the toleranced dimensioning of drawing of the test piece using your experimental R_a .
- Review and comment on your results by assigning to them a physical context as deemed necessary.
- Conclusion.

Here, we have provided the experimental results using the controls in our grade 6061 plate with a profilometer Mitutoyo 211. Next, we model them using MathCAD software.

6.6.2. Table of calibrated measurement results in [micrometer] and [microinch]

Side A, zones 1–10										
R_a	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	0.62	0.6	0.62	0.62	0.45	0.4	0.56	0.55	0.65	0.4
$\mu\text{.in}$	24	24	24	24	18	15	22	22	26	16

R_{zen}	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	2.9	3.7	2.3	3	1.3	0.3	3.3	1	1.8	0.7
$\mu\text{.in}$	114	146	91	117	51	12	130	39	71	28

R_{max}	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	3.9	5.2	3.6	6.3	2.1	0.5	4	3.1	5.5	1.6
$\mu\text{.in}$	154	205	142	248	83	20	157	122	217	63

Table 6.18a. Experiment results, face A

6.6.3. Plotting using MathCAD Software

The following charts are the results of practical experiments on the test piece shown schematically in Figures 6.31(a), (b), and (c).

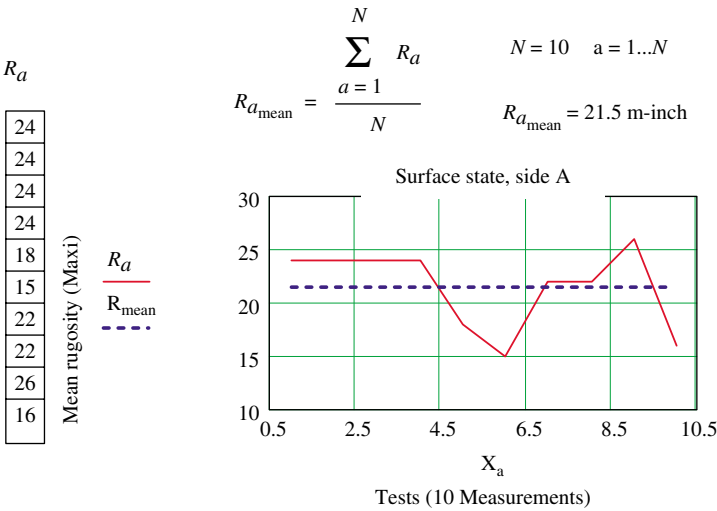


Figure 6.31(a). R_a and its mean (side A)

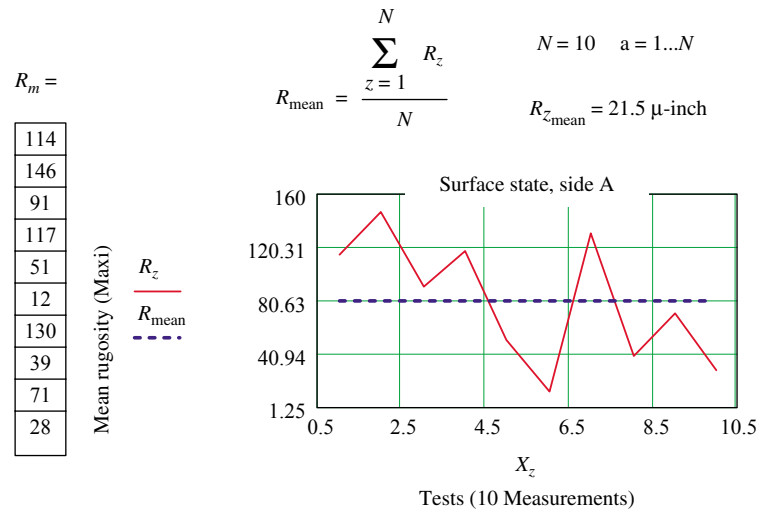
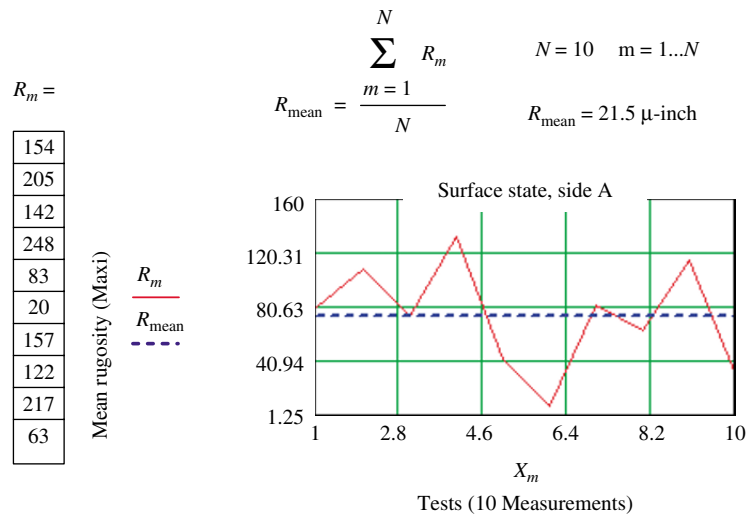
Figure 6.31(b). R_z and its mean (face A)

Figure 6.31(c). Graphical representation of tests results, side A

We have undertaken the same procedure to control the parameters R_a , R_z , and R_{max} of each side of the plate prepared in the laboratory.

Side B, zones 1–10										
R_a	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	0.46	0.45	1	1.04	0.92	0.59	0.42	0.45	0.77	0.96
$\mu.\text{in}$	18	18	41	41	36	23	17	18	30	38

R_z	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	1.7	2.2	4.8	4.5	3.5	2.7	1.6	2.4	2.8	4.7
$\mu.\text{in}$	67	87	179	177	138	106	63	94	110	185

R_{max}	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
μm	2.6	4.2	6.7	5.5	5.2	4	2	2.8	4.4	6
$\mu.\text{in}$	102	165	264	217	205	157	79	110	173	236

Tables 6.18(b). Experiment results, B-side

6.6.4. Plotting with the aid of MathCAD

The following charts were realized using the MathCAD software. Data used in these charts result from our own tests performed on the workpiece (Figures 6.32(a), (b), and (c)).

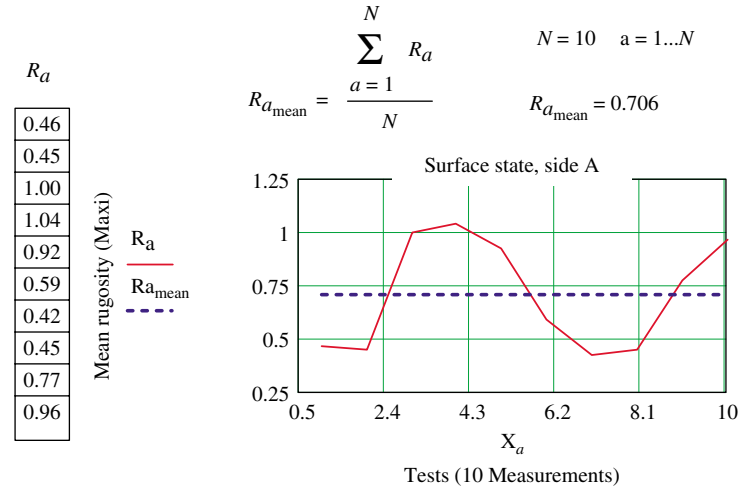


Figure 6.32(a). R_a and its mean (side B)

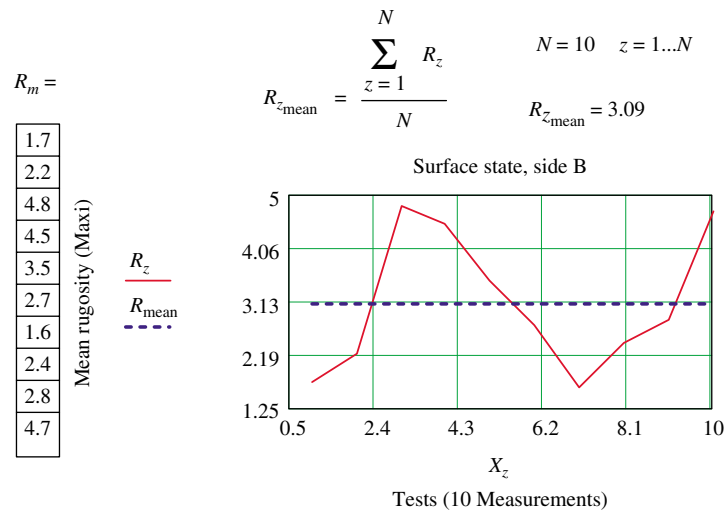
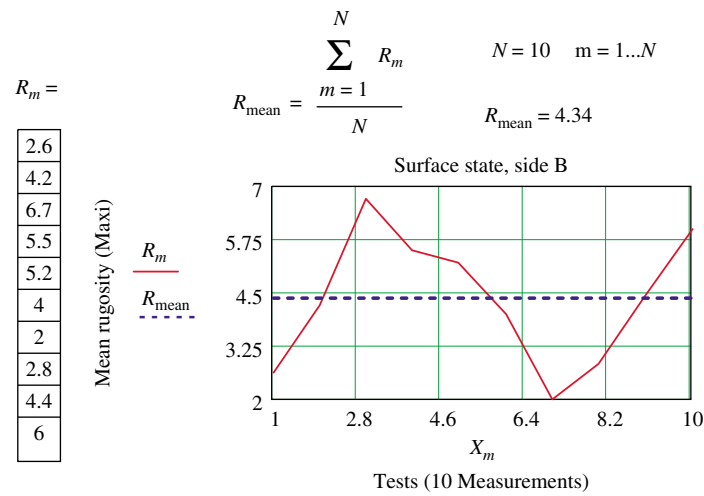
Figure 6.32(b). R_z and its mean (side B)

Figure 6.32(c). Graphical representation of the tests results (side B)

6.6.5. Graphical results of arithmetic means R_a

After conducting the tests whose results are set out above, we show the average roughness for both sides and then we compare our results with the values advocated by the industry and by ANSI B461.1.1978.

We make it a point to read the recommended value for the surface grinding (*rcp*) as presented in Tables 6.8 and 6.9. After reading this, we notice that surface roughness data belongs to the grinding range of concern for steel, however, our workpiece is made of unground aluminum alloy (6061).

6.6.6. Discussions

According to the results previously obtained on R_a and their respective means (sides A and B): 0.547 μm and 0.706 μm , we can conclude that the surface state is comparable to that which would be produced by grinding (Table 6.8). This is quite wrong when we know that the workpiece did not suffer any controlled grinding (aluminum 6061). These results are derived by pure chance inherent in good natural surface state of the piece which, incidentally, is grade 6061. According to the US standard ANSI B461.1.1978, surface roughness value R_a obtained by planing on milling is generally between 0.80 and 6.3 μm . Even assuming that these values are wrong for R_a on this grade, we still demonstrate it by calculating the uncertainties.

6.6.6.1. X_i – absolute error (systematic)

$$\Delta = \text{measured value} - \text{theoretical value} = 0.547 - 0.8 = -0.253 \mu\text{m on side A.}$$

$$\Delta = \text{measured value} - \text{theoretical value} = 0.706 - 0.8 = -0.094 \mu\text{m on side B.}$$

6.6.6.2. Relative error

$$\delta = \Delta \times 100 / \text{theoretical value} = -0.253 \times 100 / 0.8 = -31.63\% \text{ on side A.}$$

$$\delta = \Delta \times 100 / \text{theoretical value} = -0.094 \times 100 / 0.8 = -11.75\% \text{ on side B.}$$

Errors due to the operator are not quantifiable. The operator relied on SurfTest 211 of Mitutoyo (Figures 6.10 and 6.14). He took the results at the end of the course of the probe. Errors due to temperature differences are “respected” because the experiment was conducted under good conditions of temperature and humidity. We can ignore the errors due to those consequent to the usage of the apparatus because the calibration has been done systematically and in conjunction with the recommendations of Mitutoyo roughness standard (Figure 6.11).

6.7. Calculations of the overall uncertainty in the GUM method compared to the Monte Carlo method using the software GUMic

Using the software GUMic [GUI 00], we conduct the assessment of the combined uncertainty by the methods of the GUM and by Monte Carlo simulation, on the surface state resulting from conventional planing knowing the feed per tooth and radius of the tool's beak. We already know that the influence of these two parameters on the roughness is with no further proof [BAR 03]:

$$R_a = 125 \times \left(\frac{f^2}{R_c} \right) \quad [6.23]$$

The plate (Figure 6.29) has been planed on both sides by a conventional machining tool. We collected 30 values of the feed per tooth in line with the radius of the nozzle of the tool.

Quantity name	Sym-bol	Unit	Uncertainty source	Probability distribution	Mean	Standard deviation $s(X)$	Sample size (N)
Roughness	R_a	mm	Sample	Repeatability	3.5395	0.056304597	$N = 30$
feed	f	mm/tr	Sample	Repeatability	0.1005	0.024626206	$N = 30$

End result: $R_a = 1.4000 \pm 0.13 \cdot 10^{-3}$ mm; with $k = 2.0$ at the level 97% Exactitude = 97%			
Measurand R_a	GUM Method order 1	Monte Carlo method	Curves
Mean	12.55×10^{-4}	13.70×10^{-4}	See curves below (Figure 6.33)
Repeatability	$R_a = 3.5395$		
Repeatability	$f = 0.1005$		
Compound uncertainty $U_c(Y)$	4.47×10^{-4}	6.62×10^{-4}	
Effective degrees of freedom v_{eff}	57		
<i>The measurand is not linear over the range of uncertainty. Thus the GUM order 1 is not suitable and we maintain the results of the Monte Carlo method.</i>			
Standard uncertainty quantity unit $U(x_i)$	Sensitivity coefficient $\frac{\partial Y}{\partial x_i}$	Standard uncertainty measurand unit $\frac{\partial Y}{\partial x_i} \times u(x_i)$	Effective degrees of freedom v_{eff}
0.056 μm	$3.55 \times 10^{-4} \mu\text{m}$	$3.04 \times 10^{-4} \mu\text{m}$	29
0.0246 mm/tr	0.0133 mm/tr	3.20×10^{-4} mm/tr	29

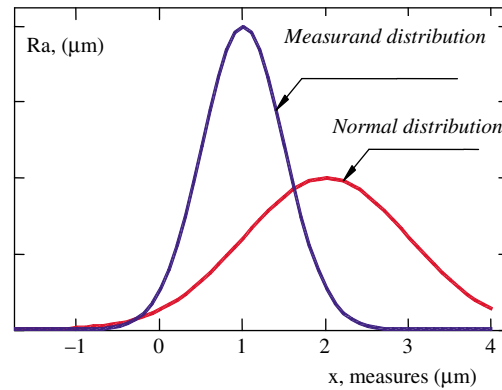


Figure 6.33. Graphical results of measurand processing using GUMic software instrument used: rugosimeter 211 Mitutoyo, 30-measurements mode repeatability

DISCUSSION.— We can conclude that the method of the GUM order 1 is not suitable. We, therefore, retain the results from the Monte Carlo. We note the fact that the GUM may be wrong and numerical methods are most appropriate in addition to being less expensive in terms of workshop. Only the computation time is invested.

6.8. Summary

Ultimately, surface control is also performed using optical means and by interferometry. In addition, the laws of optics are to be applied for such controls. We must, however, recall that the devices and measuring instruments concern isotropic objects. Special materials such as textiles and composites are controlled by some other means. For textiles [GRO 90], e.g. we cite the Kawabata chain that allows us to obtain excellent results on the “roughness” of the tissues.

To conclude, we say that the surface is an important criterion for its geometric correction. Control of surface states is carried out using various means and ways of investigation. One method is the Surftest 211. This provides a measure of physical criteria like R_a , R_z , and $R_{\max}$. However, the most common criterion is the roughness which is a fundamental statistical criterion (roughness mean deviation). The device can also protect the graph profile and the direct readings as the base length (cutoff).

Also, we should recall once again, the vocabulary pertaining to this field remains a significant factor in the control of dimensional metrology tools. For example, if we continue to use confusing language parameters for R_a , R_z , and $R_{\max}$ with those of waviness, then it would be absolutely necessary to reconsider the priorities.

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Chapter 7

Computer-Aided Metrology-CAM

7.1. Coordinate-measuring machine (CMM)

Coordinate-measuring machines (CMMs) are used in various industrial fields. They are particularly used in dimensional metrology for the measurement of *the nominal geometry, the expression of deviations, and the modeling of the real*. We believe it is wise to review the concept of nominal geometry and real entities (real axis, simulated axis, real model, real surface, etc.), and then the deviations between the real and the nominal geometry model. Some definitions of geometrical products specifications (GPS) have already been the subject of previous presentations in Chapter 2. We, therefore, limit ourselves to new topics. According to ISO 10360-2: 1994, CMM is defined as a “measuring device used in a fixed position, designed to take measurements from at least three linear or angular displacements generated by the machine. At least one must be a linear measure”.

7.1.1. Morphology of the CMM

There are various structures of CMMs: gantry structure, moving bridge, horizontal arm, and cantilever structure. Each structure meets a different need in terms of volume control and precision, but the basic principle remains the same. It has three mutually orthogonal guides spotted by three axes: X , Y , and Z . X and Y represent the horizontal axes, the vertical axis is commonly called Z .

Gantry CMM is used for the control of mechanical parts requiring high precision; while the moving bridge CMM is used for the verification of large volume and heavy parts. The cantilever-style CMM allows easy access to the test piece, often of small size. The use of the latter remains relatively marginal compared to other models. Gradually, new machines called three-dimensional (3D) system portable CMMs (poly-articulated arms) are appearing. Hardly precise when they first appeared, these CMMs soon became well-advanced tools.

Three-axis CMMs can essentially be classified into five types of configurations as follows:

1. Gantry-style CMMs having a high capacity tolerate a heavy load and are easily accessible to other parts;
2. Horizontal arm CMMs are mainly used in steel workshops. To ensure acceleration and high velocity, they are designed based on a lightweight aluminum structure;
3. Bridge-type CMMs, with a configuration equivalent to a traveling bridge, have a large dimension and tolerate significant loads;
4. Gooseneck/swan-necked CMM is the oldest configuration of CMM. The load is limited and its transmission along the axes y and z is low because of cantilevers;
5. Cylindrical-polar configuration is used for parts of revolution and for the control of aircraft engine components.

The n -axis measuring machines are hybrids of CMMs of the five configurations named above. We can cite the example of four-axis machinery composed of a gantry-type machine equipped with a turntable and a six-axis machinery consisting of two machines of horizontal-arm type mounted on the same marble for the inspection of car body. For example, the Mitutoyo *Cosmos* software includes special features of bodywork which we will be presented later. CMM is generally composed of:

- a marble (the machine base), with three straight guidance systems made by aerostatic skids and direct current motorization;
- three measuring rules and a detector consisting of photodiodes;
- either a dynamic or a static probe-head which establishes a relationship between the physical contact of the probe on the workpiece and the reading of the three displacements (Figure 7.1).



Courtesy of Mitutoyo Canada

Figure 7.1. Components of a coordinate measuring machine

Sometimes, we can install an electronic box allowing digital control of the displacement of the machine as well as counting the acquisition of the values of sensor displacements compared with the graduated rulers and a calculator with three functions, namely:

- providing assistance to the measure;
- managing displacements by numerical control of the machine (if any);
- correct, via a software component, the geometry of the machine.

7.1.2. The CMM and its environment

It is inconceivable to talk about measurements without mentioning the environmental conditions under which they are carried out. Precision tolerances provided by the manufacturers of a CMM are usually specified for a defined and

constant temperature. Although an increasing number of manufacturers are proposing sophisticated systems with temperature variations compensation (for CMMs directly implanted in workshops), it is preferable to ensure that the building where the machine is installed meets some criteria such as the absence of any heat source as well as the absence of vibration, etc. In order to avoid the occurrence of errors due to thermal expansion of the test piece and even the machinery, it is then advisable to air-condition the locale of measurement. This air-conditioning must meet certain criteria such as its ability to maintain a constant (at $\pm 0.5^{\circ}\text{C}$) and homogeneous temperature across the room. In addition, the CMM shall be set on a homogeneous and stable base to avoid undergoing deformations.

7.1.3. Advantages of CMM in metrology

Measurement using CMM provides many advantages that are very well mastered and affirming the reputation of the machine as it is known today. We mention, for example:

- real-time response in the order of small fractions of a second;
- the device is insensitive to temperature variations, within the current limits of workshops;
- the notable absence of mechanical friction eliminates the initial uncertainty. This allows a significant amplification of up to $10^8\times$ in laboratory measurements.

The CMM is usually connected to a computer that processes the data received from different probed points by an electronic collection system. The probe depends on the surface being measured. The electrical circuit (direct current) is the characteristic that varies depending on the position occupied by the measuring probe. The electronic set is designed to power the electric circuit, and then to amplify the variation in the electrical characteristic which represents the dimension and deviation. Obviously, there are a wide range of measuring models based on the electronic amplification. Classic metrology has often involved human skill. This computer-assisted practice is now carried out easily for operations such as surfacing.

The first CMM was developed in 1962 in an automobile garage (Italian company DEA). Since 1970, CMMs have evolved considerably in the field of mechanics and electronics, and especially in computer control aspects. The idea is to trigger a sensor to measure a point on any type of surface and to provide a computer tool that allows powerful calculations to compute the edges and for printing out measurement results as well. We cannot talk about CMM without including sensors. In 1970, David MacMurtry (Rolls-Royce, UK) developed a touch-trigger probe. As per this design, it is imperative to notify the system of

work coordinates. We will now briefly describe the steps to follow to construct a system of workpiece coordinates. In fact, it suffices to measure features on the test piece and to notify them to the machine as references for the piece. Knowing that three translations and three rotations generate six degrees of freedom helps us in positioning the workpiece within the machine system. We cite, for example, the measure of:

- a reference plane: by blocking two rotations and one translation;
- a circle: this is the origin. By blocking two translations;
- a line: this is the direction of the X -axis. By blocking one rotation.

The measurement results obtained by probing the concerned surfaces are recorded on a display screen showing the toleranced dimensions (if needed). A listing is provided for printing out the results. We may also transmit the measurement results to another statistical processing utility (STAT Pack) where we can proceed with the study of quality control. We may also use the command of a CNC machine tool by copying a standard piece of a given form (3D-TOL). Also it is possible to perform a tracing, which is both precise and complex. With stored systematic survey of the measurements, it is possible to follow the evolution of a manufacturing process and even to control the setting up in real time. Thus, scrap can be prevented, except for the first test piece. A systematic survey on a series of documents would also define the dispersions for a machine tool and a given machining. Before we measure various geometries using mathematical models, we will first define them.

7.2. Commonly-used geometric models in dimensional metrology

There exist many geometric models [BOU 84, 98, MAT 93]:

- CSG models (Constructive Solid Geometry);
- Boundary representation models (B-REP);
- CSG/B-REP Hybrid models (solid + surfaces);
- NURBS (Non-Uniform Rational Beta-Splines);
- TTRS models (Technologically and Topologically Related Surfaces);
- Real forms. Real geometric elements. Real geometric surfaces.

First, we note that the *nominal geometry* of parts defines the ideal form of pieces. It obeys the mathematical definitions of surfaces, lines, and points represented in a benchmark definition. We simply remind the reader that this geometry is necessary for:

- the technological description of the product throughout its life;
- the realization of technical drawing plans such as views, cross-sections, and other sections;
- the geometric characterization (GPS) is essential to support tolerancing;
- the calculation of the path (CNC) of surface cutting tools swept by them.

In this chapter, we attempt to popularize these geometries, but not to analyze them. The technical literature of this field has been an inspiration to us [BOU 84, 98, MAT 93], presenting excellent reviews on geometric modelers. We present some of them in the following text.

7.2.1. *Constructive solid geometry models*

A sphere, a cone, a cylinder, a prism, a cube, or a torus – these models allow the description of basic solid primitives of Boolean operators in a 3D assembly (Figure 7.2). In fact, the principle is based on *solid primitives + union, intersection, and subtraction* (Boolean function used in 3D software) between solid primitives – witness families of CAO models such as CSG and B-REP.

In a 3D drawing, not paying attention to the edges and the refined contact surfaces would result in “oversights”, which will not be noticed until the drawing (3D) is rejected by a CNC software; hence no simulation for tool path (Master CAM) can be created since the surfaces are not sufficiently refined (filter) in terms of the representation of their borders.

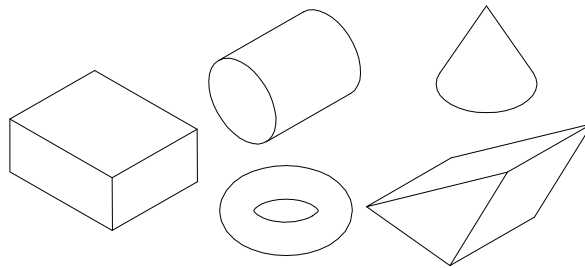


Figure 7.2. *Example of CSG primitives which constitute Boolean functions*

In terms of 3D design, the CSG is the most widely used to represent solids using transformations such as translation, rotation, and scaling.

7.2.2. Boundary representation models (B-REP)

Structure data represented by their boundaries are used in CAD/CAM because of the information derived from the topology. They are preferred for visualization of 3D solids. In short, it can be said that these models use only information from face equations of geometry and those from the oriented edges, hence the importance of their topology. This may concern vertices, faces, edges, and sometimes volumes. In these models, specialists [BOU 01, 87] say that the system keeps the *skin* of the object like the wired models of Figures 7.3(a) and 7.3(b) (Math CAD 3D and Auto DESK Pro 3D Software).

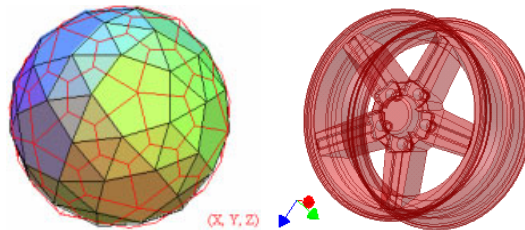


Figure 7.3. (a) Drawing of a 3D wireframe; (b) example of a polyhedron wired design

We can simply say that the coordinates of the vertices provide geometric information and that the edges are carriers of topological information. This model includes:

- *geometric* information (hence the coordinates are known parameters);
- *topological* information (hence the logical relations between the coordinates).

To describe polyhedral solids, *boundary representation (B-rep)* and *constructive solid geometry (CSG)* are operated in 3D CAD/CAM.

Specialists [BOU 01], who manipulate these solid entities, encourage the use of computation of Boolean operations in cases where approximations and rounding errors appear due to real numbers. These problems are solved using numerical approaches in an attempt to control or limit errors such as *rounding (floating intervals or by limiting geometric data to equations of planes)*. The practice of CAD/CAM involves solid and surfaced forms, commonly called hybrid models.

7.2.3. Hybrid models CSG/B-REP (solid + surfaces)

We find them in all CAD/CAM software programs in the form of surface models that allow defining curves and polynomial surfaces (B-Spline and NURBS).

The parametric or *variational* models translate the mode of construction of the object that is a function of geometrical parameters. It is possible to write relations between the parameters using various examples (graphics have been designed using Auto DESK Inventor Pro) of assembly of various surfaces, curves, B-Spline and NURBS, etc. (Figure 7.4).



Figure 7.4. Representation of hybrid models: solid + surfaces in Inventor Pro

By significantly reducing the time and offering the advantages of a 3D view, these models make an enormous contribution to CAD. Some graphics treatments may be time-consuming in terms of computation.

To speed up data processing, the B-REP structure is sometimes enriched like Baumgart structures [BAU 74] (*winged-edge data structure*). The method of doing this is clearly explained in other specialized technical literature [BOU 84, BOU 98, MAT 93].

Our goal remains simple in the study of CMM in dimensional metrology applied to the *Cégep* program. The reader can find details on the points they consider important by consulting specialized works on this topic [BOU 84, 01, CLE 94, MAT 93], which have also inspired us to popularize the concept of NURBS and SATT.

7.2.4. NURBS (Non-Uniform Rational Beta-Splines)

- Why are we trying to popularize the NURBS concept in the chapter dedicated to CMM?
- Because *Splines* are used in all software programs.

In the past, the description of complex forms was problematic with the advent of 3D modeling. It was the same for the finite element modeling, before the development of powerful computers. Now, it is necessary to have powerful calculators to compute complex mathematical formulae [BAU 74, CAS 59, CAS 63, CLE 94, MIT 00]; however, if the parameters of these functions go far beyond human imagination, they become almost unexplainable. This led to the creation of interpolated curves whose shape resembles the most complex curves but whose parameters are manipulated by a few control points. This approach is useful for CAD/CAM users whose task is not programming, hence avoiding recourse to the mathematical rigor.

Bézier curves were first used by car manufacturers [REN 08]. It is also used by graphic designers extensively. The pragmatism of the mathematical formalism used for modeling 3D shapes has led to the creation of what has been called as NURBS (*B-splines*), which use the extrapolation of control points. We will try to demystify this mathematical technique, not to calculate it but just to help popularize the concept. The reason is simple; all users of software design programs use these *Spline* functions. In fact, we should understand interpolation and extrapolation. NURBS are curves made by extrapolation of points, unlike Bézier curves which are made by interpolation (see Figure A4.1 in Appendix 4).

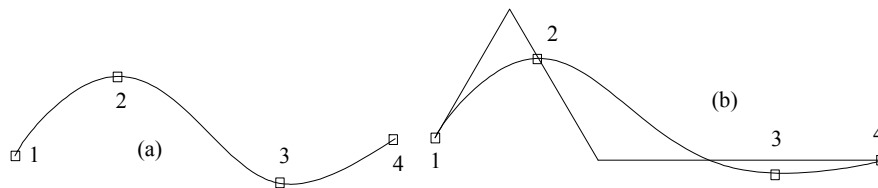


Figure 7.5. (a) *Interpolated curve (Bézier)*; (b) *extrapolated curve B-spline-style*

It was intended to create interpolated curves whose shape resembles the most complex curves but whose parameters are directly manipulated through a few control points. NURBS are curves made by *extrapolation* of points, unlike Bézier curves that are made by *interpolation* (Figure 7.5). An engineer at Renault Automobile, P. Bézier, wanted a simple way to represent a curve, which would be both manipulated by a non-mathematician, and pencil-sketched. The solution presented by Bézier curves has revolutionized many areas other than automotive design. These curves are in fact simple polynomial curves. This is not a “new type” of curve, but simply a way of representing curves, of which here are some mathematical expressions (academic standard):

$$\left\{ \begin{array}{l} \text{first-degree equation (straight line): } Y(x) = a + b \cdot x \\ \text{second-degree polynomial curve: } Y(x) = a \cdot x^2 + x \cdot b \\ \text{third-degree polynomial curve: } Y(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x \\ \text{n-degree polynomial curve: } Y(x) = \sum a_i \cdot x_i \end{array} \right. \quad \text{where } x \text{ is a real number}$$

In short, the mathematical formulation of Bézier curves based on *Bernstein* polynomials gives the equation for a curve based on $(n + 1)$ control points, which can be written:

$$Y(\alpha) = \sum_{i=1}^n \omega_i \times B_{i,n}(\alpha) \quad [7.1]$$

where $B_{i,n}(\alpha)$ are called Bernstein coefficients and are written as:

$$B_{i,n}(\alpha) = \frac{n!}{i! \times (n-i)!} \times \alpha^i \times (1-\alpha)^{n-1} \quad [7.2]$$

In this formula, α is a parameter which varies from 0 to 1, and $\omega_0, \omega_1, \dots, \omega_n$ are the control points underlying the curve. This mathematical formula is useful for demonstrations. In the literature [BEZ 86, CAS 59], it is often suggested to use algorithms designed for these purposes, such as Casteljau [CAS 63]. Further details on these methods can be found under specialized topics. It is necessary to understand the basic idea of degree and order of an equation because NURBS requires a minimum of points to extrapolate depending on their degree or order. It is suffice to know that the computational mode of NURBS uses a polynomial form, similar to the previous curves. We talk about the *order* instead of the *degree* of the curve. The order is equal to the degree +1. This is justified by the fact that a curve of:

- 1st degree and 2nd order will require at least two points to be calculated;
- 2nd degree, so of order 3, will require at least three points to be calculated;
- 3rd degree, thus order 4, will require at least four points to be calculated.

The higher the degree, the higher is the number of points to be taken into account to interpolate the curve, thus the smoother the curve we get. The lower the degree of the curve, the more likely the curve is close up to degree one where the curvature describes a broken line. In modeling, the *Non-Uniform Rational B-Splines* modify locally the shape by moving some control points or nodes. The latter are not evenly distributed hence the term *Non-Uniform* and have a weight (*rational*) that is involved in the calculation of the overall curve. The possibility to move these points and to change the weight allows creation of very complex shapes with relatively few

control elements. Here are some modeling examples using Auto DESK Inventor Pro (Figure 7.6).

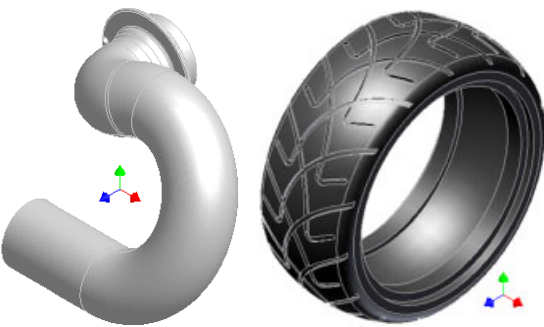


Figure 7.6. Modeling of curves and surfaces in CAD

For example, we suggest this workshop that uses the functions *Cspline* and *interp* for a 2D surface interpolation. Let us, for example, enter a matrix defining a surface; let M_z be:

$M_z =$

0.18	0.15	-0.14	-0.51	-0.3	0.33	0.15
0.93	0.17	-0.76	-0.98	-0.31	0.83	0.12
-0.65	-0.32	-0.68	0.24	0.01	0.1	0
-0.55	-0.22	0.18	0.74	-0.11	0.11	0.15
-0.98	0.17	0.37	0.81	0.39	-0.78	-0.75
-0.71	0.13	0.75	0.3	0.3	-0.18	-0.17
0.55	0.25	0.15	0.25	0.18	0.19	0.16

The number of rows of the matrix M_z must be equal to the number of columns. We identify the n vectors (X and Y) determining the mesh of the matrix:

$n = \text{rows}(M_z) \quad \text{rows}(M_z) = 7 \quad \text{cols}(M_z) = 7$

Let us identify the n vectors, X and Y determining the mesh of the matrix:

$X =$

0
1
2
3
4
5
6

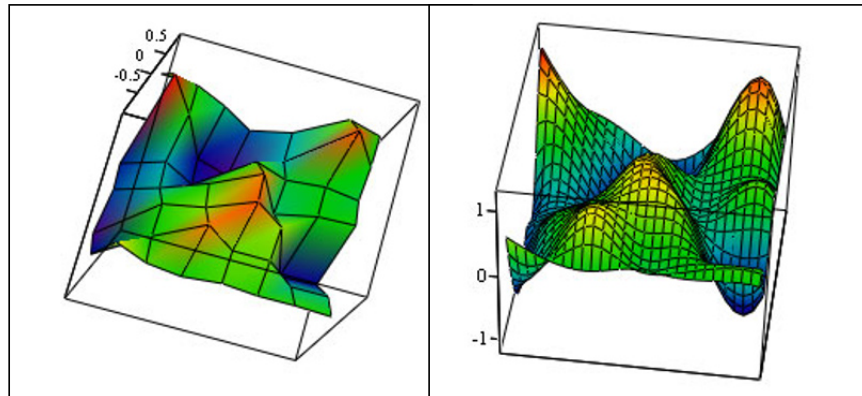
$Y =$

0
1
2
3
4
5
6

$Mxy := \text{augment}(\text{tri}(X), \text{tri}(Y))$ $\text{lignes}(Mxy) = 7$
 Spline coefficients calculated: $S := \text{cspline}(Mxy, Mz)$
 Surface adjustment function: $\text{fit}(x,y) := \text{interp}\left[S, Mxy, Mz, \begin{pmatrix} x \\ y \end{pmatrix}\right]$
 Example of interpolated values: $x_{\text{high}} := Mxy_{n-1,0}$ $y_{\text{high}} := Mxy_{n-1,1}$
 $\text{fit}(2.5, 3.9) = -0.016$ $\text{fit}(0.1, 1.7) = -0.056$ $x_{\text{low}} := Mxy_{0,0}$ $y_{\text{low}} := Mxy_{0,1}$

Density of mesh for interpolation:

$$\begin{aligned}
 xn &:= 4n & yn &:= 4n & i &:= 0 \dots xn-1 & j &:= 0 \dots yn-1 \\
 xind_i &:= x_{\text{low}} + i \cdot \frac{x_{\text{high}} - x_{\text{low}}}{xn-1} & yind_j &:= y_{\text{low}} + j \cdot \frac{y_{\text{high}} - y_{\text{low}}}{yn-1} \\
 FIT_{i,j} &:= \text{fit}(xind_i, yind_j)
 \end{aligned}$$



Mz Original surface

FIT Plot of a 2D-Spline surface interpolation

Figure 7.7. Drawing of an interpolated surface vs. an original surface

Ultimately, we are always challenged with the manipulation of surfaces using the CMM (Figure 7.7). These associated surface models give rise to planes, spheres, helices, etc. We will popularize the concept of TTRS in what follows.

7.2.5. TTRS (Technologically and Topologically Related Surfaces) models

Derived from virtual modeling, 3D structures involve time-consuming calculations. The *geometry* gives the *position*, *shape*, and *dimensions*. The *topology*

enables us to consider various *relationships between objects and space*. Topology is primarily concerned with the semantic space, that is to say, the way objects constitute a natural modeling. Topology is commonly used due to the rapid speed of execution of a 3D drawing. This applies to data sharing such as the case of two adjacent objects sharing the same border. The surface generalized in TTRS may consist of either a single surface or multiple surfaces such as the case of two coaxial cylinders.

Invariance classes [BOU 01] are of the order of seven, namely a surface shape that is *arbitrary, prismatic, of revolution, helical, cylindrical, spherical, or flat*. For example, if a defined surface is rotated around its axis while *displacing* it along this axis and the *surface* remains invariant, it is called TTRS. Specialized literature [BOU 01] defines each class by each degree of invariance, and displacements in corresponding rotation and translation (Figure 7.8).



Figure 7.8. TTRS models – technologically and topologically related surfaces

With a TTRS, we define a minimum geometrical reference element (MGDE), witness Mini–Max Chebyshev functions implemented in several CMM software programs such as Mitutoyo Cosmos:

- Min–Max or Chebyshev criterion written as:

$$\delta = (\varepsilon_{i \max i} - \varepsilon_{i \min i})_{\min i} \quad [7.3]$$

- δ is the minimum distance of deviations (in the case of the Chebyshev criterion);
- ε_i are the scatter plot of points being probed by CMM.
- Min–Max criterion on the radius $R_{\max i}$ with a constraint on each deviation $\varepsilon_i \geq 0$.

TTRSs belonging to the same invariance class are composed of *points, lines, and planes*. Hence, a cylinder TTRS will have its *real axis* as a reference; a TTRS composed of two parallel planes will have its *median plane* as a reference, etc. In Chapter 2 of Volume 1, we introduced the idea of the axis. At this stage, we remind the reader that the classical idea of the axis of a cylinder is extended here to the seven classes of TTRS grouped in Table 7.1 [BOU 01].

CI: Invariance class	Invariance Degree (DI)		MGDE: minimum reference element
1. Arbitrary shaped	0	None	Plane, straight line, point
2. Prismatic	1	1 translation along a straight line of a plane	Plane, straight line
3. Revolution	1	1 rotation around a dashed straight line	Straight line, point
4. Helical	1	1 translation and 1 rotation related (ex. drill)	Oriented straight line
5. Cylindrical	2	1 translation and 1 rotation around a straight line	Straight line
6. Flat	3	1 perpendicular rotation to the plane and 2 translations along two straight lines of the plane	Plane
7. Spherical	3	3 rotations around a point	Point

Table 7.1. Seven classes representative of TTRS (source: [BOU 01])

By combining two classes, we get another class. If we consider two blended axes, to design a coaxiality constraint, for example, we would obtain a cylindrical class. Also, two parallel axes form a prismatic class and two axes in an unspecified position give an unspecified class, etc. In 3D metrology, surfaces are formed by different positions of points, lines, and planes. Combinations between the latter generate relative positioning constraints. It should be understood that these are properly MGDE based on two classes, that is to say, distances and angles. Geometry known as “*with defects*” is defined by a nominal geometry, a modeling of the “real” and a limitation of defects of the real by “tolerancing” differences between the nominal geometry and the representation of the real. The real can be identified by deviations between a representative of the real (finite number of points or surfaces of substitution) and the corresponding nominal geometry. In the case of a finite number of points, deviations ε_i between the points of the actual surface and the nominal geometry are expressed following the normals to the nominal model (Figure 7.9).

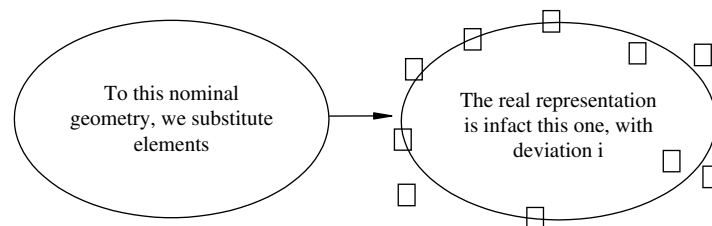


Figure 7.9. Geometry and nominal representation of its real

For a position of the nominal model, the deviations would be limited by this criterion:

$$\varepsilon_{i\min i} \leq \varepsilon_i \leq \varepsilon_{i\max i} \quad [7.4]$$

7.2.6. Real forms, real geometric elements, real geometrical surfaces

It was noted in Chapter 2 that geometric modeling covers two aspects: a modeling of the *real form* created by a manufacturing process and a modeling of the *ideal form* defined by a *drawing*. Geometric form elements constituting a workpiece are realistically defined by *finite sets of measured points* and by *geometric elements of substitution*. These elements also constitute an ideal piece with nominal geometric elements. Tolerancing allows limiting the variations of actual geometric elements compared with nominal geometry elements. Geometric modeling is based on the fundamentals of *Euclidean geometry*, that is to say those of deformable bodies where the terms *straight line*, *circle*, *plane*, *sphere*, *cylinder*, *cone*, and *torus* designate lines and surfaces of infinite extent and *ideal form*. In fact, the geometric surfaces called “real” constitute the interfaces material/environment. The geometry of these interfaces depends on the method used to obtain surfaces, on the material, temperature, degree of humidity, and the stress states of the workpiece. *Real geometric surface* takes the same designation as that used in *perfect surfaces geometry*, to which we associate the qualifier of *real*. It is then a question of *real plane* or *surface called flat*, of *real cylinder* or *surface deemed cylindrical*. The term *real surface* encompasses other types of surfaces (ruled or connecting).

7.2.6.1. Actual geometric lines

A geometric line is a real set of points connected by a continuous line which has geometric deviations from a perfect geometric line (straight line or circle). In practice, a real geometrical line can be obtained either by:

- the intersection of an ideal geometric surface and an interface material/environment;
- a set of points constructed from a real surface. For example, the real axis of a cylinder obtained by the set of all the centers of circles constructed from a surface deemed cylindrical (the same applies for a circle).

7.2.6.2. Actual geometric points

Actual geometric points are obtained from real surfaces. For example, the intersection of a line and a real geometric surface, centers of two real points.

7.2.6.3. Geometric elements captured by a CMM probe

For practical reasons, the measurement of a real geometric element can be done only via a limited number of points. Each real geometric element is known by a finite set of n captured points, from which we deduce, by compensation of the probe radius, a set of n measured points. By convention, ([CLE 94, MIT 00, REN 08]) sets of captured points and measured points are designated as the geometric element used in geometry of perfect surfaces, to which is added the qualifier “captured or measured”. In the case of lines and captured or measured points, we often specify the name of the surface(s) to which they belong. For example: a straight line L_1 measured on plane Pl_1 ; point Pt_1 measured on sphere Sph_1 , etc. The captured geometric surfaces may be the plane, the cylinder, the cone, or any surface forming an interface material/environment.

7.2.6.4. Geometric elements associated with measured elements

The combination of a perfect geometric element to a set of points is a fundamental problem in 3D measurement. This combination is necessary for each step in the interpretation of measurements. In fact, it – along with the least squares criterion – defines the normals to contact points between the probe and the surface. It then enables calculation of the sets of measured points, thus leading, according to various criteria, to a simplified representation of the actual geometry of the workpiece. The identification of a perfect geometric element, representative of a set of points, should define the nature of the geometric element, its *direction*, its *position*, and its intrinsic *dimensions*. In 3D measurement, the nature of the geometric element is always imposed. In contrast, the other characteristics are obtained by optimization. We cite five optimization criteria used by Cosmos Software:

1. Gauss criterion (least squares) where the sum of the squares of the shortest distances between the measured points and the associated geometrical element is minimal;
2. Chebyshev (or Min–Max) criterion where the longest of shorter distances between the measured points and the associated geometrical element must be minimal. In GPS we use the principle of the envelope where the Chebyshev criterion proves to be the most appropriate;
3. Tangency criterion where the associated geometric element must be located on the same side of all measured points, and be in contact with at least one measured point. The chosen side is usually that belonging to the free side of the material;
4. Minimum circumscribed criterion where the associated geometric element (circle, sphere, cylinder, and torus) must have a radius as small as possible. It is located outside of all measured points;

5. Maximum inscribed criterion where the associated geometric element (circle, sphere, cylinder, and torus) must have its radius as large as possible. It is located within the set of measured points.

The use of these criteria is *contested* by the non-uniqueness of mathematical results, by the lack of standardization, and the non-validity of the results obtained on surfaces that are measured in *very few points*. Table 7.2 below gives the possibilities offered by the five main criteria [BOU 01, MIT 00].

Criteria	Geometric parameters			Applicable to elements of limited extent	Uniqueness in the results
	Orientation	Position	Intrinsic		
Gauss	Yes	yes	yes	yes	yes
Chebyshev	yes	yes	no	Yes	no
Tangency	yes	yes	no	yes	no
Minimum circumscribed	yes	no	no	no	no
Maximum inscribed	yes	no	no	no	no

Table 7.2. Five main geometric criteria for distances calculation

Through this table, derived from the literature [BOU 01, MIT 00], we may easily understand why the use of Gauss's criterion is often recommended. The uniqueness of the results is feasible (witness the Cosmos software of Mitutoyo). The Gauss's criterion, in all cases, defines the measured points, i.e. to calculate the offsets due to the radius of the probe (compensation). It also, when used alone or together with the condition of *tangency of the free side of the material*, defines the geometric elements associated with sets of measured points.

7.3. Nominal geometric elements

7.3.1. Modeling the ideal geometric form of a workpiece

The nominal surfaces are generally parallel or perpendicular to the preferred directions. They respond to rules of connection, tangency, and intersection. The relative positions between the nominal geometric elements are defined by dimensions. Tolerancing allows limiting geometric deviations between the real geometric form and the ideal geometric form. Standardization allows limiting the deviations by defining two major classes of tolerance: dimensional tolerancing and geometric tolerancing. These points have already been extensively discussed in Chapter 2.

7.3.2. Model of real geometric elements, reference surface (SR)

The model of sets of points of contact between the sensor and the real surfaces are the measured points which are representative of real surfaces. They are deducted from the points being captured by compensation of probe radius (Figure 7.10).

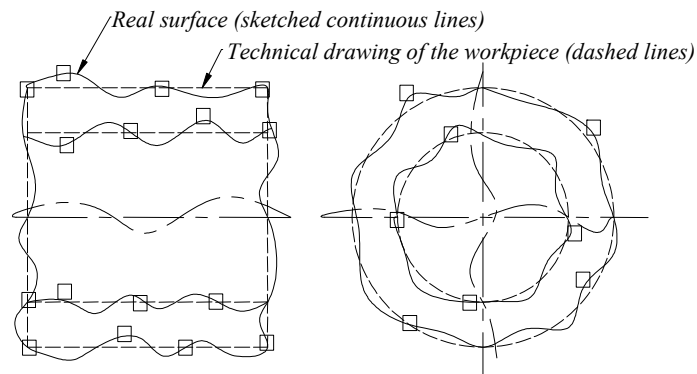


Figure 7.10. Model of contact points between the probe and the real surfaces

7.3.3. Substitution surfaces models

These are perfect surfaces, bounded by a contour, showing *orientation* and *position defects* relative to the nominal surfaces. They pass through points measured following the criterion of least squares and can be tangential to the free side of the material. Operations of control and 3D measurement can be grouped into two distinct ranges: a *range of measurement* to acquire the measured points, and a *range of treatment* enabling, by calculation, to identify, interpret, and verify geometrical specifications. The range of measurement is performed on a CMM in four steps.

STEP 1.– Inventory of real geometric elements.

The analysis of the engineering drawing presents an inventory of geometric elements involved in the specifications, and choosing, among them, the most representative sets of points to be measured. There are few rules to make these choices. Experience constitutes an essential role in this case. We set out some rules including:

- the nature of geometric elements should be chosen from a list available in the software of measurement: point, line, circle, plane, sphere, cylinder, and cone;
- the nature of geometric elements should consider the extent of the surface (that is to say the choice between a cylinder and a circle) and the distance from the

geometric characteristic sought relative to the surface (that is to say, the intersection of the axis of a cylinder of lower height with a plane that is distant from the surface of the cylinder);

- the number of points must be greater than or equal to the number of parameters of the geometric element as shown in Table 7.3.
- the dispersion of points must be on the full extent of the surface and highlight the defects of shape due to manufacturing process;
- the least-squares algorithm is sensitive to a density of points that are locally important;
- the number of points must allow a compromise between a good representation of the element and a minimum measurement time.

Nature	Number of points	Nature	Number of points	Nature	Number of points
Line	≥ 2	Plane	≥ 3	Cylinder	≥ 5
Circle	≥ 3	Sphere	≥ 4	Cone	≥ 6

Table 7.3. Number of points recommended depending on the entity to be probed [MIT 00]

STEP 2.— Probing of points and lines surfaces

7.3.3.1. Choice of probes [MIT 00, REN 08]

For each probing orientation, the stylus is chosen in a way that its length is sufficient to reach all surfaces, and the size of its ball smaller than the smallest of the cavities of the part.

To avoid collisions between the shank of the stylus and the inspected surface, the diameter of the ball of the stylus must be large enough. In fact, the stylus is never perfectly aligned with the general direction of the surface as shown schematically in Figure 7.11.

At any time, a probe calibration operations aims to estimate the two geometric characteristics necessary for calculating the registration mark, in a single contact point, of the coordinates of contact points sensor/piece. This is achieved through:

- a calibrated radius obtained by measuring the size of a sphere;
- a calibrated vector representing the origin-offset due to the variation in length and orientation of the different probes (coordinates taken by the center of the probe ball).

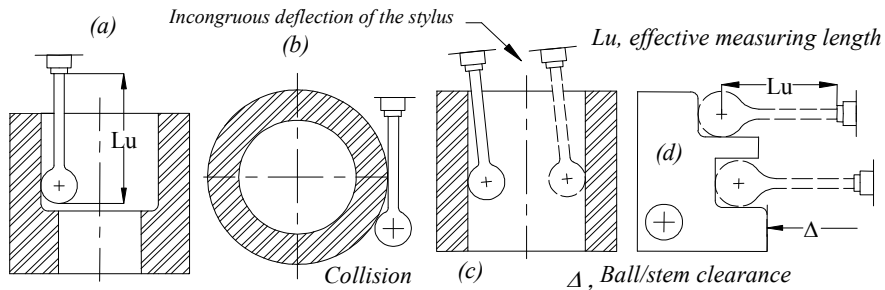


Figure 7.11. (a, b, c) Probing styli to avoid collisions; (d) large-diameter ball

7.3.3.2. Calibration of a probe–surface probing

At each contact between the probe and a surface, coordinates of the center of the sphere are collected, and expressed in a single reference measurement. It is recommended to use a short and rigid stylus because:

- the more a stylus bends or deflects, the lower will be the accuracy → For the range of control, a short stylus will be used (Figure 7.11(c));
- extensions should be avoided – use a one-piece stylus;
- styli with significant length should be made of rigid material: tungsten carbide, ceramic, or of a range: graphite fiber (GF).

Instead, it is recommended to use a probe with a large-diameter ball (Figure 7.11(d)) because:

- this will provide more clearance between the surface of the workpiece and the stem of the stylus (Figure 7.11(a));
- we gain in terms of stiffness while reducing the collision defects associated with the state of the surface being inspected. It is accepted that a large diameter ball reduces the deformation of soft surfaces;
- the term “effective length” defines the distance over which the stylus can effectively be used on a surface parallel to it before calibration with the upper tapered part including threading;
- typically a stylus with a large-diameter ball provides a greater “effective length.” However, we should check that the weight of the stylus is not prejudicial.

7.4. Description of styli and types of probing

There exist various styli [REN 08]. The international company *Renishaw* offers a wide range of styli, among which we will discuss six types. Similar to comparators, the jaws take the most appropriate form to the (surface) geometry to be inspected. This is convenient since only the form varies whereas the measuring instrument is based on a single reading technique, independent of the degree of accuracy:

- styli with ruby ball;
- star styli;
- sharp styli or with small radius;
- hemispherical-ended styli;
- disc styli;
- cylindrical styli.

The images in Figure 7.12 illustrate the essentials of CMM styli.

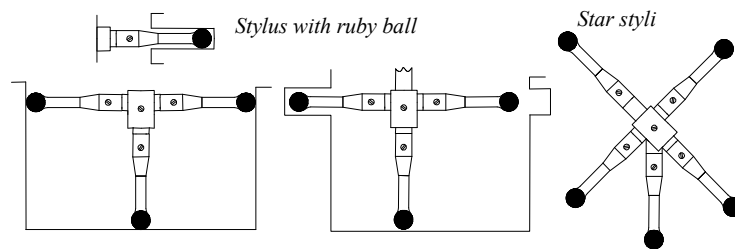


Figure 7.12. Probing styli with ruby ball and star styli

7.4.1. Styli with ruby ball

The ball with which they are made is perfectly spherical and is of synthetic ruby whose hardness reduces the stylus wear caused by docking. Its low mass minimizes inappropriate probe triggers caused by vibrations or displacements on the rapid motion mode. *Ruby balls* fit inside the interchangeable styli whose stem might be in steel, tungsten carbide, ceramic, or carbon fiber. *Star styli* are commonly used in the following cases:

- control of extreme points within pieces with walls or grooves;
- to avoid, each time, recourse to reorient the probe depending on the position of the point to be captured, thanks to its four or five directions. Each tip on the star stylus must be individually calibrated (Figure 7.13).

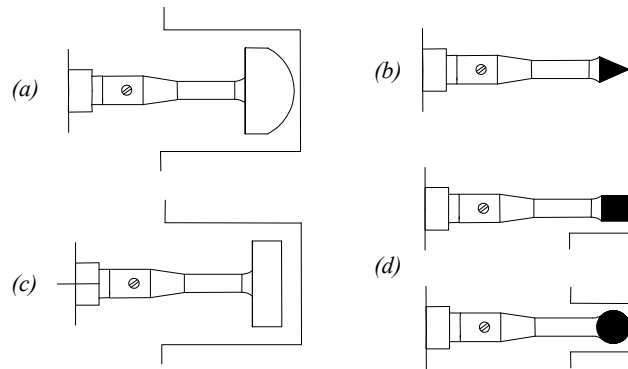


Figure 7.13. Probe stylus with hemispheric tip (a), sharp or with small radius (b), disc (c), and cylindrical (d)

7.4.2. Hemispherical-ended styli

They are known for their use justified by probing along the axes X , Y , or Z , of elements at the bottom of deep bores while having a single ball to be calibrated. In addition, the largest diameter of the sphere portion allows reaching the point of contact on, particularly, rough or uneven surfaces. Hemispherical-ended styli are recommended to probe hemispherical rubber, wax, or even moss.

7.4.3. Sharp styli or styli with small radius

Due to possible scratching, sharp styli should not be used for probing along the conventional axes X or Y . Their use is justified only to inspect thread forms, specific points, or to track tracing. In short, they are used only for applications requiring lower accuracy. Using a stylus ending with a small-radius curvature improves the accuracy during calibration. Its characteristics allow localizing and probing very small-diameter holes.

7.4.4. Disc styli (or simply discs)

They are used to probe grooves or undercuts. The disc itself is actually a portion of a sphere. The probing operation could be performed with a large diameter ball, where the only part to be used is the equatorial part. Only a small portion of the surface of this ball would be effectively used. Similarly, discs of low thickness require appropriate angular alignment with respect to the surface being probed. Some disc styli have a thread underneath that enables the fixing of a standard stylus,

hence providing the possibility of probing the bottom of bores, where access for the disc is impossible.

7.4.5. Cylindrical stylus

These are used for probing holes in a thin sheet material, to locate their centers or in order to perform a control parallel to thread or teeth features. Ruby cylindrical styli have a hemispherical end allowing for the calibration in X, Y, and Z. They are often used to retrieve a profile on surfaces without negative angle (showing an undercut).

7.4.6. Accessories and styli extensions

The extensions allow extra clearance between the part of the sensor where the styli are fixed and the effective part of it. Hence deep parts can be created. There are various accessories for mounting and fixing styli.

STEP 3.– Constitution of the database of real geometric elements (associated and measured).

The geometrical information contained in the database of a CMM relies on three basic geometric elements: the point, the line, and the plane. They are defined by the coordinates of a point and, in the case of a line or a plane, by the components of a unit vector. We give below a summary (Table 7.4).

Basic geometric elements	Position	Orientation
Point	three coordinates	
Line	three coordinates of a current point	three components of a unit vector parallel to the line of a current point
Plane	three coordinates of a current point	three components of a unit vector normal to the plane

Table 7.4. *Basic geometric elements*

Based on the captured points, we calculate the points of contact with the real surfaces as well as the parameters of geometric elements associated with points of contact. Table 7.5 shows, for each geometric element, the information contained in the database.

Type of the associated geometric element	Basic geometric elements	Intrinsic parameters	Excerpted elements
Captured point	Point		
Point contained into a plane	Point		Measured points
Least-squares straight line contained into a plane	Straight line		Set of points measured on the line deemed straight
Straight line of least-squares in 3D	Straight line		Set of points measured on the line deemed straight
Circle of least-squares Contained into a plane	Plane + Point = Center of the circle	Radius	Set of points measured on the line deemed circular
Plane of least-squares	Plane		Set of points measured on the line deemed flat
Sphere of least-squares	Point = Center of the circle	Radius	Set of points measured on the surface deemed spherical
Cylinder of least-squares	Straight line = Axis of the cylinder	Radius	Set of points measured on the surface deemed cylindrical
Cone of least-squares	Straight line + Point	Vertex	Set of points measured on the surface deemed conical

Table 7.5. Types of geometric elements associated (sources: [BOU 01, MIT 00])

STEP 4.– Standard definitions of specifications, their interpretation, and their verification. This step requires a good knowledge of standards and specifications of GPS as well as good skill in terms of computational possibilities offered by the 3D measurement software. By geometric construction, software programs allow us to determine, based on the information contained in the database, new elements such as point, line, plane, and also constructing markers. A 3D measuring software should contain at least the following operations for construction:

– *Construction of points*: The following construction of points has been briefly checked. We have tested them on CMM and the test results are given below.

Point/point	center of two points
Point/straight line	orthogonal projection of the point/straight line
Point/plane	orthogonal projection of the point/plane
Straight line/straight line	intersection of two straight lines (center of the common perpendicular)
Straight line/plane	intersection of the straight line and the plane
Point	by its Cartesian coordinates within a marker

– *Construction of straight lines:*

Point/point	straight line/plane passing through two points
Point/Straight line	straight line/plane passing through a point and perpendicular to a straight line passing through a point and parallel to a straight line
Point/plane	straight line passing through a point and perpendicular to a plane
Straight line/straight line	median of to coplanar straight lines
Straight line/plane	orthogonal projection of a straight line on a plane
Plane/plane	intersection of two planes

– *Construction of planes:*

Point	straight line/plane passing through a point and a straight line, plane passing through a point and perpendicular to a straight line
Point	plane/plane passing through a point and parallel to a plane
Straight line	Straight line/plane passing through a straight line and parallel to a straight line
Straight line	plane/plane passing through a straight line and perpendicular to a plane
Plane	plane/plane median of two planes theoretically parallel
Point	Straight line/plane passing through a point et and a straight line passing through a point and perpendicular to a straight line

– *Construction of markers:*

- The first two directions are each defined by a unit vector from the database, respectively for a straight line or a plane. These two first directions are independent and are not necessarily orthogonal;

- The third direction, perpendicular to the first two directions, is automatically calculated by the software;

- The origin is defined by a point;

- The orientation is defined by a point selected within the positive section of the reference mark.

The measurement results are calculated in terms of distances and angles between two elements contained in the database. We list, for example:

– Theoretically, six cases are possible for calculating distances: point-point, point-straight line, point-plane, straight line-straight line, straight line-plane, and plane-plane. But only the first four cases are applied, the latter two cases given in the current software are applied for zero distance;

– Three cases for calculating an angle: straight line-straight line, straight line-plane, and plane-plane.

While standardization provides geometric specifications by tolerance zone, CMM software offers only computational tools related to vectorial geometry where only the scalar and vector products are used. The difficulty will then be linked to the development of the sequence of constructions and the calculations available to check standard specifications.

7.5. Software and computers supporting the CMM

There are many interface means [MIT 00] connected to the CMM that is currently in use to perform calculations, reproduce forms (surface and volume) resulting from the manufacture, and even to assess the quality of various parts. These tools connected to the CMM ensure among other functionalities:

- geometric measurements on parts defined on planes;
- control on numerically defined parts (surface control and scanning);
- control of plastic parts for molds (3D-TOL);
- serial control and statistical processing of parts (STATPAK).

Thanks to the various software used in 3D metrology, we can establish monitoring reports both on the geometry of the parts as well as their volumetric aspect with 3D-TOL or LIMA. We can also, via *STATPAK-Win*, establish control charts, histograms, SPC calculate indices (C_p and C_{pk}), etc.

7.5.1. Geometric control

Geometric control is performed either using hard material (paper plane) or soft material: an electronic file of numerical definition provided by a system CAD (IGES, DXF, etc.). The GeoPak and CadPak software programs are designed to operate either from paper plans or from CAD files. They incorporate all ISO tolerances: tolerances of form such as the flatness, tolerances of position such as localization, and geometrical tolerances such as parallelism. The maximum material and run-out tolerancing are also integrated in the software programs.

7.5.2. Surface control

Machines are equipped with analytical software. It is therefore possible to deliver graphic control reports expressing the position of the points being measured on arbitrary-shaped surfaces. These reports are made based on files of numerical definition with various formats (IGES NURBS, IGES, DXF, SET, UNISURF, etc.)

and from the points being probed on manufactured parts. With the software *ScanPAK Win* by Mitutoyo, it is possible to reproduce the forms of a probed piece and to generate the files of points with various formats (IGES, DXF) legible by utilities of CAD–CAM. In the following sections, we will try to propose some examples on utilities previously presented. One of the software programs selectively integrated by Mitutoyo as an interface for calculation after probing is *GEOPAK-Win*, copying, digitizing (scanning), and quality control (*STATPAK-Win*). There are different probe heads [REN 08] which will not be described in this volume:

- *dynamic probing head* (the button on the probe is retractable. This probe head allows only point-by-point measurements);
- *gauging probing head* (the head allows for continuous measurements, such as statements of form – scanning);
- *static probing head* (the head consists of a mechanical leaf spring allowing for a small 3D displacement of the probe. This head performs its measurements upon the stopping of the machine, point by point);
- manual indexed heads MIP MIH and automatic PH9 PH10 [REN 08].

7.5.3. Coordinates systems and probes calibration

When using a CMM, we must identify a coordinate system defined by the axes of displacement of the machine to which a *measuring mark* or *machine mark* is associated and for enabling us to take into account the changes in markers subjected to the diversity of the probes used in that circumstance. These markers are defined as:

- *measuring markers*. Each probe generally has a spherical form and the point of contact probe/piece may be at any point of the probe's sphere. The point of contact probe/piece is unknown at the time of the measurement of a point; it is then substituted by the statement of three important informational elements such as coordinates of the center of the probe, the direction of docking, and the radius of the probe. This information is subsequently used to calculate the point of contact probe/piece. The coordinates of the center of each of the different probing spheres are expressed in a common marker defined by the calibration procedure. To this end, we coincide by calculation the center of the sphere of the probe with the center of the O of a sphere of reference fixed on the marble machine;
- *marker machine*. The three directions are defined by the directions of the three guides. The origin of the coordinate (axes) system defined by the three origins is fixed by construction on each of the three rules of measurement. This marker machine defines the values X_c , Y_c , and Z_c of the three meters measuring the relative displacements of the three detectors located in front of their graduated rulers.

The three axes of the measuring reference system are parallel to the axes of displacement of the machine. Its origin is coincident with the center O of the sphere of reference.

The procedure of calibration of the center of a probe consists in measuring, via the probe, n points on a sphere of reference, then, by calculation, identifying in the marker machine, the three coordinates U_j , V_j , and W_j of the center of the sphere of reference.

With each probe j , we associate the three constants U_j , V_j , and W_j . The coordinates X_s , Y_s , and Z_s of the center of a probe's sphere W_j , also called *captured point*, will then be expressed in the measuring marker, following the three equations (Figure 7.14):

$$\begin{cases} Y_s = Y_c - V_j \\ Z_s = Z_c - W_j \\ X_s = X_c - U_j \end{cases} \quad [7.5]$$

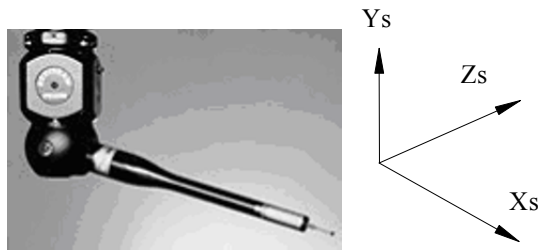


Figure 7.14. *Coordinates of the CMM probing sphere*

X_c , Y_c , and Z_c are the coordinates given by the integrating meter of the three rules of the measuring machine. The actual contact point between the sphere probed and the measured surface is unknown; it is substituted by an estimated contact point or a measured point. The latter is calculated from the coordinates of the captured point (center of the sphere probed), the direction of docking and the radius of the sphere probed. It is assumed that the contact point is at the intersection of the sphere probed and the normal to the surface, passing through the captured point. The nature of the nominal surface being known, the calculation of the measured point can be:

- a combination of a nominal surface passing, as close as possible, through the captured points;
- a calculation of the normal n_i to the nominal surface, passing through the captured point P_i and oriented in conjunction with the docking direction (outside of the material);

– a calculation of the coordinates of the measured point M_i (estimated contact point) given by the formula:

$$(\vec{OM}_i) = \vec{OP}_i - \vec{r} \times \vec{n}_i \quad [7.6]$$

where r is the radius of the sphere, P_i are the measured points and n_i a normal component.

If the nominal surface is of an unknown nature, the normal to the surface can be arbitrarily set or locally estimated. In the latter case, we measure two additional points which are close to the captured point. The normal is then defined by the plane passing through the three points (Figure 7.15).

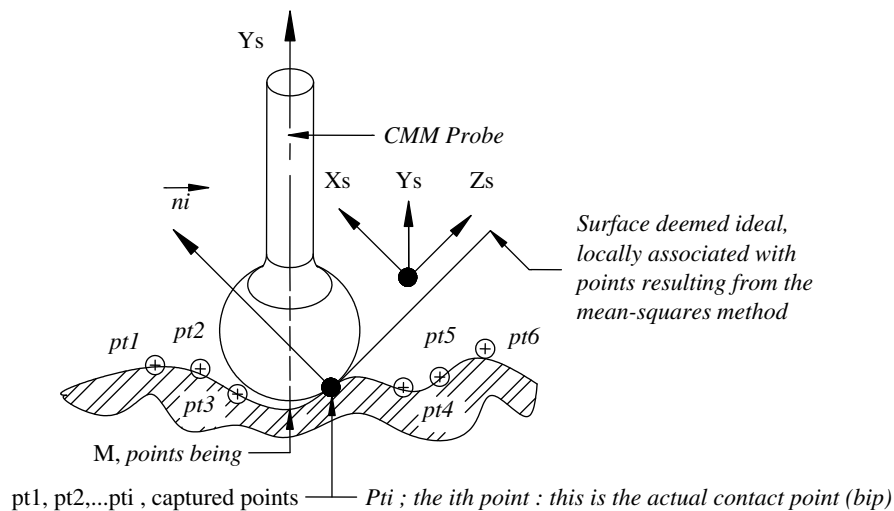


Figure 7.15. Representation of the probing points to be measured on a real surface

7.6. Starting a B504B-Mitutoyo CMM [MIT 00]

There are two methods to generate a reference for work involving a CMM. The first method is based on a simple mathematical logic by means of two principal axes XX' and YY' intercepted at the origin O . The second method is identical to the first one in that it consists of generating a reference in the same manner as described above but with the addition of a control indicator. In this regard, it is suffice to just utilize the referential models proposed by Cosmos.

1. Power with compressed air and start the computer.

2. Enter a password (login name) and a common name (user name and password).
3. Select the icon Cosmos 1.4R5 (to access the level of exploitation).
4. Give the name of the piece or the program for the session that has been started.
5. Activate the icon of the desired mode:
 - single mode and learning if the control requires it; and
 - snooze mode if the statistical or repetitive control requires it.
6. Appearance of probes' managers:
 - verification of the system unit (micrometer or inch);
 - depending on the version 1.4R5 of Cosmos, it is difficult to change the unit during an operation, except only in the case of a statistical control;
 - activate the probe from the dropdown menu;
 - select and/or calibrate the probe (e.g. PH_i) that was previously mounted; and
 - respond to the prompt (see dialog box (of) Cosmos).

Now, generate a reference of the work to be done by adopting the following approach:

- create a plane with at least three probed points out of the marble (ground part);
- align the reference plane by selecting the icon of the same name;
- create a straight line representing the axis XX' ;
- create a straight line representing the axis YY' ;
- create the intersection point by selecting the icon point. From the dialog box that opens, choose the icon illustrating the intersection of two elements XX' with YY' ; and
- create an origin for the piece under study by selecting the icon and answer the questions prompted by the dialog box.

Now, save the referential:

- select the icon “Remember Referential” and assign an identification number; and
- align the reference plane by choosing the icon having the same name.

7.6.1. Number of probing points

All the information so far obtained can be saved in a GEOPACK by assigning a file name to it. Experience, however, shows that it is often advisable to probe the sphere for five points. We reconstitute, in the calibration process, the scenario of the probe docking in relation with those to be applied on the workpiece (Figure 7.16).

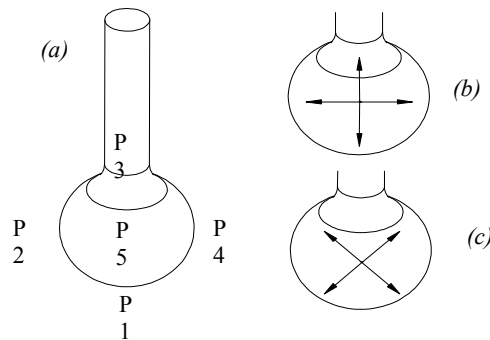


Figure 7.16. *Ideal process of probing with a sphere*

We probe, depending on the number of chosen points, and also in accordance with angular distribution such as:

- four points: three points at 120° on the equator, and one point at the pole;
- five points: four points at 90° to the equator, and one point at the pole;
- eight points: four points at 90° to the equator, zero point at the pole, or;
- eight points, three points at 120° at midway to one point on the pole;
- $(n/2)-1$: equidistant points at midway, and one point on the pole; and
- n points: $n/2$ points equidistant on the equators.

Once the stand is ready with the calibrated probes, and the piece to be isostatically controlled is well positioned, we are then ready to use the GEOPACK.

7.6.2. Key measuring functions of the Mitutoyo B504B CMM

This step is one of the most time-consuming. The nature of different measurement functions by their two-letter code gives instant access to the type of measure to be chosen. We may also use the mouse and icons corresponding to the

measure (an icon appears to show the appropriate form of the desired measure and displays at the bottom of the screen the inherent two letters, e.g. CR for circle). The different settings specific to an element must be defined as previously explained (probing, plan, etc.). Depending on the three working modes based on the nature of the element, one can choose different function keys to validate the measurement. The following are the three main working modes:

- single mode;
- learning mode; and
- repeat mode (quality control program).

Now, in the following, we discuss the main functions of each mode:

- Single mode: this mode is used during measurements of individual pieces. It is the most common mode even if the use of a CMM for such a case is not cost effective. It is obvious that the reason for using the CMM is due to the degree of accuracy it could provide;
- Learning mode: this function is programmable. In repeat mode, the computer beeps to signify the use of a new probe for the next point. The number of the new probe is then displayed; and
- Repeat mode (programmable): this mode is often used in the case of serial measurements of pieces. In this case, it is recommended to use this function repeatedly in order to sequentially measure all searched dimensions and then process the resulting data, often for statistical purposes.

7.6.2.1. *Characteristics of a toleranced element*

By calling this function, a submenu appears. The latter differs depending on the type of the element to be toleranced. For example, after a circle, one aims at tolerancing:

- a diameter → tolerance on the $\varnothing$;
- a radius → tolerance on the radius; and
- circularity → shape defects tolerance.

After a cone (or slight taper), here is what one may want to tolerance:

- a cone angle → tolerance on the angle at the vertex;
- a 1/2 (half) cone angle → tolerance on the 1/2 angle at the vertex; and
- a shape defect of a cone → shape defect tolerance.

After a plane, here is what one may want to tolerance:

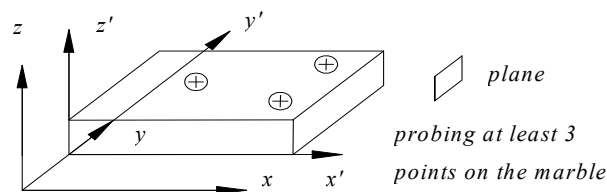
- distance from the origin → tolerance on the distance normal to the plane passing through 0; and
- flatness → shape defect tolerance.

7.7. Measurements on CMM using the Cosmos software [MIT 00]

In what follows, we limit ourselves to brief presentations relatively to examples of measurement using the CMM with Mitutoyo Cosmos software.

EXAMPLE 1.– Plane (at least three points are probed): the plane is a 3D element, but it is always possible to probe a plane regardless of the projection mode that is used. The GEOPAK calculates the plane without regard to this function, i.e. in space. Here is a simple approach to make a plane with three points. The three points are probed from the marble as explained below:

- X , Y , and Z coordinates of the intersection between the plane and the normal to the plane through the origin;
- angles of the normal to the plane with respect to X , Y , and Z of the current system. The direction of the normal to the plane influences the selection of angles relative to the axes (trace of a straight line);
- distance between the plane and the origin: perpendicular to the plane, passing through the origin; and
- form differential: if the number of probed points is higher than three (Table 7.6).



4 N004	Nb of points	X	Y	Z	Distance d to the origin	Form differential
Plane	3	0.000 90:00:00 Angle x	0.000 89:59:48 Angle y	1.475 179:59:48 Angle z	1.475	0.0030

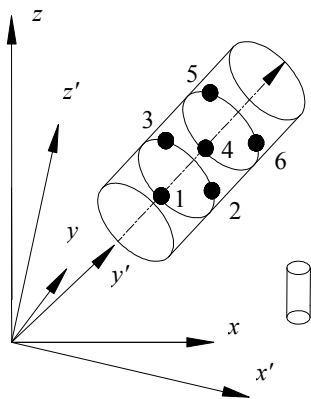
Table 7.6. Example of a plane probing

For a circle, at least three points are probed (Table 7.7).

4 N004	Nb of points	<i>X</i>	<i>Y</i>	<i>Z</i>	Diameter <i>d</i>	Form differential
Circle	4	0.001	−0.001	0.000	22.951	0.0015

Table 7.7. Example of a circle/probing

EXAMPLE 2.— Cylinder (at least six points are probed): a cylinder must be probed in space, that is to say in projection. It is recommended to probe a cylinder, at least, in eight points. From each extremity, four points will be taken in the form of circular sections to be probed one after another. From nine points, two circular sections will be taken one after the other at the ends and the remaining points will be considered randomly (Table 7.8).

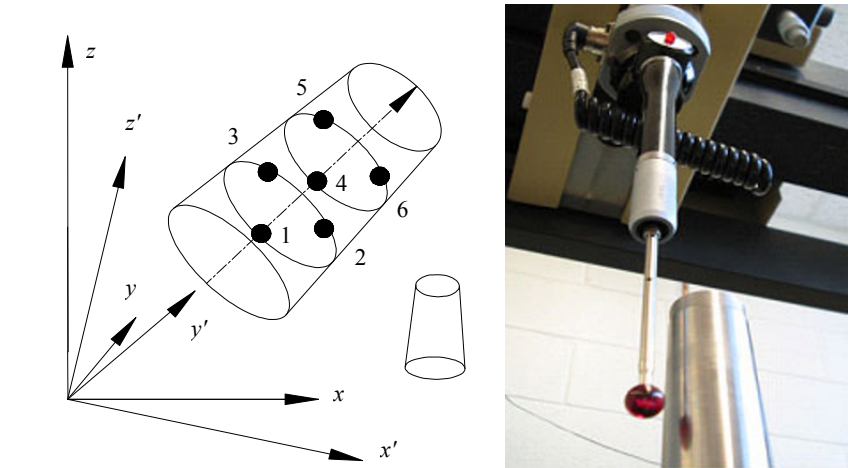


5 N005	Number of points	<i>X</i>	<i>Y</i>	<i>Z</i>	Diameter <i>d</i>	Form differential
Cylinder	6	25.000 90:00:00 Angle <i>x</i>	63.000 90:00:00 Angle <i>y</i>	2.000 0:00:00 Angle <i>z</i>	42.000	If probed points > 6

Table 7.8. Example of a cylinder probing

EXAMPLE 3.— Cone (at least six points are probed): to control a slow-taper cone, at least nine points should be probed. This function relates to slow taper cones. However, the measurement of a regular cone will be performed by probing at least six points without projection. The second probing must be done as far as possible from the first one. If we probed nine points (which is significant), three points would therefore be probed on a section at one end of the cone, then three more

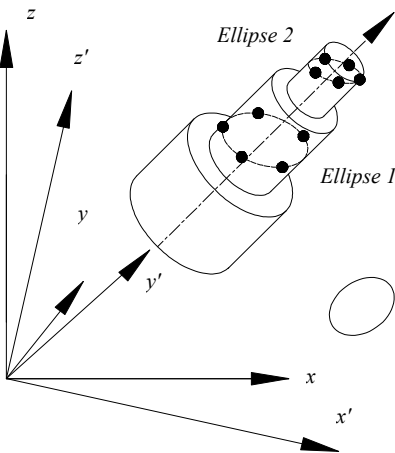
points to the other end, and the remaining points will be at the discretion of the operator (Table 7.9).



6 N006	Number of points	X	Y	Z	Angle $\alpha/2$
Cone	as one chooses	69.000	87.000	125.60	12: 00: 00
		90: 00: 00	90: 00: 00	0: 00: 00	6: 00: 00
		Angle x	Angle y	Angle z	1/2 Angle

Table 7.9. Example of a cone probing

EXAMPLE 4.— Ellipse (at least five points are probed): it is important to verify the concerned projection plane, in this case, as in the case of a circle (Table 7.10).



5 N005	Number of points	X	Y	Z	Major/Minor axis
Ellipse	5	1023690: 88:45:32 Angle x	-56.873 23:56:12 Angle y	0.000 66:05:59 Angle z	Major Axis Length = 20.004 Minor Axis Length = 10.006

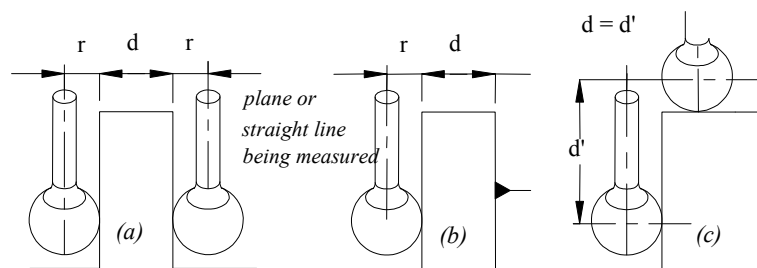
Table 7.10. Example of an ellipse probing

EXAMPLE 5.— Sphere (at least four points are probed): a sphere can be measured without projection (Table 7.11).

4 N004	Number of points	X	Y	Z	Diameter d	Form differential
Sphere	4	50.369 x	- 14.237 y	0.000 z	26.438	If probed points > 4

Table 7.11. Example of a sphere probing

The *GEOPAK-Win* software, under Mitutoyo Cosmos (V. 1.4) is no longer DOS compatible. The icons that are installed clearly illustrate the function to perform, in addition to a drop down menu. In this work, we do not present all the examples offered by the software as it would be tedious and unnecessary. However, certain technical issues related to compensation of dimensions need to be addressed here with reference to examples pertaining to distances measurement (Figure 7.17):

**Figure 7.17.** CMM distance measurement

(a) The distance between two probed points is given (by GEOPAK) by subtracting two probe radii to the distance being measured from the center of the probe;

(b) In this case the distance between the probed point and a plane or a constructed line is given (by GEOPAK) by subtracting a probe radius to the distance being measured from the center of the probe; and

(c) In this case, the distance to be measured is that between two probed points. However, GEOPAK does not compensate for the probe radius because the distance is obtained from the difference. Further, (c) is not used between the probed points and another element. In this case, the distance results from two probed points. However, GEOPAK adds two radii of the probe to the distance being measured to the center of the probe. To get the distance between a probed point and a plane or a straight line, one should be very careful because the use of the function point/point distance, that is to say, probed-point to probed-point, implies probing two points with identical diameters. This function is not recommended because it will be difficult to probe the right place of the second diametrically opposite to the first.

7.7.1. Case of circle-to-circle distance

The distance of the two elements (circles) performed by the distance function (DI) is identical to that derived from point-to-point. The two cases in Figure 7.18 illustrate the idea of, respectively, with or without probe radius compensation.

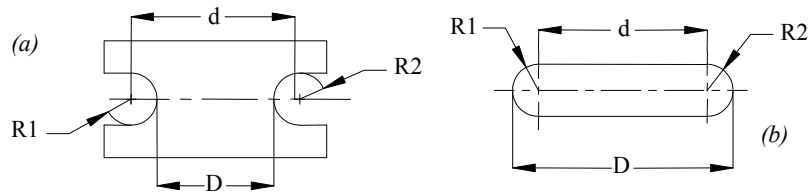


Figure 7.18. *Measuring distances on CMM*

Thus, in the first case (outer distance with compensation) GEOPAK subtracts each radius to the distance from the center (a) and the calculation is carried out as follows:

$$D = d - (R_1 + R_2) \quad [7.7]$$

Also, for the distance of the first case (outer distance without compensation) this is a simple distance from the center of the circles. In the second case (inner distance without compensation), GEOPAK adds each of the radii to the distance to the center (b) of the probe which is calculated by:

$$D = d + (R_1 + R_2) \quad [7.8]$$

The user of GEOPAK-Win should first identify his/her probe and ensure that the isotatism of the piece to be probed, will not be affected during probing. The GEOPACK offers a variety of functions ranging from simple point to solids of revolution, of which we present the main functions. In its V 1.4 version, GEOPAK-Win offers icons and a drop-down menu. Because of the Windows environment that the Cosmos uses, the function to be executed by this icon is displayed on the screen just by dragging the mouse cursor on the icon. Therefore, the operator of a CMM equipped with Cosmos can learn individually the use of the main functions. Among the icons (identical drop-down menu), we include the following (Figure 7.19).

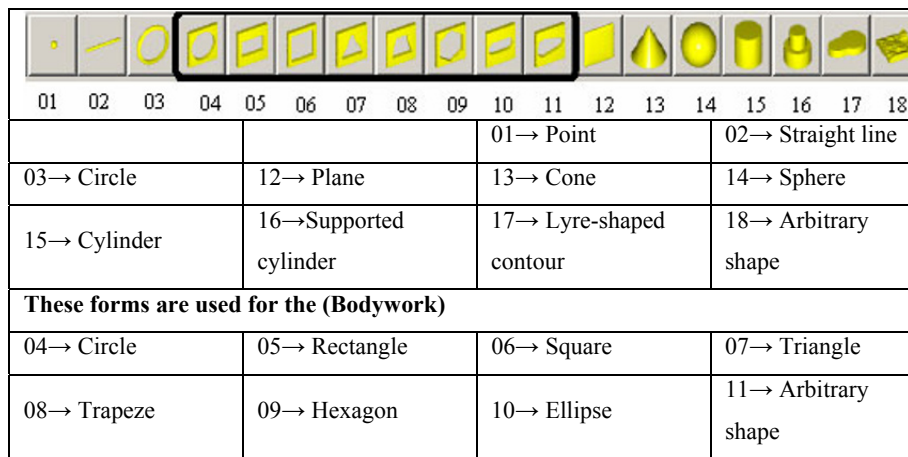


Figure 7.19. *Measuring distances on CMM from 4 to 11 for the automobile-body using the Cosmos software of Mitutoyo*

One is free to choose a system of coordinates witness of the series offered by the Cosmos for we can ourselves build up our own system of coordinates. The following coordinate systems illustrate the mode to follow in this regard:

- the circle or cylinder may be replaced by an ellipse or a cone. One should decide regarding the choice of coordinates system well in advance. It is of course possible to change or select a different model, but by convenience, it is recommended to first opt for the most appropriate coordinates system for our study;
- it should be understood that “line” is a “straight line.” We keep using the term line in the text even though it is a bit questionable (Figure 7.20).

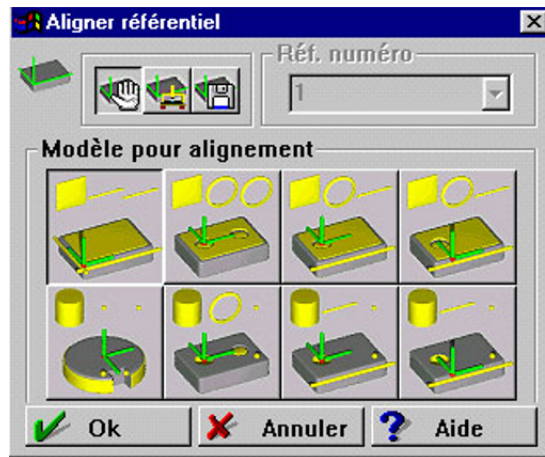


Figure 7.20. Alignment of a referential before the measurement on CMMs using Cosmos
(Source: Cosmos software-Mitutoyo)

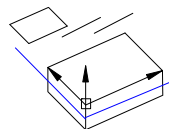
7.7.1.1. Suggestions from GEOPAK-Win to the operator during measurement [MIT 00]

NOTE.— The description of the models to follow is the result of a summary of exclusive features used by the software Cosmos Mitutoyo. We have tested all these models before recommending them to the user as follows.

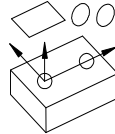
At each opened window, the GEOPAK-Win suggests the number of probing points. This is often the minimum points required, plus one for the measure that is suggested to us. One can then accept or change the proposal of the GEOPAK-Win. Usually here is what is proposed:

- the name of the element;
- the number under which the element will be stored;
- the number of points being probed, etc.; and
- the number under which the coordinates system is recorded.

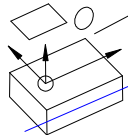
The last point may be recalled at any time and even changed as appropriate. We now define each of the coordinate systems presented above.



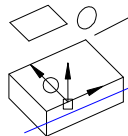
The “plane, line, line” model defines the axes in space by the plane being measured. The first line (straight line) indicates the direction of the axis XX' . The intersection of the two lines constitutes the origin of the Cartesian coordinates system. When probing points on the surfaces, a dialog box appears on the screen.



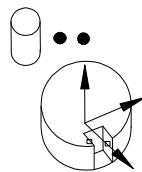
The “plane, circle, circle” model defines the axes in space by the plane being measured. The direction of the axis XX' is given by the straight line from the first circle, center of the second. The origin is located at the center of the circle.



The “plane, circle, line,” model whose origin is in the circle’s center, defines the axes in the space delimited by the plane already formed. The direction of the axis XX' is indicated by the line. The origin is established at the center of the first diameter.

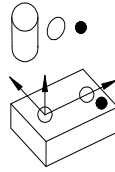


The “plane, circle, line” model whose origin is on the line is defined by the axes in the space delimited by the plane already probed. The line represents the axis XX' . The intersection of the projection of the center of the diameter on the line itself gives the origin of the piece under study.

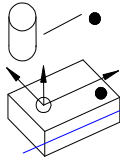


The “cylinder, point, point” model has its origin on the axis of the cylinder. It is defined by the axes in the space delimited by the cylinder being measured.

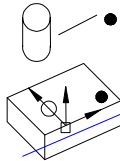
The first point determines the height of the origin in ZZ' and the second point indicates the direction of the axis XX' .



The “cylinder, circle, point” model defines the axis in the space of the cylinder being measured. The origin lies on the axis of the cylinder. The first point determines the height of the axis ZZ' from the origin O . The direction of the axis XX' is toward the origin, center of the circle being measured.



The “cylinder, line, point (the origin of the cylinder lies on the axis)” model defines the axis in the space of the cylinder being measured. The origin lies on the axis of the cylinder. Measured line gives the direction of the axis XX' .



The “cylinder, line, point” model has its origin on the line. This model is defined by the axes in space by the cylinder being measured. The origin lies on the axis of the cylinder. The point determines the height of the original ZZ' and the measured line gives the direction of the axis XX' . However, the origin located on the axis of the cylinder is projected on the line and becomes the new origin of the part.

7.7.1.2. Three types of coordinates systems

These include the following types of coordinate systems:

- Cartesian coordinates system;
- cylindrical coordinates system; and
- spherical coordinates system.

7.7.1.2.1. Cartesian coordinates

In Cartesian coordinates, the axes X , Y , and Z define the position of a point in space as shown in Figure 7.21.

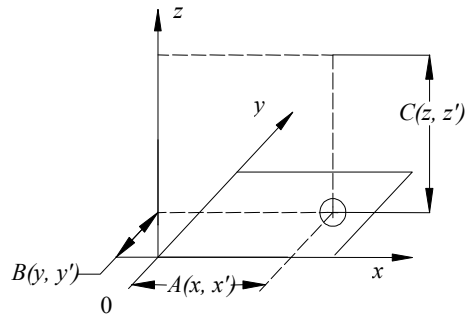


Figure 7.21. *Cartesian coordinates system*

7.7.1.2.2. Cylindrical coordinates system

In cylindrical coordinates, a point in space is defined by:

- the projected distance from the origin;
- the angle φ formed with the axis (first axis) XX' ; and
- the value of the axis ZZ' , as shown in Figure 7.22.

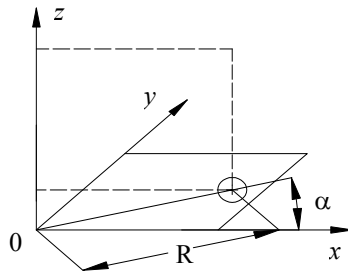


Figure 7.22. *Cylindrical coordinates system*

7.7.1.2.3. Spherical coordinates system

In spherical coordinates, the point in the space is defined by:

- the distance from the origin, in the space;
- the angle φ formed with the axis (first axis) XX' ; and
- the angle θ formed (according to the GEOPAK-Win) by the axis ZZ' to the vector of the point (Figure 7.23).

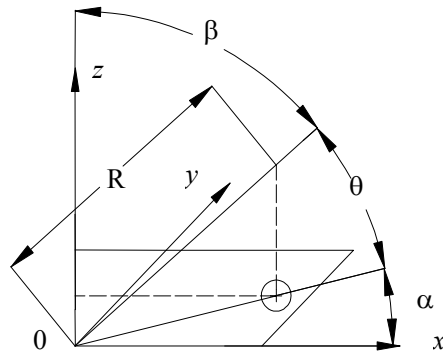


Figure 7.23. Spherical coordinates system

According to the GEOPAK-Win, one should pay attention to the fact that the angle θ can be interpreted with the classical mathematical sense from literature to literature. Sometimes, this means an elevation above XY relative to a plane.

7.7.1.3. Measures via the dialog box of Cosmos

We can measure an entity listed in the repertoire of the GEOPAK-Win [MIT 00]: point, line, circle, ellipse, plane, cone, sphere, cylinder, contour, calculation of angles, and distance. To measure a circle, e.g. simply click on the icon on the top left of the dialog box (Figure 7.24).

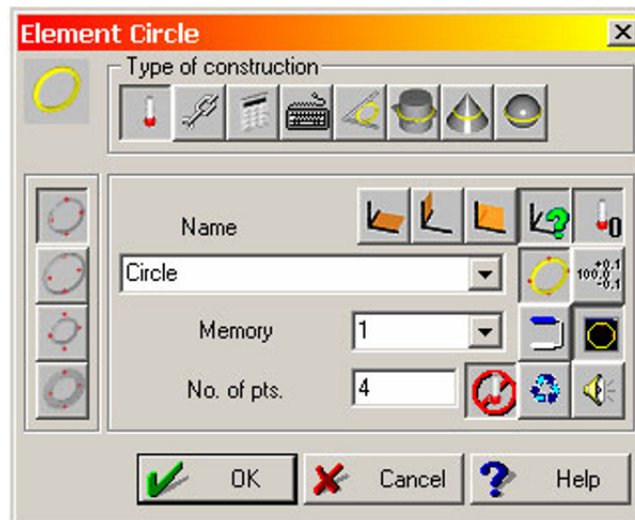


Figure 7.24. Dialog box to measure a circle (source: Mitutoyo Cosmos)

The dialog box is identical for all elements being measured via the GEOPAK-WIN. The example of measurement of a circle is a clearly demonstrative. We notice that there are five distinctly distributed data on:

1. the icons located on the first horizontal row are the elements of construction such as, measurement, connecting elements, calculations, etc.;
2. on the left, below the symbol of the icon of a circle measure, there are icons showing the types of mathematical calculations by various methods such as Gauss, minimum zone elements, circumcircle, inscribed circle, etc.;
3. on the right of the dialog box, there are two rows of icons for help;
4. in the central zone of the dialog box, there are three boxes to fill in the entry of information such as the name: circle (Cosmos, Mitutoyo often offers a name by default; however, one can always customize it), the memory: one (it also could be customized), and the number of points to be probed: four (one can enter the desired number of points); and
5. in the bottom of the box, there are three other boxes: “OK” (to validate the information), “Cancel” to abandon the information already entered, and “Help.”

7.7.1.4. Types of construction according to GEOPAK-Win

In line with the GEOPAK-Win, the approach consists of, first, selecting the element and then assessing how this element will be built.



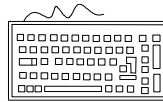
We now take a look at the calculation of the element based on its position (connected elements) relative to other elements such as the pitch diameter passing through several centers of various circles.

“Recalculation from memory,” which means that:

- the position of this element has been built in another coordinates system;
- a new element has been recalled from memory and its positions are calculated in the current coordinates system; and
- the calculation mode is changed, e.g. the button “zone of minimum element” is active while the mode of calculation has been based on the Gauss method.



We can also define any element as “theoretical element” by which we mean that the nominal value of this element will be entered through the keyboard.



7.7.1.5. Mode of calculating the elements being measured – type of calculation

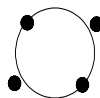
For certain types of elements, if the number of points being captured is higher than the minimum number of points, we can choose among the four methods for calculating the parameters resulting from the element. These different methods of calculation usually give perceptibly different results. We will be led to make a choice that entails the mode of calculation of the element being measured. The GEOPAK-Win [MIT 00] suggests six mathematical methods.



GAUSS: the program calculates the mean based on the points being probed and specific to the element. The sum of the squares of distant points is minimized (method of least squares). The element circle is calculated by the GAUSS method that uses the calculation of the square of the mean of the points being probed. It is virtually the only method that provides accurate calculations. The P. Chebyshev method, e.g. is applied, according to ISO 1101, to calculate the geometric errors, but the basic approach remains that of GAUSS. The latter is taking account of the compensation of the element.



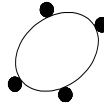
Minimum circumscribed circle: the program computes the smallest circle, which would contain all points that are being probed. This circle is well known, unique, and predefined.



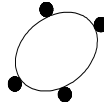
Maximum inscribed element: the program calculates the biggest circle among the points that are being probed. In this case, it is possible to obtain more than one solution.



Minimum zone element: the program calculates an element located between the two elements of ideal geometric form (Chebyshev method). The radius specified by the program (diameter) is the midpoint of the two circles. These two ideal elements contain all points between them. They are calculated in such a way that the considered zone is the smallest possible one. Irrespective of whether the circle is a maximum inscribed or a minimum circumscribed circle, it has the same center. However, the position of the center might also be different in these two circles. The radius (or diameter), therefore, is the result of an average value of two circles.



Fitting-in element: we have four methods for calculating this element. We consider only the elements that are in contact inside as shown by the icon.



Envelope element: The program calculates the points based on their smaller geometry. This method is used to calculate probably one of the four previously described elements.

7.7.1.6. Range/standard deviation and degrees of freedom

GEOPAK displays a standard deviation of $\times 4$ if we consider the graphs of roundness, straightness, and flatness. The same value can be displayed in graphical elements under “4s.” The degrees of freedom are essential for calculating the standard deviation. This depends on the minimum number of required measurement points, that is to say, depending on the type of element under study (Table 7.12).

Element type	Minimum number of points	Degrees of freedom
Line	2 number of points	2
Circle	3 number of points	3
Plane	3 number of points	3
Sphere	4 number of points	4
Cylinder	5 number of points	5
Cone	6 number of points	6

Table 7.12. Minimum number of points and degrees of freedom

7.7.2. STATPAK-Win of Cosmos, Mitutoyo [MIT 00]

The statistical evaluation of data for quality control is carried out using STATPAK-Win which is a Cosmos program (Mitutoyo, in our case). Also, control data from the 3D metrology are easily transferable to STATPAK Win-activated by the following icon:



To carry out statistical measurements, we activate the piece located on the list piece by opening the STATPAK Win-programs through the menu bar and the menu “Stats,” or using icons. We open the application data conversion of SURFPAK only through the Stats menu. The statistical applications that appear include the following:

- edit the range;
- hand tools;
- statistical analysis;
- data conversion SURFPAK;and
- data conversion STP-3-STATPAK (Figure 7.25).



Figure 7.25. Part Manager. Software PartManager-Win (Mitutoyo Cosmos)

7.7.2.1. Hand tools (calipers, micrometers, etc.)



Through this function, we activate the measurement with hand instruments:

1. Activate the part on the parts list.
2. Assign a range that tells us what measurement value belongs to which feature. (The above two points provide us an insight regarding with which measuring instrument the desired characteristic will be entered.)

3. Through the function icon that is used to edit the range, we create a range or we change it.

By using the STATPAK-Win Cosmos, it is very comfortable to carry out the tedious task of data transfer. Hence, it is obvious that the Cosmos offers the following advantages:

- all appraisals are updated if you change the database;
- several elements of statistical analysis are made in a very short time;
- STATPAK-Win records data in Cosmos. They are then accessible in all circumstances. Data are protected with a password.

However, there exists quite a difficult approach when implementing a repetitive program for quality control. Leaving the PartManager in order to return to snooze mode after transmitting data to STATPAK-Win is a cumbersome and inelegant procedure. The dialog box of STATPAK-Win is convenient to use as it is user friendly. It provides the main functions for a study of the type of statistical process control (SPC). Each of these icons is accompanied by an aid that is supportive in nature. By clicking on the icon, we obtain the related statistical functions. Before going into statistical details, it is pertinent to propose a necessary approach to generate a repetitive program that is used in quality control (Figure 7.26).

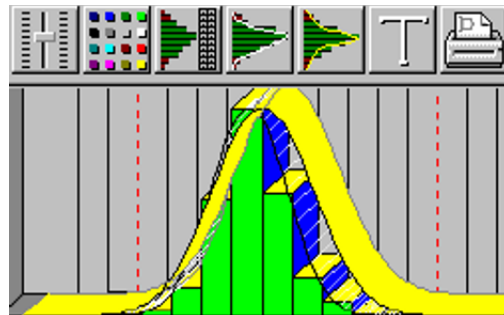


Figure 7.26. Statistical evaluation of CMM data (Source: Mitutoyo Cosmos)



At the top right position, this icon (chevrons) appears. We click on it to prompt the program (GEOPAK-Win) that data will be sent to STATPAK-Win. To do this, we must first assign a name to a characteristic. Once a letter of the name is entered, the icon “data to STATPAK” is activated.



One should then click this icon (press the button) to confirm sending the data to STATPAK-Win. The following series of steps have to be ensured once the data is sent to STATPAK-Win:

- complete all the measurements to be programmed for quality control in single mode;
- the function (circle, line, angle, etc.) should be validated. This function is represented by a yellow icon on which the tolerated element is updated;
- leave the single mode of the manager of parts PartManager;
- return, after that, to the snooze mode. A dialog box will pop up to add information on the number of repetitions and other questions through the dialog box;
- at the bottom of the screen (or on the top depending on the configuration setting), the STATPAK-Win is displayed. By clicking this button, we get the dialog box of STATPAK-Win with all the icons useful for quality control measures;
- simply click on each icon to view the statistical results, i.e. Parts-Oriented Analysis that STATPAK-Win Cosmos propose to process.

In addition to traditional statistical treatments, STATPAK-Win offers calculations of capabilities. This is a significant novelty in version V 1.4. Also, you can plug any measuring instrument to a USB port on the computer and it becomes a removable SPC. The options offered by Cosmos in terms of capabilities are summarized as follows:

- change in the estimation by standard-deviation of statistical computations;
- input limits of capability indices;
- selection of the number of groups for a preliminary analysis of capability processes (Potential Process: Pp/Ppk). Computation is done simultaneously. For each one of the input values, Cosmos uses them to propose a result in real time. After that, the important computation of Pc/Ppk (Index and coefficient of capability) is given simultaneously.

7.8. Examples of applications using CMM

We offer three examples representative of GPS and which involve Cosmos Mitutoyo software that uses the Cosmos Mitutoyo for measuring dimensions and controlling geometric constraints shown in the engineering drawing.

EXAMPLE 1.— Consider a machined part grade 6061. You are asked to measure using a CMM all the dimensions shown on the engineering drawing, to impose additional geometric constraints of your choice, and then check with the software (e.g. Cosmos) serving as an interface in your own machine.

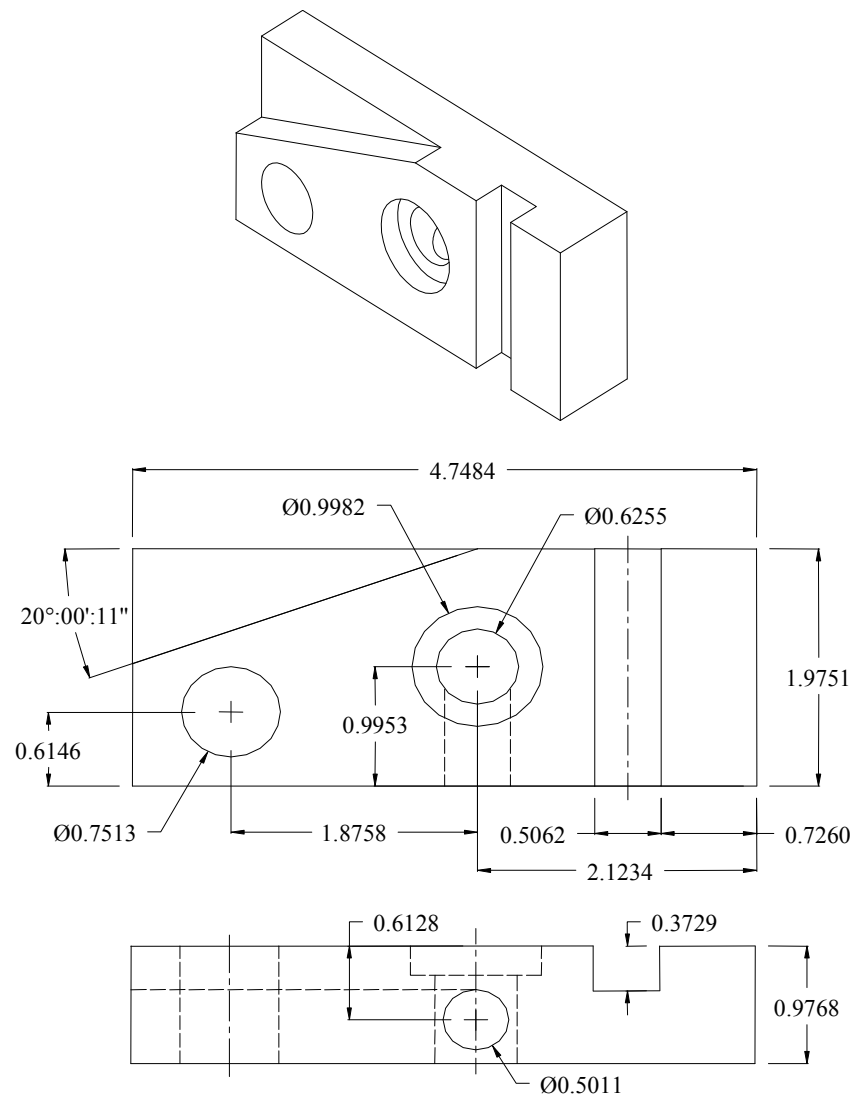


Figure 7.27. Engineering drawing of the part (1) machined 6061 to be controlled on CMM

Results (in millimeters) recorded in Table 7.13 are provided by the Cosmos. In the drawing (Figure 7.27), we have deliberately introduced the imperial unit (inch). We chose to present only the result of the dimensioned angle ($20^{\circ}:11':00''$) on the drawing.

Nb Toleranced element	Nb Probed points	Coord. <i>X</i> Angle <i>X</i> Nominal $\pm$	Coord. <i>Y</i> Angle <i>Y</i> Tol. $\pm$	Coord. <i>Z</i> Traced Angle <i>Y</i>	Diameter Dist./angle Deviation/H.T	Nb Toleranced element
1 N0002 Plane	3	-0.099 89:57:45	-0.182 89:55:52	-151.718 00:04:42	151.719	1 N0002 Plane
2 N0008 Straight line	M	-0.003 00:00:02	-257.276 89:59:58	0.000 90:00:00	257.276	2 N0008 Straight line
3 N0008 Straight line	M	-176.424 90:00:00	0.000 00:00:00	0.000 90:00:00	176.424	3 N0008 Straight line
4 N0010 Inters/Pt.		-176.424	257.27400	0.000		4 N0010 Inters/Pt.
Etc. ...						
10 N0025 Angle	9				20°:11':00"	10 N0025 Angle

Table 7.13. Result (excerpt) of a dimensional inspection on CMM

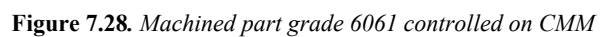
We deliberately chose to present these results in GEOPAK-DOS. The same procedure was repeated with the GEOPAK-Win and it allowed obtaining the same results to the nearest thousandth.

LABORATORY EXAMPLE ON CMM # 2.– The following example is proposed to impose geometric constraints on the positioning of the three holes, the concentricity of both diameters, the symmetry, and the skew. Based on the part provided in real scale (Figure 7.28) verify the measures. Dimension the part properly: the accepted tolerances are about ± 0.005 mm.

1. Proceed to mounting the part on marble, by explaining the isostatism.
2. Measure the required dimensions and put them on the drawing.
3. Write the report in accordance with the requirements explained above.

We also considered a dimension ($1.13 + 0.001$) – central groove on the drawing with an error of more than $2/1000$ th on the nominal. Then, we have designed a program to compute the statistical characteristics around this dimension to locate the mean value of all simulated values on the nominal dimension (1.13 in.). The results are shown on the technical drawing (Figure 7.28). In the final analysis, we conclude that:

– metrology on CMM remains an important tool for 3D measuring, with the ease, accuracy, reliability, usability, all being offered in real time;



LABORATORY EXAMPLE ON CMM# 3.– Consider the engineering drawing of a machined part 6061 (Figure 7.29). You are asked first to impose the additional and dimensional geometric constraints as it may deem necessary, then follow the approach of the laboratory procedure detailed below to build a range of control useful for dimensional metrology, and even for quality control.

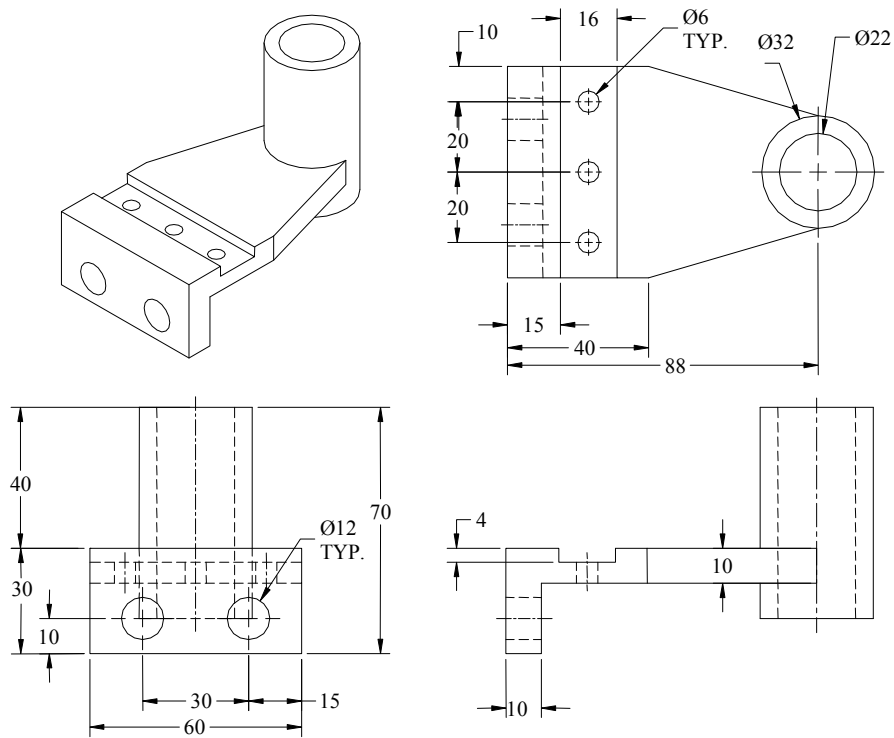


Figure 7.29. Example of a drawing to be used as a laboratory

Add geometric and dimensional constraints as stated in the title of the topic of the laboratory.

Laboratory on dimensional metrology	Student: NADIM. G	Fall 2009
1.0 Start the software program with a password, for example: Sarah G and join the range of control with all dimensional and geometric constraints. 1.1 Write the section related to the palpated elements (e.g.: PLi, CYLj, etc.). 1.2 Write the section on constructed elements. 1.3 Write the section on geometric and dimensional tolerances. 2.0 Save the control range in a file. 3.0 Start executing the full range and create a real part (see engineering drawing). 3.1 Calibrate the probes on the calibration sphere and record them. 3.2 Probe all the surfaces being already designated in the range of control. 4.0 Print the control report. 5.0 Discuss the specifications of the part based on the engineering drawing and the report.		

NOTE.– Do not forget to change the probe wherever necessary!

Discussion of the conformity of the workpiece						
Based on the engineering drawing						
What are the specifications being controlled?:						
Specification	Nominal value		Deviation max		Deviation min	Tolerance Interval (IT)
.....						
Based on the control report (unit report printed sheets):						
What are, for each specification, the obtained values?						
Specification	Without object	Value min	Value or indication	Value max	Judgments	
					Good based on the engineering drawing	False based on the engineering drawing
.....						
Is the “number of features out of tolerance” shown on the cover page of the control report consistent?:						
.....						
.....						
.....						

Instructor's Evaluation Sheet				Student: NADIM G	
				Year: 2009	
Competency assessed: Implement a procedure for measuring a 3D metrology software • Perform the measurement • Interpret the results	Machine:CMM Mitutoyo (or other) Software: Cosmos (<i>or other</i>)				
	Garding 100%				
	10%	15%	15%	50%	10%
Part I of the Control					
Define the specification					
Define the nominal of the specification					
Define the maximum and minimum deviations from the specification (see engineering drawing)					
Define the IT of the specification					
Part II of the Control					
Create the range of control Part probed elements					
Observations:					
Carry out a concise measurement procedure					
Tridimensional metrology software					
Perform the measurement					
Clean and store the work area					
Part III of the Control					
Interpret the results					
Write the report					
Defend the report					

7.8.1. *Compiling the technical file*

These include the following points:

1. Engineering drawing.
2. Range of control of the workpiece.
3. Positioning of the workpiece probed used accordingly (Table 7.14).

Laboratory of/on dimensional metrology		By NADIM G		Class of Fall 2009		
Probed element	N°	Specifications	Types of elements Others ...	Concerned Element(s)	Value	
					Distance	Angle
Example	01	{ 00.0015	Point Plane Straight line	PL_1, PL_2	25.4 mm = 1 in.	01° 25' 25"
Constructed elements	No	Specifications	Types of elements Others ...	Concerned element(s)	Value	
					Distance	Angle
Example	01	{ 00.0015	Point Plane Straight line	PL_1, PL_2	25.4 mm = 1 in.	01° 25' 25"
Example		Why such a constraint?	Why?	How?	What?	What?
Judgments	N°	Specifications	Types of elements Others ...	Concerned element(s)	Value	
					Distance	Angle
Example	01	{ 0.0015	Point Plane Straight line	PL_1, PL_2	25.4 mm = 1 in.	01° 25' 25"
Example		Is it conform?	Why?	How?	True/False	Ibid

Table 7.14. Example of laboratory tables using a CMM

7.8.2. Constitution of the CMM laboratory report under Cosmos (or other)

This includes the following points:

1. Control range and engineering drawing, with the range of GPS units.
2. Entities to be probed (physically → so no axis).
3. Dialog boxes (to be built or copied from Cosmos document).
4. Elements constructed (here the axis would be involved).
5. Calibration probe (explain the choice of the reference sample (Figure 7.20)).
6. Write, carefully, the report and propose, if necessary, future expansions.

7.9. Chapter summary and future extensions of CMMs

The CMM is, because of its large size, a laboratory machine and not a workshop and which alone replaces the conventional measuring tools: caliper or micrometer. Nowadays, computer science occupies an important part while it is not indispensable

for the measurement. Hence, all CMMs are linked to computer software allowing, in addition to the measurement of parts, to perform simulations. The SPC outputs of measuring instruments still exist but the software directly processes the statistical model. With the integration of the CMM in production facilities, manufacturers design and offer fast and robust machines. They have also added to their software programming “offline” modules to avoid the immobilization of machines while developing the ranges of control. However, there are still some disadvantageous factors such as the exchanges of data between machines and their numerical control, between the machines and the CAD, and even between machines of different brands.

Being more accurate, CMMs measure parts of various shapes and sizes. They allow both scanning complex surfaces (for quick measurements) and performing point-to-point probing. Some CMMs are used outside the metrology laboratory and are installed in production facilities with stringent environmental conditions. CMMs are adopting new materials and optical sensors offering temperature-compensation algorithms of higher performance. Half of the CMMs are now used in the automotive industry and over a quarter of them in aeronautics. In our view, a CMM should not be considered just as a probe, a movable arm, and a marble support in granite. One should also consider the digitally controlled driver, its constituting materials, its integration into the environment, and its relationship to other units in the laboratory or workshop.

The CMM, more robust and less sensitive to dust and especially to temperature variations in workshops, were made of invar (iron–nickel alloy) because of its very low thermal expansion (of the order of $10^{-6}/^{\circ}\text{C}$ 11.10 against $11.10^{-6}/^{\circ}\text{C}$ for a typical steel). Also, it is possible to consider the behavior of the machine between 17 and 35°C . Generally, CMMs use a material that expands significantly with temperature. They then incorporate temperature sensors to compensate for the expansion of the structure around 20°C . Aluminum is a homogeneous material having excellent thermal conductivity. Mathematical corrections that are applied to aluminum machines are powerful tools, but they do not master all the strains. With a granite structure, however, there is less risk, because the machines are less likely to dilate.

On some CMMs, we think of the axis movement to be conducted on linear guides in braided carbon fiber. Aluminum alloys that are used are not sufficiently rigid hence a work based on correction matrixes is necessary. The mechanical stress imposed on the machine leads to causing deformation on the machine, and at the end requires rebuilding a compensation matrix. With the braiding of carbon fiber, the structure deforms less easily. To withstand the stringent environmental conditions of the workshop, the CMMs must also be less sensitive to vibration. This led manufacturers to design machines whose center of gravity is as low as possible. For this reason, one should consider the shape and angle of the gantry. Mitutoyo offers a software program (Correct Plus) for correcting in real time the drift of machine tools

based on the measurement result, and without operator's intervention. "It also identifies the various stages of elaboration of the piece, the different machine tools being involved, and thus it corrects their drift without changing the drift correction tool."

7.10. Bibliography

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Chapter 8

Control of Assembly and Transmission Elements

8.1. Introduction to the control of components for temporary assembly and elements for power transmission: threads, gears, and splines

This chapter addresses both temporary assembly components, such as screw threads, and mechanical transmission components, such as gears and keys. Although the words “measurement” and “control” are frequently combined in metrology, measurement and control by comparison are two distinct practices. The practice of control and measurement does not presuppose an enforced combination between them. We may control without measuring. In workshops, the control and the measurement are often carried out by using a comparator with mechanical, pneumatic, optical, or electronic amplification. In the industrial sense, the understanding of threaded elements is important. In screw threads, dimensional control is normally done on the *base diameter* and rarely on the profile. On power transmission components (lead-screw, milling table screw), the constancy of the pitch is important. Yet, this is what is inspected in machinery maintenance. As for measuring components such as micrometric screws and screw threads gauge (contact), it is imperative to control the constancy of the pitch (P) and the profile shape as well to prevent clearances that may result from premature wears.

8.1.1. *Method of obtaining threads and tapping in mechanical manufacturing*

Screw threads are principally intended to temporarily assemble parts. For this, we may opt for an internal and an external thread fitting each other to be associated

based on a given *pitch* and a specific material. To design holes intended to receive bolts or studs, a *standardized bore* should be drilled so that the tapping fits the size (tools dimensions) of taps. Depending on the destination of the holes, we may drill a complete hole known as through hole or inversely a blind hole (see Tables A5.1–A5.3 in Appendix 5). The *through hole* is a plain hole with a diameter equal to that of the drill. The *blind hole* is the result of a plain hole whose bottom is a taper with an opening of 118–120°. It is the trace left by the end of the tap drill. On a flat representation, a blind hole is designated, in sectional view, by two solid lines which delimit the diameter. The tapered part is represented by solid line. A dotted line indicates the axis of the hole.

Tapping operation. The hole drilling diameter is chosen depending on the expected thread diameter. This machining results in a helix characterized by its dimension and shape. In the case of a through hole, tapping may be either through or of limited length.

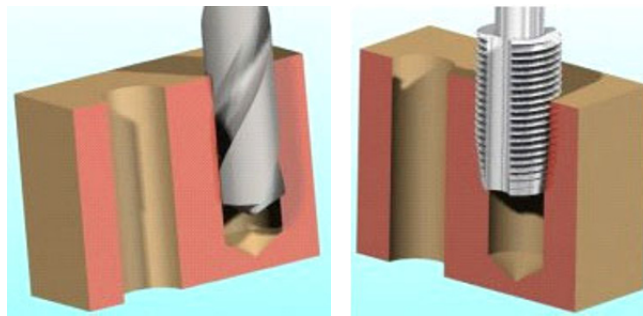


Figure 8.1. Conventional representation of the drilling holes and tapping

In the case of a blind hole, tapping never reaches the bottom of the hole (Figure 8.1). In a flat representation, the diameter of a *through* hole, in sectional view, is designated by two solid lines representing the diameter of drilling. Two thin lines indicate the diameter of tapping. A dotted thin line indicates the axis of the hole. The end of tapping is represented by a solid line. In both cases, the hatchings, indicating a cut, cross the thin lines and end at thick lines. On a sectional view, tapping is represented in solid lines, by three quarters ($3/4$) of a circle in thin line outside the drill. Under the form of hidden edges, all the lines have the same width.

NOTE 8.1.– Upon assembly of the parts, the drawing of the threaded part takes priority over the tapped part.

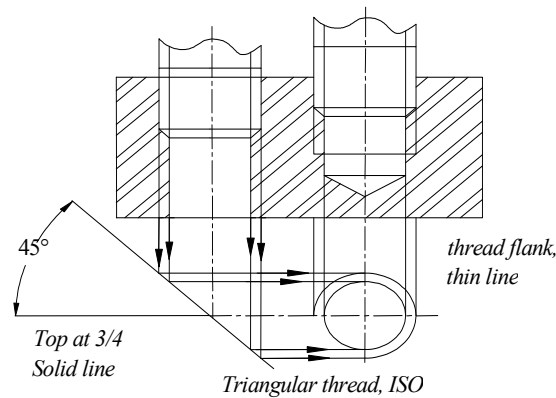


Figure 8.2. Conventional representation of threads

8.1.2. General description of thread dimensioning

Screws are machine components serving for assembling parts, transmitting motion, or exercising pressures. A thread is obtained by cutting a helical groove, depending on the shape of the desired profile. For a greater advance per turn, we use a multiple-thread screw. To do this, two or more identical threads are manufactured side by side and rolled in parallel (as chords wound spirally around a single cylinder). In this case, p represents the pitch which is the distance, measured parallel to the axis, between two peaks of consecutive threads. We also distinguish what is called axial pitch. It represents the distance traveled, measured parallel to the axis, by the screw when it rotates one turn (one revolution). For a single-thread screw, the axial pitch is equal to the pitch ($p_x = p$); for a screw with two threads, it is equal to twice the value of the pitch ($p_x = 2p$), and for a n -thread screw, it will be equal to $p_x = np$.

In hardware fasteners, the coarse pitch is the most commonly used type. The fine pitch is used for the case of low-height nuts of impacts and expensive structures. Threads can be triangular (screws and bolts), trapezoidal (screw maneuver), square, or round. To ensure interchangeability, standardized dimensioning should be respected. Any breach in this condition would lead to a questionable assembly. The dimensions and type of screw thread must be marked and designed following these specifications:

- M (for triangular thread ISO), G (for pitch of gas), Tr (trapezoidal thread), etc.;
- nominal diameter d ;
- if necessary the helical pitch L ; the profile pitch P ; the direction of the helix (LH designating left-handed screw thread and RH a right-handed screw thread); tolerance class; engagement length (S for short, L for long, and n for normal); and finally the number of threads.

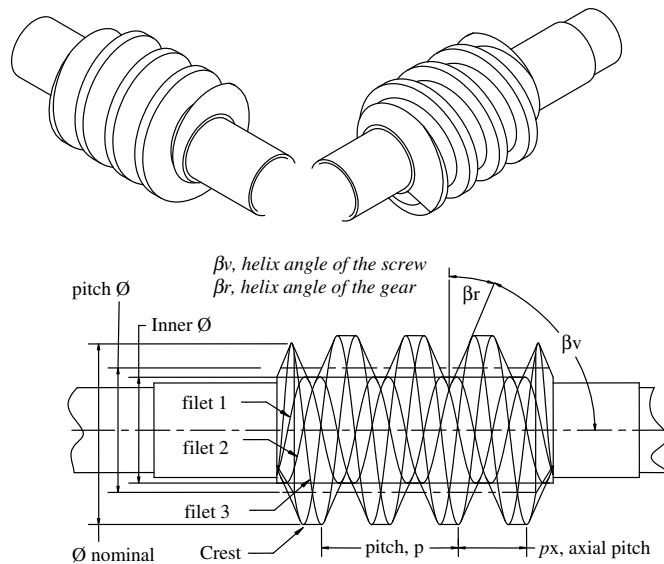


Figure 8.3. Key parameters of screw with n -entry threads

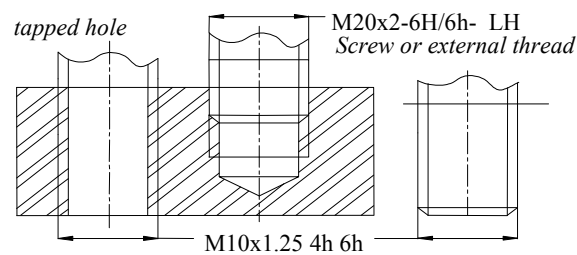


Figure 8.4. Standardized dimensioning of threads (internal and external and triangular thread; ISO)

Examples: M20x2-6H/6h-LH
M20xL3-P1.5-6H-S
G0.5A
Tr 40x7

Reading: M, screw thread symbol, according to ISO;
10, nominal diameter, $d = 10$ mm;
1.25, pitch of 1.25 mm;
4h, precision class of the pitch diameter (d_2);
6h, precision class of nominal diameter, d .

Canadian indications are generally under the standard ANSI/ASME B.7M-1984 (R 1992). We will refer to either the standard or the Machinery's Handbook (25th edition, pp. 1631–1676). The dimensioning of screws obeys standards to facilitate interchangeability and manufacturing. The ideal case would be to converge all standards worldwide in terms of dimensioning. The diversity of technological processes is not a major problem in dimensional metrology; it is more advantageous to converge toward unified designations. The examples offered here are deliberately stated in metric for universality purposes.

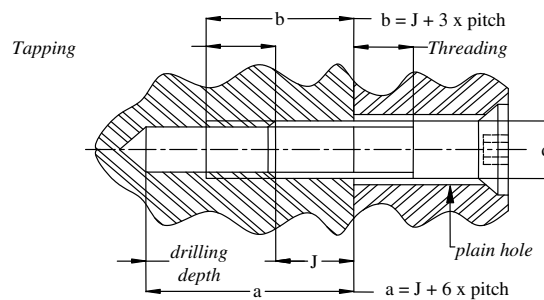


Figure 8.5. Normalized dimensioning for (external) screw threads

Reading: J is the location of the screw. For hard metals: $J > d$ and for soft metals: $J > 1.5d$.

8.1.3. Designation of threads and tapped holes for blind holes

This function limiting the formed threads is not due to aesthetics choice. There is an important function of resistance linked to this dimension. Blind holes are used for many cases in mechanical engineering. The length of the thread, its nominal size, and orientation are prerequisites for rigid assemblies. The designation can be done in two ways.

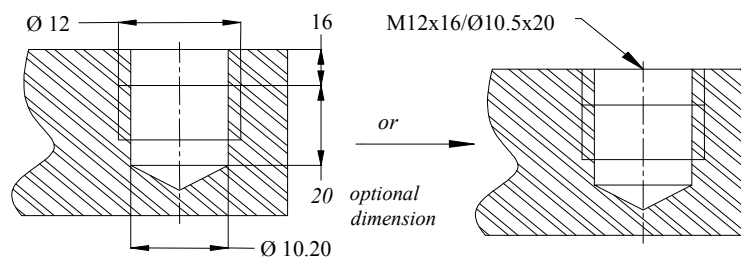


Figure 8.6. ISO standardized dimensioning (blind hole)

Mounting screws into a tapped blind hole. To further substantiate the foregoing, we present an example of mounting screw into a tapped blind hole. The choice of taps, depending on the pilot-hole, is not random. It obeys the calculations that are made beforehand and we will present them later in this chapter. Screw thread may also be produced using a die. Table 8.1 shows some values of the dimension thread root a_c (mm).

Values of thread bottom clearance a_c				
Pitch p (mm)	1.5	2 to 5	6 to 12	14 to 44
a_c (mm)	0.15	$\frac{1}{4}$	$\frac{1}{2}$	1

Table 8.1. Value of thread bottom clearance (metric) a_c in accordance with the pitch

If we need to interchange from one machine to another, screw threads should obey international conventions [ACN 84, OBE 96]. Screw threads may be either right or left handed. Right-handed threads are the most common, especially in hardware fasteners. Left-handed threads are used exceptionally.

Right-handed thread. A right-handed screw thread penetrates a threaded hole if it is turned clockwise. A nut gets closer to the head of the screw if it is turned in the same direction. If the screw is placed vertically, we would observe that the thread wound up going from left to right. If the screw is placed horizontally, the slope of the thread would correspond to the position of the thumb of a right hand. It is marked by the letters RH (right hand).

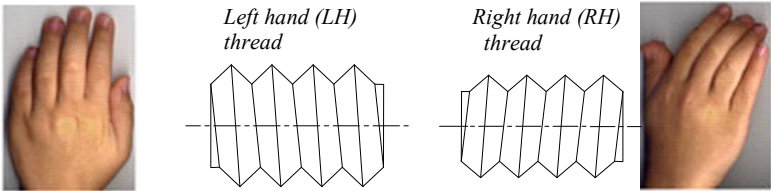


Figure 8.7. Representation of thread directions

Left-handed thread. The left-handed thread is identified by the letters LH (left hand). A screw with left-handed thread goes out of its threaded hole by turning clockwise. A nut moves away from the head if it is turned in the same direction. If the screw is placed vertically we observe that the thread wound up going from right to left. If the screw is placed horizontally, the slope of the thread corresponds to the position of thumb on the left hand.

8.2. Helical surface for screw threads

A helical surface is characterized by the *profile* and the *helix* commonly called helical lead which is itself defined by the diameter of the cylinder D_{pitch} of the basic cylinder, constituting the screw thread. It is marked by its pitch P .

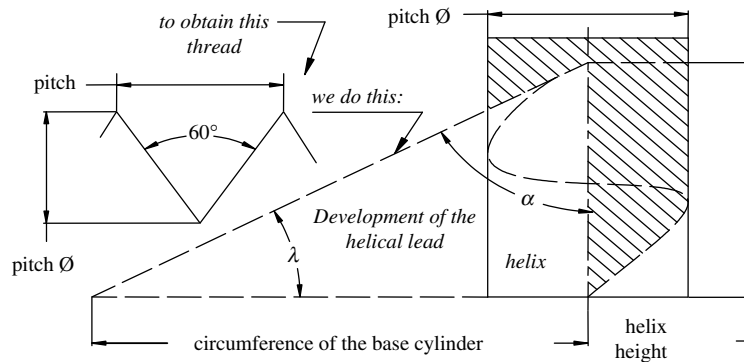


Figure 8.8. Helical surface illustrating the development of a thread (ISO) at 60°

The profile form is always read in the plane containing the axis of the helix. This form is dimensioned based on the helical lead. The helical surface is generated by the combination of rotational motion of the piece and translational motion of the tool shaping the profile and is located in the radial plane (λ).

To find the helix angle for a given thread, we apply this formula:

$$\tan(\alpha) = \left(\frac{\text{Helix height}}{\pi \times D_{\text{pitch}}} \right) \quad [8.1]$$

This approach is classic but still in operation. It is necessary to refer to the diameter recommended by the standard. There are many other methods of obtaining a screw thread. There are various technological processes for obtaining external threads. To consult them, we should refer to the specialized technical literature [CHE 66, FAN 94, OBE 95].

8.2.1. Technological processes for tapping and its control (Go – Not Go)

Tapping is an operation that consists in screwing a screw in a material which is initially plain. For this, we must make a threaded hole and screw-down the screw into the hole via taps. The screw thread is carried on a cylindrical workpiece which

will be transformed into a screw by using a tap and die. For example, a tap 3×0.5 allows screwing a screw of 3 mm outer diameter and 0.5 mm pitch. A hole is produced using a drill. The instruments used to handle taps and dies are called, respectively, die handle and “tap wrench.” The latter receives the tap in its middle. The tap will be inserted in the material and turned using a tap wrench according to the following procedure:

1. mark the center of the future tapping with a bradawl;
2. drill the female workpiece with a high-quality drill of suitable diameter (e.g. 2.5 mm for a tap of 3×0.5 mm). The diameter is indicated on the tap;
3. *deburr* the hole using a reamer or a drill bit of big size;
4. put a drop of oil on the tip of the tap;
5. begin tapping using tap number (1) by placing it perpendicularly to the hole and by pressing while turning as to screw the tap into the hole. Move forward gradually using forward-back motion to break off the chips;
6. get out the tap and clean it and blow out the hole to break off the chip;
7. begin again with tap number (2) (two strokes) and finally the tap without a stroke;
8. check the thread with a screw (“Go,” “Not Go”).

The classical principle of tapping (a set of three taps numbered depending on the profile, pitch, and depth) necessitates, beforehand, squaring taps. This is a control that requires a square and a marble (perfectly flat surface). To repair/remake a damaged tapping, we follow this procedure:

1. If it is a through hole, we use the tap (3) through the hole opposite to the one damaged.
2. If it is a blind hole, we drill at 3 mm from the beginning of tapping and use a longer screw to exploit threads from the bottom. In both cases, there is deterioration.
3. If the whole thread is damaged or that the resistance of damaged threads is too low, we rethread the hole using a larger-diameter screw by following the steps normally.

It is recommended to use a significantly larger screw (4 mm for a damaged thread of 3 mm). To start screwing it is recommended to begin by positioning the screw and unscrew it until running the screw pitch, then, go in a clockwise direction. This avoids damage to the pitch of the female screw when going in the wrong way.

NOTE.— Never force the tap: it is made of high-speed steel (very strong but brittle). If it jams, add oil. When tapping in aluminum, we could use directly and solely tap number 2.

8.2.2. Tapping (by hand) with tap wrench and set of taps

In this chapter, our goal is not to redo the technological processes of screw threads or tapping production. There are excellent handbooks dealing with these subjects. We should consult them advisedly. Our goal is to address the procedure and means of controlling these items (screw threads, tapping). We will start by addressing possible defects on the spiral/helical helix. For conventional tables on the choice of appropriate tap drill for a pilot hole that should receive the screw, refer to Table A5.1 in Appendix 5.

1. Find the $\varnothing$ of the tapping drill bit required to drill the hole of a nut with 11/8"-7 NC.

2. Find the $\varnothing$ of the drill bit required to drill the hole for receiving a tapping screw 1½"-13 NC. Then, compare your results with those presented on the tables of the manufacturer.

SOLUTION FOR QUESTION 1. – $\varnothing$ of the tapping drill = $\varnothing$ nominal – $(0.75 \times 1.299p)$
 $= 1.125 - (0.75 \times 1.299 \times 1)/7$
 $= 0.9860" = 63/64"$

therefore 63/64" is the drill bit closest to the result, thus it is accepted.

SOLUTION FOR QUESTION 2. – the tap's outer $\varnothing$ = $\varnothing$ nominal = ½" = 0.500".

Thread pitch = 1"/N = (1/13) = 0.077" = 0.986" = 63/64"

$\varnothing$ drill for tapping = $\varnothing$ nominal tap – 1"/N = 0.5000" – 0.077" = 0.423".

→ 0.423" ≈ 27/64" is the commercial drill bit closest to the result.

For example, for the 27/64", the proposed value for 1½"- 13 NC gives rise to 0.4219 for a nominal $\varnothing$ of ½". We see easily that our result, which is 0.423, is close to 0.4219, conventional value. The same explanation applies to the case 63/64". For 11/8"-7 NC, the table shows 0.9844 while our result is 0.9860. Before starting to drill and later tapping the resulting pilot holes, it is imperative to square taps (and drills). Sometimes this operation is not necessary if the straightness of tools has been checked.

8.3. The main threads in the industry

There are various types of threads in the world [ACN 84, FAN 94, EBO 96]. The main ones are listed below. Each thread has specific characteristics but which are sometimes found in others as well. We will try to present the main ones:

- ISO thread;
- cylindrical GAS thread;

- gas taper thread;
- Whitworth (BSW) and Whitworth (BSF) thread;
- Briggs tapered thread;
- thread Sellers American standards (NC coarse pitch series = *National Coarse* and NF series = fine pitch series, *National Fine*).

There are many other types of threads used in the industry such as “atypical threads”, that is to say parts with threads suitable for special cases. We shall not discuss these cases here because of their non-interchangeability. In the industrial world, the case of ISO thread is the most commonly used. Threads called “American” are also used in Canada and in the United Kingdom. These standardized threads are also known under the acronym ABC (American, British, and Canadian).

8.3.1. ISO Threads

Triangular thread is the most commonly used in hardware fasteners and bolts. The profile is defined or built based on an equilateral triangle with each side being equal to the pitch P . It offers an excellent combination between strength and ease of manufacturing. The finest the pitch, the more the tolerances are reduced and the more the manufacturing process is likely to be expensive. The coarse pitch is part of the base series to be used first. They are mainly used in hardware fasteners and bolts for all ordinary purposes (ferrous and non-ferrous metals). The vibrations are in this case avoided. As for fine pitches, they are recommended in the case of threading on thin tube, of short thread-engaging length (low height nut, etc.), impacts, vibrations, and when the constructions are expensive (automotive, aeronautics, space, etc.).

Tolerance system used for threads is similar to that in the adjustments system (Figure 8.9). While in the case of adjustments, only the nominal diameter is taken into account, for screw threads, both the diameter ($d1$ or $D1$) over the thread crests, and the pitch diameter ($d2$ or $D2$), hence a dual dimensioning. The value of the tolerance interval is represented by a number (or grade 3–9). The values four, six, and eight are the most common; six is the most commonly used value for screwed assemblies (e.g. 6H/6g). The higher the grade, the higher the tolerance. Values below six are recommended for fine or precise performances as well as for short-length of engagement between threads. Above six, larger tolerances are recommended for a bit more “coarse” requirements with the thread engagement that is long enough.

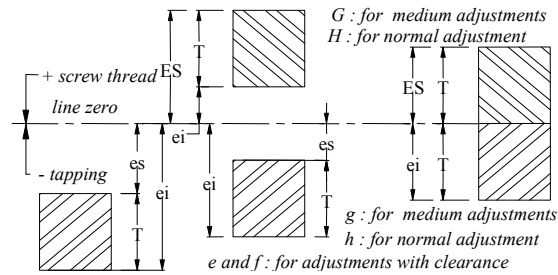


Figure 8.9. Tolerances with respect to zero line (e.g. ISO metric thread)

Deviation (or position of the IT): The position of the tolerance interval is specified, as for adjustments, with a lowercase letter for the shafts, rods or screws (e , g , h , etc.), and a capital letter for the threaded holes (G , H , etc.). The letters h and H have a zero deviation. Basically, an assembly of type e/H will give a large clearance; g/H will bring a small clearance, and (h/H) an assembly without clearance, etc.

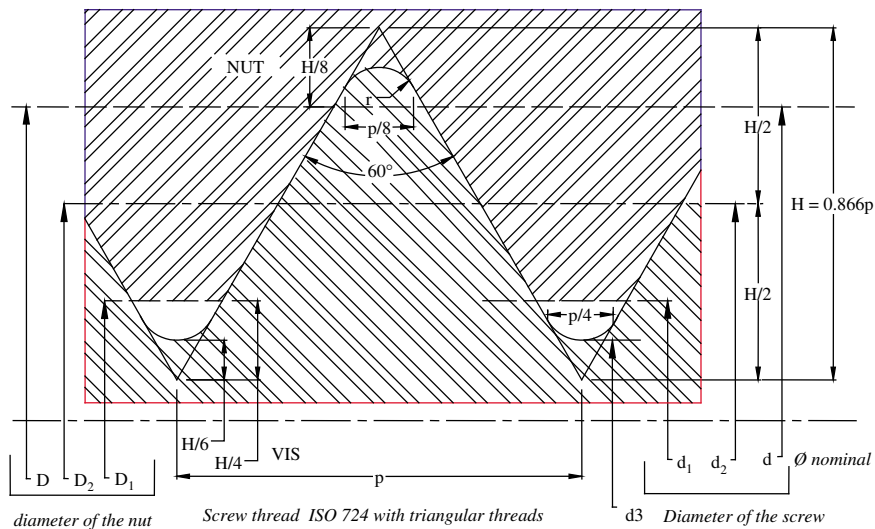


Figure 8.10. Triangular ISO profile as a replacement for SI profile (see Tables A5.4–A5.6) in Appendix 5

The characteristics of threads and detailed expressions of their calculations are:

- h_3 : screw thread depth;
- H : nut thread depth; primitive triangle height;

- D, d : nominal diameter;
- d_3 : screw core diameter;
- D_1 : minor diameter of the nut;
- d_1 : diameter of the bore of the nut;
- $r_{\max}$: root radius of screw thread;
- $r_{1\max}$: root radius of the nut thread;
- D_2, d_2 : thread pitch diameter;
- p : pitch;

The “universal” terms that follow are used to calculate the characteristics of threads. They are found in manuals dedicated to technical drawing and mechanical engineering. They are presented here for demonstration purposes.

Pitch diameter:

$$D_2 = d_2 - 2 \times \left(\frac{H}{2} - \frac{H}{4} \right) = d - 0.6495 \times p \quad [8.2a]$$

In some handbooks, the expression of D_2 takes the following form:

$$D_2 = d_3 = d_1 - 2 \times \left(\frac{H}{2} - \frac{H}{6} \right) = d - 1.2268 \times p \quad [8.2b]$$

$$D_1 = d_1 = d - 1.10825 \times p = \text{inner diameter of the tapping}$$

The height corresponds to the pitch diameter D_2 , (see Figure 8.10) also written as:

$$H_2 = \left(\frac{D - D_1}{2} \right) = 0.5412 \times p \quad [8.3]$$

Screw thread depth: ($D = d$: nominal diameter and $p = \text{pitch}$):

$$H_3 = \left(\frac{d - d_3}{2} \right) = 0.6134 \times p \quad [8.4]$$

Theoretical height of the triangle:

$$H = P \times \frac{\sqrt{3}}{2} = 0.866 \times P \quad \text{with} \quad r = \left(\frac{H}{6} \right) = 0.1443 \times p \quad [8.5]$$

Drilling diameter of the nut (pitch diameter):

$$D_2 = d_2 = \left(d - \frac{3H}{4} \right) = d - 0.6495 \times p \quad [8.6]$$

The characteristics of the ISO screw thread are: a screw is a cylindrical (taper) rod on which helical grooves were dug leaving the thread in relief. The threaded rod thus obtained is screwed into a hole with grooves corresponding to the screw threads. Screw threads which are adopted universally and used worldwide today, in hardware (bolts) are called ISO, in reference to the organization that has defined them. English speaking countries, unconditional of the non-metric, have also adopted this standard. But before the ISO screw thread, the thread SI was defined by Zurich Congress in 1898. The SI thread is formed by an equilateral triangle whose side is equal to the pitch of the screw. From this characteristic together with the form of the bottom land and the crest of threads, a very simple rule follows: to create a tapped hole at x mm, it is necessary to first drill the pilot hole to a value equal to the difference between x and the pitch (Figure 8.11).

Interchangeability SI/ISO: there are no significant problems between the two systems, since the shape of thread, the pitch and the diameters are common to both standards. There is total interchangeability between the SI and the ISO. As, generally, the diameter of the tapping of pilot hole is greater than the diameter D_1 of the SI profile, there is *de facto* a configuration ISO. We can, *without problem*, mount an ISO screw into an SI nut. We must not forget that developers of standards are industrialists, and interchangeability is certainly not an arbitrary result:

- the coarse pitch, for example, $\varnothing 10 \rightarrow$ pitch = 1.50 mm;
- the fine pitch is commonly used for having the advantage of thread engagement (Figure 8.11). For some $\varnothing$ of crew, there are several relevant fine pitches. For example, for the diameter 10, we find 0.75, 1, and 1.25 mm.

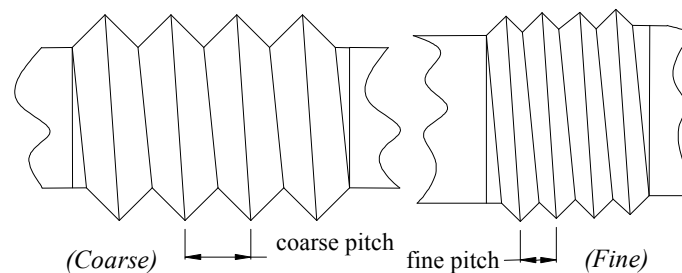


Figure 8.11. Fine pitch and coarse pitch screw threads

The SI thread was replaced in 1959 by the ISO thread. The shape of the thread is also an equilateral triangle, and almost all the SI characteristics are identical except for the truncation at the top of the threads of the nut, which is doubled (increased from $H/8$ to $H/4$. H is the theoretical height of the thread). The inner diameter of the nut, at the crest of the thread, is a bit larger under ISO specifications. This allows to increase the diameter of the core of the screw and to improve its mechanical strength. In accordance with ISO 1207, we offer standardized threads for machine screws. For slotted pan head machine screws, ISO provides the appropriate table.

8.3.1.1. Graphical comparison of different profiles: ISO, SI, and SI_m

We note that the main difference lies in the height of the threads.

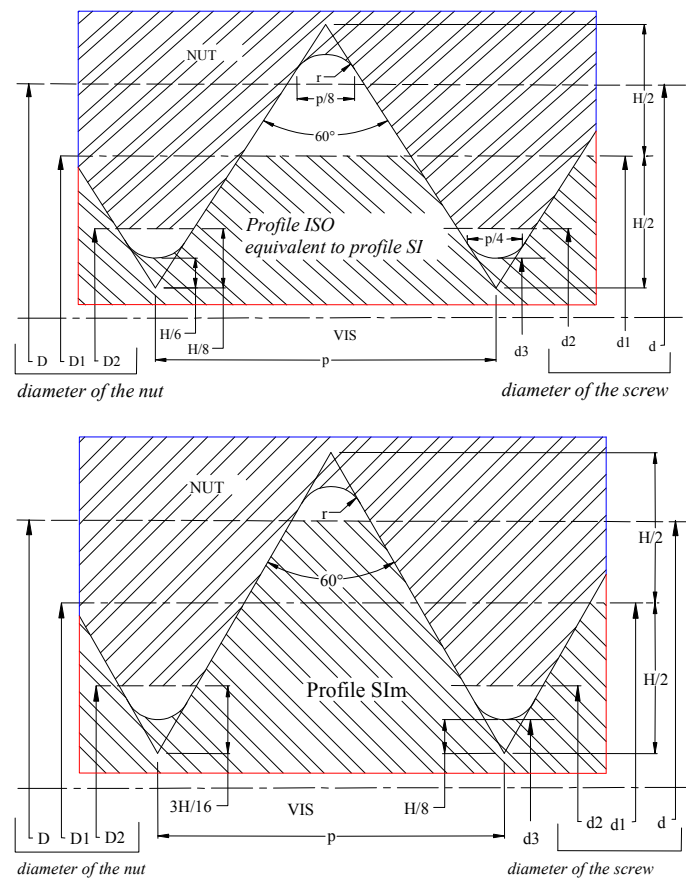


Figure 8.12. Graphical comparison of three threads under ISO, SI, and SI_m

8.3.1.2. ISO metric trapezoidal screw threads

Not being used as hardware fasteners (screws and bolts), trapezoidal screw threads allow performing screw maneuvering or screws for power transmission to one or more threads. They are more receptive to heat treatments than triangular threads. Screw threads are usually irreversible. Thus, the helical pitch P_h corresponds to the axial displacement during tightening. The profile pitch P is equal to the axial distance between two consecutive flanks.

For example, for a screw: with a single thread, $P_h = P$, with two threads $P_h = 2P$, etc. with n threads $P_h = nP$. Thus, ISO 2901 standard suggests two possibilities of profiles:

- profiles with “empty” thread root and without clearance on the flanks (allow a better centering between the screw and the nut or an improved concentricity);
- profiles with “empty” thread root and with flanks clearance (in general).

Single-thread screw or tapping: For example, Tr 40 × 7-7e (a right-handed thread screw of nominal diameter 40 mm and pitch $P = 7$ mm; quality 7e).

Multi-thread screw or tapping: For example, Tr 40 × 14 (P7) LH – 8H, for a two-left-handed thread tapping with nominal diameter 40 mm, pitch $P = 7$ mm, helical pitch $P_h = 14$ mm, and quality 8H.

8.3.2. American Standard pipe threads

Cylindrical gas threads are identical to the Whitworth profile:

- cylindrical GAS thread, symbol G ;
- BSP.F cylindrical thread (British Standard Pipe-Fastening).

When D is the outer diameter and P the pitch and (1 inch = 25.4 mm)/number of threads per inch:

$$D_1 = (d_1 - D) - 1.28 \times P \quad [8.7]$$

$$H = 0.96049 \times P \quad [8.8]$$

$$h = 0.64033 \times P \text{ with } r = 0.13733 \times P \quad [8.9]$$

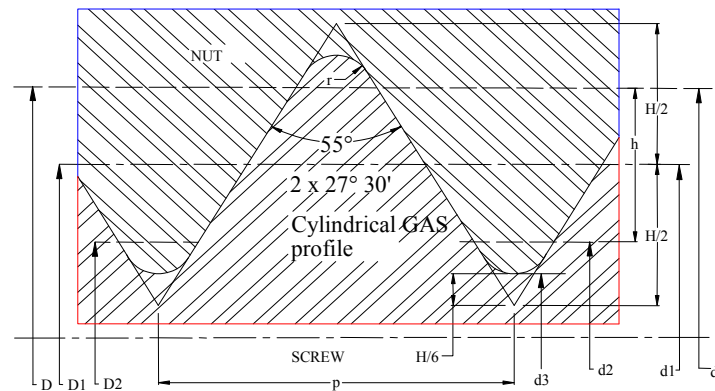


Figure 8.13. Cylindrical GAS screw thread (see Tables A5.7 and A5.8 in Appendix 5)

8.3.3. The Whitworth thread

The Whitworth thread is the National thread used in the United Kingdom (Figure 8.14). It differs from the thread called “American” by its angle and the rounded shape of the crest and the thread root. It is characterized by a thread with a non-equilateral triangle-shaped pattern as in SI and ISO, but rather an isosceles with vertex angle of 55° . Additionally, the pitch is not given directly but indicated based on the number of threads per inch. This leads to difficulty in terms of the calculation of the pilot hole diameter, a multitude of different pitches, and total incompatibility with ISO threads. The table of a Whitworth thread characteristics is presented in Tables A5.8 and A5.9 in Appendix 5. The pitch is the distance between two consecutive threads of a screw measured parallel to the axis of rotation. The thread is the spiral projection.

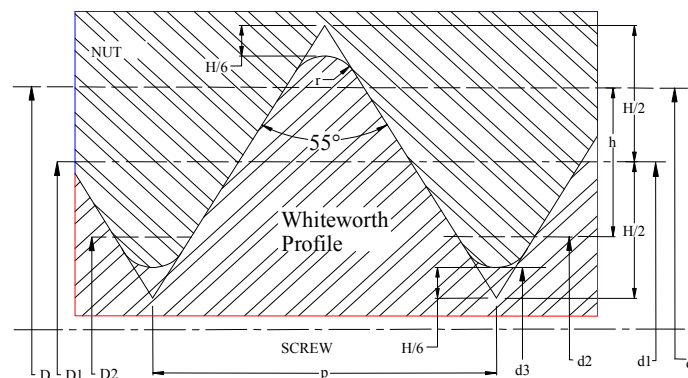


Figure 8.14. Whitworth screw thread (for the table of values, see Table A5.9 in Appendix 5)

8.3.3.1. Whitworth thread, Series BSW, with pitches

The schematic and formulas are identical to those in Figure 8.13, however, for a Whitworth thread (BSW series, with common pitches) we refer to the values in Table A5.9 in Appendix 5. Despite the advantages of ISO thread (simplicity, cost, ratio strength/weight, etc.); the Americans and English have long been reluctant to abandon their standards [OBE 96] so much so that American manufacturers have proposed a third standard.

8.3.3.2. Whitworth thread, BSF series, with fine pitch

For Whitworth thread (BSF series, fine pitch), the design is identical to that of the previous figures for the specifications table. *Tapered thread* is still used for tight connectors (gas). Different threads specific to certain professions such as watchmaking, optics, and precision instruments subsist. Wood screws and those called self-tapping screws, for which the problem of compatibility with the nuts does not arise and which are subjected to specific standards. Only drilling diameters differ.

8.3.4. BRIGGS tapered threads; cone 6.25%

These threads are ordinarily given according to the American Standard ASA-B2-1-1960 [OBE 96]. We find them in some cases of refinery plumbing and, in general, in high-pressure fluids pipelines. The table of tapered thread BRIGGS characteristics is presented in Tables A5.10 and A5.11 in Appendix 5. The following schematic shows an assembly case highlighting the pitch and the angle as important characteristics of this type of thread (Figure 8.15).

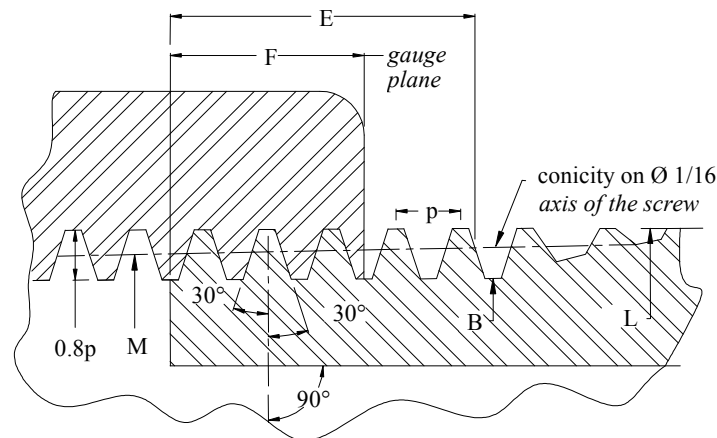


Figure 8.15. BRIGGS tapered threads, cone 6.25% (see Tables A5.10 and A5.11 in Appendix 5)

8.3.5. American Standard thread, NC and NF series

8.3.5.1. American Standard thread, NC series

Key characteristics of the NC series are the nominal diameter D and the pitch P :

$$H = p \times \cos(30^\circ) = 0.866 \times p = \left(\frac{0.866}{\text{number of threads per inch}} \right) \quad [8.10]$$

Figure 8.16 is an exhibit of Sellers American Standard threads, NC and NF series.

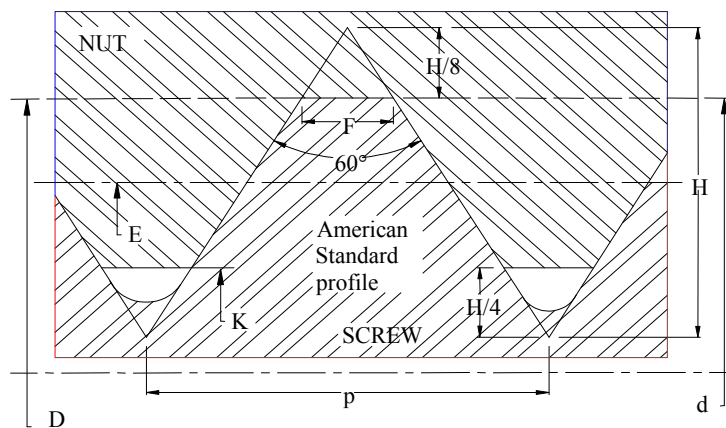


Figure 8.16. American Standard thread, NC series (+F thread root)

8.3.5.2. American Standard thread fine-pitch, NF series

The American National thread, formerly the Sellers Standard or United States Standard thread (USS), is derived from the sharp V-thread. It differs from the latter only by the flat shape of both the top of the thread and the bottom land of the groove. The thread flank angle is equal to 60° , as for the ISO thread. The schematic is identical to that of Figure 8.16, for a thread called Sellers American Standard NF Series. The characteristics are the nominal diameter D and pitch P . H is calculated using equation [8.10]. Complete tables of the characteristics of a Sellers American Standard thread, fine-pitch NF series are presented in Tables A5.12–A5.15 in Appendix 5.

8.3.6. Pipe threads called “GAS”

They are used in various industries (pneumatic, hydraulic, valves, etc.). The main dimensions of the profile are linked to ISO 228 whose original profile is the *Whitworth* profile. There are two main cases. Pipe threads without seal (pressure tightness),

since screw and nut have both cylindrical threads. In the case of sealing pipe threads with sealing, the screw is taper-threaded (taper 1/16) and the nut is cylindrical-threaded. When tightening, there is wedging between the tapered part and the cylindrical part. The seal may be enhanced by interposing a joint compound for example. Exceptionally, it is possible to find an internal tapered thread, designated by R_c . Note that the designation does not match the size in inches:

- threads without sealing: letter G ; designating the thread; letter A or B corresponding to the tolerance class of the external thread; reference standard;
- threads with sealing: letter R ; letter p in the case of cylindrical internal thread; designation of screw thread; reference standard.

8.3.7. Main threads implemented in Canada

Screw threads presented here are all used in Canada [CSA 84]. However, the most commonly used is the so-called unified screw thread. It is realistic to say that a thread is preferred over another depending on its proper use. In any case, hardware and fasteners industries are promoting interchangeability.

8.3.7.1. ANSI/ASME B1.1–unified profile or UST thread (ABC)

Standardized in the United States, Great Britain, and Canada (ABC), it has the same basic profile (equilateral triangle) as the ISO triangular metric thread. The screw thread is characterized by a flat thread crest and a rounded thread root. Essentially, it is differentiated from ISO profile by nominal sizes in inches, different tolerances and the designation.

Main series:

- Coarse-pitch series (designated by UNC): it is the basic series to be used first, it is primarily used in hardware fasteners and bolts for all ordinary purposes (to avoid vibrations) and in the case of non-ferrous metals.
- Fine pitch series (designated by UNF): it is regularly used in the automotive industry and aeronautics. It is more resistant to vibration.
- Extra-fine series (designated by UNEF): it is used in instrumentation, particularly in aeronautics, and also when there is severe vibrations and shocks.
- Complementary series: they complement the previous series. We can easily find their respective tables in the technical literature;
- 8-thread series (8N): this series uses eight threads per inch, replacing the coarse-pitch with greater diameter (>1 inch), initially developed for high pressure sealing.

Classes 1A, 2A and 3A are applied to the screws (or external threads) and classes 1B, 2B and 3B for nuts (or internal threads). Quality increases going from class 1 to 3, for example:

– 3/8–24 UNF-2B (tapped hole, class 2B, $d = 3/8$ inch, 24 threads/inch, fine pitch);

– 0.500–13 UNC-2A LH (LH = left-hand screw, class 2A, $d = 0.5$ inch, coarse pitch, 13 threads/inch).

The *Unified* thread is recent, compared with other threads presented here. It makes the difference between the English National thread and the American National. Its creation is due to the fact that the American National thread did not allow its use on English machines and vice versa. Yet, it was by agreement between the United States, the United Kingdom, and Canada that it was decided, in 1949 to unify (hence the name *unified*) both thread forms, thus giving rise to the unified thread shown schematically above.

The characteristics of the thread (ABC) are the pitch P ($=1''/\text{number of threads per inch}$):

$$D = 0.61134 \times P \quad (D \text{ represents the depth}) \quad [8.11]$$

where F = thickness = $(p/8)$ for the unified thread. F = flat = $(p/8)$ for the American National thread. Note that American National threads are similar to the unified thread but have a flat root ($F = p/8$) in thread root. The related calculations are identical:

– thread pitch:

$$P = \frac{1}{N} = \frac{1 \text{ inch}}{\text{Number of threads per inch}} \quad [8.12]$$

– depth of thread:

$$D = 0.6495 \times P = \frac{0.6495}{N} \quad [8.13]$$

– flat root of thread:

$$F = \left(\frac{P}{8} \right) \quad [8.14]$$

– inner diameter ($\varnothing$ at the root) called “minor thread diameter”:

$$D_{\text{inner}} = D_{\text{outer}} - (1.299 \times P) \quad [8.15]$$

APPLICATION EXAMPLE.— Find the thread pitch, depth, the flat root, and the minor diameter of the screw designated: $1\frac{1}{4}" - 7 \text{ NC}$:

$$P = \left(\frac{1}{N}\right) = \frac{1}{7} = 0.142857 \text{ inch}; D = 0.6495 \times \left(\frac{1}{N}\right) = 0.092786 \text{ inch}; F = \frac{P}{8} = 0.017857 \text{ inch}$$

$$\begin{aligned} \text{Minor diameter} = \varnothing_{\text{inner}} &= \varnothing_{\text{outer}} - (1.299 \text{ in}) = \varnothing_{\text{outer}} - 2 \text{ depths} \\ &= 1.250" - (1.299 \text{ p}) \\ &= 1.250" - (1.2991"/7) = 1.250" - 0.185" \\ &= 1.065" \end{aligned}$$

8.3.7.2. Symmetrical trapezoidal profile at 29° ACME threads

This is the American equivalent of the trapezoidal thread according to ISO 2901. It is characterized by a profile angle of 29° (versus 30° for ISO Profile) and by nominal dimensions in inches. These profiles are developed for motion and power transmissions. The characteristics are the pitch, P ($= 1/\text{nombre of threads per inch}$), (see Table A5.16):

$$D = \left(\frac{P}{2} + 0.010 \text{ inch}\right); D \text{ is the thread depth (inches)} \quad [8.16]$$

$$F = (0.3707 \times p); F \text{ is the thread crest, in inches} \quad [8.17]$$

$$C = (0.3707 \times p) - 0.0052; C \text{ is the thread root, in inches} \quad [8.18]$$

There are two types of possible threads:

- one type for general application with three classes of tolerance $2G$ (common use), $3G$, and $4G$ with reduced clearance;
- another type with centered threads allowing centering of the nut relative to the screw (controlled concentricity) with five tolerance classes $2C$, $3C$, $4C$, $5C$, and $6C$ precision increasing from two to six).

EXAMPLES OF DESIGNATIONS.—

– $1\frac{3}{8}-4 \text{ ACME}-2\text{GA}$ (screw for letter A, $d = 13/8 = 1.375$ inch, four threads per inch, class $2G$).

– $1\frac{3}{4}-4 \text{ ACME}-2\text{GB}-\text{LH}$ (tapped hole for letter B, “LH” left-handed screw, $d = 1.75$ inch, four threads per inch, class $2G$).

– $2\frac{3}{4}-3 \text{ ACME}-3\text{GA}-2-\text{START}$ (a 2-thread screw, $d = 2.75$ inch, three threads per inch, class $3G$).

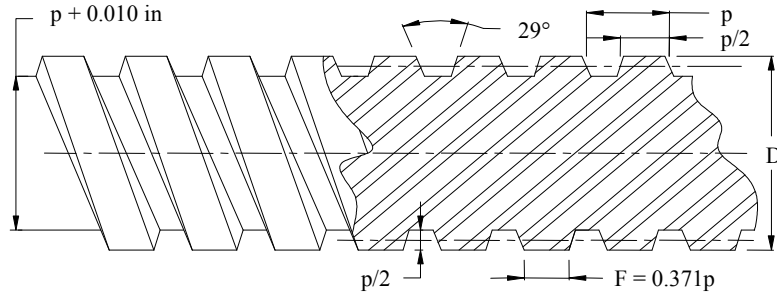


Figure 8.17. *Acme thread (see Table A5.16 in Appendix 5)*

The Acme thread is stronger than the square thread (Figure 8.17). It is mainly used for motion transmission to different components of machines (lathe lead-screw and milling machine table screw). Its control is done using thickness gauge.

8.3.7.3. Worm screw thread

The worm thread is identical to the Acme, however, it is deeper. It is used in worm gear transmission mechanisms. This thread is characterized by the pitch, p ($= 1/\text{number of threads per inch}$) and:

$$D = (0.6866 \times p); D \text{ is the thread depth, in inches} \quad [8.19]$$

$$F = (0.3100 \times p); F \text{ is the thread flat root, in inches} \quad [8.20]$$

$$C = (0.3350 \times p); C \text{ is the thread crest, in inches} \quad [8.21]$$

Characterized by a height reduced by 40% compared to the usual Acme profile, this profile is designed for coarse-pitch applications of shallow depth (thread on thin tube) requiring heat-treatment.

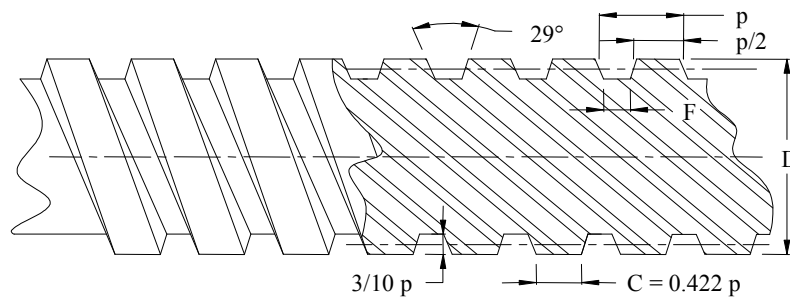


Figure 8.18. *Worm thread*

8.3.7.4. Symmetrical trapezoidal profile with reduced height (29°), Stub Acme

One possible class 2G (see dimensions, pervious tables). Note that the US standards suggest a Stub Acme version at a 60° angle (instead of 29°) see [8.21]. The asymmetrical trapezoidal *Buttress thread* (ANSI BI.9) (Table A5.8 in Appendix 5) is characterized by the pitch, p ($= 1/\text{number of threads per inch}$):

$$D = \left(\frac{3}{4} \times p \right); D \text{ is the thread depth, in inches} \quad [8.22]$$

$$F = \left(\frac{1}{8} \times p \right); F \text{ is the thread flat root, in inches} \quad [8.23]$$

The buttress thread is a very strong thread which is used in cases requiring high pressures, in a single direction. It is designed to withstand high loads in one direction. It is, in equal size, more resistant than other threads. This increased resistance results from a greater thickness of the thread root. The face supporting the load is tilted by 7°. This low inclination generates, under the effect of axial load, even if it is high, a moderate radial load, which approximates it to the square thread. This thread is easy to perform. The main applications are: pipe threads, mechanisms of breech-loading weapon, airplane engine hubs, etc.

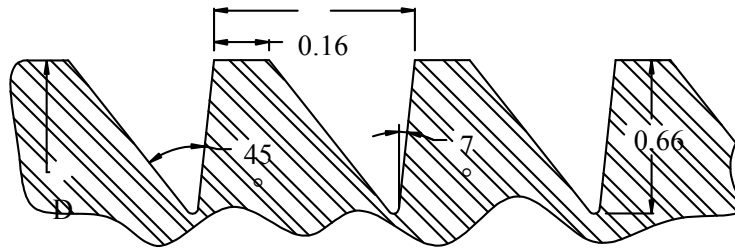


Figure 8.19. *Buttress thread (see Table A5.8 in Appendix 5)*

8.3.7.5. Square threads and modified square threads

The square thread, also known as Sellers thread (see Table A5.16 in Appendix 5) is, of all the threads, with the least friction. But this thread is difficult and expensive to produce. Thus, a modified version of a slightly trapezoidal form a (10° angle), is proposed by the US standards:

$$P = \left(\frac{1}{\text{Number of threads per inch}} \right); p \text{ is the pitch, in inches} \quad [8.24]$$

$$D = \left(\frac{p}{2} \right); D \text{ is the thread depth, in inches} \quad [8.25]$$

$$F = \left(\frac{p}{2} \right); F \text{ is the width of the thread, in inches} \quad [8.26]$$

$$S = \left(\frac{p}{2} \right); S \text{ is the groove width, in inches} \quad [8.27]$$

$$d_{\min i} = d_{\text{nominal}} - p \quad [8.28]$$

$$d_{\text{pitch}} = d_{\text{nominal}} - \left(\frac{p}{2} \right) \quad [8.29]$$

$$\alpha = \left(\frac{\text{screw pitch}}{\pi \times d_{\text{nominal}}} \right); \alpha \text{ is the helix angle at the thread root} \quad [8.30]$$

$$\beta = \left(\frac{\text{screw pitch}}{\pi \times d_{\text{nominal}}} \right); \beta \text{ is the helix angle at the crest of thread} \quad [8.31]$$

The square thread is used to transmit pressures parallel to the axis of the screw (die block, control jack, etc.).

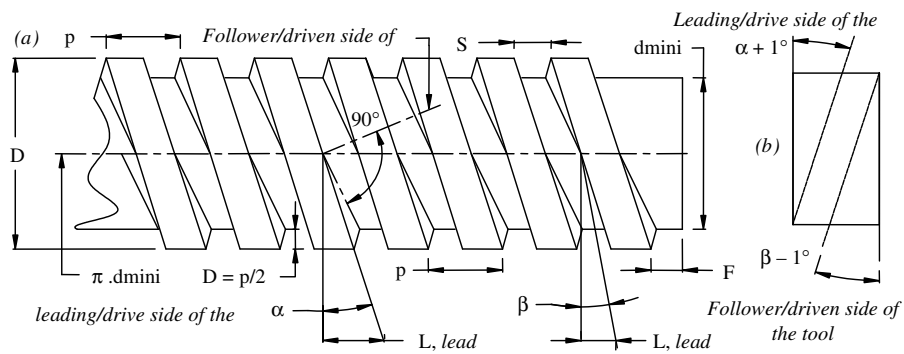


Figure 8.20. (a) geometrical characteristics of the tool and helixes of a square thread;
(b) checking lateral reliefs of a tool (square thread)

EXAMPLE.— What will be the thread pitch, depth and minor diameter for a four-square thread screw, per inch, if the nominal diameter is $1\frac{1}{2}$ "?

SOLUTION.—

Pitch $P = 1''/N = 1''/4 = 0.250''$, hence $L = \text{lead screw} = 0.250''$.

Depth $= D = (P/2) = 0.250/2 = 0.125''$.

At thread root $= D = \varnothing \text{ nominal} - P = 1.5 - 0.250 = 1.250''$.

Thread helix angle $= \alpha = (\text{lead screw})/\pi \text{ thread minor diameter}$.

$\alpha = L/\pi \nu \text{ min} = 0.250/(3.14 \cdot 1.250) = 0.0635$, thus $\tan(\alpha) = \tan(0.0635) = 3^\circ 38''$.

Helix angle at the thread crest, $\beta = (\text{lead screw})/\pi \cdot \varnothing \text{ nominal}$.

$\beta = L/\pi \cdot \nu \text{ nominal} = 0.25/(3.14 \cdot 1.5) = 0.053$ thus $\tan(\beta) = \tan(0.053) = 3^\circ 2'$.

8.3.7.6. Other types of special threads - round profile

This thread is the most resistant to impacts because it can withstand considerable strain. Its rounded shape opposes limits to the phenomenon of stress concentration. Nominal diameters are the same as those of the ISO triangular thread. The pitch is an integer (2-3-4 and 6 mm). Their main uses are: screwed base light bulb, railway equipment, etc. They are designated as follows: Rd 25x3, left-handed, 2 threads (for 25 mm, pitch = 3 mm, two left-hand threads).

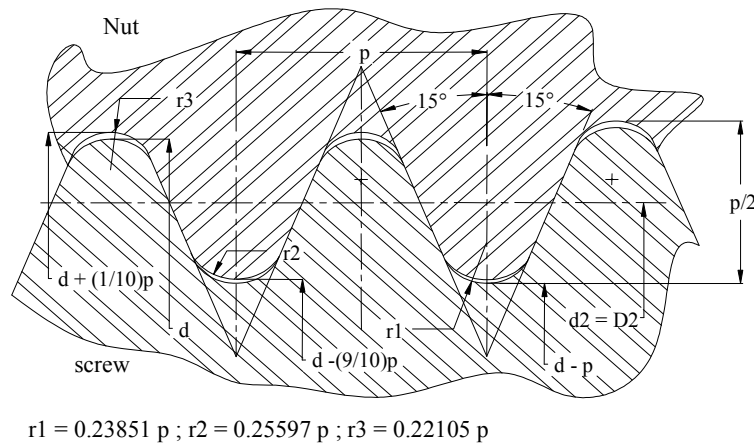


Figure 8.21. Round thread

8.4. Principles of threads control

To study the processes of threads control [CAS 78, EBO 96], it is necessary to analyze the helical surface from which the thread is derived. Workshop experiments remain one of the best places of assessment as to appropriate measurements. Facing the continuing duality of methods and techniques used in metrology, we should seek the best possible combination, given the means and the existing methods. In mechanics, the helical surface is often used:

- On clamping or transmission elements such as fasteners, cam grooves, the wedging ramps, etc. It, then, requires qualities of resistance. Specifically, the base diameter and profile are often inspected.
- On the components whose motion transformation is accurate, we mention the lathe lead-screw and the milling machine table screw where the constancy of the pitch will be verified.
- On mechanical elements of measuring instruments: micrometer screw for which it is imperative to inspect not only the constancy of the pitch but also the perfect profile correction. This helps avoiding issues due to premature local wears.

By reading both Figures 8.12 (ISO) and 8.16 (unified, USA, UK, and Canada), we notice that the difference is almost non-existent, or even negligible. Both present a sharp 60°V-thread which forms an equilateral triangle whose edge is equal to the thread pitch. By truncating this equilateral triangle at its vertex and at its base, with a width equal to one eighth (1/8) of its side length, its height is reduced to the same proportion and becomes equal to $\frac{3}{4}$ of its original height. However, if the height of an equilateral triangle is equal to 0.866 multiplied by its side, the height of the truncated triangle constituting the American National thread becomes equal to the thread pitch multiplied by $\frac{3}{4}$ of 0.866, i.e. 0.6495. Thus this number forms the constant that allows the depth of any American thread to quickly be determined.

In fact the international metric thread has similar form as the “American National.” However, the thread root (ISO) can be straight or round. The profile generator of a thread, according to ISO, is also a truncated equilateral triangle as shown in Figure 8.16. The truncation at the crest of the screw threads is equal to 1/8 of the height H of the triangle and the crest truncation of the nut is equal to $H/4$. Depending on the pitch p and the nominal diameter d , measured outside of the screw, the theoretical height of the triangle, for this thread is given by expression [8.32]. The height of contact between the threads is written as follows:

$$h = H - \left(\frac{H}{4} - \frac{H}{8} \right) = 0.866 \times p \times \left(\frac{5}{8} \right) = 0.51412 \times p \quad [8.32]$$

The expression of the screw thread depth is already given by the relationship of H_3 in equation [8.4]. It takes the following form:

$$H_1 = H - \left(\frac{H}{6} - \frac{H}{6} \right) = 0.866 \times p \times \left(\frac{17}{24} \right) = 0.6134 \times p \quad [8.33]$$

The drill diameter will be calculated by expression [8.6]. It is equivalent to the nominal diameter.

8.4.1. Defects of the helical surface

The helix drive may present variations in the pitch often due to poor synchronization in generating movements or poor rigidity of the assembly. Therefore, the profile may show the following defects:

1. defects of its own form due generally to an ill-suited shape of the tool because; under a sectional view, it is no longer on the radial plane;
2. defects of position relative to the axis of the helix due to poor adjustment of the tool and which will be reflected by differences in diameter or a wrong direction of the profile;
3. defects in material grades (of the workpiece) poorly adapted to the cutting;
4. defects due to poor choice of cutting conditions (not optimized).

Type-3 defects are “sneaky” because it was sometimes believed (wrongly) that the material has no influence on the profile resulting from a thread machining. It is a fundamental error because the material, ill-suited to temperature changes, is at the origin of a broken, brittle, discontinuous, or poorly formed profile, hence material is responsible of adaptation of a part to temperature variations to which it is subjected.

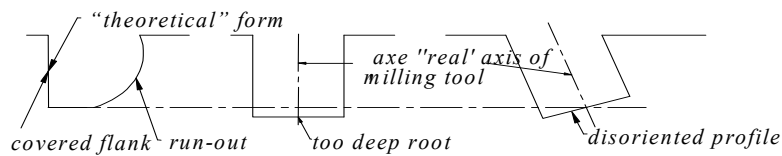


Figure 8.22. Major profile defects

Tools manufacturers often insist on the importance of cutting conditions to obtain a good result for the finished piece. They offer optimized models to guarantee

the lifetime of their tools. Typically, the four major [FAN 94] geometric deviations (defects) are here as listed by order:

- *Defects of order 1* (or 1st level): They correspond to the geometrical defects of surfaces such as flatness, straightness, roundness, etc.;
- Exploration methods applied to the metrology of electronic sensor apparatus (inductive);
- *Defects of order 2* (or 2nd level): They are linked to the undulations, the kinds of successive hills and valleys included in the profile caused by vibrations, deformations of machines, chattering, heat treatment etc.;
- Exploration methods applied to electronic sensor (inductive);
- *Defects of order 3* (or 3rd level): They are the roughness ridges, kinds of furrows traced with of regularity on the undulations relief by cutting tools;
- Exploration methods applied to electronic devices with piezoelectric sensor, etc.;
- *Defects of order 4* (or 4th level): More irregular, sometimes accidental, they correspond to pull-outs, cracks or fissures in the material, etc.;
- Exploration methods applied to electronic devices with piezoelectric sensor, etc.

8.4.2. Control, without measurement, of threads

For regular control [CASE 78] without measurement, we use:

- thread ring and thread plug for simple control;
- thread limit gages (“Go, Not Go”);
- thread snap gauge (“Go, Not Go”).

A snap gauge with grooved rollers fitting the profile of the screw thread (Figures 8.23 and 8.24), ensure easy engagement with the screw to be controlled, as well as low wear. Striations on the side (Go) have theoretical profile and the length of the rollers is equal to the nominal diameter d . The rollers on the side (Not Go) have a special profile limited to the control of the flanks and they do enclose a single thread. The grooves of the rollers, facing each other, are shifted a half-pitch and a slight lateral clearance prevents any misalignment. The adjustment of the spacing of the rollers, depending on the nominal diameter, d , is obtained by rotating their off-centered axes relative to the support. The tapping is, in this case, controlled by the threaded plug gauge “Go”. The screw is controlled by the threaded ring.



Figure 8.23. Ring and plug gauge for easy control of ISO thread

In medium quality hardware (fasteners), the control should ensure easy fitting with a clearance less than a given value. We use, for this, fixed limit thread gages (see Figure 8.24).

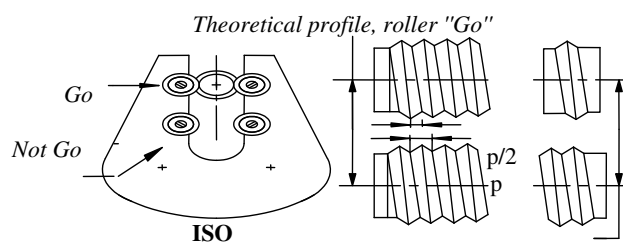


Figure 8.24. Control using fixed limit thread gages (profile side "Not Go")

8.4.2.1. Direct measurement with a thread micrometer

Direct measurement [CAS 78] is given by means of a screw thread micrometer as previously discussed. The contact points of the micrometer are suited to engage, respectively, the hollow and the crest of thread as shown below:

$$d_4 = d - \left(\frac{1}{8} \times H \right) + \left(\frac{7}{8} \times H \right) = d - 0.866 \times \left(\frac{6}{8} \right) \times p = d - 0.6650 \times p \quad [8.34]$$

The V-shaped contact point, moving in rotation, fit over the thread in contact. This method of measurement is satisfactory only for threads of average quality. Figure 8.25 gives a schematic presentation of the foregoing explanation.

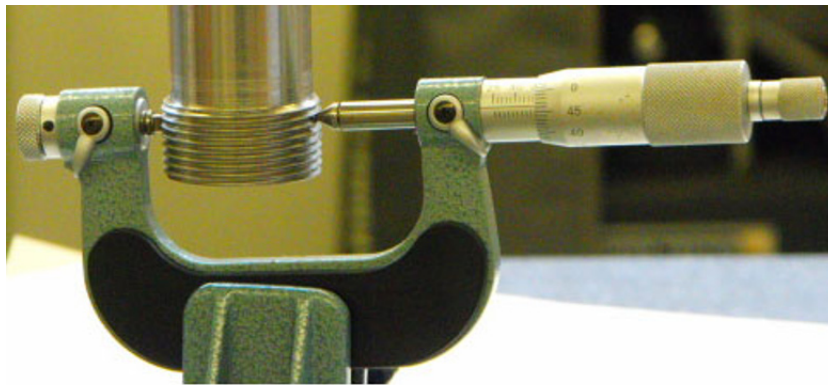
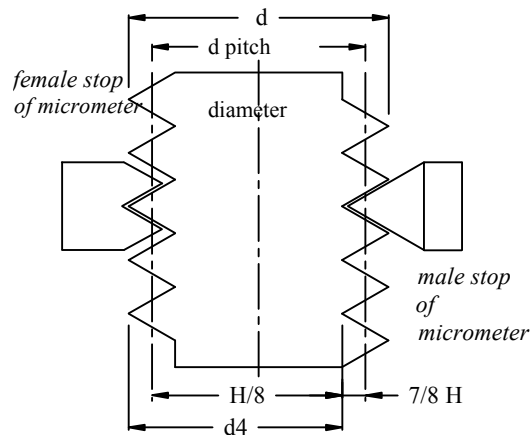


Figure 8.25. Thread micrometer to measure the pitch circle diameter

8.4.2.2. Indirect measurement using thread measuring wires

Indirect measurement [CAS 78] is given for ISO threads. The three measuring wires have the same diameter $\varnothing$ such that the measuring wire is tangent to both the flanks and the theoretical base of the pitch triangle untruncated. As the point of tangency lies in T (Figure 8.26) and mid-action of a_c : and that $r = H/3$, we will then get:

$$2r = \left(\frac{2 \times 0.866 \times p}{3} \right) = 0.5770 \times p \quad [8.35]$$

The distance on measuring wires is expressed:

$$D_e = \left(d + \frac{2H}{8} \right) = \left(\frac{2 \times 0.866 \times p}{8} \right) = d + 0.216 \times p \quad [8.36]$$

When the measuring wires have a diameter d_p different from the previous diameter, the distance on measuring wires is written:

$$D_e = d - 1.5150 \times p + 3 \times d_p \quad [8.37]$$

It is worth noting the linguistic sprain made to the terminology used to characterize threads. The “diamètre primitif” in French (pitch diameter) is used to designate what is termed “diamètre sur flancs” (i.e. diameter over the flanks). This translation from the American term: pitch diameter is actually wrong because the “diamètre primitif” is reserved to the case of gears. The threads may, sometimes, be fine (NF). In this case, we can also use a goniometer.

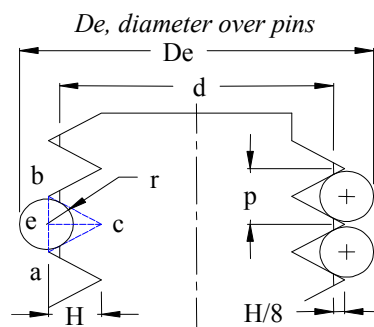


Figure 8.26. Indirect measurement using cylindrical measuring wires

8.4.2.2.1. Measurement over the wires for buttress threads¹

The application of the method of the three measuring wires [OBE 96] also applies to the case of buttress threads. This allows us to write the following:

$$D_{hp} = d_p - \left[\frac{P}{\tan(\beta) + \tan(\alpha - \beta)} \right] + D_p \times \left[1 + \cos\left(\frac{\alpha}{2} - \beta\right) \times \csc\left(\frac{\alpha}{2}\right) \right] \quad [8.38]$$

$$\text{with } D_p = P \times \left(\frac{\cos(\beta)}{1 + \cos(\alpha)} \right)$$

where D_{hp} is the measurement over the wires in inches; d_p is the pitch diameter in inches; D_p is the diameter of wires in inches; P is the pitch in inches; α is the angle of inclination of the thread, in degrees (45° or 50°); β is the frontal angle of the thread in degree (45° or 50°). The latter is measured from the perpendicular of the thread to the axis.

For buttress threads of 45°, we can use this relationship derived from the technical literature [OBE 96]:

$$D_{hp} = d_p - P + (3.4142 \times D_p) \quad [8.39]$$

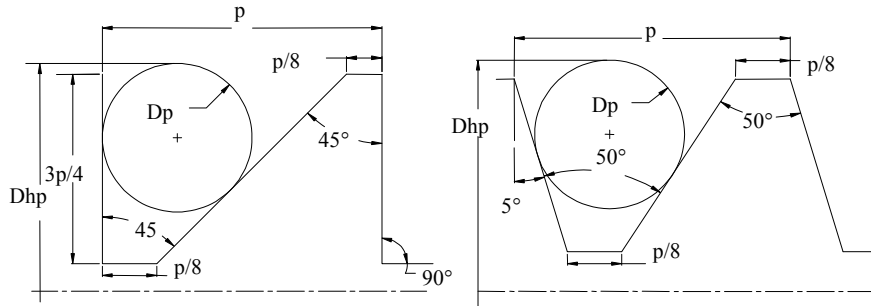


Figure 8.27. *Measurement over the wires of buttress threads*

For buttress threads with a 50° angle, we can use this equation [OBE 96]:

$$D_{hp} = d_p - (P \times 0.91955) + (3.2235 \times D_p) \quad [8.40]$$

¹ Measurement over pins, or often call measurement over wires.

In general, for *American National Standard Buttress Threads*, the ANSI B1.9-1973 standard proposes this relationship derived from technical literature—buttress thread of 52° —[OBE 96]. C is a correction factor < 0.0004 . The standard recommends using a measuring wire with a diameter $W = 0.54147 \times P$:

$$D_{hp} = d_p - (P \times 0.89064) + (3.15689 \times D_p) + c \quad [8.41]$$

8.4.2.3. Control of threads' profile and pitch

Indirect measurement is given for fine quality threads. Figure 8.27 gives a schematic presentation. The ultimate goal is to detect both deformities and defects of the profile position, asymmetric related to a plane normal to the axis.

- Align thread crests on the fixed reticle.
- Measure the angles α and β of flanks inclination using moving reticle.
- Calculate the value of the apex angle $\gamma = 180^\circ - (\alpha + \beta)$ and interpret the result.

Here is a schematic of the amplification and its interpretation:

- if the deviation observed on the angle α is, for example: $\delta_1 = 5'$;
- if the deviation on the angle β is, for example: $\delta_2 = 25'$.

The interpretation of the observation of the foregoing, inspired from the literature [AGM 08, CAS 78] gives:

$$\gamma = 180^\circ - [(\alpha \pm \delta_1) + (\beta \pm \delta_2)] = 180^\circ - [(60^\circ 20') + (59^\circ 35')] = 60^\circ 20'$$

- Figure 8.27(a) shows that the apex angle γ is higher by $20'$ since $\gamma = 60^\circ 20'$;
- the profile is asymmetric and coated by: $(90^\circ - 59^\circ 35') - (60^\circ 20')/2 = 15'$;
- there is a profile defect and a position defect resulting in a defect in mean $\emptyset$;

When a microscopic goniometer is inaccessible, the verification of tapping will be carried out by molding (see Figure 8.28(b)) thread in soft wax held by a metallic support. The contact is maintained throughout the duration of the hardening of the wax and the mold is, then, released while avoiding any rotation. This method is similar to mechanical control by comparison because it is copying the mold on the threads and reading the goniometer or by other appropriate means. This verification is done in rare cases where it is impossible to get access to a goniometer or a profile projector. It is identical to that used in fracture mechanics [GRO 94] when measuring the geometry of weld seams.

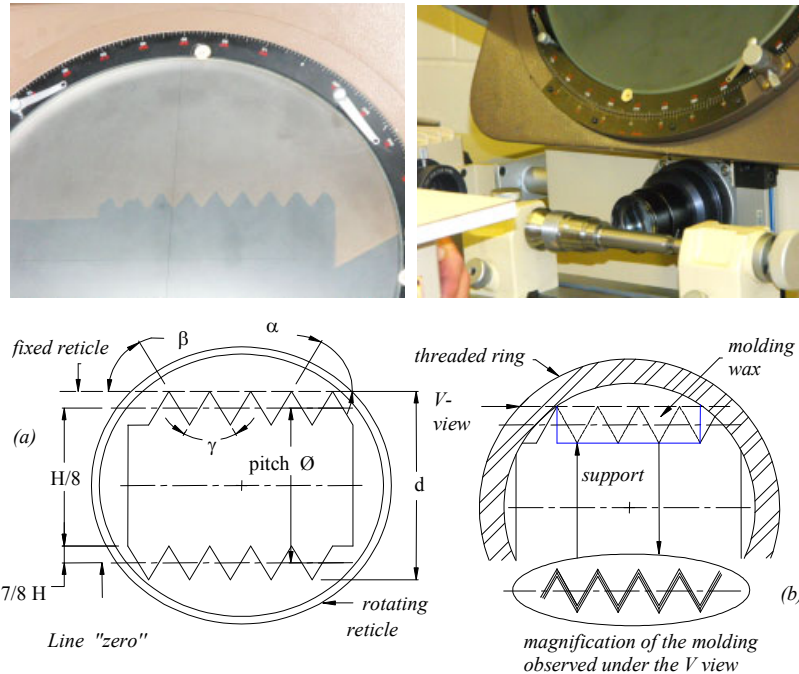


Figure 8.28. (a) Reading the profile of a thread form on projector;
(b) molding wax pattern of an internally threaded ring

8.4.3. Control of a thread pitch using ruler and gauge

This method is classic, simple and accurate. For example, the number of screw threads per inch is the number of threads counted within one inch length. This is, by analogy, the same for metric counting of threads over a given length. The pitch is determined by placing the ruler on the screw or by using a screw pitch gauge as shown in Figure 8.29.

8.4.4. Checking the straightness of tapping tools by squaring

To obtain a good tapping while preventing breaking the taps, it is recommended to:

- Drill a pilot hole to a dimension calculated as previously indicated.
- Choose a wrench that is proportional to the diameter of the tap. Do not take a wrench too long for the tap because the torque will certainly be larger and will facilitate operating manual effort, however it could break the tap.

- Gently engage the tapered tap.
- The thread guide uses a clamp to maintain the axis of the tap collinear with that of the axis of the hole. We may also use the centering spindle the concerned machine tool.

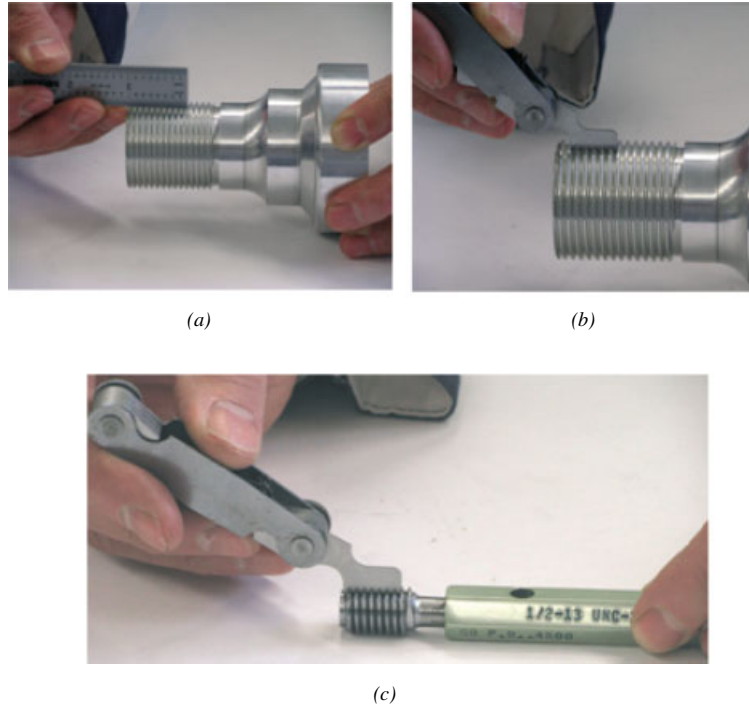


Figure 8.29. Hand-checking of the number of threads (per inch and in mm) (a) counting the number of threads per inch; (b) counting the number of threads stamped on the thread pitch gauge; and (c) counting the number of threads stamped on the thread pitch gauge

8.5. Screws resistance and quality classes

8.5.1. Minimum torques for screws with diameters of 1 to 10 mm

The torques indicated [TEI 00] in Table 8.2 concern screws with 1 to 10 mm in diameter and quality classes (8.8 to 12.9). They are not valid for Hexagon socket set screws (Figure 8.30) and are calculated as:

$$M_{\min} = \frac{K \times \pi \times R_m \times d_3^2}{16} \quad [8.42]$$

where M_{min} , minimum break torque; R_m , resistance to tensile failure of the material of the screw; d_3 , minimum inner diameter of the screw thread (or hub $\varnothing$); k , coefficient (see table below) depending on the class quality.

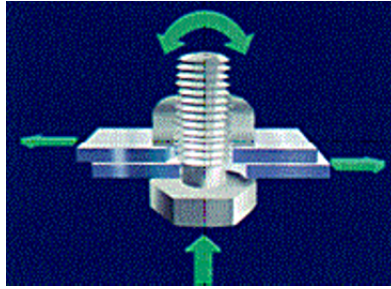


Figure 8.30. Forces applied to a screw

Quality class	8.8	9.8	10.9	12.9
k	0.84	0.815	0.79	0.75

Table 8.2. Coefficient k as a functions of quality classes of screws. Source: [DIE 00]

8.5.2. Example of calculations of efforts on threads (North American concept)

The effort (W) applied to the outer diameter d_0 is designed to have the pitch p . The friction on the threads is μ_t and on the helix will be μ_c . If the diameter of the helix is d_c , let us calculate the time required to ensure the condition of resistance. The schematic illustration of the assembly is shown in Figure 8.31.

- W is the applied load: = 5 000 N;
- Nominal diameter (outer) = d_0 = 20 mm;
- Helix diameter = d_c = 30 mm;
- Thread pitch p = 4 mm ;
- Coefficient of friction (threads) = μ_t = 0.075;
- Coefficient of friction (helix) = μ_c = 0.095;
- λ is the angle of helix;
- L is the height of the helix;
- d_m is the pitch diameter of threads in contact.

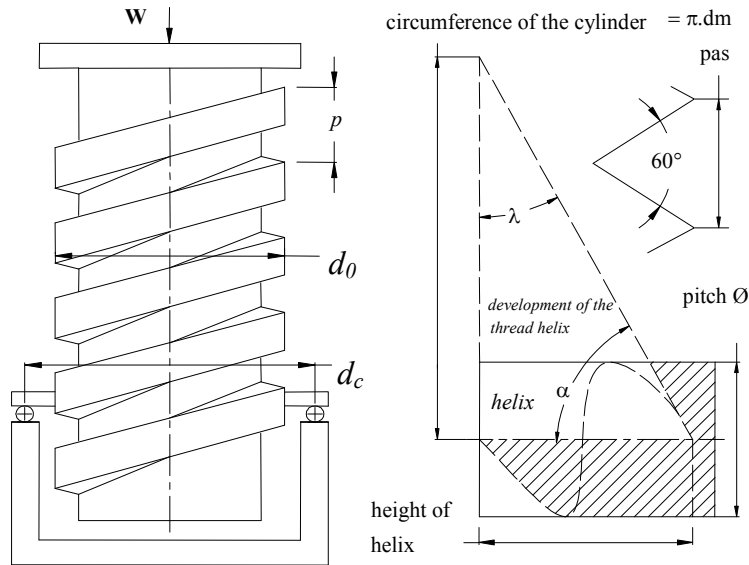


Figure 8.31. Minimum torques. Geometric illustration of the helix for threads

SOLUTION.— For a simple thread, the pitch will be equal to the height, ($L = p$). For a square thread, the mean thread will be approximated by using the following equation:

$$d_m = \left(d_0 - \frac{P}{2} \right) \quad [8.43]$$

$$d_m = 20 \text{ mm} ; L = 4 \text{ mm} ; \tan(\alpha) = \left(\frac{L}{\pi \times d_m} \right) = 0.637 \text{ rad} = \left(\frac{3}{38} \right) \text{ DMS} \\ \left(59.068 \right)$$

The moment required to raise the effort will be expressed by the following equation:

$$M = \left(\frac{W \times d_m}{2} \right) \times \left(\frac{\mu_t \times \pi \times d_m + L}{\pi \times d_m - \mu_t \times L} \right) + \left(\frac{W \times \mu_c \times d_c}{2} \right) \quad [8.44]$$

APPLICATION EXERCISE.—

Given: $W = 5000$, $d_0 = 20$; $p = 4$; $\mu_t = 0.075$ and $\mu_c = 0.095 \Rightarrow M = 1.4091 \times 10^4 \text{ N} \times \text{m}$

The moment required to decrease the effort will be expressed as:

$$M_1 = \left(\frac{W \times d_m}{2} \right) \times \left(\frac{\mu_t \times \pi \times d_m - L}{\pi \times d_m - \mu_t \times L} \right) + \left(\frac{W \times \mu_c \times d_c}{2} \right) \quad [8.45]$$

NUMERICAL APPLICATION.—

Given: $W = 5000$, $d_0 = 20$; $p = 4$; $\mu_t = 0.075$ and $\mu_c = 0.095 \Rightarrow M = 7.689 \times 10^3 \text{ N} \times \text{m}$

The work ratio (effectiveness) of the screw, when the frictions are neglected, is:

$$\text{eff} = \frac{1 - \mu_t \times \tan(\lambda)}{1 + \mu_t \times \tan(\lambda)} = 0.060 = 60 \% \quad [8.46]$$

Because the efficiency of the screw depends on the helix angle, the effort applied to the screw will block the latter if the coefficient of friction of threads μ_t is $\geq$ to the tangent to the helix angle, that is to say: $\mu_t = 0.07$ and $\tan(\lambda) = 0.064$. For a range λ_1 given as $\lambda_1 = 0.0001$ degree, 2 degrees, up to 85 degrees, we plot the efficiency curve as in Figure 8.32.

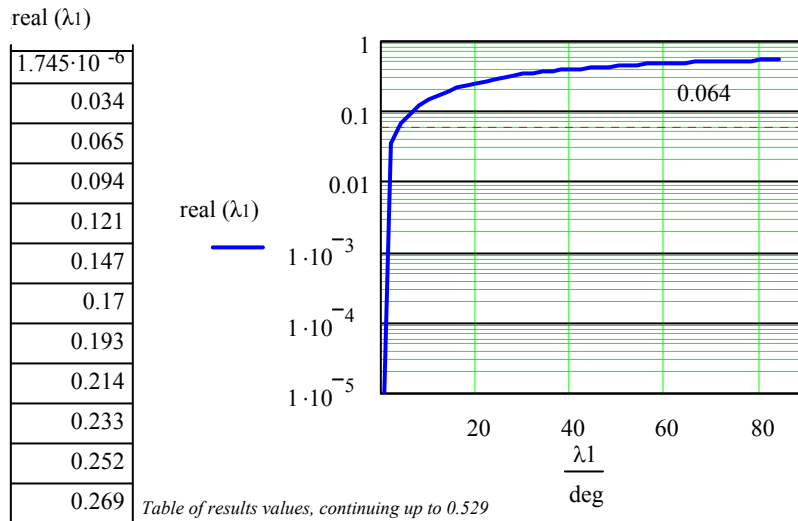


Figure 8.32. Efficiency curve of screw threads

8.6. Control of screw thread by mechanical and optical comparison

Obviously, the control of threads using a micrometer is advantageous on a machine tool. Among the advantages of a thread micrometer, we quote:

- Simplicity and usability offered by the use of calibrated micrometer allow a reading that does not affect the isostatism imposed (during the assembly) to the workpiece, since the link between the workpiece and the thread micrometer is not permanent. The only link, also temporary, is significantly performed by the mobile and fixed contact points. It is used to control the pitch diameter of thread.
- The control is done on the machining bench/worktable. This is still very advantageous because we can simultaneously monitor the real evolution of the dimension during being worked.

Among the inevitable drawbacks, we mention:

- Both the micrometer and the workpiece are sensitive to temperature changes, within the current limits of production workshops. Expansion or contraction of the workpiece to be inspected on the machining bench/worktable affects inevitably the machining accuracy.
- Additional installation of cylinders and control for the control of threads, inevitably adds uncertainties to the measurement.
- The choice of cylindrical rods is probably tainted with uncertainties.
- Manipulations are often subject to the degree of the operator's skill.
- Both the cleaning of contact surfaces and hygrometry conditions affect the accuracy of control, even measuring.

8.6.1. *Laboratory example on threads control*

Objectives and purpose of the laboratory control threads:

- to manipulate geometric and physical concepts on threads;
- to inspect the threads using micrometer;
- to observe the measurement accuracy resulting from mechanical metrology and compare it with the same measurement resulting from profile projector control;
- to understand the principle of reading threads (the importance of the helix angle, β).

Control of threads on GEOPAK profile projector (Mitutoyo): briefly explain the basic principle of control and measurement of thread resulting from a machined tapered mandrel. We use, for this purpose, the following device(s):

1. thread measuring wires;
2. shadow projector (GEOPAK);
3. a graduated ruler of your choice;
4. gauges of your choice and gauge provided for this purpose;
5. the thread micrometer, once calibrated by yourself (mm and inch);
6. draw a table comparing the measurement results above (from one to five) and highlight the advantages and drawbacks of optical metrology measurement with GEOPAK over other methods of metrology;
7. dimension properly the piecework, taking into account the tolerances required for a correct drawing.

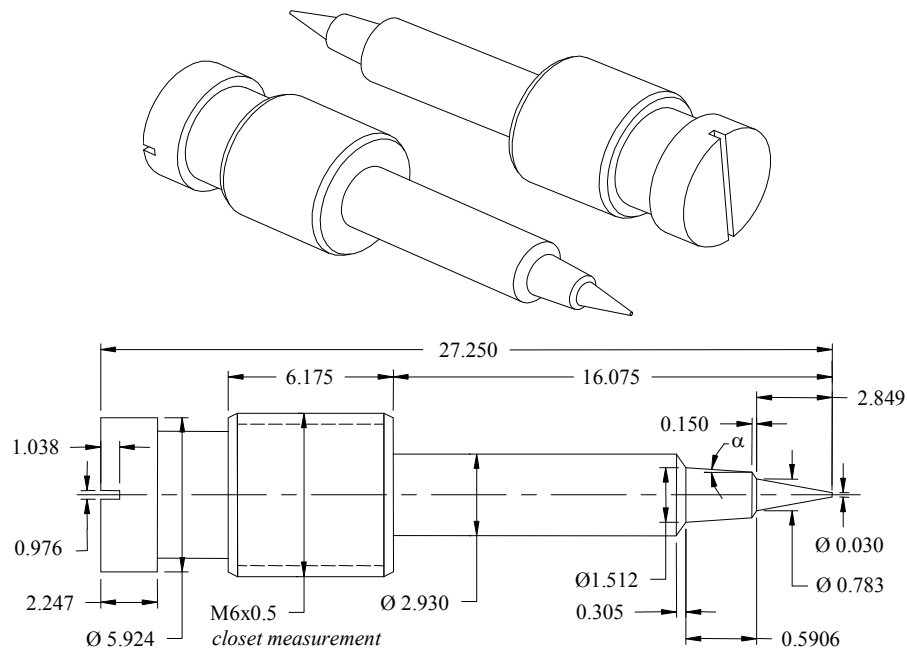


Figure 8.33. Drawings of a threaded screw (unknown characteristics of the thread)

After identifying the relative and absolute errors and their uncertainties, it is desirable to calculate them.

Technical drawing of the screw to be identified: We have manufactured, at the machine shop, a screw thread. Then we had deliberately distorted this thread so that it no longer matches this “normalized” thread. Consequently, it becomes a special thread (commonly called *bastard*). The next step will be to identify the closest thread to this thread.

Table of results of measurements with form projector: Threads in Figure 8.33 are made in the workshop. You are asked to check the veracity of these dimensions of the threads by using at least three means of control: CMM, form projector, and micrometer (if possible).

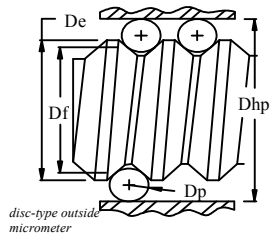
The following formulas summarize briefly the calculation of diameters over the wires depending on the type of screw thread. The helix angle does not affect the measurement of $D_{\text{(over the wires)}}$ or D_{hp}. In fact it is the pitch diameter that is targeted. It is, then, obtained as a function of $D_{\text{(over the wires)}}$ surface $D_f = f(\text{Pitch}, D_e, D_p, D_e) = 0.5 \times \text{pitch} \times \sec(0.5 \text{ thread angle})$ Example: $(\alpha) = 60^\circ, \rightarrow D_{hp} = 0.57735 \times \text{pitch}$	
Useful formulas for determining the pitch diameter (D_f) corresponding to diameter over the wires (D_{hp}) (value read) as a function of the diameter of chosen wires (D_p). Formula (a) if the diameter over the wires is known. Formula (b) if the pitch is known.	Type of screw thread Form of thread
We use the same formula as for the American UNF thread	Standard international SI
$D_f = D_{hp} + 0.86603 \times \text{pitch} - 3 \times d_p \quad (a)$ $D_{hp} = D_f - 0.86603 \times \text{pitch} + 3 \times d_p \quad (b)$	UNF thread American Unified National Standard
$D_f = D_{hp} + 0.86603 \times \text{pitch} - 3 \times d_p \quad (a)$ $D_{hp} = D_f - 0.86603 \times \text{pitch} + 3 \times d_p \quad (b)$	V-Shaped thread
$D_f = D_{hp} + 0.9605 \times \text{pitch} - 3.1657 \times d_p \quad (a)$ $D_{hp} = D_f - 0.9605 \times \text{pitch} + 3.1657 \times d_p \quad (b)$	Withworth- Thread British Standard Whitworth
$D_f = D_{hp} + 1.1363 \times \text{pitch} - 3.4829 \times d_p \quad (a)$ $D_{hp} = D_f - 1.1363 \times \text{pitch} + 3.4829 \times d_p \quad (b)$	British Association Standard

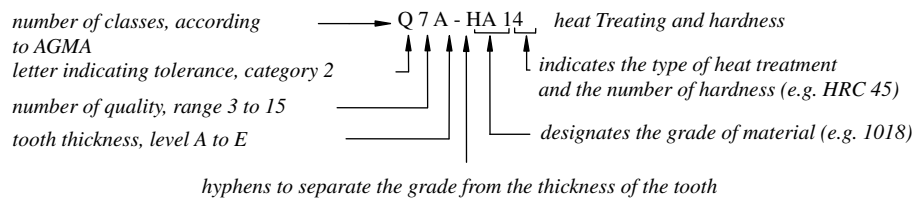
Table 8.3. Summary table of main wire thread measurements [OBE 96]

Summary of section one (threads): First, we consider a thread as a helical surface essentially defined both by its helix lead traced on the base cylinder and by its profile whose form will be read in the plane containing the axis. All kinds of threads are standardized and shall conform to specifications. Screw threads control is performed through various ways and with different means. Among these means, we retain the screw-thread micrometer, the set of thread measuring wires with a classical micrometer, profile projector or gauges, etc (Table 8.3).

8.7. Introduction to gear control

Given the diversity of gears [AGM 08], we limit ourselves to the presentation of involute spur gears control. Spur gears (parallel) are the most common. Bevel gears (or concurrent) enable transmission between perpendicular shafts. Screw gear pairs allow both irreversibility and a significant reduction with one torque of gears (their low performance exclude them from major powers). In most applications, the trains run by means of reduction gears (reduce speed and increase the torque). They are standardized and have the advantage of being interchangeable. They allow economic production possibilities. In the case of gearing for very long series (automobiles), manufacturers deviate from these standards to optimize costs.

The ISO 1328 standard provides thirteen precision classes, class 0 being the most accurate. The progression from each class is $2^{1/2}$. This standard provides definitions, rules, formulas, areas of validity and provides the permissible values of deviations (Tables similar to those of adjustments) for flanks of similar gear teeth as well as those relating to the compound radial deviation and radial runout. The main standards used in this chapter are ISO and AGMA (*American Gear Manufacturers Association*). Here is a model of designations:



This section presents the nomenclature of spur gears. It also presents some calculation formulas for this type of gears according to ISO and AGMA (www.agma.org). In ISO diameter designation, we use the letter d for the pinion and the letter D for the gear (Table 8.4).

CLASS	Domain of use
Classes 0 to 4	Either for gear teeth of exceptional precision or for high speeds ($V > 30$ m/s): gears, etalons, turbines, etc.
Classes 5 to 6	Gears under high speeds ($V < 20$ m/s): machine tools, measuring apparatus, automobiles and turbines, etc.
Class 7	Case of teeth cut by hobbing (ground), $R_a = 0.8$ to 3.2 μm ; Good quality in general mechanics
Classes 8 to 9	Common quality with hardened gears unground 3.2 μm , $V < 7$ m/s
Class 10	In addition to conventional methods: extrusion, sintering, plastic injection, etc.
Classes 11 to 12	Slow gears $V < 2$ m/s, and large-module gears, etc.

Table 8.4. Domain of use, by class, of gears according to ISO 1328

AGMA Standard classifies tolerances as shown in Table 8.5

Tolerance class for a gear tooth thickness <i>Tooth Thickness Tolerance—Tolerance Classes for Spur, Helical, and Herringbone Gears</i> <i>All tolerance values in inches</i>						
AGMA Quality Nb#	Pitch diameter	Class				
		A	B	C	D	EC
Quality Nb# AGMA 7	00.5	0.07400				
	01.2	0.03100				
	02.0	0.01900	0.00930	0.00480		
	03.2	0.01200	0.00600	0.00300		
	05.0	0.00750	0.00370	0.00190		
	08.0	0.00500	0.00250	0.00125	0.00063	
	12.0	0.00300	0.00180	0.00090	0.00044	
	20.0	0.00240	0.00120	0.00060	0.00030	0.0001600
	32.0	0.00160	0.00080	0.00043	0.00020	0.0001000
	50.0	0.00120	0.00060	0.00030	0.00014	0.0000700
	80.0	0.00080	0.00045	0.00022	0.00011	0.0000555
	120	0.00067	0.00034	0.00017	0.00009	0.0000450
	200	0.00050	0.00025	0.00013	0.00006	0.0000300

Table 8.5. Tolerance classes for gear tooth thickness according to AGMA (United States)

8.7.1. Parallel spur gears

According to ISO 701 straight cylindrical gear (Figure 8.34) can be easily compared with cylindrical friction gears. Similar to them, it ensures the transmission of a circular motion between two parallel shafts close together, the ratio of angular

velocities of shafts determining the ratio of gear diameter. If the friction gear drive made by the grip, is unable to forward a great effort and it is inseparable from a certain shift. In the case of a gear, binding, obtained by barriers provides an absolutely fixed gear ratio and can transmit considerable torque.

– The pitch cylinder of rolling wheel gear is a cylinder determined by the instantaneous axis of relative motion of the gear coupled with respect to the considered gear.

– External diameter is the diameter over the top of teeth.

– Root diameter is the diameter of the circle coinciding with the bottoms of tooth spaces.

– Flank of tooth is the portion of the surface of a tooth between the pitch circle and the bottom land.

– Thickness of a tooth is width of the toothed portion of a gear, measured on the pitch diameter.

– Profile is the section of a flank. The profile form is said involute.

– The pitch is the arc length of the pitch circle between two consecutive profiles counterparts. It is equal to the product module by the number π .

– The module (m) constitutes the basis for gear tooth dimensioning calculations.

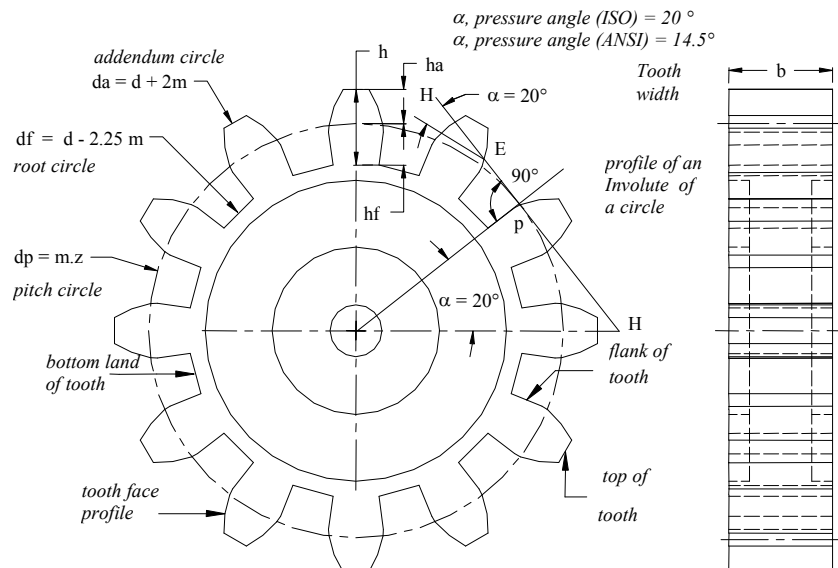


Figure 8.34. Characteristic of a straight-toothed wheel

To be able to mesh together, the two gears must have the same module (same size of teeth). Two different module gears are not compatible. The dimensions of a straight-tooth cylindrical gear are defined following the number of teeth Z , and the module m . To ensure continuous meshing engagement, a couple of teeth shall mesh before another couple of teeth stops engaging. In the case of a small number of teeth, a teeth correction will be necessary to avoid running interference between several pairs of teeth.

8.7.1.1. Possibility of correction of teeth for gears of 8–17 teeth

A correction of teeth on the gears [FAN 94] can be performed in order to avoid interferences that undermine the basis of teeth. This process changes the pitch circle diameter of the gear, thus the centerline distance. From a geometric viewpoint, the module is found on a tooth. This is the distance (in mm) between the pitch radius and the outer radius. For example, for a module 2, there are 2 mm between pitch radius and outer radius. Reported to diameter, it is necessary to add twice the pitch diameter to find the inner diameter. Because of their relative simplicity the spur gears are often used to introduce the kinematic relations. The original circumference, with perimeter (πd) must imperatively contain an integer number of teeth Z all placed at successive intervals equal to the pitch (p). This results in a pitch circumference perimeter $p = \pi d/Z$, and by definition this result is known as the Module $m = \text{pitch}/\pi = d/Z$, thus $d = mZ$.

The pitch p : On the circular pitch, it measures the distance (the length of the arc of the pitch) between the corresponding profiles of two adjacent teeth. In other words, the pitch is equal to the pitch circumference (πd) divided by the number of teeth Z , note that $d = mZ$, and consider:

$$\pi \times d_{\text{pitch}} = \pi \times (\pi \times d_{\text{pitch}}) \times p = \pi \frac{m \times Z}{Z} = 3.14159 \quad [8.47]$$

English-Speaking countries use the concept *diametric pitch*: $P = \pi/\text{in inches}$.

Module m : It is a number whose values are normalized.

8.7.1.2. *Formulas on the spur gear (standard teeth, mm)*

Designations, units in mm	Symbols	Formulas
Module	m	P/π
Pitch	p	$m \times \pi$
Pitch circle diameter	d	$Z \times m$
Number of teeth	Z	P/m
External diameter	D	$m \times (Z+2)$
Addendum	ha	m
Dedendum for module 0.25 to 1.25	hf	$1.40 \times m$
Dedendum for module 1.5 to 8	hf	$1.25 \times m$
Tooth height for module 0.25 to 1.25	h	$2.40 \times m$
Tooth height for module 1.5 to 8	h	$2.25 \times m$
Tooth (circular) thickness	e	$\pi/2m$
Corrected pitch circle diameter	dc	$m (Z+1)$
Corrected external diameter	Dc	$m (Z+3)$

Table 8.6. *Key characteristics of a spur gear (mm)*

Gear tooth with correction: A teeth correction can be performed on gears to avoid interferences which may weaken the base of the gear teeth (for gears with 8 to 17 teeth). However, it should be noted that this process changes the pitch diameter of the gear, thus the center distance “a” with a tolerance of 0/0.05 (Table 8.6). For the respective pitch circles diameters d_1 and d_2 (gear and pinion), the transmission ratio (i), or velocity ratio is expressed by:

$$i = \left(\frac{\omega_1}{\omega_2} \right) = \left(\frac{Z_2}{Z_1} \right) \quad [8.48]$$

where ω_1 is the input speed and ω_2 the output speed; Z_1 is the number of input teeth and Z_2 the number of output teeth.

Whatever the number of gear teeth, all gears having the same module and the same pressure angle α , can be manufactured with the same cutting tool. A series of modules has been standardized in order to limit the number of tools and measuring systems. The thickness of the gear tooth and its strength depend on the choice of the module. This choice should not be improvised. Center distance a is the shortest distance between non-intersecting axes of mating gears. This distance, in the case of spur gears, is worth:

$$a = \left(\frac{d_1 + d_2}{2} \right) = (r_1 + r_2) = m \times \left(\frac{Z_1 + Z_2}{2} \right) \quad [8.49]$$

Among the works on this subject, we find modeling, based on finite elements of this kind of entities as a fundamental element in power transmission. This profile is

defined by the trajectory of a point on a straight line, which rolls without slipping on a circle. This profile has many merits because it allows:

- tolerance on the center distance without affecting the operation;
- only one tool per module allows cutting all gears;
- unlike any other profile, the wear on active surfaces is evenly distributed;
- unlike other profiles, vibrations are lower.

8.7.1.3. Graphical presentation of the involute of the circle

The geometric drawing of a gear flank can be done in various ways (involute). We easily find the method in literature [FAN 94, OBE 96] dedicated to drawing and dealing with surfaces developments. If, for example, a line is rolled without slipping on a circle, each point on that line describes, in relation to the circle, a curve called an involute. We may also think of a live wire unrolled from a circle: the end of the wire describes the involute with respect to the circle from which it is unrolled. Here is the North American approach:

$$\tan(\varphi) - \varphi = \text{inv}\varphi \quad [8.50]$$

with $\text{inv } \varphi$ the involute angle φ ; the basic pitch P_b and the circular pitch P ; the pressure angle of the rack φ_c ; the pitch diameter D and the base radius R_b .

$$P_b = P \times \cos(\varphi_c) \quad [8.51]$$

The circular pitch p is the length of the arc of the circle between corresponding points of two consecutive teeth along the pitch circle. It corresponds to:

$$P = \frac{\pi \times D}{N} = \frac{P_b}{\cos(\varphi_c)} = \frac{\pi}{m} \quad [8.52]$$

The diametric pitch P (not circular) is the number of teeth per inch of diameter (Figure 8.35). It increases as the size of the teeth decreases and vice versa. It is a quantity that is used in the AGMA. The module m depends on the ratio of the width of teeth to diameter. It increases with the size of the teeth and is, therefore, the inverse of p by the constant 25.4 nearly (Table 8.7).

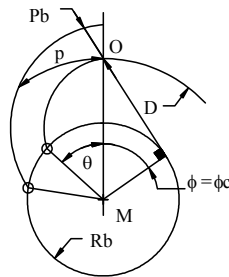


Figure 8.35. Plotting of flank of the teeth by involute

AGMA	ISO	ONVERSION AGMA - ISO
$P = \left(\frac{N}{D} \right) = \left(\frac{1}{m} \right)$	$m = \left(\frac{d}{Z} \right) = \left(\frac{P}{\pi} \right)$	$P = \left(\frac{25.4}{m} \right)$ [8.53]

Table 8.7. Conversion AGMA-ISO

Pressure angle ϕ_c : For an arbitrary contact position between the pinion and the gear on the line of meshing engagement, pressure angles, respectively, of the gear ϕ_r and the pinion ϕ_p are different. However, when this contact point is made on O (pitch point), then pressure angles become equal to ϕ_c which is also pressure angle. The base circle D_b is the circle developing the involute of the circle. This is the circle at which the involute profile is generated. The line normal to the profile is tangent to the base circle and, therefore, the line of action is also tangent to the base circle of, both, the pinion and the gear:

$$D_b = D \times \cos(\phi_c) \quad [8.54]$$

The pitch circle is the locus of points where there is rolling without slipping between pinion and gear. Gear meshing is assimilated to two gear pitch circles rolling without slipping on one another (Table 8.8).

AGMA (inch)	ISO (mm)	CONVERSION: AGMA-ISO
$D = N \div P$	$D = m.Z$	$D_b / \cos(\phi_c)$
N is the number of teeth D is pitch ϕ	Z is the number of teeth D is pitch ϕ	ϕ_c is the pressure angle D_b is the pitch diameter
Formulas	AGMA	ISO
Addendum	$a = 1/P$	$h_a = m$
Dedendum	$b = 1.25/P$	$H_r = m$
Backlash	$c = 0.25/P$	$c = 0.25 \times m$

Table 8.8. Conversion AGMA-ISO

To get a proper engagement meshing between pinion and gear, their modules must necessarily be the same. The center distance C (or a) represents the distance between the centers of the pinion and the gear. In normal operation, its value is equal to the sum of the pitch radii of both the pinion and the gear. The center distances may vary depending on the temperature of the housing and gears especially when the materials of the gears and housing are different. A clearance between a pair of teeth is necessary for the proper functioning of the gear. It allows effective lubrication and flexibility in case of expansion due to temperature variation. A decrease in the center distance leads to a decrease in the clearance. It is, therefore, possible to use the variation of center distance to control the clearance. To calculate the center distance C (AGMA) or a (ISO), we use Table 8.9.

AGMA	ISO	Formula in common AGMA-ISO
$m = \left(\frac{1}{P}\right) = \left(\frac{D}{N}\right)$	$m = \left(\frac{d}{Z}\right) = \left(\frac{P}{\pi}\right)$	$C = \left(\frac{d + D}{2}\right)$ [8.55]
$C = \left(\frac{N_p \dots \pm \dots N_p}{2p}\right)$	$a = m \times \left(\frac{Z_1 \dots \pm \dots Z_2}{2}\right)$	

Table 8.9. Correspondence between ISO and AGMA

The sign (+) for a gear with external teeth and the sign (–) for a gear with internal teeth. Involute traced from the same base circle are all geometrically identical or superimposed. The profiles of the flanks and sides of the teeth follow strictly the geometry of the involute. If we place, keeping it taut, a wire wound on a disk or spool, the end of the wire describes an involute of the circle. The profile of the involute of the circle is the most used (universal). Being insensitive to variations in center distances, it is cut using relatively simple tools. The cycloidal profile, which is also a common form of gears, is mainly used in micromechanics. This principle allows us to obtain gears with small numbers of teeth without interference at the time of gear cutting. Its disadvantage is that it is sensitive to center distance variations.

Interference phenomenon: there is interference when the tip of the tooth of a gear is in contact with the bottom land of a tooth of the mating gear. At the time of gear cutting this defect is characterized as an *interference machining at the bottom of the tooth* (French: *usinage parasite du pied de la dent*) where the interference is avoided. For the center distance (a) we consider:

$$r_a = \sqrt{(r_a)^2 + a^2 + \sin(\alpha)^2} \quad [8.56]$$

where $\alpha = 20^\circ$ ($= \varnothing_c$). If both gears have more than 17 teeth, there is no risk of interference. For a rack and pinion system, interference is avoided if $Z_1 \geq 18$.

A number of teeth Z_1 less than thirteen has to be avoided. Corrections of teeth with or without variation in center distance and shortening of teeth, allows avoiding *interferences*. We try in what follows to present another approach to simply make the involute without too many equations.

As already described, the involute of a circle is a curve that characterizes the path of a point on a straight line rolling without slipping on the base circle of radius r_b . This curve shows a cusp P_{reb} foot of the involute. This point is located on the base circle. In fact it is a curve plotted by the right hand dropping a coil of wire held in the left hand. The involute of the circle (also known as *anti-clothoïde* in French) is a plane curve involute, that is to say that its normal lines are the tangents to the circle. It was first studied by Huygens when trying to design clocks without pendulum for their use on a boat on the sea. He used the involute of the circle in an attempt to force the pendulum. Here is a set of equation accordingly:

$$\begin{cases} X(t) = k \times \cos(t) + t \times \sin(t) \\ Y(t) = k \times \sin(t) - t \times \cos(t) \end{cases} \quad [8.57]$$

Based on $k_{\text{iterations}} = 0.01$ and $t = 0$ to 20, the curve is defined *parametrically* as in Figure 8.36(a).

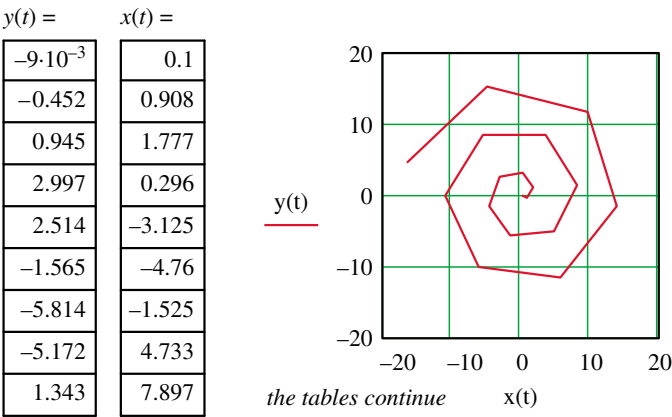


Figure 8.36a. Plotting an involute of circle with the MathCAD software

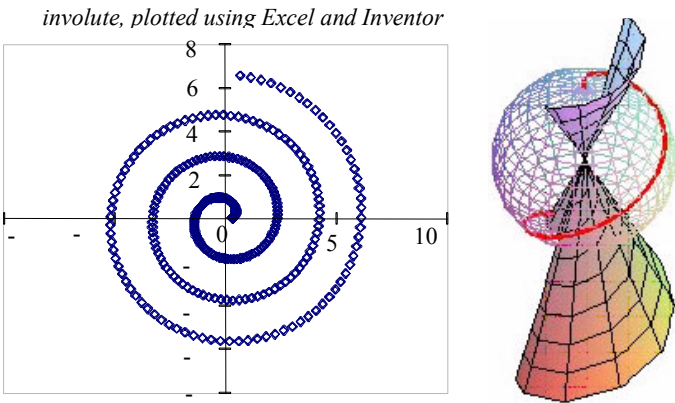


Figure 8.36b. Plotting an involute using Excel and MathCAD (3D)

It can also be defined by an intrinsic equation:

$$R_c^2 = 2 \times k \times s \tag{8.58}$$

where R_c is the radius of curvature and s the curvilinear abscissa to swing following the path of a cycloid (Figure 8.36(c)).

Moreover, in physics, we can also address the involute in a kinematic approach. To do this, it suffices to consider a curve, crossed by a uniformly varied motion, is such that the rotational speed is constant ω .

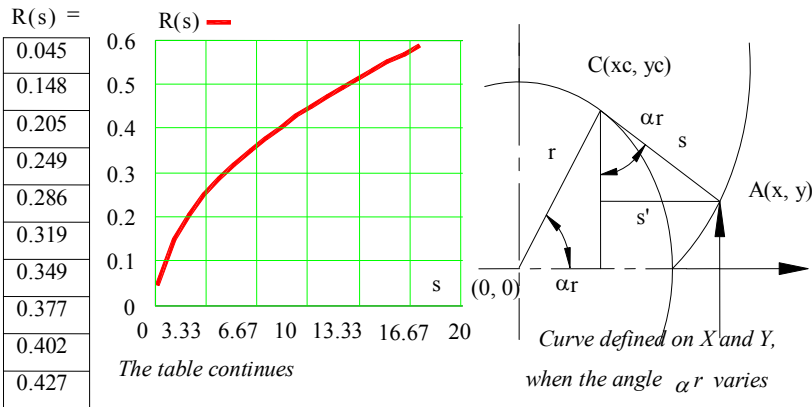


Figure 8.36c. Another plotting of an involute of the circle with the MathCAD software

8.7.2. Metrological control of the main types of gears

Gear measurement: different methods of measurement (Figure 8.37) allow dimensional inspections on gear teeth to be performed (range, depth, projection, etc.). The most accurate method, but also the longest, is to use a CMM with a fine probe. Here we present two other methods that are quick, widespread in the industry of spur gears or helical gears, inexpensive, and allowing measurement of certain characteristics of a gear.

The measurement is carried out using two balls (d_b) or two cylinders with a diameter (d_c). The distance M is measured according to the following formula:

$$M = \left(\frac{d_b}{p} \right) \times \left(\frac{1}{\cos(\varphi_c) + \frac{\pi}{2N}} \right) + d_c \quad [8.59]$$

For a given gear (having the cutting angle of the cutting tool φ_c , the number of teeth N , and the base diameter d_b), we can determine the theoretical distance M using the diameter d_c of the balls or cylinders. If we prefer the contact to be located on the pitch circle of the gear to be measured, then the diameter d_c should be equal to:

$$d_c = d_b \times \left(\tan \left(\varphi_c + \frac{\pi}{2 \times N} \right) - \tan(\varphi_c) \right) \quad [8.60]$$

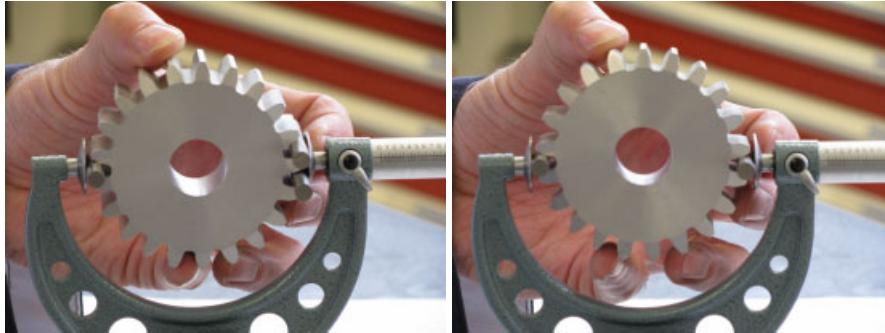


Figure 8.37. Measurement over the wires with a disc type outside micrometer

8.7.2.1. Spacing of a micrometer during measurement

The measurement is performed using a micrometer. A distance M is measured over a number of teeth Z . The two points of contact between the micrometer and the

“anti-homologous” faces of the gear are on the same circle of diameter D_c . The spacing M will be equal to m :

$$M_{\text{deviation}} = \cos(\phi_c) \times \pi \times \left(Z - \frac{1}{2} \right) + N \times \phi_c \quad [8.61]$$

For a correct measurement, the following condition should be met: the point of contact between the micrometer and the tooth must be above the pitch circle. Considering the diameter D_c of the contact point, this condition is then written:

$$D < D_c < D_0 \quad [8.62]$$

The minimum number of teeth to be measured is calculated using the following equation:

$$Z_{\min} = \left(\frac{1}{2} \right) + \left(N \times \frac{\phi_c}{\pi} \right) \quad [8.63]$$

8.7.2.2. Interpretations of the measurement carried out through these two methods

– Comparison between $M_{\text{theoretical}}$ and M_{measured} gives us qualitative information on the gear being measured. By making a measurement at different locations, we can for example check that the module m (the spacing between the teeth) is constant.

– However, be cautious with respect to the interpretation of measurements. We can, for example, obtain the same measurement values for M in two different locations whereas the module m and the diameter d of the gear are different. Errors on these two parameters can be offset and distort interpretations.

8.7.3. Spur gears with helical teeth

Having the same use as the foregoing; they are widely used in power transmission. Gear teeth are tilted with respect to the axis of rotation of the two shafts. With identical cutting, they are more efficient than the previous ones to transmit power and torque. *The helix angle*, constituting the inclination of the teeth, generates an axial force which increases rapidly when the angle of inclination also increases. It is, therefore, necessary to counter the force by adding, to the system, thrust bearings. Because of the angle of inclination, the basic formulas used for spur gearing dimensioning shall be slightly changed accordingly:

$$\tan(\alpha) = \left(\frac{\pi \times D_{\text{pitch diameter}}}{\text{helix height}} \right) \quad [8.64]$$

8.7.4. Helical gears with parallel axes

The inclination of the teeth generates an axial force that increases rapidly when the angle of inclination increases. This force should, therefore, be countered by adding, to the system, thrust bearings. In these circumstances, values chosen for α should not be greater than 25° for helical gears with parallel axes. This angle is not harmonized among the different manufacturers. Some manufacturers have chosen (17° $45'$). In summary, the same helix angle should be chosen for the two gears, for compatibility reasons. Additionally, the helix angle has an influence over the pitch diameter, thus the center distance. Finally, the helical gears, with parallel axes inclined to the left, mesh with helical gears with parallel axes inclined to the right.

8.7.5. Parallel spur gears with helical teeth

Similarly to straight-tooth gears, they enable the transmission of motion between two parallel shafts. The angle of inclination of the gear teeth, the helix angle, is the same for both gears, but in opposite direction. Some applications are mounted on non-parallel shafts (in this case the gears are called in French: *engrenages gauches*). Among the advantages of helical teeth, we quote:

- more supple transmission; more gradual and less noisy;
- greater bearing contact/engagement (two, three or four pairs of teeth still engaged);
- transmission of significant loads, high speeds;
- easy realization of an imposed center distance (by varying the value of helix angle).

Drawbacks: additional structural stress due to the helix angle (axial force on the shaft bearings and accentuation of the shaft flexion torque) less efficiency. They cannot be used as portables (some gearboxes, etc.); these gears must always remain engaged.

Helix angle β : it measures the inclination of either the teeth or the helix, relative to the axis of the gear (normally, values ranging between 15° and 30°). High values of β provide more smooth-running and progressivity but result in higher axial forces. A spur gear is a helical gear with $\beta = 0^\circ$.

Actual magnitudes (or normal): p_n , m_n and $\alpha_n = 20^\circ$. They are ISO standardized and are measured perpendicular to the helix. The values of an actual module m_n are to be chosen among the standardized values of the module m , which are indicated for the straight-teeth. $p_n = \pi \cdot m_n$ (note that $p_{n1} = p_{n2} = p_n$).

Apparent magnitudes (or tangential): p_t , m_t , and α_t are not standardized and depend on the value of β . They are measured in the plane of rotation of the gear (an analogy with straight teeth):

$$\tan(\alpha_n) = \tan(\alpha_t) \times \cos(\beta) \quad [8.65]$$

$$m_t = \frac{m_n}{\cos(\beta)} \quad [8.66]$$

$$p_t = \frac{p_n}{\cos(\beta)} = \pi \times m_t \quad [8.67]$$

The center distance depends on the value of the angle β . By varying β , we can obtain any desired center distance, which may be interesting for the gear trains:

$$a = (r_1 + r_2) = \left(\frac{d_1 + d_2}{2} \right) = (Z_1 + Z_2) \times \left(\frac{m_t}{2} \right) = (Z_1 + Z_2) \times \left(\frac{m_n/2}{\cos(\beta)} \right) \quad [8.68]$$

Width b : for reasons of continuity and progressivity, the gear width b must be greater than the axial pitch p_x .

Forces generated by meshing gears: for helical gears with crossed axes, the inclination of the teeth generates an axial force. This axial force increases rapidly when the inclination angle α increases. Among the advantages, disadvantages and tips, we quote:

- the meshing is silent, vibrations are reduced;
- a wide choice of reduction ratios is possible;
- the performance is low (40–70%), and depends on the materials and the type of lubrication;
- the bearings should withstand significant axial forces.

8.7.6. Bevel or concurrent gears

They are used to transmit motion between concurrent shafts, whether they are perpendicular or not (gears whose axes are concurrent). The gear teeth are cut into a conical surface. The teeth may be straight but also helical or spiral. The bevel gears (or concurrent) are a large group used to transmit motion between two non-parallel shafts whose axes are concurrent. Axes at 90° are the most common. Pitch surfaces are no longer cylinders but cones (pitch cone). The cones are tangent to a contact line MM' and their common apex is the point S , which is also the intersection of the axes of rotation of both gears as designated in Figure 3.38(b). This type of gear allows us to make a bevel gear. Unlike spur gears (/), bevel gears of the same

module are not interchangeable. A pinion can mesh only with the gear with which it has been calculated and manufactured. The pairs of bevel gears are used for power transmission or motion at 90° .

This type of gears runs in inseparable pairs. A bevel gear is designed to work with another complementary bevel gear. A bevel gear is compatible with one particular bevel.

Among the advantages and disadvantages, we note that:

- The bearings must withstand an axial force. There is an allowance for bevel gear.
- Bevel gears are made to be engaged (to mesh) in pairs.
- The apex of the cone shall coincide with the point of intersection of the axes (precise fitting).

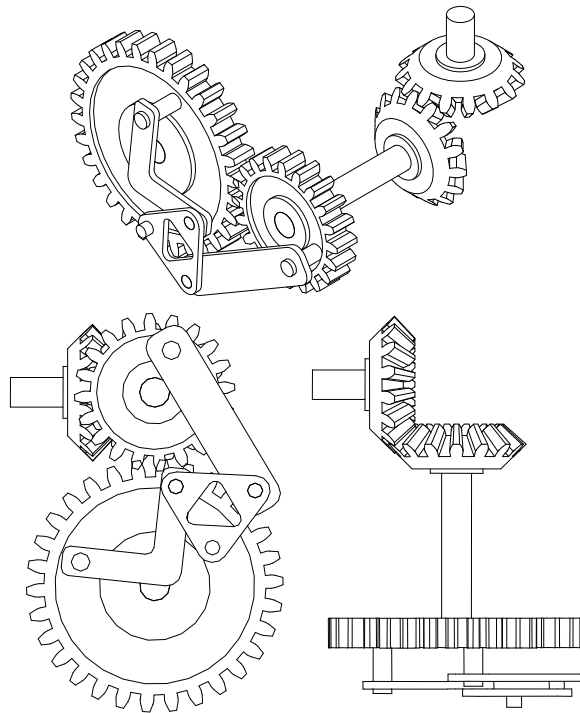


Figure 8.38a. *Presentation of bevel and cylindrical gears*

NOTE.— Why address all these issues about the various gears in a work devoted primarily to dimensional metrology?

First of all, the virtues of the profession as instructors/trainers encourage us. Additionally, we cannot control or measure *accurately* unless we master the origin of defects and errors. However, the classic mistake made by some “measurers” is to restrict explanations of measurements to a mere reading of a measuring instrument even if it was the best in the world. Certainly, there is a risk of “pedagogical” fragmentation when more details are given on certain aspects of such modules.

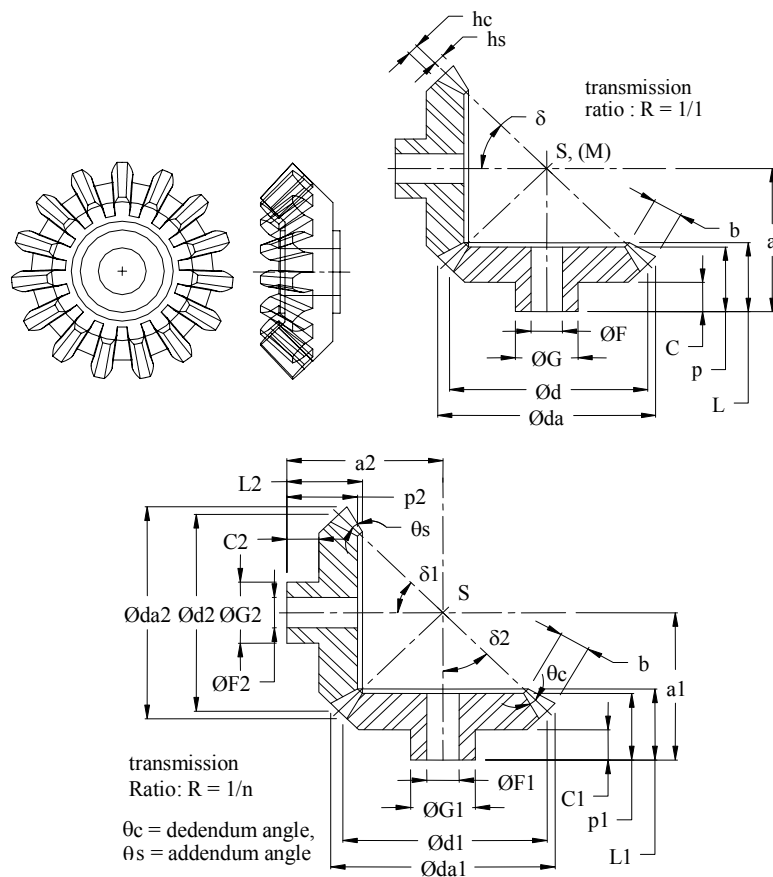


Figure 8.38b. Presentation of bevel gears

The dimensions recommended in gears industry are grouped in an example of two tables (Tables 8.10 and 8.11).

Dimensions of bevel gears, reduction ratio 1/1										
m	z	d	d _a	b	C	p	L	G	F, H7	a
1	19	19	20.4	4.5	9	12.5	14	16	6	19.8
1.5	19	28.5	30.6	7	11.5	17	19	25	8	27.6
2	19	38	40.8	10	15	23	26	33	10	37
3	19	57	61.2	17	25	37.5	42	50	15	57.2

Dimensions of bevel gears, reduction ratio 1/2										
m	z2 / z1	d2 / d1	d _{a2} / d _{a1}	b	C2 / C1	p2 / p1	L2 / L1	G2 / G1	F2/F1, H7	a2/a1
1	20/40	20/40	20.4	6	9/8.5	13.5/12	15/14	17/20	6/6	29.32/20.67
1.5	20/40	30/60	30.6	8	11/10	18/15	20/18	26/35	8/8	42.35/28.41
2	20/40	40/80	40.8	10	15/12	24/20	27/24	34/45	10/10	57.4/ 38
3	18/36	54/108	61.2	17	17/17	23/29	36/34	45/55	15/15	73.8/ 51.5

Table 8.10. Examples of bevel gears dimensions, according to the documentation "Prud'homme Transmission"

Gear	Symbol	Formula
Module	m	p/π
Pitch	p	$m \times \pi$
Pitch circle diameter	d	$Z \times m$
Number of teeth	Z	d/m
Tip circle diameter $\varnothing$	d_a	$d + mZ\cos(\delta)$
Base circle diameter $\varnothing$	d_b	$d - 2.5 m\cos(\delta)$
Addendum	h_s	m
Dedendum	h_c	$1.25m$
Pitch angle	δ	$\sin(\delta) = d/2L$
Addendum angle	θ	$\tan(\theta_s) = m/L$
Tip angle	δ_s	$\delta_s = \delta + \theta_s$
Tooth width	b	$b = km; 4 \leq k \leq 6$

Table 8.11. Main formulas for bevel gears calculations

8.7.7. Worm gears

The transmission is performed between two orthogonal shafts (perpendicular to each other but not concurrent). One gear resembles a screw and the other a right helical gear. The direction of rotation of the gear depends on that of the screw but also the inclination of the teeth (right hand thread or left). These gears allow large reduction ratios with a single pair of gears (up to 1/200) and offer possibilities of irreversibility (a screw thread). They give smoothest transmission of all gears, with no noise and no impact:

- this gearing system allows an important speed reduction;
- in some circumstances, the system may be irreversible;
- the performance of the system is low. It increases the angle of inclination of the teeth;
- the helix direction is the same for both the screw and the gear;
- there is no standardization for worm gears. In case of failure, the faulty component should be replaced by a strictly identical one.

Downsides: a sliding and significant frictions leading to poor performance. As a result, they are limited to moderate power; require proper lubrication and torques with low friction materials (e.g. steel screws with bronze gear). There is irreversibility when the screw is likely to carry the gear and not the reverse. However, if the gear can also entail the screw (case of screws with several threads), then there is reversibility. The main characteristics are:

- This system allows a significant reduction. Under certain conditions, the system may be irreversible. The performance is low and increases as a function of the angle of tooth inclination. The direction of helix is the same for the screw and the gear.
- There is no standardization for the worm gear.
- In case of failure, the faulty component should be replaced by a strictly identical element. In most cases, the part should be manufactured based on a model (Table 8.12).

Gear	Symbol	Formula
Module	m	$p \div \pi$
Pitch	p	$m \times \pi$
Pitch diameter	d	$Z \times m$
Number of teeth	Z	$d \div m$
Dedendum diameter (root circle)	dc	$(Z + 2) \times m$
Worm	Symbol	Formulas
Outside diameter	D	$(2 \times m) + d$
Tangent to the angle of inclination	β	$(m \times Z) \div d$
Pitch diameter	d	$p \times Z = \pi \tan \beta$
Center distance	a (or C)	$(d(\text{gear}) + d(\text{screw}))/2$

Table 8.12. Major formulas for worm gear calculation

8.7.8. Racks

A rack is a gear with an infinite pitch diameter. Therefore, the remarks that apply to gears also apply to racks. To be able to mesh, the rack must have a module

identical to the mating gears module. A helical gear with parallel axes can mesh with a helical-tooth rack, if the module is identical and if the helix angle is complementary. We cannot find a rack with a helix angle of 45° . It does not present any interest because the helix angle is too closed, and the ensuing losses are too large. The rack presents a translational motion. In this case, it is important to know the pitch p and the module $m(p = \pi \cdot m)$. To get a long rack, we should assemble, one by one, pieces of racks. Under this form, they are called “aboutable” in French, i.e. butt joint (Table 8.13).

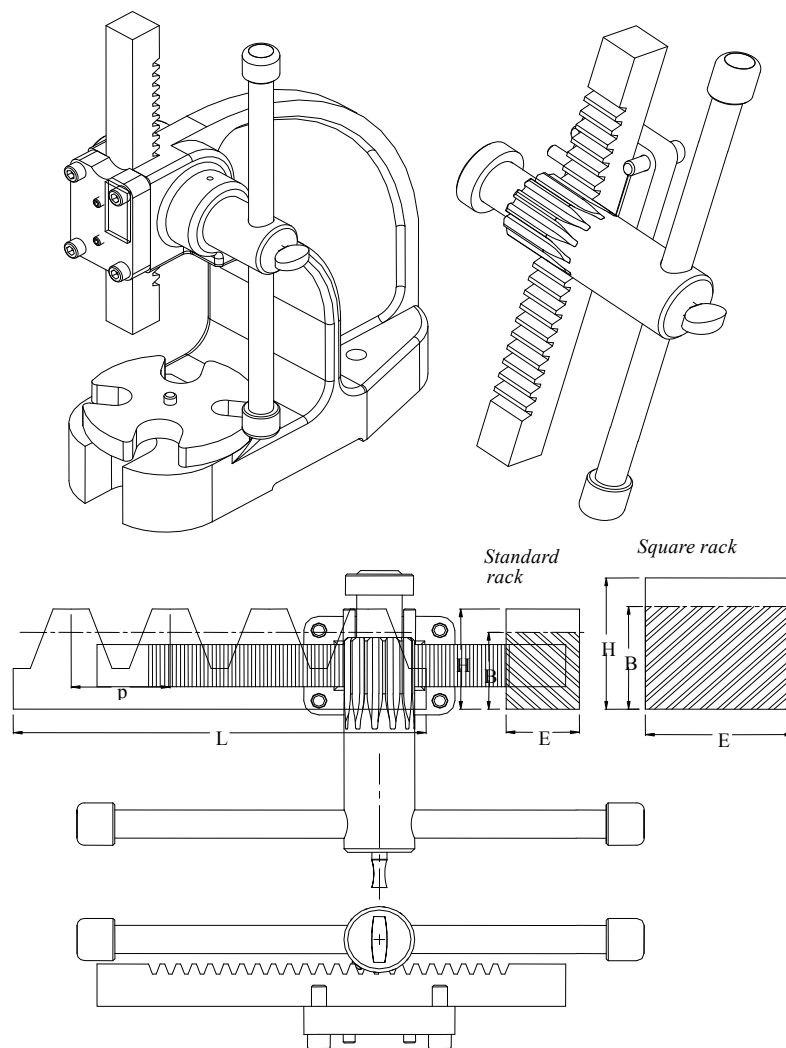


Figure 8.39. Prefabricated rack. Butts joint of rack

Module m	Z, # of teeth	$L^{(+)}$	Pitch, p	E	H	B	E Square	H square	B square
1	153	480	3.14	10	8	7	10	10	9
3/2	102	480	4.71	15	10	8.5	15	15	13.5
2	72	452	6.28	20	15	13	20	20	18
	108	678	6.28	20	15	13	20	20	18
	162	1 017	6.28	20	15	13	20	20	18
3	72 108	678	9.42	30	20	17	30	30	27
		1 017	9.42	30	20	17	30	30	27

Table 8.13. Dimensions of racks according to the documentation “Prud’homme”.
Butt-joint of two racks, possible without alteration, $L^{(+)}$

8.7.9. Control of gears with a vernier calipers

Gear tooth vernier caliper (see Chapter 3, Figure 3.39) is used in dimensional metrology [OBE 95] during the inspection and measurement of the sizes of spur gears. It is mainly used during manufacturing on machine tool stage, where the assembly does not require disassembly of the part being machined. The size of the gears is variously measurable. The instruments can hand-held, mechanical by comparison, electronic or optical. Vernier compass for gears measurement is part of the first category. One of the advantages of using the vernier compass is that it is not required to separate the gear from the assembly.

Number of teeth to avoid the interference phenomenon ($\alpha = 20^\circ$, teeth uncorrected)					
Number of teeth of pinion Z_1	13	14	15	16	17
Maximum number of teeth for the gear Z_2	16	26	45	101	1 309

Table 8.14. Uncorrected gear teeth

For uncorrected teeth, here is a table useful for the choice (gear/pinion) (Tables 8.14 and 8.15).

Indicative number of teeth to avoid interference								
Z_i Nb. of teeth on pinion	$\beta = 0^\circ$	$\beta = 5^\circ$	$\beta = 10^\circ$	$\beta = 15^\circ$	$\beta = 20^\circ$	$\beta = 25^\circ$	$\beta = 30^\circ$	$\beta = 35^\circ$
08								12
09							12	34
10						12	26	∞
11					13	23	93	∞
12			12	16	24	57	∞	
13	16	17	20	27	50	∞		
14	26	27	34	53	207	∞		
15	45	49	69	181	∞			
16	101	121	287	∞				
17		∞	∞					

Table 8.15. Indicative number of teeth to avoid interference

The performance is significant in transmitting power, it should be chosen appropriately as suggested in Table 8.16.

Performance variations η , given: $f = 0.05$ and $\alpha_n = 20^\circ$									
β (degrees)	1	2	3	5	8	15	25	30	40
η	0.25	0.40	0.49	0.62	0.72	0.82	0.88	0.89	0.90

Table 8.16. Variation of the performance η

Figure 8.40 is an example of schematization of module cutting in the median plane which is offset from the centerline of the gear.

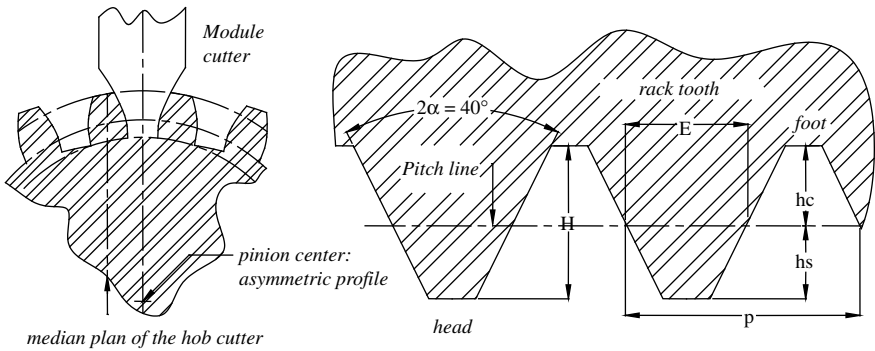


Figure 8.40. (a) Module cutter; (b) tool rack

Deterioration of gear teeth: there are three main categories of deterioration of the teeth surfaces. Alterations characterized by small metal particles that break off the teeth. The centering or positioning of gears depends on the machining of bearing rolling drivers, or shaft bearing, allowing the positioning the shafts and gears. Generally and in normal conditions (device properly designed, constructed, assembled and “set up”) that centering is automatically ensured. If an incorrect centering occurs, it results in abnormal and fast wear of rolling bearings and gears. If this is rapidly detected, the center distance between the gears must be checked and also the backlash between the teeth (clearance of gauges, for example). The relative position should, if necessary, be readjusted. The parallel faces of gears teeth in contact must be controlled (blue marking), etc. There are three main methods for measuring tooth thickness: chordal thickness measurement, the *measurement of the bearing* and measurement with by pins or balls.

8.7.10. Chordal thickness measurement

This method uses a compass for gears measurement which is referenced on the outside diameter of the tooth. The thickness S is measured on the pitch diameter [DIE 85, LON 85, OBE 95].

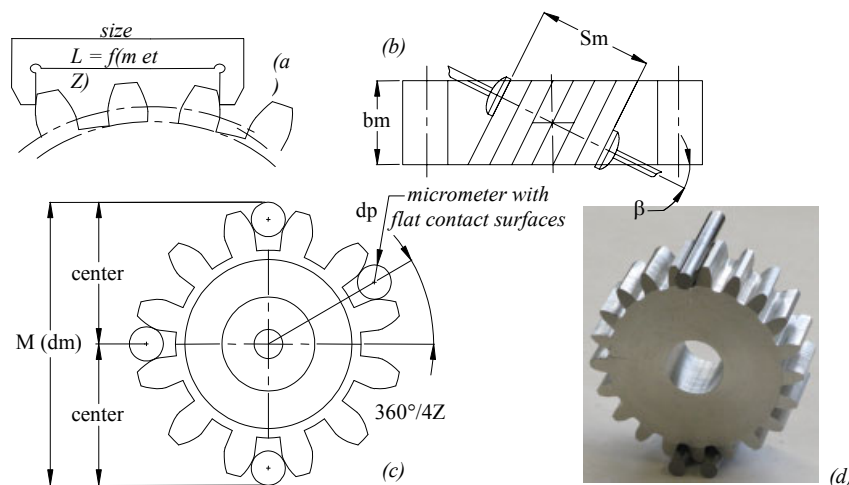


Figure 8.41. Controls: by size (a); disc type outside micrometer (b); by pins/measuring wires (c) and (d)

To measure the diameter of contact points ($S_m = \text{Spur measurement}$) of teeth using the micrometer, a reference point is required to allow measurement of the

helix. Let $b_{\min}$ be the space (of reference) or the required width. β_b is the helix angle of the base cylinder. To make a stable measurement of S_m , it is appropriate to apply this:

$$b_{\min} = S_m \times \sin(\beta_b) + \Delta b \text{ where } \Delta b \geq 3mm \quad [8.69]$$

8.7.11. Over wire measurement

As shown in the preceding figures, the measurement over wires, as the term implies, is made out of measuring wires inserted in diametrical opposition. The procedure for measuring the arc with a measuring wire, as shown in the preceding figures, consists of placing a pin (ball) in the space between the teeth and, then, uses a micrometer between them (measuring wires) and the reference surface. To measure the diameter over the wires M when the measuring wires are at equal distribution of teeth of the pinion, the following expression is applied:

$$M = 2 \times \left(\frac{\text{distance to the center of the pinion}}{\text{measuring wire}} \right) + (\text{wires diameter}) \quad [8.70]$$

To measure the distance over the wires, in the second case, the following formula applies:

$$M = 2 \times \left(\frac{\text{distance to the center of the pinion}}{\text{measuring wire}} \right) \times \cos\left(\frac{\pi}{2/Z}\right) + (\text{wires diameter}) \quad [8.71]$$

The internal gears are measured in the same way, except that the measurement is performed between the wires (internal). The helical gears can be measured only with balls. In the case of worm screw, three pins are used. The following equations allow the calculation of tooth thickness, regardless of the location of points of contact of wires. For consistency of monitoring forms below, it is recommended to refer to Figures 8.37 and 8.41. For a given number of teeth, we set the expression of mean diameter:

$$d_m = \left(\frac{d \times \cos(\varphi)}{\cos(\varphi_1)} \right) + d_p \quad [8.72]$$

For a number of teeth with given size, the d_m will take the following form:

$$d_m = \left(\frac{d \times \cos(\varphi)}{\cos(\varphi_1)} \right) \times \cos\left(\frac{\pi}{2 \times Z}\right) + d_p \quad [8.73]$$

Where the value of diameter is obtained from:

$$\text{inv}(\varphi_1) = \left(\frac{S}{d}\right) + \text{inv}(\varphi) + \left(\frac{d_p}{d \times \cos(\varphi)}\right) - \left(\frac{\pi}{2}\right) \quad [8.74]$$

When the thickness of the tooth S is calculated based on that known over the wires dm , the following equations can be used in order to find S :

$$S = d \times \left(\frac{\pi}{2} + \text{inv}(\varphi_c) - \text{inv}(\varphi) - \frac{d_p}{\cos(\varphi)} \right) \quad [8.75]$$

$$\cos(\varphi_c) = d \times \frac{\cos \varphi}{2 \times R_c} \quad 8.76]$$

Worm gears profile is of type III. They are the most used, with a pressure angle $\alpha_c = 20^\circ$.

8.7.12. Measuring thickness of rack teeth

The gear teeth of rack can be measured using disc-type outside micrometers [OBE 95] or a special vernier [STA 97], a CMM or a profile projector. However, there are two ways of measuring the thickness of teeth which do not require special tools. To do this, it suffices to place two pins in the spacing between teeth and measure the distance over the pins (or over the balls), thus we can read the thickness of the teeth. This measurement using a ball with a diameter = 0.4200 inch gives a diameter over pins of 6.5388 inches. The use of pins of small sizes is not appropriate because the pins drop to the hollow between teeth and jam in the root of tooth. Thus we cannot properly use the micrometer. There are two standards for the pitch circle diameter:

$$D_{\text{pide}} = \frac{1.680}{D_p} \quad \text{or} \quad D_{\text{pide}} = \frac{1.728}{D_p} \quad [8.77]$$

Note that, the problem is not due to the disc-type micrometer that is used, but to the measuring pins themselves. These pins are not chosen randomly; they are standardized pins. The mathematical calculation of the *involute* is essential in this case. Moreover, if the pins have large size; we will not be able to measure the diameter using a micrometer as shown schematically in Figure 8.42.

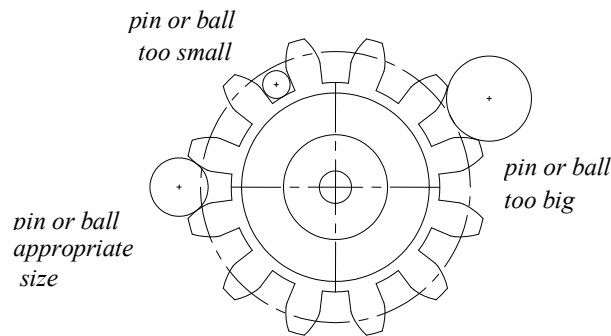


Figure 8.42. Measurement and control depending on the size of balls and pins

8.8. Introduction to spline control

We present the control of the inner diameter value d , the outer diameter D_e and the number of splines and their width b . The typical example of a single spline is the key shaft. There are various cases of splines. The difference from one spline to another one is the force that is supposed to be withstood or transmitted. We can get the centering grooves through the inner diameter D by setting a clearance on the outer diameter and a clearance at the base of the splined shaft. We can also do this by the outer diameter D_e by setting a clearance on the inner diameter. Figure 8.43 illustrates the foregoing [OBE 96, TEI 00].

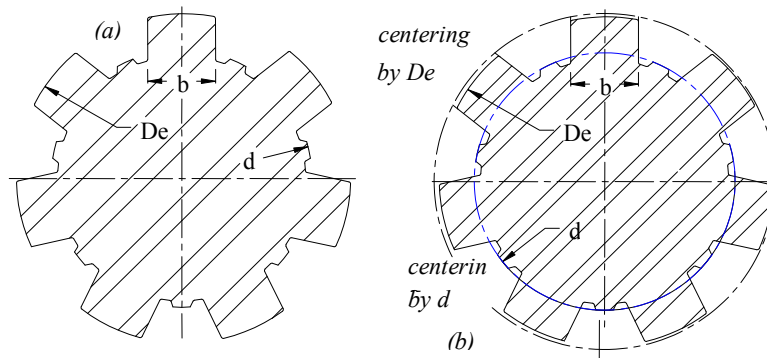


Figure 8.43. (a) Splined shaft; (b) centering splined shafts and hubs

Splined shaft (DIN ISO 14-B): with parallel flanks, the splined shaft is also used to transmit significant forces (see Figure 8.44).

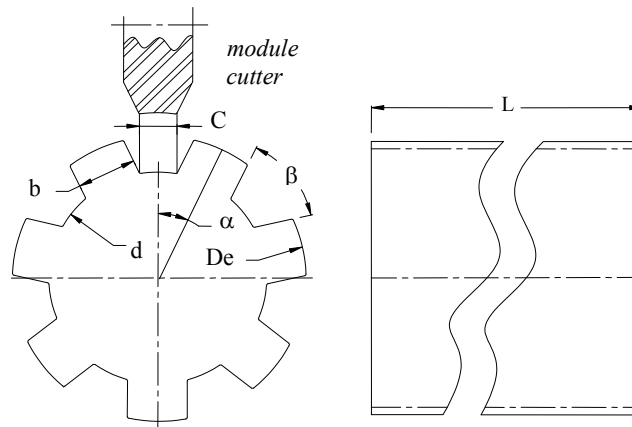


Figure 8.44. *splined shaft (external splines)*

Splined ring (DIN ISO 14-A): see Figure 8.45 hereinafter.

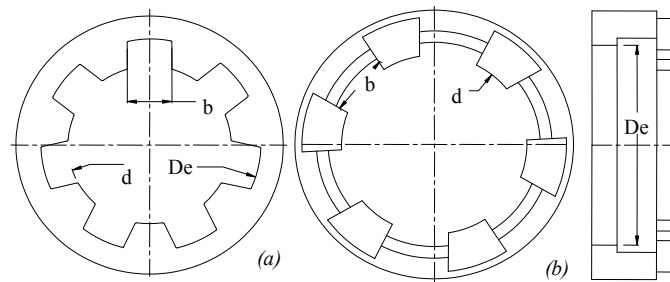


Figure 8.45. *Splined Ring (a, internal splines); parallel flanks (b)*

Industrial applications using fluted shafts and rings are among many other varied examples. We will not discuss this issue from the aspect of mechanical engineering, but under the aspect of dimensional metrology. Using some examples, we will demonstrate, how to control grooves by mechanical comparison.

Standard processes of splines manufacturing: the splines, even if they do not present the same technological interest as in the case of threads or gears, their use is important in cases of multiple parallel keys or the transmission of significant power. When there is significant load/force, a groove or a thread may not be sufficient to support the phenomenon of matting applied in the grooves. For this reason and for many others, both internal and external splines are used. We opted to present the technological processes of mechanical manufacturing of splines for the simple

reason that their manufacturing method creates a curiosity about the instruments of control of the forms that would result.

8.8.1. Dimensional control of splines

Rings and buffers: The rings and buffers are used as tools of control, by mechanical comparison, of grooves. They are used as maximum and minimum limits. Figure 8.46 illustrates an example of control instruments.

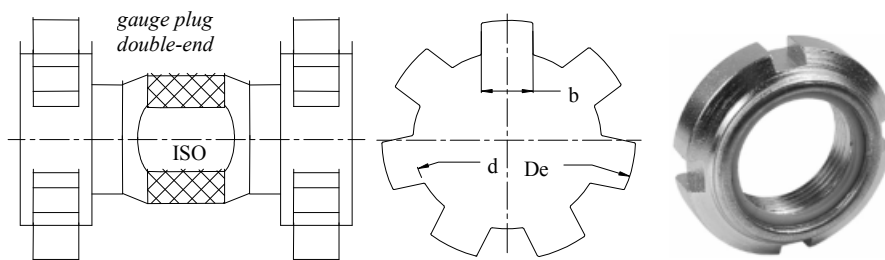


Figure 8.46. double-end splined gauge plug

8.8.2. Control of the geometric correction of splines

In mechanical serial manufacturing, when adjusting the machine, it is useful to take some parts and to control their dimensions, on the splined shafts [CAS 78, DIE 85]. Ordinarily, the hub results from a broaching process; this means that the passage of the gauge plug remains a sufficient safeguard. Figure 8.47 illustrates the foregoing.

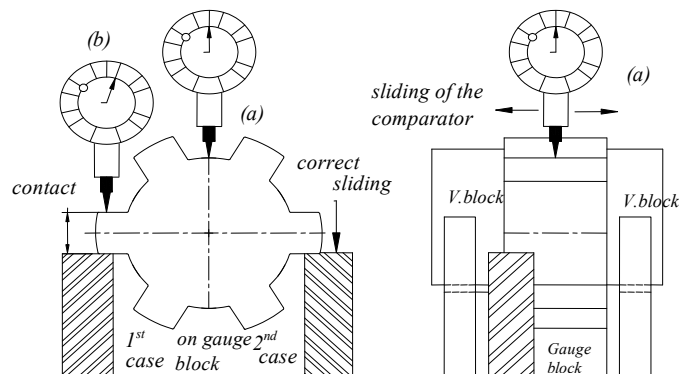


Figure 8.47. Total control of the correction of a splined shaft

The shaft is placed on two V-blocks with its cylindrical bearings. We do adjust two gauge-blocks (case 1 and case 2) under two pins. One of the two blocks is used to calibrate the comparator. The sliding of the comparator on “ a ” will show if there was an error along the width of the spline b . Meanwhile we check the defect of parallelism to the axis. The errors of division of the spline are also highlighted by a clearance or forcing the gauge-block (case 2). Concentricity is also controlled using comparator, in b for both the inner diameter d and the outer diameter D_e , depending on the centering mode that is adopted.

8.8.3. Woodruff key – standardized ANSI B17. 2-1967 (R1998)

This type of keys is defined by the standard as “demountable machinery part which, when assembled into key-seat provides a positive means for transmitting torque between shaft and hub”. Its identification number allows easy reading of the key dimensions [LON 85, OBE 96]. The keyway is circular (Figure 8.48). We shape them using the Woodruff keyseat milling cutter keyway bars stretched to the profile. Then, the parallel faces are ground. Once assembled into the milled keyway inside the shaft, the key will play the same role as usual parallel key. Among the advantages of *Woodruff* key, we mention the simple realization of keyway, easy fitting to any taper in the mating assembly. Among disadvantages of this key is that the keyway weakens the resistance of the shaft. The *Woodruff* keyway dimensions are defined by ANSI B17.2-1967 (R1998).

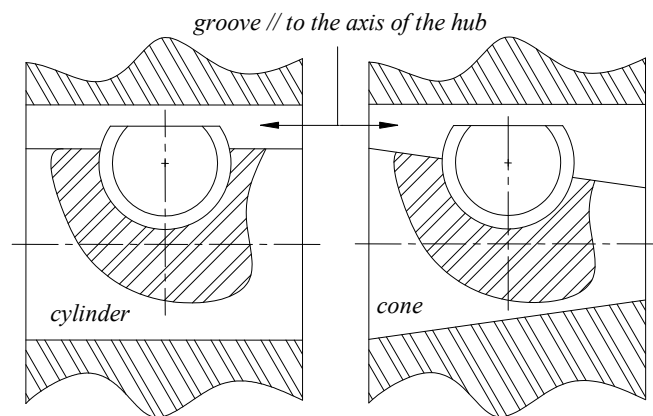


Figure 8.48. Woodruff keys

It is preferable not to use Woodruff keys to transmit significant torques. Recommended tolerances in dimensional metrology for keys are: $h9$ to $h11$. We should be careful in choosing the tolerance, since, depending on the location,

we may also choose E9, P9, and H13. Therefore, it is advisable to refer to handbooks [EID 85], [OBE 96], dedicated to mechanical engineering.

8.8.4. Control of key-seats

We address this topic according to the American standard ANSI B17.1-1967, approved in 1973, revised in 1989 [OBE 96]. This standard establishes a uniform relationship between shaft sizes and key sizes. A key is a demountable part which, when retained into a key-seat, transmits torque between a shaft and a hub. The keyway is located in a shaft or hub parallel to its axis. ANSI B17.1-1967, R1989 provides the use of two shanks called “Key shank” (or key stock) one of which is recognized with a tolerance always positive, while the other one, called normal, recognized with a negative tolerance. As a result of these two types of key shanks (key stock), two classes of parallel keys are recognized, one of which is not recommended:

- *Class 1*: Tolerances of this class allow obtaining a lateral adjustment with clearance. This type parallel keys applies only to parallel keys.
- *Class 2*: Tolerances within this class allow obtaining lateral adjustment with clearance or interference. This type is suitable for forced keys.
- *Class 3*: Tolerances within this class correspond to degrees of non-standardized interference. The ANSI standard recommends, then, the use of data from class 2.

In case the shaft has several diameters and other shoulders, the dimensions of the key, corresponding to the diameter where the key will be encased, it is however recommended to meet the condition of nominal depth of the shaft housing [FAN 94, OBE 96]. In metrology, the geometric tolerances to be controlled are focused on parallelism of housing with respect to the axe of the shaft or hub. For example, the ANSI standard provides 0.002 inch for housings with up to 4 inches in length.

8.8.5. Calculating the depth of the housing (groove) and the distance from the top of the key

Regarding the milling of the key-seat, the total depth of the cut from the outer side of the shaft up to the base of housing is given by $(M + D)$ where D is the depth of the housing. In metrology, we inspect the shaft/key assembly by measuring J between the top of the key and the outer side (opposite) of the shaft (see Table A5.16 in Appendix 5) [OBE 96, TEI 00].

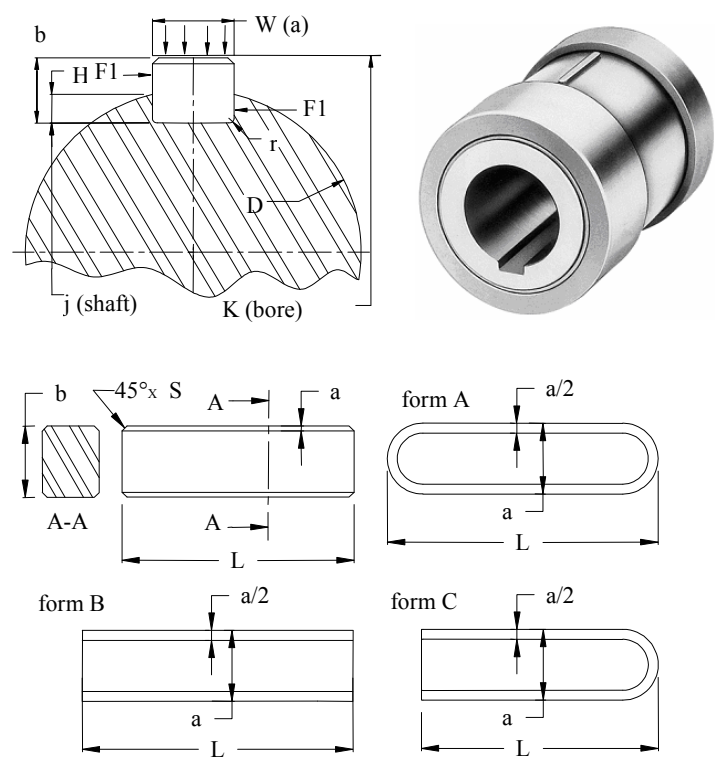


Figure 8.49. Illustration of keys (shaft-hub) in metric and imperial units

Main forms of keys in imperial units (inches)

Keyway (or keyseat)	on a	On shaft → On hub →	N9 Js9
	on j	If $d \leq 22$	+0.00 -100
	on j	if $22 < d \leq 130$	+0.00 -200
	on k	If $d \leq 22$	+100 -0.00
	on k	if $22 < d \leq 130$	+200 -0.00
Key	on a	H9	
	on b	H9 if $b = a$	H11 if $b \neq a$

Table 8.17. Tolerances on keys and keyways (in μm)

The foregoing results may be expressed as (Table 8.17):

$$J = D - (A + H) + b \quad [8.78]$$

where b is the height of the key. To facilitate the control, we can calculate the value of A :

$$A = \left(\frac{D - \sqrt{D^2 - W^2}}{2} \right) \quad [8.79]$$

where D is the shaft diameter (in inches), W (or a) is the key width. There is a simple formula to calculate A to the nearest thousandth of an inch. Here is an example:

NUMERICAL APPLICATION.—

$$\text{given } W = 0.0099 \text{ in, } D = 0.4060 \text{ inch} \Rightarrow A = \left(\frac{D - \sqrt{D^2 - W^2}}{2} \right) = 6.036 \times 10^{-5} \text{ inch}$$

Ordinarily, the key is subjected to contact pressure on the half-flank in contact with the hub. Only the criterion of resistance to overlaying should be inspected, because the energy criterion is not often significant due to the low sliding velocity between the hub and the shaft. Experiment has shown that, depending on the type of the installation, the length of the key or the spline become key factors, in order to meet the condition of resistance. We will estimate the length (L in mm) of the key which meets the condition of resistance: $P \leq P_{\text{max permissible}}$. We use the diagram in Figure 8.49 and Table 8.18.

Ø Nominal	Shaft and hub				Parallel key: A , B , and C forms			
	shaft	bore					(+)	
d	j	k	r	$W(a)$	a	b	L	S
$6 \leq d \leq 8$	$d-1.2$	$d+1$	8/10 to 16/100	2	2	2	6 to 20	16/100 to 1/4
$8 < d \leq 10$	$d-1.8$	$d+1.4$	8/10 to 16/100	3	3	3	6 to 36	16/100 to 1/4
$10 < d \leq 12$	$d-2.5$	$d+1.8$	8/10 to 16/100	4	4	4	8 to 45	16/100 to 1/4
$12 < d \leq 17$	$d-3$	$d+2.3$	8/10 to 16/100	5	5	5	10 to 56	16/100 to 1/4
$17 < d \leq 22$	$d-3.5$	$d+2.8$	8/10 to 16/100	6	6	6	14 to 70	16/100 to 1/4
$22 < d \leq 30$	$d-4$	$d+3.3$	8/10 to 16/100	8	8	7	18 to 90	16/100 to 1/4

$30 < d \leq 38$	$d-5$	$d+3.3$	8/10 to 16/100	10	10	8	22 to 110	16/100 to 1/4
$38 < d \leq 44$	$d-5$	$d+3.3$	8/10 to 16/100	12	12	8	28 to 140	16/100 to 1/4
$44 < d \leq 50$	$d-5.5$	$d+3.8$	8/10 to 16/100	14	14	9	36 to 160	16/100 to 1/4
$50 < d \leq 58$	$d-6$	$d+4.3$	8/10 to 16/100	16	16	10	45 to 180	16/100 to 1/4
$58 < d \leq 60$	$d-7$	$d+4.4$	0.25 to 0.4	18	18	11	50 to 200	4/10 to 6/10
Etc. ...	(+ length of keys = 10-12-14-16-18-20-22-25-28-32-36-40-45-50-56-63-71-80-90-100-110-125-140-160-180-200 up to 500							

Table 8.18. Dimensions of parallel keys (steel, resistance > 600 MPa)

The radius is imperative to ensure resistance at the root of the cutting. The sharpest the angle, the more likely a rupture occurs (we will discuss this point in Chapter 9).

Problem on an A-shaped parallel key (ISO): the connection transmits a torque $M = 69 \text{ Nm}$ and the permissible pressure $\sigma_{\text{applied}} \leq [\sigma] = 20 \text{ MPa}$; $a = 14 \text{ mm}$; $b = 10 \text{ mm}$ and $\delta = (d-j) = 8 \text{ mm}$; shaft diameter $d = 42 \text{ mm}$, $L = 42 \text{ mm}$.

Solution: on the surface (S , in mm^2), an effort will lead to a medium contact pressure (Hertz) that is written as follows:

$$F_{\text{hub}} = \left(\frac{M}{r} \right) = \left(\frac{P}{S} \right); \text{ where } S = L \times H \quad [8.80]$$

Since H corresponds, in our case, to the half-flank: $H = (b - \delta) = b - (d - j)$, in mm. The effort is applied along the length L called effective length (mm). Using equation [8.45], we calculate the applied pressure: $\sigma_{\text{applied}} \leq [\sigma_{\text{admissible}}]$.

NUMERICAL APPLICATION.—

$$\text{Given } M = 69\,000 \text{ N.mm}; r = 21 \text{ mm}; \text{ using [8.77]: } F_{\text{hub}} = \left(\frac{M}{r} \right) = 3.286 \times 10^3 \text{ N}$$

Based on the admissible conditions derived from the fundamental laws of materials' resistance, let us consider the adequate expression to our problem:

$$L \geq \left(\frac{F}{H \times \sigma_{\text{admissible}}} \right) \quad [8.81]$$

We know from the technical literature [DIX 00] that the mounting conditions require minimum dimensions to the key. Depending on the shape of the key (see Figure 8.50), its actual length is then written:

- form A: $L_{\text{actual}} = L + a$;
- form B: $L_{\text{actual}} = L$;
- form C: $L_{\text{actual}} = L + a/2$:

$$\sigma_{\text{admissible}} = 20 \text{ MPa}; b = 16; \delta = 8; H = (b - \delta) = 2(H = 1/2 \text{ flank})S$$

$$= L \times H = 2L(H = L)$$

$$F = 1.643 \times 10^3 \text{ N} \Rightarrow L \geq \frac{F}{H \times \sigma_{\text{admissible}}} \geq 41.075; \text{ with } a = 14,$$

$$L_{\text{admissible}} = L + a = 55.075 \text{ mm}$$

$L_{\text{actual}} = 55.075 \text{ mm}$ and it is concluded that the key's effective length L is chosen accordingly as it satisfies the condition of permissible resistance (should not be exceeded) that is 20 MPa.

Conclusion: $L_{\text{actual}} \geq L_{\text{effective}}? \rightarrow \text{Yes, since } 56 \text{ mm} \geq 42 \text{ mm}$

Similarly, we can check the length of the spline and thus compare it with the actual length which will be measured in dimensional metrology. If $d = 25 \text{ mm}$; $D = 34 \text{ mm}$; S is the tamed surface by unit of $10 \text{ mm}^2/\text{mm}$ in length; M the moment (torques) = 300 000 Nmm, the admissible constraint $[\sigma] = 80 \text{ MPa}$:

$$F = \left(\frac{M}{r} \right) \rightarrow F = 1.765 \times 10^4 \text{ N}$$

Figure 8.50 illustrates a case of calculation and control of splines.

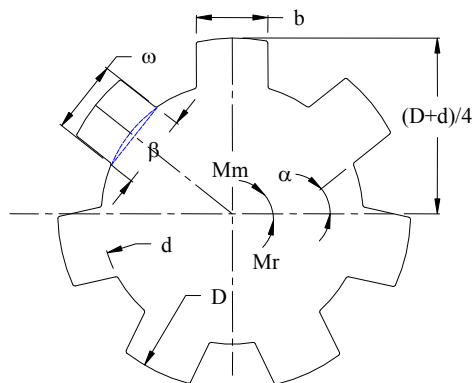


Figure 8.50. Calculation and control of the width of the spline (inches)

The spline transmits a torque $M = |M_m = Mr|$. The pressure is uniform over the length L . The surface subject to matting is then equal to: $S \times L$

The pressure applied on the half-flank of the spline is written as:

$$\sigma = \left(\frac{F}{S \times L} \right) \leq [\sigma_{\text{admissible}}] \Rightarrow L \geq \left(\frac{F}{S \times [\sigma_{\text{admissible}}]} \right) \Rightarrow L \geq 22.059 \text{ mm}$$

Considering, then, $L = 24 \text{ mm}$, we will now calculate the spline width using equation [8.47] by deliberately choosing imperial units:

$$B = 1.25 \text{ in}; n = 6; \alpha = 15^\circ \rightarrow \omega = \sin \left(\frac{(360/n) - 2\alpha}{2} \right) \times B = 0.813 \text{ in.}$$

B is the diameter of the shaft body in inches; n is the number of splines and ω the width of the groove (inches).

8.8.5.1. Square parallel key: $\varnothing 3 \frac{1}{4} \text{ in.}$

Calculate adjustments (*Max and Min clearance*) for square parallel keys $\varnothing 3 \frac{1}{4} \text{ in.}$ (Table 8.19).

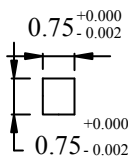
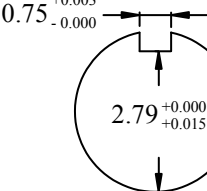
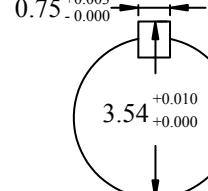
			
High and low	Clearance	Side shaft	Clearance
$J_{\text{max}} = (B_{\text{max}} - A_{\text{min}}) - C_{\text{min}} =$ $(3.596 - 2.816) - 0.748$ $= 0.78 - 0.748$ $= 0.032 \text{ inch}$	$J_{\text{min}} = (B_{\text{min}} - A_{\text{max}}) - C_{\text{max}} =$ $(3.586 - 2.831) - 0.750$ $= 0.755 - 0.750$ $= 0.005$	$J_{\text{max}} = A_{\text{max}} - C_{\text{min}}$ $= 0.755 - 0.748$ $= 0.005$	$J_{\text{min}} = A_{\text{min}} - C_{\text{max}}$ $= 0.753 - 0.748$ $= 0.750 - 0.750$ $= 0$

Table 8.19. Calculations of the adjustment of a square parallel key in μm (case 1)

8.8.5.2. Square parallel key- Class 1: Ø1-7/8 in.

Calculate adjustments (Max and Min clearance) for the square parallel key Ø1-7/8 in (Table 8.20)

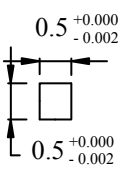
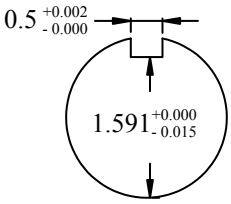
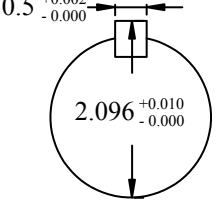
			
High and low	High and low	High and low	High and low
$J_{\max} = (A_{\max} - B_{\min}) - C_{\min}$ $= (2.106 - 1.576) - 0.498$ $= 0.53 - 0.498$ $= 0.032 \text{ inch}$	$J_{\max} = (A_{\max} - B_{\min}) - C_{\min}$ $= (2.106 - 1.576) - 0.498$ $= 0.53 - 0.498$ $= 0.032 \text{ inch}$	$J_{\max} = (A_{\max} - B_{\min}) - C_{\min}$ $= (2.106 - 1.576) - 0.498$ $= 0.53 - 0.498$ $= 0.032 \text{ inch}$	$J_{\max} = (A_{\max} - B_{\min}) - C_{\min}$ $= (2.106 - 1.576) - 0.498$ $= 0.53 - 0.498$ $= 0.032 \text{ inch}$

Table 8.20. Calculations of the adjustment of a square parallel key (μm) (case 2)

8.8.5.3. Rectangular parallel key Ø6 3/4 inches

Calculate the adjustments (Max and Min clearance) for the square key Ø6 3/4 in (Table 8.21).

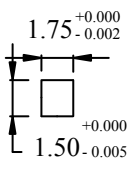
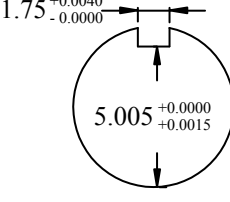
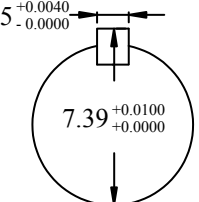
			
High and low	Clearance	Side shaft	Clearance
$J_{\max} = (B_{\max} - A_{\min}) - C_{\min}$ $= (7.4 - 5.87) - 1.495$ $= 0.035 \text{ inch}$	$J_{\min} = (B_{\min} - A_{\max}) - C_{\max}$ $= (7.390 - 5.885) - 1.500$ $= 1.505 - 1.500$ $= 0.005 \text{ inch}$	$J_{\max} = A_{\max} - C_{\min}$ $= 1.754 - 1.745$ $= 0.009 \text{ inch}$	$J_{\min} = A_{\min} - C_{\max}$ $= 1.750 - 1.750$ $= 0$

Table 8.21. Calculations of the adjustment of a square parallel key in μm (case 3)

8.8.5.4. Rectangular parallel key Ø56mm–tightened

Calculation of the *adjustments (Max and Min clearance)* for rectangular parallel key Ø 56 mm (Table 8.22).

High and Low	Clearance	Side shaft	Clearance side bore
$J_{\max} = (A_{\max} - B_{\min}) - C_{\min}$ $= (60.5 - 49.8) - 9.91$ $= 10.7 - 9.91$ $= 0.79$	$J_{\min} = (A_{\min} - B_{\max}) - C_{\max}$ $= (60.3 - 50) - 10$ $= 10.3 - 10$ $= 0.3$	$J_{\max} = B_{\max} - C_{\min}$ $= 15.982 - 15.957$ $= 0.025$ $J_{\min} = B_{\min} - C_{\max}$ $= 15.939 - 16$ $= -0.061$	$J_{\max} = A_{\max} - C_{\min}$ $= 15.982 - 15.957$ $= 0.025$ $J_{\min} = A_{\min} - C_{\max}$ $= 15.939 - 16$ $= -0.061$

Table 8.22. Calculations of the adjustment of a rectangular parallel key (μm)

To deduce the keyways width, we can use the appropriate respective tables (see literature on mechanical engineering).

8.9. Summary

It seems obvious that the elements of power transmission constitute an important part in dimensional metrology. Throughout these sections dedicated to threads, gears and other splines, we discussed the characteristics of the main entities and the means of their dimensional control. We have focused on the classic control (mechanics and mechanical comparison). This in no way excludes the fact of using other means such as the CMM and optical metrology (goniometer) to mention only these two means.

A screw thread is a helical surface defined by both its helix traced on the base cylinder and its profile. There are several types of threads, among which we have presented the most commonly used, with an emphasis on the ISO thread characterized by its truncated equilateral triangular shape. Ordinarily, hardware (fasteners and bolts) is controlled with thread plugs and rings: “Go,” “Not go.” As for medium quality, it requires thread rings and plugs with Max-Min limits. We can also use thread snap gauges.

In the section addressing gears, we emphasized the control of gear teeth after having presented all the various characteristics therein. For the case of splines, it is necessary to consider, additionally, the control of the assembly accuracy in terms of parallelism, concentricity of gear teeth and the axis. Yet this is the purpose of the calculations of resistance giving rise to parallel keys lengths.

The goniometer (goniometric microscope, not covered in this handbook) is also used to control the shape and position of the profile, as well as the regularity of the pitch. The profile projector is also a safe, accurate and reliable instrument to measure the same parameters of threads, splines, keyseats, or (fine) gears. In Mechanical metrology, the control of gear tooth shape is performed with a generating tool. Unfortunately this control is limited to the thickness of the tooth and the outer diameter. The vernier caliper, which was presented as part of this work, is an accurate and easy-to-use instrument.

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Chapter 9

Control of Materials Hardness Testing

9.1. Introduction to non-destructive testing

The hardness measurement is used to evaluate the influence of a surface treatment, the wear resistance of a material, and also the quality of a coating. Given the applications [AST 99, GRO 98, GUR 78, ISO 85, ISO 95, ISO 98, ISO 99a, b, ISO 00, KNO 39, LAS 92, KAL 06, OBE 96] for which the mechanical measurement is designed, the offer in terms of durometers is varied.

In schools, the problem of choosing an apparatus does not arise with the same intensity as in a firm. The rationale for device selection is based on measurement processes, in addition to relatively simple criteria. We should bear in mind that in terms of hardness, each case is a special case.

There are many methods of measuring hardness of a “material”. Some are based on measuring the dimensions of the indentation left by an indenter, others are based on the depth of penetration, and a few others on the rebounding/bouncing ball to the material surface. Usually, these are the first two tests that draw attention.

To select a device for measuring hardness, one should first select the testing method. To do this, other criteria, such as type of material, its mechanical strength, or the thickness of parts to be tested, should be taken into account. Some selection criteria are exhibited in Table 9.1.

Selection criteria	
We should consider this ...	To choose that ...
The type of the part to be analyzed: the nature of the material, strength, thickness, surface texture, size, weight, etc.	The testing method (Brinell, Rockwell, Vickers, Knoop, Shore, etc.) and the loads to be applied
The weight and bulk of the piece	A portable device (or not)
The frequency of measurements	A device operating manually or automatically
The type of analysis to perform (a quick check or a precise measure of hardness)	A portable device (or not), with automatic reading of the size of the imprint (or not), with different penetrators (or only one)
The precision required in the application of the load	The mode of application of load (deadweight or ball screw with load cells)
Measurement of hardness and knowledge of material behavior under load, modulus of elasticity of the material, etc.	A test method “classic” (Rockwell, Brinell, Vickers, etc.) or instrumented hardness test (Martens)
The investment	A portable device or not, operating automatically or manually

Table 9.1. *Criteria for choosing the appropriate test method*

Hardness is not a directly measurable value. To define hardness, it is often necessary to assess the size of the indentation left by an indenter. Some devices allow visualizing and measuring the hardness value automatically by image analysis. A piece of steel is “hard”, a thermoplastic tire is less hard, and a rubber product is even much less harder. Evaluating hardness might sound a simple approach but in fact it is a bit more complicated than what it seems to be. Intuitive and common, even trivial, the concept of hardness is likely to be one of the simplest issues to comprehend in dimensional metrology.

To evaluate the hardness of a material, we often tend to exert pressure to “feel” its strength and deformation or the imprint left at the cessation of stress test. We now understand that, unlike primary quantities such as length and weight, the hardness is not directly measurable. Henceforth we will discuss the characterization of the “resistance to penetration of a harder body”. Thus, the hardness value makes sense only if we specify the conditions under which it is measured, that is to say, the force F that is exerted on N , the type of penetrator, its shape (ball or diamond in millimeter) or the time of penetration (t in second).

To this end, the adopted methods have been technically evolved. Originally designed to measure the hardness of metallic materials, measuring devices have far exceeded this scope and now control a wide variety of materials such as ferrous alloys, plastics, glass, plaster, and even asphalt.

Canada, specifically, is experiencing a particularly growing interest because of harsh climates and “potholes” in Montreal and Ottawa. Depending on the circumstances, measuring the hardness value may help in assessing the effectiveness of heat or chemical treatment of a workpiece (e.g. cementation), to characterize its coating (e.g. paint, varnish), to evaluate its resistance to wear, or to better understand its mechanical behavior and its aging (possible cracks or hardening, or even damage in whole bridges). Among so many processes, we can distinguish the most common.

The hardness tests which have been long reserved for the strength of materials – non-destructive and destructive testing – are in fact a discipline in the field of metrology; the test depends on the means of measurements, calibration, and readings related to errors and other uncertainties. We will not explore in detail the properties of materials if it does not involve intrinsic factors of the material or other factors that would influence the measurement such as the fatigue or the flow of the material. Material characteristics, as we know, influence the choice of measurement and control method. Hardness is primarily an intuitive notion characterizing the resistance that a body offers to local strains. The main methods of measuring the hardness of a material can be classified into three categories: static penetration, rebounding/bouncing, and impact penetration.

Unlike minerals whose hardness is historically characterized by the scratch test (e.g. Mohs scale), the hardness of metals is generally characterized by using rebounding/bouncing or penetration tests. We consider the materials studied here as homogeneous and isotropic, and assume that the state of the material depends only on the values of constraints which are applied at a given time.

Tests commonly used in metrology laboratories to study the behavior of metals under the effect of mechanical stress are of varying importance, depending on their use. Therefore, we can be satisfied with a limited number of selected tests to be sufficient for the understanding of the most important dimensional factors. The methods on which we focus here are: Brinell method (steel ball), the Vickers method (diamond pyramid), and the Rockwell method (steel ball and/or diamond cone). We will also discuss other methods, without going into much detail.

9.1.1. *Measurements of hardness by indentation*

The principle is that a non-deformable indenter always leaves an imprint in the material being tested. The size of the imprint is then measured and thus the hardness value is deduced (in index or number). In a first approach, we can fairly relate simple yield stress R_e with the surface of the indentation; that is to say, the more penetrating the object sinks, the more likely the stress surface S expands, therefore the more the force F is constant, the more the stress decreases. When the stress is no

longer sufficient to plastically deform the solid being tested, then the penetrating object stops. We then have:

$$R_e = \frac{F}{S} \quad [9.1]$$

where F is the constant force in Newtons (N) and S the stressed surface in square millimeter.

9.1.2. *Presentation of the main hardness tests*

Tests of hardness and micro-hardness [KAL 06, OBE 96] are diverse and varied. They are governed by ISO standards that can be read in Tables A6.1–A6.4. Sometimes we can use, for the same material, more than one type of test and finally compare the results. We quote here the essence of these tests and then define some of the commonly used such as:

- Brinell Hardness Test (HB);
- Vickers Hardness Test (HV);
- Rockwell Hardness Test (HR);
- Knoop Hardness Test (HK);
- Shore Hardness Test;
- Microhardness Tests;
- Barcol Hardness Test;
- Mohs Hardness Test;
- International Rubber Hardness Degrees (IRHD).

9.1.2.1. *Brinell test (ISO 6506)*

This test is used on parts or rolled coarse molds [ISO 99]. It consists in indenting the surface of the test material using a hardened steel or carbide ball which is then maintained for a given period of time (about 20 s) by applying a fixed load ranging from 150 to 30,000 N. When the indenter is removed, the diameter of the indentation left in the test material is measured by reading the Brinell hardness, expressed in units or numbers (HB) of the material.

9.1.2.2. *Vickers test (ISO 6507)*

This test is not recommended for coarse castings [ISO 98]. Differences between the lengths of the diagonals may exist for very anisotropic materials. The need for

very careful preparation of surfaces is a drawback of the Vickers test, although it is the most accurate and most used for measuring the hardness of spherical or cylindrical parts. The test consists in measuring the size of indentation in the test material by a diamond indenter with an apex angle of 136° . The loading time and the applied test force are defined in accordance with ISO 6507. The two diagonals of the indentation left in the surface of the material after removal of the load are measured in Vickers units (HV). In fact, it is the same principle as the Brinell test, but with a diamond pyramid apex angle of 136° applied for a defined time and with a stated test force.

9.1.2.3. Rockwell test (ISO 6508)

In this test, we use a diamond cone or hardened steel ball at 120° on the top. In the Rockwell test [AST 00], which consist of using an indenter, we measure depth of penetration of the indenter instead of measuring the size of the indentation as in Brinell and Vickers tests. The test is performed in three distinct stages: the application of a preload (using a cone or a ball), and then an additional charge (surcharge), and finally a return to preloading phase. The indenter is then pressed to a certain depth (greater than the depth obtained under a simple preload). By measuring it, we get the Rockwell hardness (HR) of the material. This test has several variations of scales: Rockwell *A, B, C, D, E, F, G, H, K, N, T*, etc. depending on the underlying conditions, that is to say, the type and size of the indenter, the preload, and the test force. Other typical applications of this test are:

- *HRB*: Coarse parts cannot be tested this way since the imprints left on the test part are small. So, the reading is directly taken.
- *HRC*: This test is reserved for steel.

9.1.2.4. Knoop test

In this test, a pyramidal diamond with a rectangular base is used. Knoop hardness (HK) is a variant of Vickers hardness. A pyramidal indenter with a diamond base creates a more elongated imprint. This test will be detailed in section 9.5.

9.1.2.5. The Shore hardness test

This test [ISO 85] consists in measuring the depth of penetration of an indenter. The Shore test *A* (Table A6.2), for example, is dedicated to soft elastomers such as rubbers and *D* to stiffer elastomers such as plexiglass. Besides the use of indenters, other measurement methods are based on the principle of a rebounding/bouncing ball on the test piece (the harder the material, the higher is the rebound height). This consists in measuring the height of the rebound/bounce, or the ratio of the rebound velocity on impact velocity. This provides the hardness value HL. Finally, let us mention other methods, less common than the previous, but still

preferred in some very specific applications: IRHD hardness for rubber or Barcol hardness, particularly used in coatings and composite materials:

- *Shore A and C*: truncated cone;
- *Shore D*: blunt cone. The Shore hardness is used to evaluate the hardness of soft materials (elastomers) or rubber derivatives.

Most methods presented above do not coexist without being mutually distinguished. Characteristics such as the test strength and the nature and shape of the indenter or the size of the imprint make them particularly appropriate for one or the other applications. If there is one rule to remember in terms of hardness, this would be that *there is no rule*. The type of material, its strength, size and test piece weight, thickness, homogeneity, surface condition, the kind of results expected and the required accuracy are variables that may favor one method over another.

9.1.2.6. *Microwordness*

In these cases [ISO 95, ISO 00, MIT 00] the indenter is of Vickers type with a pyramidal shape and a diamond base. Very low loads and the measure of the imprint are read using a microscope. The penetration is less important to measure the superficial hardness of perfectly polished surfaces. The most common tests in durometry are: Brinell (HB), Vickers HV (macro and micro), Rockwell (HRC), and Knoop (KN) specifically for the microhardness of well-polished surfaces. There are various scale conversion tables, derived from various tests, applied to ferrous alloys (steel) and partially to those called soft alloys. The balls are standardized and are subjected to periodic inspection as per ISO 2039-1. They are made of polished steel with a defined diameter. These balls are pressed against a surface (the standard tests of at least 4 mm thick). If the applied test force is 358 N, after 30 s of application of the load, the indentation is measured and the hardness number is calculated based on the surface. The ball is then marked H358/30 and the result is given in MPa. Hardness tests performed using specific methods ensure:

- quality control of materials and other inspections of materials;
- evaluation of welds and other metal welded alloys [GUR 78];
- assessment of failures by cracking [ASK 89, GRO 94, LAS 92].

Hardness indices (or numbers) are often dimensionless. The test is normally conducted at ambient temperature. The load is applied gradually, without shock or vibration and is maintained at its final value for 10–15 s. The surface should be previously prepared in order to prevent any alteration. The test piece should be put on a rigid support. The hardness depends on the crystalline structure, the interatomic distance of the network, the valence (double bond), and the cross-linking of the

plastic material. Moreover, the penetration techniques vary depending on the material and the shape of the indenter used.

Hardness is not an intrinsic property of the material. It constitutes a value resulting from a procedural measure. The hardness of a material has long been associated with resistance to scratching, or cutting, in machining processes. Note that conversion between the different values of hardness scales is *not advisable* because of the large area of the material and the non-automatic correlation between their properties. Doing a mathematical conversion such as rule of three is not recommended. Different loads, different shapes of indenters, homogeneity of specimens (standards), and the properties of cold working are factors that sometimes complicate the measurement.

9.2. Principle and description of the Rockwell hardness

Rockwell hardness tests are tests of penetration. There are actually several types of indenters that are made of a diamond cone or polished ball made of hardened steel. The Rockwell hardness value is obtained by measuring residual penetration of the indenter on which a light load is applied. The test is conducted in three distinct phases as follows:

- a preload $F_0 = 10 \text{ kgf}$ is applied on the indenter. The indenter penetrates to an initial depth E that is measured as the initial Rockwell hardness value;
- applying an additional test force F_1 . The indenter penetrates to a depth P ;
- releasing the load F_1 and reading the penetration indicator. The value e represents the residual indentation depth obtained by applying and later releasing the force F_1 .

The Rockwell hardness value (B and C) is formulated according to the following scale:

- scale B , E , and F : $\text{HRB} = 130 - r$;
- scale C : $\text{HRC} = 130 - r$.

The two most frequently used scales are B and C . Evidence shows that one unit of Rockwell hardness corresponds to a penetration of 0.2 mm. The different scales are shown in Table 9.2.

These scales are appropriate to very thin products and are used in hardness measurement of coatings. The two scales used in these cases are the N scale (diamond cone) and the T scale (steel ball). In both cases, the initial load F_0 is 29.4 N. Each of them can be used with a total load of 147 N, 294 N, or 441 N. Note that other scales exist: W scale (ball diameter 3.175 mm), X scale (ball diameter 6.350),

and *Y* scale (ball diameter 12.70 mm). In this case a unit of Rockwell hardness corresponds to an indentation depth of 0.001 mm (Table 9.3).

Scale	Test	Indenter	Total force ($F_0 + F_1$) N	Applications
<i>A</i>	HRA	Diamond cone with circular section and a rounded spherical tip of 0.2 mm radius	588.6	Carbide, thin steel
<i>B</i>	HRB	Steel ball of 1.588 mm (1/16 in.) diameter	981	Copper alloy, mild steel, aluminum alloy. Materials having a tensile strength ranging from 340 to 1,000 MPa
<i>C</i>	HRC	Diamond cone circular section with a 0.2 mm radius spherical tip	1,471.5	Steel, cast iron, titanium. Materials with a hardness tensile strength > 1,000 MPa
<i>D</i>	HRD	Diamond cone with circular section and a 0.2 mm radius spherical tip	981	
<i>E</i>	HRE	Steel ball of 3.175 mm (1/8 in.) diameter	981	Aluminum foundry alloy and Cast iron
<i>F</i>	HRF	Steel ball of 1.58 mm diameter	588.6	Annealed copper alloy, thin sheet metals
<i>G</i>	HRG	Steel ball of 1.588 mm diameter	1,471.5	Cupro-nickel, Copper-nickel-zinc alloy

Table 9.2. Table indicative of Rockwell B and C scales (source: [NEW 06])

Scale	Symbol	Indenter	Total force value ($F_0 + F_1$) N
N	HR15N	Diamond cone with circular section and spherical tip of 0.2 mm radius	15
N	HR30N	Diamond cone with circular section and spherical tip 0.2 mm radius	30
N	HR45N	Diamond cone with circular section and spherical tip 0.2 mm radius	45
T	HR15T	Steel ball, 1.588 mm diameter	15
T	HR30T	Steel ball, 1.588 mm diameter	30
T	HR45T	Steel ball, 1.588 mm diameter	45

Table 9.3. Indenters of hardness testing applied under the Rockwell test (source: [NEW 06])

Applicable standards, both international (ISO) and European (CEN), are:

- ISO 2039-2: Plastics – determination of hardness – Part 2: Rockwell hardness;
- ISO 6508-1: Metallic materials: Rockwell hardness test – Part 1: Test method (scales *A, B, C, D, E, F, G, H, K, N, and T*).
- American Standard ASTM E1: Standard methods for Rockwell hardness and Rockwell superficial hardness of metallic materials.

9.2.1. Comparison of indentation methods (Table 9.4)

Test	Preparation of the test piece	Mainly used in	Comments
Brinell	The surface of the test piece does not require an extremely careful preparation.	Workshop	Among the other three testing methods, this one is the easiest in terms of implementation.
Rockwell	Good surface preparation (with sandpaper OO for example). The presence of scratches leads to underestimated values.	Workshop	The test is well suited for high hardness (above 400 Brinell) and is used for small test pieces (perfectly stable piece). The Rockwell has the disadvantage of having a relatively large dispersion.
Vickers	Very carefully prepared surface condition resulting in small imprint; the presence of defects may disturb reading.	In laboratory	This versatile test is suitable for both soft and very hard materials. It is used for small test parts. The reading of diagonal lengths is slow.

Table 9.4. Comparative table of hardness testing methods (source: [NEW 06])

As earlier explained, the Rockwell test (from Stanley Rockwell who made his first testing machine in 1921) consists in indenting an imprint on the test piece by a diamond conical indenter or a steel ball having specific dimensions. It is a method for determining the hardness value by static indentation using a spherical diamond-tipped cone of 120° angle and of 0.2 mm tip radius and a diamond whose apex angle is 120° , terminating in a circular bottom of 0.2 mm radius. This method derives, by convention, a linear function of the variation in cone penetration (or ball) when the cone is placed on the test surface to be studied under a preload F_0 ; the same load is applied again, after exerting a higher load F . A graduated dial is used to directly find the Rockwell hardness value, in accordance with the predetermined scale, depending on the material under test. The needle in the dial of the durometer stops by itself on reaching equilibrium, and during the preload F_0 , another additional charge is applied called major additional load F_1 , thus further pushing the indenter into the sample being controlled as shown in Figure 9.1.

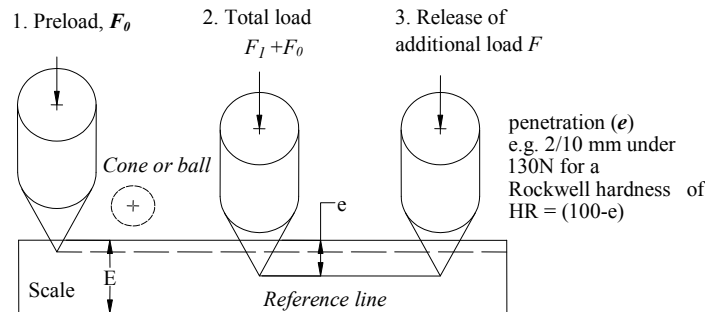


Figure 9.1. Rockwell (A, B, and C) test – diamond

- D : diameter of steel ball in mm;
- F_0 : preload in N;
- F_1 : major additional load in N;
- F : total load in N.

When equilibrium has again been reached, the additional major load is removed but the preload F_0 is still maintained. Removal of this second load leads to a partial recovery of the material, thereby reducing the depth of penetration as shown in Figure 9.1. The permanent increase in depth of penetration is calculated using the depth of penetration due to the application and later removal of load. The increase in depth of penetration (e) is used to calculate the Rockwell hardness number (HRC) using the following equation:

$$HRC = E - e \quad [9.2]$$

where e : permanent increase in depth of penetration due to major additional load F_1 , measured in units of 0.002 mm; E : a constant depending on form of the indenter: 100 units for diamond indenter, and 130 units for steel ball indenter (polished); HRC : Rockwell hardness number (or index).

The Rockwell hardness scales are grouped in Table 9.5.

9.2.2. Typical applications of Rockwell scales

The Rockwell hardness test is advantageous since it gives a fast and direct reading after the test. The disadvantage is that it allows the inclusion of several arbitrary scales, thereby adversely affecting the calibration. The trick is to try to put a cigarette paper under the block test and then observe the effect on the needle.

The Brinell and Vickers hardness test methods do not have this drawback. Here are the types of Rockwell tests (Table 9.6).

Summary table for different Rockwell hardness scales					
HR scale	Indenter	Minor load F_0 in kgf	Major load F_1 in kgf	Total load F in kgf	Values of E
<i>A</i>	Diamond cone	10	50	60	100
<i>B</i>	1/16 in. steel ball	10	90	100	130
<i>C</i>	Diamond cone	10	140	150	100
<i>D</i>	Diamond cone	10	90	100	100
<i>E</i>	1/8 in. steel ball	10	90	100	130
<i>F</i>	1/16 in. steel ball	10	50	60	130
<i>G</i>	1/16 in. steel ball	10	140	150	130
<i>H</i>	1/8 in. steel ball	10	50	60	130
<i>K</i>	1/8 in. steel ball	10	140	150	130
<i>L</i>	1/4 in. steel ball	10	50	60	130
<i>M</i>	1/4 in. steel ball	10	90	100	130
<i>P</i>	1/4 in. steel ball	10	140	150	130
<i>R</i>	1/2 in. steel ball	10	50	60	130
<i>S</i>	1/2 in. steel ball	10	90	100	130
<i>V</i>	1/2 in. steel ball	10	140	150	130

Table 9.5. Scales of Rockwell hardness testing (source: [NEW 06])

Test type	Applications
HRA	Case-hardened steel and thin sheet (metals)
HRB	Copper alloy, mild steels, aluminum alloys, malleable iron
HRC	Steel, hard cast irons, hardened steels and hardened materials HRB 100
HRD	Sheets, some cases of hardened steel and pearlitic malleable iron
HRE	Casts, aluminum and magnesium alloys, metal beads
HRF	Annealed copper alloy, metal sheet of thin plates
HRG	Phosphor bronze, copper-beryllium, malleable iron
HRL, HRM, HRP, HR HRS, HRV, HRK	Soft metals and plastics

Table 9.6. Rockwell hardness scales by application type (source: [NEW 06])

9.2.3. Rockwell superficial hardness test [AST 99, AST 00, ISO 00]

The test method, previously applied in Figure 9.1, is still applicable. The indenter is pressed into the test piece by applying a preload load F_0 (usually 3 kgf). The different scales of Rockwell superficial hardness are presented in Table A6.5

in Appendix 6. We present below three complementary pictures (Figure 9.2) that represent Rockwell hardness testing on both ferrous alloys, soft and plastic.

NOTE.— Only one side (specified by the manufacturer) is valid for testing.

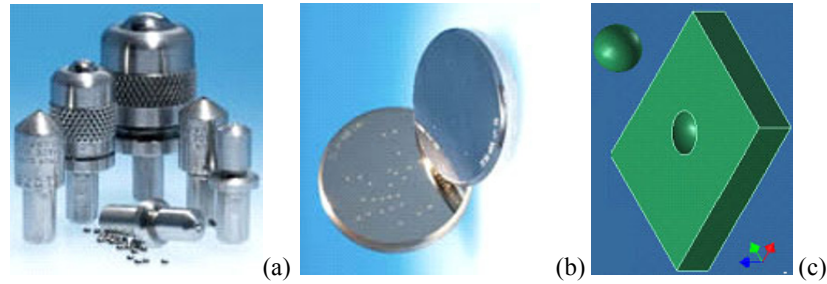


Figure 9.2. (a) Steel balls, diamond cone; (b) two standards; (c) Rockwell on plastic

9.2.4. Rockwell hardness tests of plastics

The hardness of plastics is measured by either the Rockwell or the Shore test. Both tests allow measuring the resistance of plastic based on the imprint resulting from the applied pressure. The hardness value read cannot be correlated to other properties or characteristics. The Rockwell test is usually chosen for harder plastics such as nylon, polycarbonate, polystyrene, and acetal. The ASTM-D785 standard is also applied to determine the Rockwell hardness of plastics and electrical insulating materials. A minor preload of about 10 kgf is then applied. This preload F_0 is then applied without shock for 10–15 s. We should then immediately implement the minimum load that will be the basis of machine settings (set up). The applied load F_1 is removed after 15 s of application. The Rockwell hardness is then read in accordance with the appropriate scale.

9.2.5. Comparison between Shore and Rockwell hardness ball testing

The Rockwell hardness test also determines the hardness of plastics after taking into account the elastic recovery of the test sample. This test is different from ball impression hardness tests as well as the Shore test. In these tests, the hardness is determined based on the depth of penetration at partial load, thus excluding any elastic recovery of the material. Rockwell values can then be directly linked to the values of the ball. Shore scales A and D can be compared with the value scales resulting from a ball hardness test. However, linear correlation does not exist between these two tests. A polished hardened steel ball, 5 mm in diameter, is pressed against the surface of a test sample (at least 4 mm thickness) with a force of

358 N (according to ISO 3029-1). After 30 s of load application, we measure the depth of impression, based on which the indented surface is calculated. Ball indentation hardness H358/30 is calculated using equation [9.2]. We propose the below approach:

- choice of indenter: diamond cone with 120° apex angle or steel ball whose hardness is likely to exceed 80 HRC;
- securing attachment of the indenter in its housing (a retainer screw is required);
- cleaning the anvil (support) and the test piece and then appropriately position the latter. Turn the lever to “Rockwell” to disconnect the *optical*¹ system (on some durometers) and connect the needle of the comparator to the indenter;
- placing the loading lever in the lower position (check before the test);
- choice of the load to be applied according to the scale: $F_0 = 10 \text{ daN}$, $F_1 = 140 \text{ daN}$, $F = F_0 + F_1 = 150 \text{ daN} = 1,500 \text{ N}$;
- bringing the workpiece surface in contact with the indenter by rising the anvil using a wheel or lever knob (depending on the apparatus);
- the indenter is pressed without shock using a load F , always by rising the anvil. The needle of the comparator then turns back to zero. The load $F = (F_0 + F_1)$ is then applied gradually;
- at cessation of the movement of needle of the comparator, release the loading lever while conforming to the load application time limit (stated by the applicable standard ISO/ASTM) – CSA suggests 10–15 s;
- load F_0 is removed (hence the lever, usually located at the rear of the device, will descend gradually) but load F is maintained. Due to the elasticity of the metal, the penetrator moves slightly back. The depth of penetration (e) read on the comparator indicates the HR.

Recall that a cone Rockwell unit represents an average depth of 2μ . The result is written and recorded as HR (Rockwell units). The method described here is called “Rockwell C.” It is used for hardened steel materials with case-hardened layers hardening higher than 0.6–0.7. The hardness value is read on the graduation in black. The Rockwell *B* procedure is used for testing unhardened steel and all non-ferrous metals. The recommended load is 100 kg; the indenter is a steel ball

¹ Some machines are simply used for hardness testing and are not equipped with optical system for microhardness, and are therefore without direct opportunity to perform a Vickers or Knoop test. The specimen should be moved over the apparatus toward an optical displayer in order to magnify and thus read the dimensions of the indentation.

1/16 in. in diameter (see Tables 9.1–9.3). The reading follows the numbers in red, the point “0” is obtained on the numbers in black.

9.2.6. Overall description of the Rockwell hardness testing machine

The appliances for testing the hardness are many but the principle and purpose are substantially the same. For example Figure 9.3(a) represents an old apparatus that is still in practice, and Figure 9.3(b) represents a modern apparatus.



Figure 9.3. Rockwell hardness and microhardness testers [MIT 00]

The latter has advantages in terms of database and statistical processing because it is equipped with an output for statistical process control. The device (*Mitutoyo Hardness Tester Rockwell & Superficial HR-521 Series*) consists of a cast iron frame with embedded devices as exhibited hereinafter.

9.2.6.1. Charging device

A load selector with a push button allows loads to be chosen by weight from 1 to 250 kg. By pushing the corresponding button, there will be a direct shift from one load level to another. The Mitutoyo HR-521 device, shown in Figure 9.3(b), allows the testing of samples in both Rockwell hardness and superficial hardness in real time, with a viewing screen. The application of the load is electronically controlled. The working stroke of approximately 2 mm produced by beam balance is covered by the indenter, virtually without any loading adjustment. A hydraulic brake allows regulation of the applied loads.

9.2.6.2. *Optical system*

Some devices [MIT 00] are equipped with a projector optical system (Figure 9.3(b)) by which the imprint of the indenter (ball or diamond pyramid) is projected on the frosted glass with a magnification of 20, 44, 70, or 140×. The distance from the objective piece is:

- 1.50 mm nearly, with a magnification: 140×;
- 6.80 mm nearly, with a magnification: 70×;
- 8.80 mm nearly, with a magnification: 44×;
- 26.5 mm nearly, with a magnification: 20×, etc.

Indentations can be measured quickly and accurately on the ground glass with the door/slide strip accommodating interchangeable scales.

9.2.6.3. *The comparator*

The comparator provides a direct reading of the hardness value of the tested object.

9.3. Brinell hardness test

This test is named after a Swedish engineer Brinell (1849–1925), who developed a method to evaluate the hardness of materials. The Brinell hardness test is a simple test which measures the indentation surface of the impression of a ball applied to the material. It is also known as the “ball test.” Concisely, the Brinell hardness value is calculated by dividing the load applied by the surface area of the indentation. Among the applicable international standards (ISO and ASTM), we quote:

- ISO 6506-1: Brinell hardness test – Test method;
- ISO 6506-2: Brinell hardness test – Verification and calibration of testing machines;
- ISO 6506-3: Metallic materials – Brinell hardness test – Calibration of reference blocks;
- American standard ASTM E10: Standard Test Method for Brinell Hardness of Metallic Materials.

The Brinell test consists in indenting the test material using a steel ball of 10 mm in diameter subjected to a load of 3,000 kgf. For softer materials, this load may be reduced to 1,500 kgf or 500 kgf to avoid excessive indentation. The whole load is

normally applied for 10–15 s, in the case of iron and steel, and for at least 15 s in the case of other materials. The indentation left is usually measured using a low powered microscope. The Brinell hardness number is then calculated by dividing the load applied by the surface area to the indentation. The material to be tested is generally made of polished carbide, of known dimension πD . The surface must be flat and clean (without lubricant, oxide, or calamine). The test material must have sufficient thickness so that the indenter does not deform the material. Otherwise, the measure would be unreliable. Here is a schematic illustration in Figure 9.4.

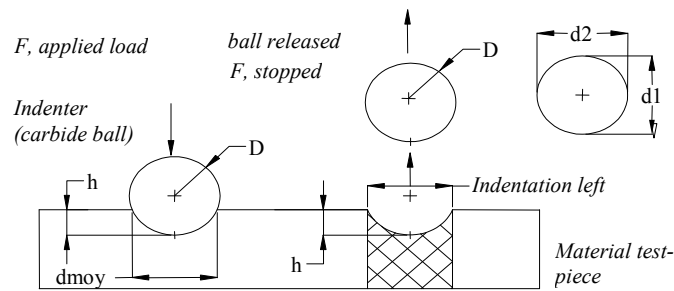


Figure 9.4. Principle of Brinell hardness test

The thickness of the test-piece should be at least eight times the depth h of the indentation.

$$\text{For } d_1 = 2.998 \text{ mm; } d_2 = 3.001 \text{ mm} \rightarrow d = \left(\frac{d_1 + d_2}{2} \right) = 2.998 \text{ mm}$$

$$\text{For } F = 500 \text{ N; } S = \frac{\pi \times d^2}{4} = 7.059 \text{ mm}^2 \rightarrow HB_{\text{Brinell index}} = \left(\frac{F}{S} \right) = 70.83$$

where F : test load (Newton); D : diameter of the indenter (millimeter); d_1 and d_2 : diameter of the indentation at 90° (millimeter); h : depth (millimeter).

The test result is read “75 HB 10/500/30” which means “a Brinell hardness of 75 was obtained using a 10 mm diameter ball under a 500 kgf load applied for 30 s”. In cases of a Brinell test of extremely hard metals, a tungsten carbide ball is preferred. Compared to other hardness tests, the HB test reveals a wider and deeper indentation. This fact makes it possible to have more accurate results of hardness values due to the multiplicity of structure grains, and irregularities in the uniformity of the material. This method is advantageous since it helps in performing the buckling of the macrohardness of the material, especially those materials with heterogeneous crystalline structure.

9.3.1. Applied load and diameter of the ball

Several works [AST 99, AST 00, ISO 00] have proven that there is an exponential relationship between the load F and the diameter of the impression:

$$F = a \times D^n \quad [9.3]$$

n is a factor that is independent of the diameter D of the ball, for a given metal, and a depends on D through the following relationship:

$$K = a \times (D^{n-2}) \quad [9.4]$$

where K is a value depending on the material. Using [9.3] in [9.4], we obtain:

$$F = \left(\frac{K}{d^{n-2}} \right) \times (D^n) \quad [9.5]$$

Using [9.3] and [9.4], and by eliminating d from equation [9.5], we find:

$$HB = \frac{1}{1 - \sqrt{1 - \left(\frac{F}{K \times D^2} \right)^{\frac{2}{n}}}} \times \left(\frac{2 \times F}{\pi \times D^2} \right) \quad [9.6]$$

Through equation [9.6], it appears that the Brinell hardness number depends on F and D due to the quotient (F/D^2) . Therefore, if we vary F and D so that the quotient (F/D^2) equals a constant, the results of hardness measurements would essentially be the same. Generally, we agree to take as a value of that ratio (F/D^2) , 30 for ferrous alloys (i.e. steel and iron), F in kilogram and D as 12.5 mm; 10 and 5 for non-ferrous alloys. Here are some comparative examples:

EXAMPLE 1.— $F = 200$ N; $D = 4$ mm; $K = 40$; $n = 1$ and by applying equation [9.6], HB will be:

$$F = 200 ; D = 4 ; K = 40, n = 1 \text{ of [9.6]} \Rightarrow HB = 158.894 \text{ MPa}$$

EXAMPLE 2.— $F = 200$ N; $D = 4$ mm; $d = 3.46$ and using equation [9.6] simplified, HB will be:

$$F = 200 ; D = 4 ; K = 40, d = 3.46 \text{ of } HB = \frac{2 \times F}{\pi \cdot D \times (D - \sqrt{D^2 - d^2})} = 159.972 \text{ MPa}$$

Both methods are similar to each other because the ratio $HB/HB = 158.894/159.972 = 0.995 \approx 1$. However, it is easier to calculate the Brinell hardness using equation [9.6].

9.3.2. Thickness of the tested metal

The metal to be tested is presented as a sample having a certain thickness. If this thickness is low, we would observe impressions characterizing strain hardening on the opposite side. The hardness of the support is then involved which may distort the measurements. The sense of the error introduced is difficult to predict. The minimum thickness required for an accurate measurement is given in Table 9.7 (according to Rajakovico and Meyer).

Brinell hardness	Thickness $E_{\min}$ for			
	$D = 10 \text{ mm}$	$D = 5 \text{ mm}$	$D = 2.5 \text{ mm}$	$[E/D]_{\min}$
Less than 50	8	4	2	0.8
From 50 to 100	5	2.5	1.5	≈ 0.5
More than 100	3	1.5	1	≈ 0.3

Table 9.7. Brinell hardness as a function of minimum thickness

9.3.3. Meyer hardness test (named after Rajakovico and Meyer)

The indenter is similar to the Brinell hardness indenter. In more general terms, it uses the same durometer as that used for Brinell hardness. The measurement is performed with the same principles as the Brinell hardness. The Meyer hardness values are found by:

$$HM = 0.102 \times \left(\frac{4 \times F}{\pi \times D^2} \right) \quad [9.7]$$

NUMERICAL APPLICATION.— Based on the same parameters as those used in the previous test (Brinell):

$$D = 10; d_1 = 2.998; d_2 = 3.001; F = 500; \text{ with } d = \left(\frac{d_1 + d_2}{2} \right) = 2.9995$$

$$\Rightarrow HM = 7.217 \frac{\text{daN}}{\text{mm}^2}$$

Measurement of impression diameter: the measurement of the impression diameter shall allow a relative error of less than 0.25%. A projection optical microscope is

used in order, both, to achieve a magnified projection of the imprint image in a known ratio, and to measure with a graduated ruler, the diameter of this projection. Irrespective of the method applied, we should measure the two framed diameters of the rectangle; the average value of the two measures is to be considered.

9.3.4. Operating procedure for Brinell hardness test

The following approach is proposed as a model [ISO 98]:

- the test piece, the support, and the indenter as well are cleaned with non-inflammable detergent (i.e. carbon tetrachloride). The test piece is placed upon the support;
- check the condition and size of the indenter and then fix it in its housing;
- check the conformance of the magnification of the objective, the ruler, and the scale HB;
- set the test selector on Brinell position;
- adjust the test piece position to obtain a clear image on frosted glass screen;
- choose the test load in line with the material to be tested. The selection criteria of the load are the nuance of the metal, example for steel $(F/D^2) = 30$, and the ball diameter, if for example $D = 2.5$ mm, thus we would obtain:

EXAMPLE. – $D = 2.5$ mm $\rightarrow$ with $F = (30 \times D^2) = 187.5$ daN

Once the appropriate load is chosen, we approach the test piece to the penetrator and we trigger the loading lever. It is recommended to wait until the lever ends running (autonomously); wait for around 15 s and then gradually suppress the load by slowly bringing the lever to its lower position. The opposite diameters measured on the impression being d_1 and d_2 (diametrically), we consider an average diameter $d = (d_1 + d_2)/2$. To determine the Brinell hardness, we either use ordinary appropriate tables or calculate the HB using formula [9.8]. For example, if we choose a 2.5 mm diameter ball, with a previously calculated load $F = 187.5$ daN, with a piece thickness of $e = 1.5$ mm then:

$$D = 2.5 \text{ mm} \rightarrow \text{with } F = (30 \times D^2) = 187.5 \text{ daN with } e = 1.5$$

$$S = \left(\pi \times \frac{D^2}{4} \right) = 4.909 \text{ mm}^2 ; \quad HB = \frac{F}{S} = 38.197 \approx 382 \text{ MPa}$$

9.4. Principle of the Vickers hardness test [ISO 98, ISO 99, ISO 00]

The Vickers hardness measurement is performed with a standardized diamond pyramid tip with a square base and an apex angle of 136° . Hence the impression is square shaped; the two diagonals d_1 and d_2 of this square are measured using an optical device (Figure 9.5). The value d is obtained by calculating the arithmetic mean of d_1 and d_2 . The value of d will then be used in the calculation of the hardness value. The imposed load and the period of time are also standardized. The applicable standards are:

- ISO 6507-1: Metallic materials – Vickers hardness test – Test method;
- metals, E92: Standard Test Method for Vickers Hardness of Metallic Materials;
- Ceramics, C1327: Standard Test Method for Vickers Indentation Hardness of Advanced Ceramics.

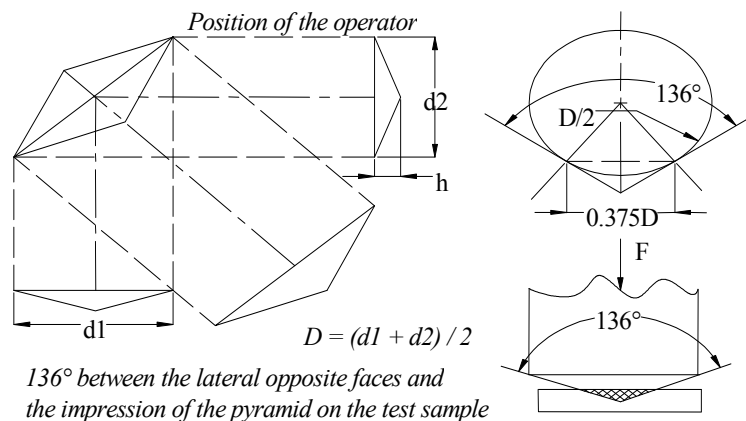


Figure 9.5. Vickers test principle

The degree of hardness HV is read through a chart (or table). The opposite faces are subjected to a load ranging from 1 to 100 kg. The whole load is usually applied for 10–15 s. We have already seen that the accuracy of the Brinell test depends on the ratio (d/D) of indentation diameter to the ball diameter. It is deemed *optima* if $1/4 < (d/D) < 1/2$, i.e. close to $3/8$. When $(d/D) = 3/8$, the planes tangential to the sphere of the ball, along the circle limiting the imprint, envelop a cone with an apex angle of 136° . Here is a schematic illustration of the Vickers test.

- HV: Vickers hardness;
- d_1 and d_2 : measure of the indentation made at 90° (two diagonals of the square of the indentation in millimeter);
- F : test load (Newton).

Load application is done under the same conditions previously stated in the Meyer test:

$$HV = 0.102 \times 2 \times F \times \left(\frac{136^\circ}{2} \right) = 0.189 \times \left(\frac{F}{D^2} \right) \quad [9.8]$$

To obtain optimum results using the Vickers test, it was decided to take a diamond pyramid with a square base whose faces are opposite each other at an angle of 136° . The preparation and the procedure are the same as in the Brinell test, with the only difference being the free choice of the test load. However this chosen load should be neither too large (to avoid the deterioration of the test piece) nor too small (appropriate imprint for good accuracy):

$$HV = 1.854 \times \left(\frac{F}{D^2} \right) \quad [9.9]$$

If a denotes the face, and D the diagonal of the imprint, we would obtain exactly the same formula as in [9.9]. Here is the proof:

$$S = \left(\frac{a^2}{\sin(68^\circ)} \right) = \left(\frac{D^2}{2 \times \sin(68^\circ)} \right) = \left(\frac{D^2}{1.854} \right) \quad \text{and from } HV = \left(\frac{F}{S} \right) = 1.854 \times \left(\frac{F}{D^2} \right)$$

The two diagonals of the indentation left on the surface of the material after removal of the load are measured using a microscope and then averaged. The area of the flank angle sloping surface (indentation) is then calculated using equation [9.9] which becomes:

$$HV = \frac{2 \times F \times \sin\left(\frac{136}{2}\right)}{D^2} \quad [9.10]$$

For the same reasons set forth above, we must consider the average diameter. Equation [9.9] will also take the following form:

$$HV = 0.189 \times F \times \left(\frac{d_1 + d_2}{2} \right)^2 \quad [9.11]$$

where F : test load in kgf; D : arithmetic mean of the two diagonals, d_1 and d_2 in millimeter.

It should be noted that several tests of Vickers hardness results in hardness numbers that are almost identical, based on a uniform material. This is much better, compared to the variation in scales found in other hardness testing methods. The advantage of the Vickers test is that *extremely* accurate readings can be achieved, with a single indenter that is suitable for all types of metals and surface conditions (tempered) and metals. Furthermore, this test is accurate on soft materials. The only drawback is the price of the machine. We can perform both tests simultaneously and make a statistical treatment in line with expected quality standards, thanks to three goals and penetrators for both types of tests (HV and HK). This set allows measuring and reading of Vickers hardness scales as well as Vickers and Brinell microhardness numbers. The resulting calibrations are appropriate. The hardness value is directly displayed. There is a definite advantage regarding the repeatability of measures, witness the micro-Vickers MVK-H. The manufacturer Mitutoyo [MIT 00] has developed the software "I.M.A.G.E"² of computer-assisted hardness measurement. This is an image analysis system, part of which is reserved for the Vickers, Knoop, and Brinell tests.

In the context of a work on the fatigue fracture of materials, we carried out mechanical tests (HV) on a cross-welded steel alloy [GRO 94, LAS 92] (see photograph in Figure 9.6 showing imprints performed through our own tests).

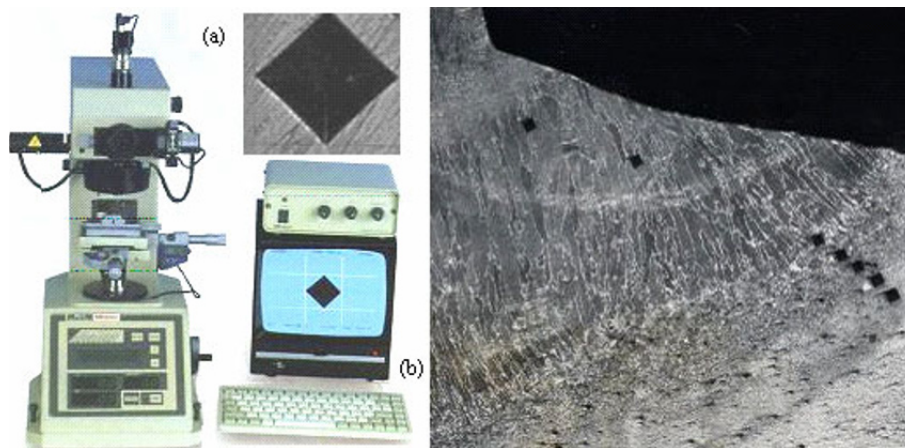


Figure 9.6. (a) Vickers and Brinell microhardness tests. Application on micro-Vickers: MVK-H; (b) micrograph of alloy welding outlet on a cross-welded steel

² I.M.A.G.E stands for (in French): indentation, mesures, analyses, gestion des empreintes, i.e. (in English): indentation, measurement, analysis, management of impression.

9.5. Knoop hardness (HK)

This test was proposed in the United States, in 1939, by F. Knoop and the team of the National Bureau of Standards [KNO 39]. The Knoop test [AST 99, GUI 70, ISO 95] is dedicated to *brittle materials such as glass and ceramics*. This test allows measuring the hardness by measuring the imprint left by the penetration of a diamond applied with a given imposed force. This test is similar to the Rockwell hardness test. The diamond indenter is pyramidal with a rectangular base, with an angle of 170° between two opposite faces and 130° for the two other faces (Figure 9.7). The standard for Knoop microhardness E384 in 1969. The penetration resistance of optical glass is defined using an apparatus for testing the microhardness. A sample, with the appropriate thickness, is polished. The diamond used is prismatic so that the angle between the axes vertically opposite (vertex angle) is $172^\circ 30'$ or 130° . The loading time is 15 s, a load $F = 0.98$ N is applied at five points of the sample. l is the length in millimeter of the long diagonal, the hardness is expressed as:

$$HK = 1451 \times \left(\frac{F}{l^2} \right) \quad [9.12]$$

The relative microhardness of materials is also determined by the Knoop hardness test. In this test the diamond pyramid constitutes the indenter with atypical angles 130° and $172^\circ 30'$ as shown in Figure 9.7.

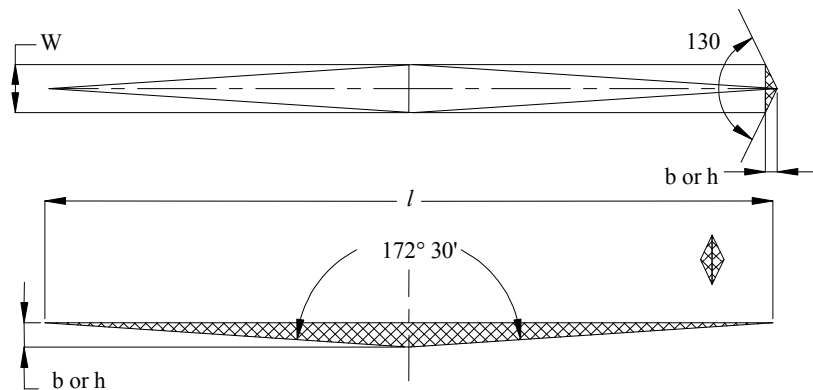


Figure 9.7. Knoop microhardness test

The pyramidal diamond, known as Knoop indenter, is pressed against a material (sample), which leaves an *octal polyhedral* imprint after removal, with a diagonal seven times longer than the others. The hardness is determined based on the

indentation depth [AST 99] left by the Knoop indenter. The diamond indenter used in the Knoop test has an elongated pyramid shape, with an angle of approximately 170° between two of its opposite faces and an angle of 130° formed by the two other faces. When pressed into the sample and then removed, the indenter leaves a four-sided imprint with a size of about 0.01 to 0.1 mm. The length of the imprint l is equal to nearly seven times the width w , and the depth is 130 the length. Given these dimensions, under an applied load F , the surface area S (square millimeter) can be calculated after measuring the length of the corresponding longest side using a microscope. The Knoop hardness (HK) is expressed as follows:

$$HK = 14.229 \times \left(\frac{F}{S} \right) \quad [9.13]$$

ASTM D-1474 standard is applicable to the case of organic coating materials. In this test, Knoop hardness (HK) is determined under the conditions of temperature and humidity: temperature at $23 \pm 2^\circ\text{C}$ and $50 \pm 5\%$ relative humidity. The apparatus is calibrated to apply a load of approximately 25 g; the contact time should be around 18 ± 0.5 s.

$$KHN = 0.0025 \times \left(\frac{1}{C_p \times l^2} \right) \quad [9.14]$$

where 0.025 is the load F (in kilogram) applied to the indenter; l is the length of the longest diagonal of the indentation (in millimeter); C_p is the indenter constant equal to 7.028×10^{-2} .

The term microhardness refers to a static impression caused by a load not exceeding 1 kgf. The indenter can be a diamond pyramid as used in the Vickers test or an elongated pyramid-shaped diamond as used in the Knoop test. The procedure is similar to that conducted for the Vickers hardness test, except that the microhardness is measured based on a microscopic scale, which requires high precision instruments. The test surface requires special preparation and a metallographic finish. A small load is applied and the reading is expanded to a magnitude of about ($\times 500$). It is measured with an accuracy of $\pm 0.5 \mu\text{m}$. The Knoop hardness index NHK is also expressed by the ratio of the applied load P to the area S that is not covered with the imprint:

$$KHN = \left(\frac{F}{S} \right) = \left(\frac{P}{C_p \times l^2} \right) \quad [9.15]$$

where $F(P)$ = the applied load in kgf; S = the projected area not covered with the imprint in square millimeter; L = length measured along the diagonal of the indentation in millimeters;

$C_p = 0.07028$ is the indenter constant, connected to the projected area multiplied by the square of the length along the diagonal.

The indenter is a pyramid-shaped diamond. The relationship between the long and the short diagonal is in a ratio of 7:1. The depth of the impression is about one thirtieth ($1/30$) of its length. When measuring the Knoop hardness, only the longest diagonal of the indentation is considered in the formula [9.15]. Microhardness tests carried out under very low loads allow very localized measurements (about $100 \mu\text{m}^2$). By using a microhardness tester (micro-durometer), we can, for example, determine the hardness of a given phase within a multiphase sample or the hardness of a very fragile and thin sample.

9.6. Barcol hardness

The Barcol hardness test [ISO 85, ISO 95] is ideal for measuring resistance to penetration of a sharp steel point under the pressure of a spring load, on certain polymers. The instrument used for this test is portable. The reading of the hardness is directly given on a range of scale from 0 to 100 units. According to *ASTM D2583*, the test is used for both reinforced and non-reinforced rigid plastics. The specimen is positioned under a sharp, pointed indenter, with a uniform pressure applied to the specimen until there is indication of the completion of penetration. The result is then converted into a Barcol index (absolute Barcol number). The Barcol test is also used to determine the radii of curvature of resin materials. The device used is usually a portable Barcol that may be mounted on a console (which looks like a sheep) as in Figure 9.8.



Figure 9.8. Barcol apparatus



Figure 9.8. (continued) Barcol apparatus for measurement of the hardness of plastics, elastomers, and natural synthetics (rubber, acrylic, acetate resin, polyester, thermoplastic, PVC, and neoprene)

9.7. Rebound hardness test by Shore test (scleroscope)

In this test [ISO 85], a diamond-tipped small mass of steel is allowed to fall straight down from a known height. When falling, the mass is guided by a smooth tube. The hardness is then evaluated based on the height of the rebound/bounce. In simplified terms, this test measures the energy of plastic deformation. Theoretically, if the impact is perfectly elastic (very hard test piece), the tip bounces up to release its height (neglecting frictions); the difference in height h can be linked to the kinetic energy ΔE_c absorbed upon impact. If m is the mass dropped and g is the acceleration of gravity (9.807 s^{-2}), we can write the following:

$$\Delta E_c = m \times g \times h \quad [9.16]$$

In the case of an extremely soft object, the tip sinks and does not bounce. The specimen is normally calibrated to obtain a hardness of 100 for hardened steel

at 0.9% carbon, and about 35 for mild steel. Note, however, that the results of the Shore test depend greatly on the surface condition of the test piece. The specimen must be vertically maintained in order to avoid friction that may distort the measurement. The mass of the test part should be much larger than the mass used in the measuring device.

The Shore scleroscope measures the hardness in terms of elasticity of the material, and the hardness number depends on the height of rebound. The advantage of this method is that there will be no trace left after the test. The Shore durometer is the typical apparatus used in measuring the hardness of iron alloys, lightweight, plastics and rubbers. The most common durometer is model *A* used for measuring the hardness in soft materials. The *D* scale is commonly used for plastics and harder rubbers (fluoropolymers and vinyls). The material is subjected to a pressure defined by the calibration of the spring which is connected to the indenter in the form of a cone or a sphere. The result is indicated by reading the depth of the impression left after indentation for a defined time. There are other scales for Shore hardness such as *O* and *H*. They are rarely used as scales as they are dedicated to some engineering plastics materials. The Shore durometer is generally a portable device [MIT 00].

Example of Shore hardness.— $\rightarrow 80$ = very hard; $\rightarrow 60$ = hard; $\rightarrow 45$ = medium; $\rightarrow 20$ = soft.



Figure 9.9. Standard etalons for Shore hardness for plastics; portable Shore durometer (Mitutoyo)

The hardness value is determined by the penetration of an indenter that is pressed on the sample as shown in Figure 9.9. Because of the resilience of rubbers and plastics, the hardness reading may change over time. Therefore it is worth paying attention to the penetration time. The ASTM D2240 standard defines the procedural method. This standard includes ISO 7619 and ISO 868, DIN 53505, and even JIS K 6301 which also was amended by JIS K 6253. Although the Rockwell hardness may be sometimes used, the Shore hardness remains the most appropriate to test such

materials. The results obtained from the Shore hardness test allow us to assess the relative resistance to indentation of various grades of plastics. However, it is important to note that this test is not suited for testing the resistance to scratches, abrasions, or constraints. As shown in Table 9.8, the correlation between the two scales of Shore hardness is low. It is inadvisable to make mathematical conversions, as if it were a Rockwell test.

Shore durometer	Indenter	Applied load F (in mN)
Type A	Hardness of a steel rod $\varnothing$ 1.10–1.14, with a truncated cone at 35° on 0.79 mm $\varnothing$	$F = 550 + 75HA$
Type D	Hardness of a steel rod $\varnothing$ 1.10–1.14 with 30° tapered tip, on 0.79 mm $\varnothing$	$F = 445 HD$

Table 9.8. Shore hardness scale and strength tests (source: Mitutoyo Canada)

The Shore hardness units are linked to the range, from 0 for the complete penetration of an indenter of 2.50 mm up to scale of 100. The pressure load is applied as quickly as possible and without shock. The hardness reading is taken after a period of $15 \text{ s} \pm 1 \text{ s}$. If an instant reading is specified by the manufacturer of the device, the scale is read within 1 s of applying the load. There is no correlation between the results obtained with different types of hardness testers (durometers).

9.7.1. Comparison of the indenters for the Rockwell and Shore tests

The Rockwell test is applied to plastic materials after allowing time for recovery of the material's elasticity. This is the main difference between the Rockwell hardness test and the Shore test. The Rockwell number is derived from the depth of the impression after penetration, under an applied load. *A*- and *B* Shore hardness scales can be compared to those resulting from the ball indentation. There is no correlation between them.

9.8. Mohs hardness for minerals

It is not easy to measure the resistance of a smooth surface to abrasion. The Mohs hardness [GUI 70] is therefore used to indicate the resistance of a substance to scratching caused by another substance. For minerals, the mineralogist Friedrich Mohs (1773–1839) proposed a hardness scale devised from 1 to 10. This scale is used to distinguish the hardness of minerals (noble metals such as diamonds). This method consists of 10 minerals arranged in ascending order from 1 to 10. Diamond is rated as the hardest with an index of 10. The following table shows that talc is the

softest with index 1. The figures do not represent a quantitative indicator. The 10 minerals covered by this scale are shown in Table 9.9. We do not intend to address measurement of hardness in this book, which is dedicated to dimensional metrology. We have just made an attempt to popularize the general concept of Mohs hardness.

Mineral	Hardness index	Simple test: scratched with
Diamond	10	Scratches the glass
Corundum	9	Scratches the glass
Topaz	8	Scratches the glass
Quartz	7	Files, hardly
Orthoclase (Feldspar)	6	Files
Apatite	5	File steel, hardly
Fluorite	4	File steel, hardly
Calcite	3	A Copper coin
Gypsum	2	With a nail
Talc	1	With a nail, easily

Table 9.9. Ten minerals hardness indexes

The surface hardness is the ability of enamel surfaces to resist abrasion and incision. The standard EN 176 provides that the glazed tiles should have hardness not less than five. For example, the tile with a surface of the enamel that has the highest Mohs hardness is less incised than other materials (Figure 9.10).

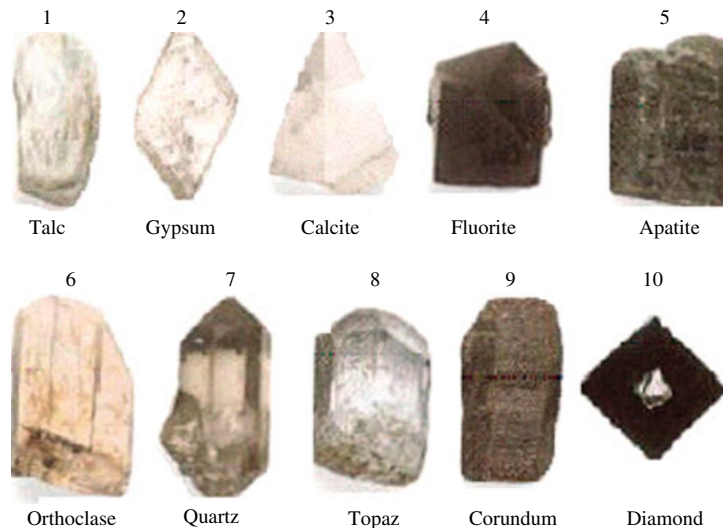


Figure 9.10. Samples of the five minerals included in the Mohs test scale
(Source: "Guide visuel Roches et minéraux," Mondo, Vevey 1994, Chris Pellant)

9.8.1. *Mohs scale of hardness minerals*

The Mohs hardness scale is ordinal (it is neither linear nor logarithmic), one should therefore proceed by comparison (the ability of a material to scratch another one) with two other minerals of known hardness. For example, a human fingernail has a hardness of 2.2 to scratch gypsum but not calcite. There is also a scale with five classes, designed to address the lack of regularity of the original scale. The term hardness, which expresses the resistance of a body to scratch, should not be confused with the tenacity (tensile strength), which is the resistance to shock. For example, quartz has a hardness of seven and the diamond has 10. They are easily broken with a hammer, even if they are harder than steel.

9.8.2. *How should the hardness of a mineral be measured?*

To measure the hardness of a mineral, we successively try to scratch a one cent coin (penny), for example. We may also check if it scratches glass. We specifically examine which mineral on the Mohs relative hardness scale is the most hardly scratched by the mineral studied. The following remarks will help avoid errors:

- After a fresh break of the mineral being tested, we rub a sharp edge against a flat surface of the mineral belonging to the Mohs scale. We dry the trace and check if it persists. Do not press strongly to avoid crushing the mineral.
- By rubbing a soft mineral against a hard mineral, a mark is left on it and it is the soft mineral that is scratched.

9.9. IRHD rubber hardness tester

Unfortunately we do not report laboratory cases for these tests. We will then attempt to succinctly provide general information.

Standard methods for measuring the hardness of plastics and rubbers use the Shore A and D hardness tests, as previously mentioned. They also use the method IRHD (International Rubber Hardness Degrees) with scales *N*, *H*, *L*, and *M*. These test methods [AST 99] are specified by ISO 868 and ISO 48. The hardness tests are highly accurate and thus promising in dimensional metrology. The hardness IRHD provides four methods for determining the hardness of vulcanized thermoplastic and rubbers, at *N*, *H*, *L*, and *M* scales (see Table A6.2 in Appendix 6). There are also four methods that provide the apparent hardness of curved surfaces (vehicle tires): CN, CH, CL, and CM. The test consists in measuring the difference between the depths of imprint by the ball inside the area of rubber under a small contact force, followed by a large total force. The methods

differ in terms of the diameter of the ball indenter and the force of the imprint chosen as shown in Table 9.10.

Test method	Indenter diameter (millimeter)	Contact load N	Additional load N	Total load N	Applications
N	2.50 ± 0.01	0.30 ± 0.02	5.40 ± 0.01	5.70 ± 0.03	Thickness: = 4 mm, Scope: 35 – 85 IRHD, or: 30 – 95 IRHD
H	1.00 ± 0.01	0.30 ± 0.02	5.40 ± 0.01	5,70 ± 0.03	Thickness: = 4 mm, Scope: 85 – 100 IRHD
L	5.00 ± 0.01	0.30 ± 0.02	5.40 ± 0.01	5.70 ± 0.03	Thickness: = 6 mm, Scope: 10 – 35 IRHD
M	0.395 ± 0.005	0.008 3 ± 0.000 5	0.145 ± 0.000 5	0.153 3 ± 0.001	Thickness: < 4 mm, Scope: 35 – 85 IRHD, or: 30 – 95 IRHD

Table 9.10. Scale of hardness IRHD and load tests (source: ISO 868)

The relationship between the depth of the imprint D , the hardness of rubber and the IRHD number (index), is based on the following experimental formula:

$$\frac{F}{E} = 0.0035 \times (r^{0.65}) \times (R^{1.35}) \quad [9.17]$$

where F = force of penetration (in Newton); r = radius of the indenter (in millimeter); E = Young's modulus (elasticity) in MPa. Here is a numerical example:

$$F = 5.7 \text{ N}; \quad r = 2.50 \text{ mm}; \quad E = 200 \text{ MPa} \Rightarrow D = 1.35 \sqrt{\frac{F}{E \times 0.0035 \times (r^{0.65})}} = 4 \text{ mm}$$

9.9.1. Control of rubber and other elastomers by IRHD and Shore test

Based on the Young's modulus value E and by using the curve resulting from experimental testing, we can read the numbers IRHD. The unit of measurement is the IRHD. ISO 48 defines the conditions of the international hardness testing of vulcanized elastomers having a hardness ranging between 10 and 100 IRHD. The hardness tolerance is ± 5 points compared to baseline (initial value). The Shore A is always used as the unit of hardness measurement for elastomers. The test conditions are very different between the Shore A and IRHD, depending on the thickness, time, apparatus, load, and the shape of the indenter. The correspondence table is valid only for tests on plate specimens. The measure on the joints/gaskets involves other parameters such as thickness and shape. The two methods react differently. Usually

the Shore A and IRHD are considered to be equivalent. In fact, there is a difference because the mixtures are black due to the effect of carbon black (CB) which is, among other things, a conductor of electricity. Again, we do not report laboratory cases on these tests, in this first edition. Ongoing work will be presented later.

9.10. Comparison of the three main hardness tests and a practical approach for hardness testing: Brinell HB, Rockwell HR, and Vickers HV

Table 9.11 provides a comparison of the hardness levels obtained based on the employed treatments. Several scales exist; however, the most used are the three scales herein presented. It should also be noted that this is not a very precise representation but will help the user have a figurative idea. For further details, it becomes imperative to refer to real and well-dimensioned scales. Correspondences (approximate) of some major hardness indexes (Rockwell, Brinell, Vickers) with each other and with R_m allow comparative approach between the different hardness tests.

Note, also, that it is important to always:

1. take the arithmetic mean of several tests results. The first test is called “test for nothing.” It consists just in starting the device;
2. place the indenter in a clean and unmarked location on the sample to be tested;
3. when reading, the large black needle should not exceed 1 to 1¼ times. If that happens, we should double check the scale, or even repeat the test (Table 9.12).

HV	HB	R_m	HV	HB	R_m	HV	HB	HRC	R_m
80	76	255	155	147	495	230	219		740
85	80	270	160	152	510	235	223		755
90	85	285	165	156	530	240	228	20.3	770
95	90	305	170	162	545	245	233	21.3	785
100	95	320	175	166	560	250	236	22.2	800
105	100	335	180	171	575	255	242	23.1	820
110	105	350	185	176	595	260	247	24.0	835
115	109	370	190	181	610	265	252	24.8	850
120	114	385	195	185	625	270	257	25.6	865
125	119	400	200	190	640	275	261	26.4	880
130	124	415	205	195	660	280	266	27.1	900
135	128	430	210	199	675	285	271	27.8	915
140	133	450	215	204	690	290	276	28.5	930
145	136	465	220	209	705	295	280	29.2	950
150	143	480	225	214	720	300	285	29.8	965

HV	HB	HRC	R_m	Notes on the conversions of hardness values Based on the fact that the hardness tests are not aimed at investigating the materials properties, two main ideas are important to bear in mind: the conversion of hardness indexes measured by different tests is approximate; the measurement of hardness is not a direct reading of the stress of the material.
310	295	31.0	995	
320	304	32.2	1,030	
330	314	33.3	1,060	
340	323	34.4	1,095	
350	333	35.5	1,125	
360	342	36.6	1,155	
370	352	27.7	1,190	
380	361	28.8	1,220	
390	371	39.8	1,255	
400	380	40.1	1,290	
410	390	41.8	1,320	
420	400	42.7	1,350	
430	410	43.6	1,385	
440	418	44.5	1,420	
450	428	45.3	1,455	

Table 9.11. Approximate correspondences of some of key hardness ratings after the German Standard DIN 50-150 [NEW 06]

Scale	Hardness number	Interpretation
Rockwell Scale	60 HRC	Rockwell Hardness "C" scale with "60" as the value of the test
Brinell Scale	80 HR15N	Rockwell Hardness, N scale, "15N" with "80" as the test value
	200 HBS10/3000/15	"200" value of the test using a steel ball (HBS) of 10 mm in Ø, under a load of 300 kg, for 15 s
	500 HBW 1/30/20	"500" value of the test using a tungsten ball (HBW) of 1 mm in Ø, under a load of 30 kg, for 20 s
	HB 200	Informal report assuming a 10 mm steel ball, under a load of 3000 kg. "200" result of the value of the test
Microhardness	200 HV 500/15	"200" value of the Vickers test with a load of 500 g, for 15 s
Durometer	A/50/15	Durometer type A, "50" value of the test, during 15 s
	D/50/15	Durometer type D, with "50" value of the test, during 15 s

Table 9.12. Procedure of reading the results of nuanced hardness

9.11. Main mechanical properties of solid materials

We discuss at this stage the definition of the characteristics of the following mechanical tests: flow, tensile, resilience, and fatigue. We need these definitions for the understanding of errors to be qualified and then quantified in metrology.

9.11.1. Flow testing

It is a slow deformation of a body subjected to an extended and constant load. Flow testing reveals the viscoelasticity of plastics. However, when using resins, the duration of stress is too short to give rise to a flow. The test method allows us to determine, as a function of time, the flow elongation of a specimen under a temperature T and subjected to a load most often in tension, rarely in compression. The load is applied at the extremity of the specimen which is suspended by the other extremity; the elongation is then measured as a function of time.

The flow is a slow, progressive and *irreversible* deformation, caused to a material under pressure or tension, constant or frequently repeated, but with intensity of stress below the proportional limit of Hooke's diagram. Under stress, the dislocations begin to move and act on each other. Their movement is more or less limited upon termination of plastic deformation and this leads to hardening or consolidation. The combined effects of stress and temperature lead to the establishment of a dynamic balance between consolidation and softening. The deformation curve shows three stages of deformation characteristics, namely:

- primary or transient flow: flow velocity is reflected by the slope over time;
- secondary or stationary flow: the flow rate remains constant. The velocity is a function of the stress level and temperature;
- tertiary or transient flow: flow rate increases over time and ends with the breaking of the sample. This time (lifetime) flow decreases with increased temperature or stress.

9.11.2. Tensile testing of solid materials

The principle consists in subjecting a specimen material to a tensile testing and measuring the corresponding elongation. The tensile stress (σ) is expressed in MPa. The elongation is expressed as a percentage of initial length of the specimen. It is the most common test in the study of the mechanical properties of materials (see Table A6.6 in Appendix 6). Cylindrical specimens of circular section are often used in this test (Figure 9.11). This does not preclude the use of specimens of different shapes as

in the case of plastics, textiles [GRO 90] (*Yarn test*) or light alloys. The main standards of the tensile test are ISO/R 527; ASTM D 638; DIN 53-455. The test piece has a section S_0 and a length L_0 . The shape of the curve defines the behavior of ductile material, namely:

- L is the length of the specimen, in millimeters;
- S section of the specimen, in square millimeters;
- R the exerted traction expressed in MPa;
- ε the relative elongation expressed in percentage.

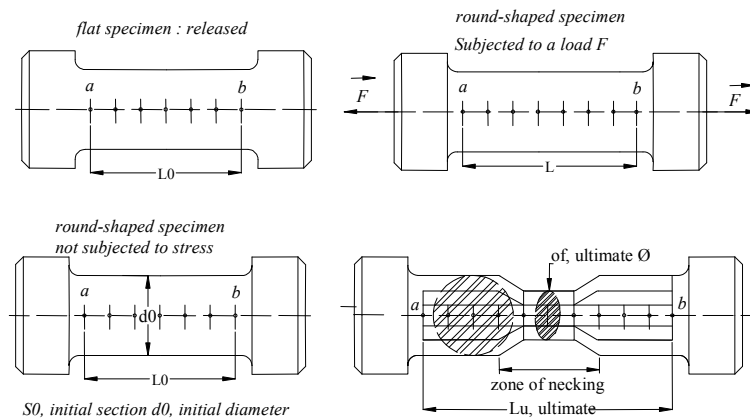
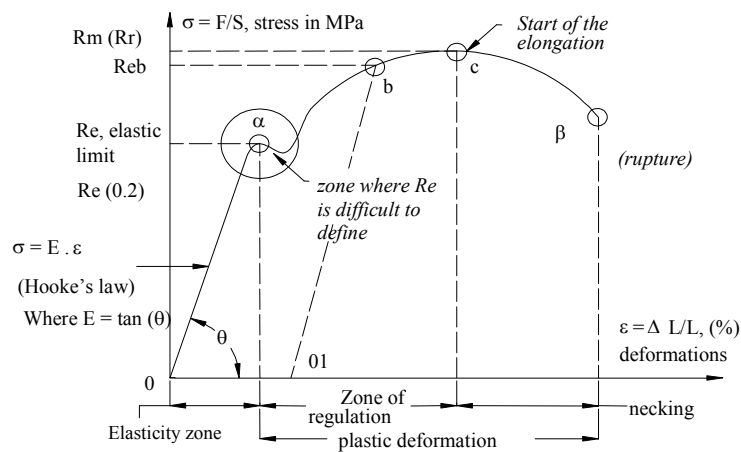


Figure 9.11a. Traction curve of a ductile material (steel)

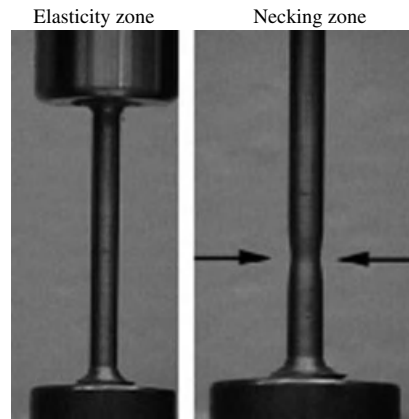


Figure 9.11b. Traction curve of a ductile material (steel)

9.11.2.1. Brittle behavior

The material has no plastic range. The fracture occurs in the elastic range, that is to say, glass, gray cast iron, ceramics, etc.

9.11.2.2. Ductile behavior

Beyond the elastic deformation (reversible), there is a permanent plastic deformation. This plastic deformation is usually accompanied by a hardening as most metals and certain thermoplastic polymers.

9.11.2.3. Nonlinear elastic behavior

The elastic deformation is not proportional to the stress that imposed to it (like in certain thermoplastic polymers and elastomers). The use of traction curves yields the following characteristics:

- elasticity limit (R_e or $R_{e 0.2}$);
- tensile strength (R_m);
- elongation of the rupture (A);
- necking of the fracture (Z).

9.11.2.4. Elasticity limit

R_e or $R_{e 0.2}$ (i.e. 0.2% of deformation) is the constraint, yielding the first plastic deformation. In practice, this point is difficult to determine as there is a yield strength $R_{e 0.2}$ for which there is a plastic deformation of 0.2%. This represents the

stress level at which the material deviates from Hooke's law. In other words, it is the tensile force such that any failure of this effort has the effect of causing irreversible residual deformations in the material subjected to it.

In the case of three-dimensional loading, criteria of elasticity limits define the elastic domain within the stress space: Tresca criterion, von Mises criterion. The most used criterion is the von Mises criterion for which we define an equivalent stress. This criterion is widely used in finite element modeling in continuum mechanics.

9.11.2.5. Tensile strength R_m

This represents the maximum stress R_m , during the test. In the case of brittle materials (no plastic deformation), R_e and R_m coincide. It is the traction effort at which the material breaks into two parts.

9.11.2.6. Elongation after fracture $A\%$

This is the percentage of elongation after fracture compared to the initial length. It is a measure of ductility (brittle materials $A = 0$). It shows the value of the elongation obtained at fracture. It can be read on the curve (plastics), or measured on the ruptured/fractured specimen. This is an indication of the ductility of the material.

9.11.2.7. Necking to the fracture Z

It shows the change in section (denoted Z in % of the initial section) of the specimen after the break/fracture/rupture. The metal begins to elongate elastically with increasing efforts until reaching to the value R_e . Then, it continues to elongate, although traction is lower with the loss of elasticity and with the reduction in the diameter (constriction). The elongation continues with increasing stress until reaching the value R_m , after which a little elongation is still observed while the traction is low, then there is a break with a relative elongation $A\%$ and a coefficient of necking ($Z\%$ coefficient of necking at fracture in percentage): this is the percentage of reduction in section after fracture compared to the initial section below:

$$e\% = \left(\frac{L - L_0}{S_0} \right) \times 100 \quad [9.18]$$

where S_0 is the initial section of the specimen and $S_{ultimate}$ its section after rupture.

9.11.3. Impact test for steels

In 1982, an offshore platform was broken (Norway) and ignited by brittle fracture. On January 16, 1998, at the crossing in Rotterdam, the "FLARE" was

broken into two (the islands of Saint Pierre and Miquelon, Quebec). It took half an hour for the rear section to sink. Her bow section was then sunk off the coast of Nova Scotia; here is a photograph from the “Mail technical” OTUA (No. 64) (Figure 9.12).



Figure 9.12. Aerial photo of the “FLARE” taken from the rear side (Source: Investigation report on a marine (excerpt), according to “Courrier technique” de l’OTUA n° 64, Breaking and sinking Bulk Carrier “FLARE” (Cabot Strait in January 16, 1998)

The analyzed characteristics are typical of a fast brittle fracture. Pendulum impacts are built on the model described by Charpy in 1904 (Figure 9.13). The term “resilience” is reserved for the energy of fracture obtained with a U-notch specimen and divided by the sub-notch section in kgm/cm^2 . It is now denoted KCU (in J/cm^2). It is, in fact, a shorthand term to designate “impact energy absorbed by the bending due to the impact on Charpy specimen.” Standards consider that this designation applies to both U-shaped notch and V-shaped specimens. Evoking resilience when it comes to values in Joules seems to be inappropriate language in mechanics. To avoid confusion, it would be better not to use the word “resilience.” This is hard to apply in practice, because of the widespread testing on V-notch specimen, where

this is still known as Charpy-V resilience test and K.V. resilience. “Resilience” was considered as a quality of steel, a synonym for “tenacity,” before the advent of fracture mechanics, and as a measure of “energy of fracture.” In mechanics of materials and structures, this is known as a “S.I.F” (*Stress Intensity Factor*).

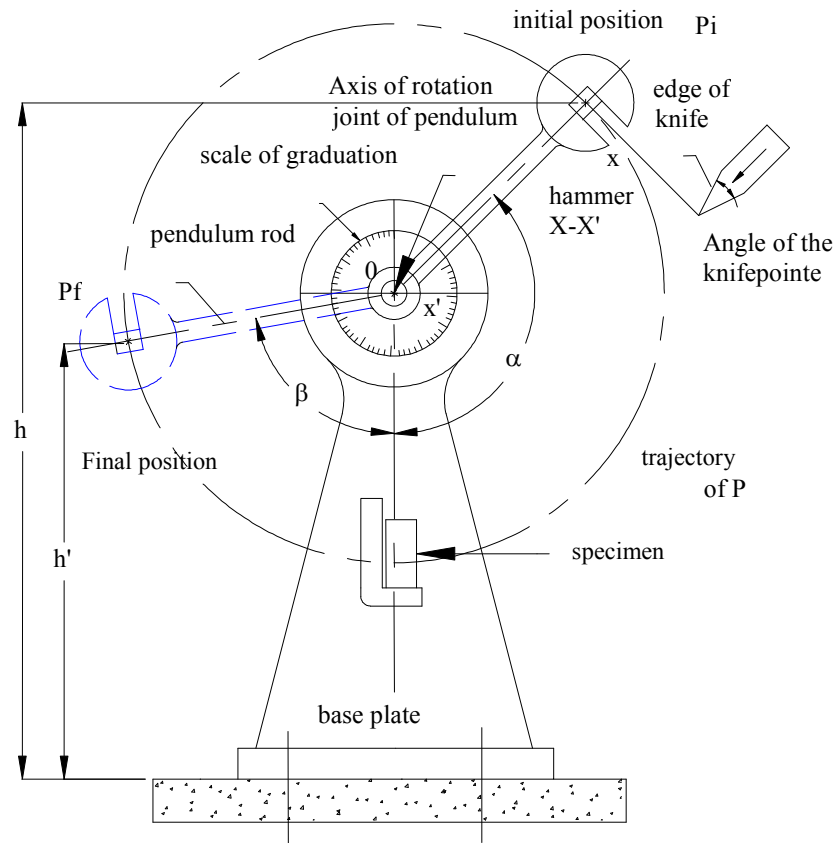


Figure 9.13. Schematic of the Charpy test (shock or resilience)

Moreover, we may also mention the ASTM E23-96 standard which, in addition to the classic Charpy test, describes the Izod impact test. In the latter, the piece has a length of 75 mm (55 mm for the Charpy test) and the V-notch is applied with 28 mm of one extremity. A special impact knife strikes the notched side at the free extremity.

We will not go into more detail on these aspects in the context of this edition. Currently, we prefer to limit ourselves to dimensioning and control aspects of

metrology but before that we will first attempt to answer some questions on current issues.

9.11.3.1. *Why notched impact testing is important?*

Notched impact tests were first introduced to characterize the impact resistance of steel. We know that ductile steel in tension may sometimes be less tenacious under impact. Tests on non-notched bars show very poorly discriminatory and inappropriate methods of manufacturing. The notched bar has been adopted in order to increase their selectivity. The reason is to make the notched impact test more severe (weapons). This test is rarely operated in basic dimensional metrology. It is therefore representative of real situations where variations in section, and other geometrical discontinuities, introduce effects of stress concentration [ASK 89]. Notched specimens allow evaluation of the notch sensitivity of shipbuilding steels. The geometry of the notch becomes an important factor in the field of fatigue failure mechanics.

9.11.3.2. *Criterion of steel quality*

In metrology, energy failure often serves as a benchmark of quality, more sensitive than the tensile properties, which enables comparison of flows having the same shade and detecting possible drifts in the manufacturing process. This is a characteristic inherent to quality assurance, thus inducing the traceability requirement. The KU criterion was often used in the context of engineering construction based on heat-treated steel. Currently, it is rather energy KV that is taken into consideration for relevant steels [ASK 89]. When the inclusions are elongated, the failure energy becomes lower in the transverse direction than in the same direction and the anisotropy along/across accentuates as the sulphur content increases (blister). To examine the fragility [GUI 70] of materials, we start from the fact that non-brittle breakage is influenced mainly by fractures, while brittle breakages induce only a minimal fracture. One way to characterize the ability of a material to break due to its fragility would be to measure the energy absorbed by the fracture under given conditions of a specimen having a particular form of material: this is the “resilience” or impact testing.

9.11.3.3. *Why address these aspects of continuum mechanics, in the frame of a work that is, a priori, used in dimensional metrology?*

In metrology, accuracy of measurements and control is of main concern. Yet experts in fracture mechanics used to call this “dimensioning”. Although the calipers-based metrology is left in favor of “microscopic dimensioning”, the concern and rigor of metrology remain/subsist. In both mechanical cracking failure as well as in dimensional metrology of calipers, digital simulation methods have emerged

as useful approaches for the calculation of uncertainties, showing the statistical method of the GUM and Monte Carlo [GRO 94] and the GUMic [GUM 08] discussed in Chapter 1.

9.11.3.4. *Principle of the energy absorbed by the test specimen during failure*

We drop a mass m (kilogram) from a height h_0 (meter) on a test specimen of section S (square centimeter). After the impact characterizing the ductility of the material, the load reaches a height h_1 that should be measured (in meters). To make an impact test, the lenticular mass is positioned so as to further engage the retaining hook. The needle is brought back to its starting position and the mass is released. The latter, when falling, leads to breaking the specimen and then goes up while carrying the needle. Based on the geometrical and mechanical characteristics of the apparatus, the value of the absorbed, and ultimately resilience, is deduced.

9.11.3.5. *Resilience and reliability criterion for structures*

In all cases, the energy absorbed by fracture is denoted KU or KV and is expressed in Joules. The radius of the edge of the impact knife is 2 mm. The ISO standard also includes the ASTM knife whose edge has a radius of 8 mm.

The most commonly used feature in this context is the fracture energy KV . The notch impact test consists in measuring the effort required to break, with a pendulous mass, a machined square-shaped rod with a 1 cm^2 section, cut in its middle by a notch and based on two supports. This test is performed on a drop hammer pendulum that stores the energy absorbed by the fracture. The test allows characterizing the resistance of a material to impact:

- ISO specimen: V-shaped notch, notch depth 2 mm, angle = 45° , notch-root radius 0.25 mm
- ISO test: U-shaped notch (UF), depth of notch: 5 mm, notch-root radius = millimeter.

9.11.3.6. *Specimen*

We use standard specimens (ISO). We will take the specimen of size $10 \times 10 \times 55$ having a transverse cut of 5 mm deep in the middle, ending in a rounded 1 mm radius (Figure 9.14). In this test, the specimen is clamped on two supports separated by 40 mm, symmetrically arranged to the notch. It receives in its middle, on the side opposite to the notch, the impact of the “drop hammer” which tends to cause the opening of the notch and the fracture of the specimen by bending.

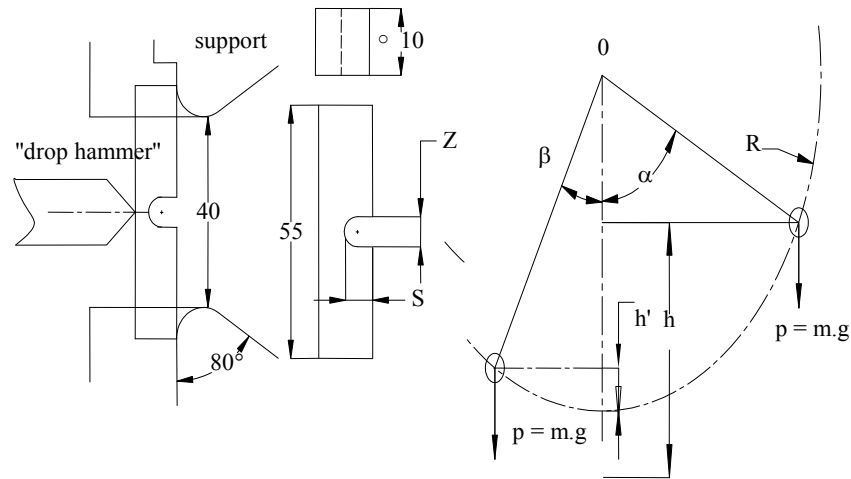


Figure 9.14. Positioning of specimens ISO of resilience of a welded joint

9.11.3.7. Calculations

The knife is vertically removed from a height corresponding to the initial energy W_0 . We then release the knife that breaks the specimen and measure the height of rebound of the pendulum to determine the amount of the non-absorbed energy W_1 . Finally, we calculate the energy absorbed at different positions:

$$W = (W_0 - W) = \text{weights} \times (h_0 - h) = m \times g \times h \quad [9.19]$$

Considering the last simplified diagram of Figure 9.14, where R is the distance between the axis O and the center of gravity of the pendulum, α is the initial angle formed by the rod of the pendulum with the vertical, β , will be the angle raised from the pendulum after the impact. Based on [9.19], we therefore consider:

$$\text{Initial energy before the impact} \rightarrow m \times g \times h_1 = m \times g \times R \times (1 - \cos(\alpha))$$

$$\text{Final energy after the impact} \rightarrow m \times g \times h_2 = m \times g \times R \times (1 - \cos(\beta))$$

$$\begin{aligned} \text{Energy absorbed by the rupture} &\rightarrow (m \times g \times h_1) - (m \times g \times h_2) \\ &= m \times g \times R \times (\cos(\beta) - \cos(\alpha)) \end{aligned}$$

9.11.3.8. Correction

In fact, part of the energy is lost because the machine is continuously operating. Part of the energy is used to move the needle indicating the angle on the graduated dial and part of the energy is lost by mechanical friction at the level of axis of the pendulum. Also, there is a loss of energy due to resistance to the ambient

environment, while additional loss of energy is caused by the displacement of the specimen after the impact. We will try to evaluate these energy losses and, as such, will proceed as follows:

– without placing the specimen, set the needle to zero, release the pendulum normally, β_1 is the angle indicated as a result of this operation (e.g. $\beta_1 = 160.1^\circ$);

– without touching the needle, release the pendulum again. Since normally a small amount of energy has first been used to move the needle, this time the needle will be pushed a little further because the pendulum is not delayed by the needle, except toward the end of its movement. Repeat the process (without touching the needle) until the needle stops permanently. We denote the angle indicated β_2 (let $\beta_2 = 160.2^\circ$);

– leave the needle in position β_2 and release the pendulum normally so as to make five complete oscillations (or 10 beats): five in one direction and five in the opposite one. At the beginning of the 11th beat, we set the needle so that the pendulum pushes it just 1° . Do not touch the pendulum during these 11 beats. Note the angle β_3 (let $\beta_3 = 156.6^\circ$). The energy required to move the needle at an angle β_2 is written:

$$\text{if } \beta_2 = 160.2^\circ; \beta_1 = 160.1^\circ \rightarrow \text{ then } \beta_2 - \beta_1 = 0.1^\circ$$

The angle β of mean value of increase between readings β_2 and β_3 is:

$$\text{for } \beta_2 = 160.2^\circ; \beta_3 = 155.6^\circ \rightarrow \text{ then } \beta = \frac{(\beta_2 + \beta_3) \times 2}{2} = 316.8^\circ$$

The energy lost by air drag and mechanical friction during a half oscillation, i.e. a beating, will be:

$$\text{for } \beta_2 = 160.2^\circ; \beta_3 = 155.6^\circ \rightarrow \text{ then } \varepsilon = \frac{(\beta_2 - \beta_3)}{10} = 0.36^\circ$$

The energy lost ε during an angle β_2 move is then:

$$\text{for } \beta_2 = 160.2^\circ; \beta_3 = 155.6^\circ \rightarrow \text{ then } \varepsilon = \left(\frac{\beta_2 - \beta_3}{10} \right) \times \left(\frac{\beta_2}{\beta_2 + \beta_3} \right) = 0.182^\circ$$

$$\text{The effective angle of drop will be: } \alpha = \beta_2 + \left(\frac{\beta_2 - \beta_3}{10} \right) \times \left(\frac{\beta_2}{\beta_2 + \beta_3} \right) = 160.3825^\circ$$

The total energy lost (drag, friction of pendulum, and friction of the needle) will be represented by the angle: $(\alpha - \beta_1)$, i.e. by $160.4^\circ - 160.1^\circ = 0.3^\circ$, hence the correction on the angle of increase β after the rupture of the specimen is written:

$$\text{for } \beta_2 = 160.2^\circ; \beta_1 = 160.1^\circ \rightarrow \text{ then } \beta = \beta_1 + (\alpha - \beta_1) \times \left(\frac{\beta_1}{\beta_2} \right) = 160.382^\circ$$

The potential energy transmitted to the specimen after rupture will be written:

$$\text{from [9.19] consider: } \Delta W = m \times g \times R \times (1 - \cos(\beta)) \quad [9.20]$$

where m is the mass of the specimen, R is the distance between the specimen and the axis of rotation of the pendulum (Figure 9.14), and β is the angle of rebound of the pendulum after fracture/rupture of the specimen of dimensional metrology.

9.11.3.9. Laboratory report

The laboratory report must include the following:

- type, model, and capacity of the machine being used (Figure 9.13);
- type, size, and standard of the specimen (CSA, ISO, DIN, etc.);
- maximum linear velocity v of the pendulum (at impact) and the energy loss;
- energy absorbed by the specimen due to rupture;
- energy of the pendulum just before the impact;
- temperature of the specimen;
- aspect of the surface (facies) in the fracture site;
- number of samples that have not undergone complete fracture.

Note that the energy absorbed by the specimen due to the fracture (point 5) corresponds to a work surface ($1 \times 1/2 = 1/2 \text{ cm}^2$). Actually the absorbed energy will be presented per unit of surface.

9.11.3.10. Temperatures of resilience test

Resilience tests are carried out mainly at room temperature and at temperatures below 0°C , that is to say between -20°C and -196°C . The ductile structure of the specimen becomes brittle with low temperature during the test. The coolants used to decrease the temperature of specimens are shown in Table 9.13.

Alcohol	up to -80°C
Freon	up to -155°C
Liquid nitrogen	from -155°C to -196°C

Table 9.13. Example of three coolants

9.11.3.11. The curve of ductile–brittle transition

This curve is determined by experimenting with three sets of specimens of resilience at different temperatures. It can highlight, if it exists, a zone of ductile–brittle transition of steel. Facies of the surfaces of fractured specimen are either 100% grain or crystalline in the area of brittle fracture, either mixed facies in the zone of ductile–brittle transition, or even without grain facies in the zone of ductile. The more the grain size of a metal is low, the better are the values of resilience at low temperature (for a graphic translation, see Figure 9.15).

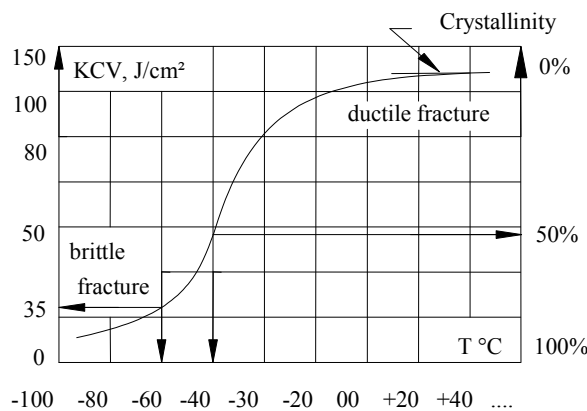


Figure 9.15. Theoretical curve of ductile–brittle transition

9.11.3.12. Endurance

Endurance is the resistance to repeated stresses. Endurance is characterized by fatigue limit σ_D , maximum stress fatigue $\sigma_{\max}$, and lifetime N .

9.12. Mechanical tests on plastic materials

9.12.1. Tensile strength, strain, and modulus ASTM D638 (ISO 527)

To understand material performances, it is important to know how it will react in the presence of a load. If we know the importance of the deformation caused by a given load (stress), we will be able to predict the reaction of the application in operating conditions. The ratios on stress–strain under stress are the most common mechanical properties used to compare the materials or design an application.

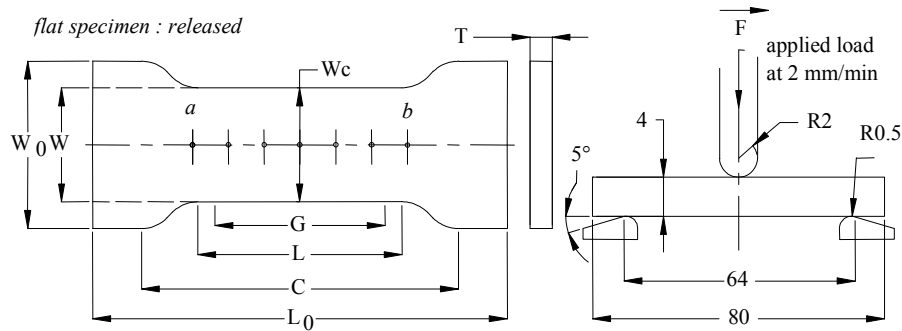


Figure 9.16. ASTM D 638. (a) Specimen for tensile molded plastics test; (b) impact test specimen for molded plastics

- W : width of narrow section and W_0 is the total width minimum;
- L : length of the narrow section and L_0 is the minimum total length;
- G : gauge length;
- R : radius conjunction.

9.12.2. Flexural strength and modulus ASTM D 790 (ISO 178)

Resistance to bending allows us to measure the degree of resistance of a material to bending or to measure its consistency. Unlike the tensile load, in a bending test the stress applied is in one direction. A simple beam resting freely on two supports is loaded mid-span.

On a standard test machine, the loading spout is pushed onto the sample at a velocity of 2 mm/min. To calculate the flexural modulus, a bending-load curve is plotted using the data recorded. The latter reflects the initial segment of the linear curve. The flexural modulus (ratio stress/strain) is most often included among the flexural properties. This concerns the part of the curve where the plastic is not distorted. The values of flexural stress and flexural modulus are expressed in MPa (or psi in imperial units).

9.12.3. Impact test

As previously presented, we use tests such as tensile strength and flexural strength in cases where the material absorbs the energy slowly. In fact, the materials very quickly absorb the energy applied on them. The Izod and Charpy methods are

used to study the behavior of samples subjected to specific stress impacts, and to assess the fragility or robustness of the samples. Metrologically speaking, it is possible to obtain information about the typical behavior of a material by testing different samples prepared under different conditions and by changing the radius of notch and test temperatures. Both tests are performed on a pendulum impact test. The sample is locked (clamped) in a vise; the pendulum is dropped from a predetermined height, thus causing the shearing of the sample due to the sudden load imposed. The residual energy of the *drop hammer-pendulum* pushes it up. The difference between the height of the drop and the height of increase represents the energy required to break the sample. The test is performed at room temperature or at lower temperatures to test the embrittlement under low temperature conditions. Test specimens vary depending on the size of cuts.

9.12.4. Interpretation of resistance to impacts – ASTM compared to ISO

The impact properties can be very sensitive to the thickness of the sample and the molecular orientation. Thickness differences in the sample invoked by ASTM and ISO methods may strongly influence the resistance to impacts. A change of approximately 3 mm in thickness could change the type of failure of ductile behavior into a brittle behavior. Materials already showing a brittle fracture mode with a thickness of 3 mm, such as the reinforced grades, are not affected. However, it is necessary to understand that only the test methods have changed, not the materials. The ductile–brittle transition, mentioned earlier, does rarely occur in real conditions.

9.12.5. Izod impact strength ASTM D 256 (ISO 180)

The notched Izod impact test has become the standard for comparing the impact resistance of plastics (see Figures 9.16(a) and 9.16(b)). However, this test has little to do with the reaction of a molded piece to a real environmental impact. The sensitivity of notched material can vary; therefore this test will penalize certain materials more than others.

In metrology, the *notched* Izod test serves primarily to determine the impact resistance of parts with many sharp angles, such as ribs, intersections, and other elements of increased stress.

The unnotched Izod test uses the same geometry of loading except that the sample does not contain notches. This type of test indicates always values higher than the values given by the notched Izod test. This is due to the absence of a stress concentrator (S.I.F). The ISO designation indicates the type of the sample and the type of notch, for example:

- ISO 180/1A designates a type-1 sample and type-A notch. As shown in Figure 9.16, the dimensions of a sample of type 1 are usually about 80 mm in length, 10 mm in height, and 4 mm thick;
- ISO 180/1U designates the same type-1 sample but fixed in the opposite direction (unnotched). The samples used in the ASTM method have the same size, the same notch radius, and the same height but their length and particularly their thickness are different: 63.5 mm in length and 3.2 mm in thickness;
- the ISO results are defined as the impact energy expressed in joules, used to break the test sample, divided by the sample surface at the notch level. The results are expressed in kJ/m^2 . The ASTM results are defined as the impact energy expressed in joules divided by the length of the notch in meter. This gives a final result in J/m .

9.13. Fatigue failure and dimensional metrology for the control of the dimensioning of materials assembled by welding

9.13.1. Fatigue testing

Metallic parts that are subjected to repeated or alternate stresses may break even if the maximum stress is below the elastic limit. The lifetime of these parts is even longer when the stresses are lower (Wöhler curves). We carry out fatigue tests [GRO 98] by subjecting a metallic specimen to tensile/compression or alternate bending. For most steels, there is a critical stress below which fracture occurs only after a considerable time. This stress is the fatigue limit of steel. The origin of the fracture lies in a tiny crack that extends smoothly and results in a sudden break. We calculate the metallic parts subjected to repeated stresses so that no stress point, per square millimeter, exceeds the fatigue limit. This implies fitting different parts by ensuring the spacing of large curvature radius and taking care of the surface texture.

9.13.2. Tenacity

It is the resistance to deformation and fracture. It is characterized by the limit of strength R_e , the tensile strength R_m , and hardness HB, HRC, or HV for the resistance to deformation. Works on cross-welded joints [GRO 94, GRO 95, LAS 92] with different welding processes showed that the fracture occurs at the foot of the weld. Vickers tests showed that the heat affected zone is located at the foot of the weld seam for the following four welds (SAW = *Submerged Cored Arc Welding*, FCAW = *Flux Cored Arc Welding*, SMAW 57, and SMAW 75 = *Submerged Metal Arc Welding*) (Figure 9.17). Calculations of the fracture of the crack led to the correlation between the intrinsic factors of the material (C and m) as the tenacity

depends on them through ($\Delta K = fh$) stress intensity factor. Thus, as per the law of propagation of cracks by Paris-Erdogan, we can write:

$$\frac{da}{dN} = C \times (\Delta K)^m \quad \text{with } (\Delta K) > 0 \quad [9.21]$$

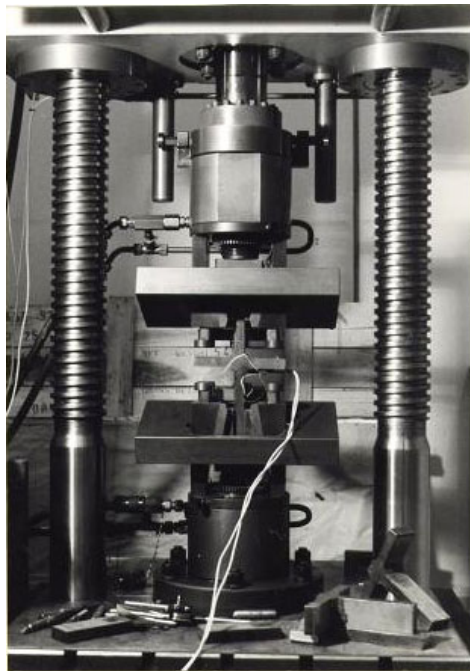
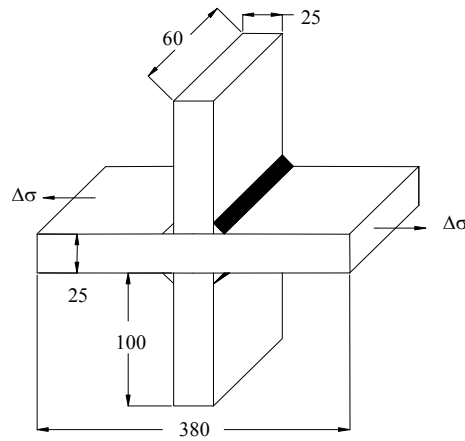


Figure 9.17. Test of resilience in an enclosed structure cross welded

C and m are the intrinsic parameters of the material and (da/dN) expresses the ratio of crack propagation. The linearization of the expression [9.21] can be easily written (C and m) thus:

$$\log\left(\frac{da}{dN}\right) = \log(C) + m \times \log(\Delta K) \quad [9.22]$$

Knowing the expression of toughness (that is to say, stress intensity factor or toughness), written as:

$$\Delta K = \Delta \sigma \times g\left(\frac{a}{T}\right) \times \sqrt{\pi \times a} \quad [9.23]$$

By replacing ΔK (ISO 12737: 1996) by its expression in [9.23], we get:

$$\frac{da}{dN} = g\left(\frac{a}{T}\right)^m \times C \times \Delta \sigma^m \times (\sqrt{\pi \times a})^m \quad [9.24]$$

where N is the number of cycle (loading); a is the crack length in millimeter (or micrometer), and T is the thickness of the sheet requested, in millimeter. Here is the final result of the average of (C , m) intrinsic parameters of the material for the four welding processes:

$$C_{\text{average}} = \frac{6.069 \times 10^{-8}}{24.64 \times m} \text{ MPa} \times \sqrt{m} \text{ with a correlation } R^2 = 0.963 \quad [9.25]$$

Here (Figure 9.18) is the result of a photographic work [LAS 92] on this matter. The fracture mechanics allow us to quantitatively predict the risk of rupture. In other cases, we cannot rely on previous data or correlations between material properties and fracture behavior [GRO 94] on constructions.

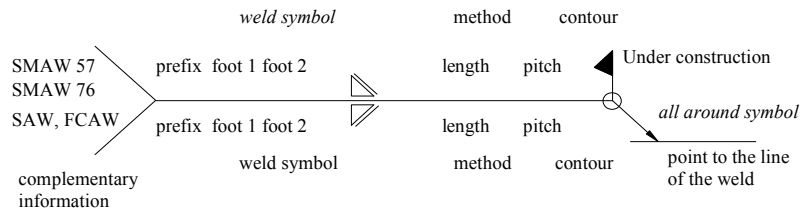


Figure 9.18a. Dimensioning according to ISO 2553 for the four cross-shaped welded joints testing



Figure 9.18b. Dimensioning according to ISO 2553 for the four cross-shaped welded joints testing

The procedure for measuring the dimensions of the welds (Figure 9.18) is based on the copying of the size modeled using a dentist powder.

Figure 9.19 illustrates the results (average) from four welding processes. The method used (the Gurney method, UK 78) to measure the geometry of the weld seam is a toothpaste into which we had driven the cord. Then, we measured the imprint left by the cord. Thus, we deduced the evolution of the geometry $g(a/T)$ used to calculate the fracture parameters (Figure 9.20) [GUR 78].

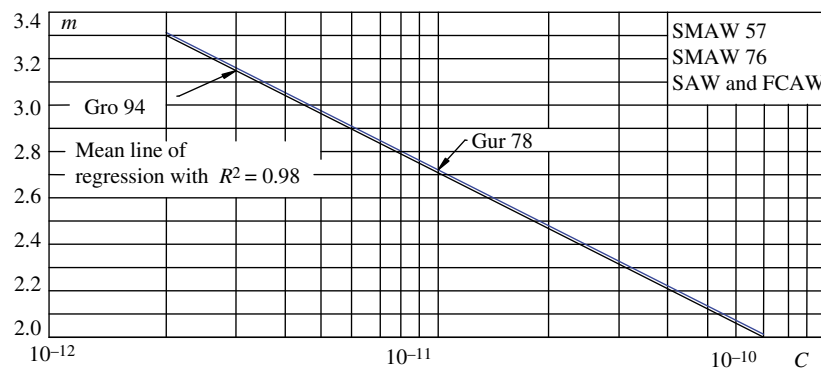


Figure 9.19. Average relationship between intrinsic parameters (C and m) of welded material; see formula [9.25]

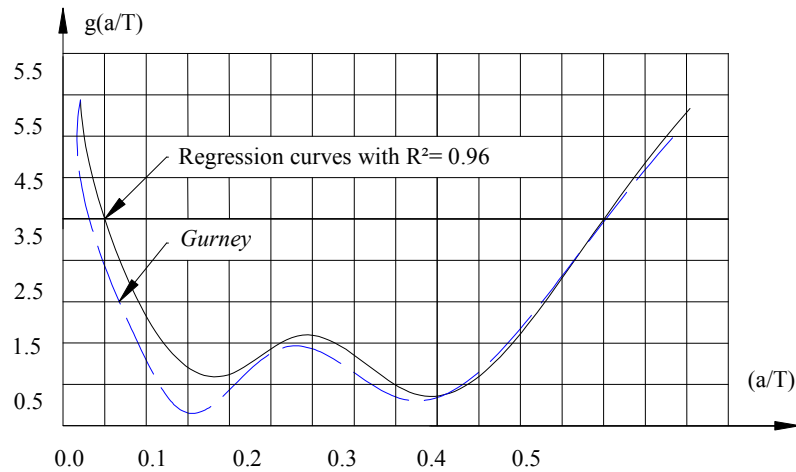


Figure 9.20. Evolution of the geometry correction factor by welding process

9.13.3. General tolerances for welded structures according to ISO 13920

The welded joints and other structures have tolerances of four per class (CL1, CL2, CL3, and CL4). Although large, the tolerances are in force in the workshops. We present a table from ISO 13920 (Table 9.14).

Tolerance class	Linear dimensions: example length L in mm							
	2–30 mm	> 30 up to 120	> 120 up to 400	> 400 up to 1,000	> 1,000 up to 2,000	> 2,000 up to 4,000	> 4,000 up to 8,000	> 8,000 up to 12,000
CL1	± 1	± 1	± 1	± 2	± 3	± 4	± 5	± 6
CL2	± 1	± 2	± 2	± 3	± 4	± 6	± 8	± 10
CL3	± 1	± 3	± 4	± 6	± 8	± 11	± 14	± 18
CL4	± 1	± 4	± 7	± 9	± 12	± 16	± 21	± 27
Tolerance class	Angular dimensions: example $\Delta\alpha$ in degrees, min							
	± 20'	± 15'	± 10'					
CL1	± 20'	± 15'	± 10'					
CL2	± 45'	± 30'	± 20'					
CL3	± 1°	± 45'	± 30'					
CL4	± 1° 30'	± 1° 15'	± 1°					

Table 9.14. Tolerance of welds according to ISO 13920

Tests on materials are carried out either in destructive or non-destructive ways. The first leaves the trace of the impression or the facies break (fracture). The second consists in revealing, without altering the piece, the defects that can affect its behavior when in service.

In metrology, there are several methods that adopt the second way. Such as, for example, the control of ultrasound, magnetoscopic, radiometallographic control, control by X-ray (Bragg's law), control by gamma ray or gammagraphy, and penetrant testing (liquid penetrating the flaws, cracks, fissures, etc.). These methods will be presented in a future work which will deal with the metrology of materials and structure and their reliability.

9.14. Summary

We presented two ways of testing, non-destructive and destructive. The first deals with hardness tests and the second with several mechanical tests. The first does not focus on the mechanical properties of materials, whereas the second is designed to investigate the limitations of materials. We have presented six types of tests but only four of them are major tests: Rockwell, Brinell, Vickers, and Knoop are generally used for solids and isotropes. Other tests: Barcol, Shore, IRHD, and Mohs are used for minerals, plastics, and rubber products. Both the first four types of hardness tests and the other four tests do present scales of measurement. We have seen that some tests have correspondences over other test scales; however, a systematic conversion or a mathematical research of systematic equivalence is less desired. The scale of measurement of Mohs test is specific to minerals and does not concern conventional machining materials such as steel, cast iron, and some plastics.

The mechanical tests we have presented mainly included the tensile and impact tests. They can be applied to ferrous alloys, non-ferrous metals, and plastics. The aim was to popularize concepts about the properties of materials that metrology measures daily in workshops and laboratories of metrology. We have often heard that this subject is not a part of metrology, but rather of the strength of materials and mechanics of continuous milieus. This way of thinking about issues of control and measurement is simplistic, in many respects. Just entering into any laboratory or manufacturing workshop will make us aware of the fact that control and dimensional measurements are inevitable and the metrology instruments indispensable.

Sometimes, neither control nor dimensioning – as suitable as they might be – is sufficient. Calculations by computer simulation are becoming increasingly indispensable in metrology. Thus, we have presented some mathematical formulas necessary for the verification measurements. We have avoided presenting the results ensuing from the micrographic analysis of materials; not because they are moving

away from dimensional metrology of precision, but, on the contrary, because this deals with high-precision optical metrology. The amount of data and themes addressed in that case is so large that we found it useful to give priority to some topics over others.

In the chapters dealing with the interference and study of profile projector, we have addressed these issues. The reader may refer to them and adopt applications appropriate to their own case. As for microscopy for the analysis of structures, the literature suggests many cases and it would be wise to refer them.

9.14.1. *There is seriously no universal solution to conduct hardness tests*

Brinell, Vickers, and Rockwell tests, for example, are most commonly used with metallic materials (that is to say, in most applications). Shore and IRHD (International Rubber Hardness Degrees) tests are reserved for plastic materials and rubbers.

The Brinell test is frequently used in workshops. As the applied loads are relatively large (the only test of its kind allowing up to 3 tons) and they generate significantly large imprints on the piece, the surface does not require special treatment. This test is frequently used to measure the hardness of heterogeneous materials or raw materials such as cast iron, castings, the elements of piping, hot-rolled metals, and aluminum parts. The method, however, is poorly suited for tests on small specimens or pieces of low thickness.

As for the Vickers test, it is the most reliable and most accurate of all tests. "It is also the one whose application scope is wider". It is used in the field of surfaces treatment and in the automotive and aerospace industries. Often performed in the laboratory, the Vickers test can be extended to very low loads (up to 10 g). It is therefore suitable for parts of lower thicknesses. The drawback is that the test is more difficult to implement. "The lower the loads to be applied, the more the part's surface state should be cared for."

The Knoop hardness test is better suited, than that of the Vickers, to finest specimens or coatings. For a diagonal of the same length, the impression is in fact much less profound (of factor 4), under the condition of obtaining a sufficient surface quality.

The Rockwell test, in its turn, is less accurate than the Vickers test, but it allows quick and easier controls. Unlike previous tests, it provides the value of hardness just after penetration, without necessarily providing any reading device (optical or manual). The result of a Rockwell test is independent of the operator. It is therefore appropriate for high hardness materials (due to relatively significant test loads) and

it requires less surface preparation. These advantages make it a relatively common procedure. Yet it is prevalent in almost all laboratories of schools. Let us also mention its limitations, especially the application of low loads (i.e. for the finest pieces) since it does not allow loads as low as that used in the Vickers.

In the famous Formula 1 (motor sport), one of the most widely used tests to distinguish between soft and hard tires is the Shore test. It may also be used for measuring the hardness of a large number of non-metallic materials (rubber, polyester, PVC, leather, glass, plastic, etc.) We know, based on the processes of materials and rheology, that the more an elastomer is likely to get older, the more likely it hardens and becomes more tenacious. The Shore test is often used to track the aging of these materials. As for the hardness IRHD, it is preferred to use the Shore test when dealing with small parts (O rings, small pipes, etc.) The rebound method is simple and even faster to apply. It is applicable to portable devices, which show directly the value of hardness. The method is *less destructive* than Vickers or Rockwell tests. It requires no special preparation of the surface, but it quickly finds its limits on thin and relatively light specimens. The parts must be massive enough to not vibrate and potentially distort the measurements.

9.14.2. Some criteria for choosing hardness testing apparatus

An important criteria for choosing the apparatus is its portability. The choice of devices also depends of course on the weight and dimensions of parts (machines table tests show necessarily limited capacity) as well as on the type of tests to be performed. Portable solutions are suitable for rapid inspections, to validate a material when it is received, or to check inventory parts on site, but they will not give the accuracy of a testing machine. Portable devices typically employ the method of rebound hardness (HL), but they can also incorporate small penetrators (Shore IRHD or Rockwell).

When a portable device gives the result in HL hardness units, it has also in most cases a function of conversion in HV, HR, or HB units. We should be very careful about using empirical conversion scales (valid for very specific materials) between Rockwell, Brinell, and Vickers units, since there is no universal reliable conversion scale. The comparison of results between different methods is nothing but an approximation. One should keep skepticism toward appliances expected to perform several types of tests. In many cases, they use only one indenter, and thus only convert the result in different units. Yet, all manufacturers, or almost, have their own conversion tables. At some point, we noticed that students tend to convert their units to avoid laboratory tests. This practice is questionable and therefore misleading in terms of results. The fact that some tables compare scales side by side does not necessarily mean that their equivalence is verified *de facto*.

It is often difficult to distinguish between two hardness machines performing the same type of tests. *A priori*, no difference can be noticed: they have the same indenter and provide the result with the same accuracy (since it is normalized). Furthermore, they offer a range of similar loads. The principle essentially remains the same. The difference is in the criteria such as the degree of automation of the machine, the mode of recognition of the imprint, or the automatic moving of table. Hardness measuring devices are becoming more “miniaturized.” They also have integrated all kinds of automatic functions to overcome the influence of the operator, or to simplify the task: tactile screens, cameras and image analysis systems, powered turrets allowing launch of automatic sequences in changing the position of the indenter or lentils, etc.

9.14.3. Indentation reading mode

In a Rockwell test, for example, the hardness value is given immediately after indentation, by measuring the residual depth of the cone or the ball being used. However, this is not the case in the Vickers or Brinell hardness tests which require measuring the size of the indentation left. To this end, there are several methods: the use a device equipped with a microscope (we then use the thumb wheels to move and overlay graduated features with the diagonals of the impression), a camera (allowing to automatically measure the dimensions of the imprint by image analysis), or a simple frosted glass allowing measurement of the diameter with the naked eye. Manual methods are of course less expensive, but their outcome is strongly influenced by the operator who performs the test. It also depends on the frequency of measurements. The load exerted on the indenter can be applied by a set of dead loads associated with a lever system, or by pressure exerted by a ball of a ball screw, and controlled by a force transducer. Both solutions offer comparable accuracy; however, the ball screw is the most flexible method.

9.14.4. The expected result

In most applications, a simple measure of hardness is sufficient to characterize the resistance to wear of a material or the influence of surface treatment to which it is subjected. Sometimes the accuracy of the measurement is not even considered. What excites industrialists (and students) is their ability to quickly control their workpiece well below or above a certain degree of hardness that they have initially fixed. Nevertheless, there are also cases where we must go further and more accurately characterize the material’s behavior under load. One solution for this is the instrumented hardness HM (Martens, formerly universal hardness HU) which allows continuous recording of the test force, time, and depth of indentation. The resulting curves – representing the charging and discharging cycles as a function of

time or the displacement of the indenter – are therefore very instructive. They allow determining the elastic modulus of the material, knowing the boundary between the elastic and the plastic behavior and its flow. This is the only reason that made us to limit this manual to simple definitions of destructive tests.

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Chapter 10

Overall Summary

Constructions are sometimes made *without calculation*, hence apparent useless control or even measurement.

Then, crafts, but not arts, first began to assess, followed by control and finally measure. Arts, from the time of *Maestro* Leonardo Da Vinci, had already given *measurement*, even though abstract, to be formalized or at least without the current popular scope. Gradually, the measurement was formalized and more accurate means, if not reliable, were integrated into the function metrology. The multiplicity of measurements in space and time imposed contingencies that were then classified as errors or uncertainties.

Products, non-conforming to what is commonly called the standard, or the refusal of conform products, may have an adverse impact on the industry. One way to guard against this risk is to master the dimensional metrology and thereby to master the metrology in the enterprise by calibrating without falling into the “*étalomania*,” and by inspecting the measuring instruments to detect the uncertainty of measurement.

As such, metrology is a real tool for the quality that has been integrated in the same way in all reference frames of existing quality standards with ISO 9000 subject to question, for that matter. The dimensional metrology needs are evolving with the imperative of quality. This is due to many reasons. First is the development of quality assurance in production activities as well as testing and analysis laboratories accordingly, with the impact of recognitions led by the certification and accreditation. The importance of testing and analysis in the global exchanges has led to an increase of mechanisms for mutual recognition. This resulted in a harmonization of practices leading to profound changes.

Given the economic risks involved in accepting a bad product or, alternatively, refusing a conforming product, we understand the need for reliable and good quality measurements. For this, two basic criteria are to be taken into account: *accuracy* of measuring instruments and the confidence that can be placed in the measurement result, which is quantified by the uncertainty of measurement. Every enterprise must therefore.

- quantify the errors revealing the measurement uncertainty by objectives;
- ensure the quality of measurement results that it realizes.

This involves many people and requires us:

- to involve engineering firms that specify the characteristics of measurement means;
- to refer to the industry benchmarks to analyze the measuring means necessary to ensure the control of manufacturing operations;
- to define calibration procedures (internal or external);
- to ensure the technical supervision of the measurement procedures to avoid faults;
- to train personnel to manage the measuring instruments;
- to study how to ensure traceability of measurements.

This last requirement addresses two concerns over the globalization of the economy: to ensure harmonization of measurements in Canada and internationally, and to enable a benchmark of measurement results over time. This requirement can be met by calibrating instruments at national laboratories level (NRC, Canada) or in accredited laboratories (Mitutoyo, Canada). To fit a measuring instrument, we perform a calibration consisting in comparing the measuring instrument to recognized etalon standards, to determine the deviation from the etalon standards.

Unlike the calibration, which is a technical operation, the inspection is an “administrative” operation that allows a decision to be made: if the results were within the limits of acceptance, then the apparatus is re-enabled. In the case of an adjustment or repair, then re-calibration and verification become compulsory, to ensure the conformity of the measuring instrument.

One of the factors involved in the decision process for a product acceptance is the uncertainty in measurement. It is considered that uncertainty in measurement is an interval, centered on the measurand value, in which the true value is more likely to be situated. So, it is perfectly conceived that the more likely the uncertainty is to be low compared with the tolerance to be measured, the lower the risks to be taken

in terms of acceptance or rejection of a product. Hence, it becomes important to know the uncertainties of measurement, to select a device and a method which would be adapted to the measurement to be carried out. It is therefore necessary to know, first, the needs in terms of measurement. This means that – based on the specifications of manufactured products, or the measurement's motive during a manufacturing process – we could determine the uncertainties of the required measurement. Then, we shall analyze the measurement process and estimate the ensuing uncertainty.

Choose carefully the material to be inspected. Should one apply the same monitoring methodology to all measuring instruments? No, above all, for cost purposes. What selection criteria should be maintained? When the measurement accuracy is likely to be determinant for the quality, safety, and security of the product, the measuring means should be monitored rigorously. Other devices may only be listed in an inventory. In this case, one should consider the possible consequences of an undetected drift, assess the risk in terms of probability, and compare it with the total cost.

Determine the frequency of monitoring. It is impossible to determine a time interval that is short enough so that no risk of a measuring apparatus drift would occur. A high frequency of calibration is costly, mainly because the operation itself is expensive in addition to taking into account shortfall resulting from the immobilization or the replacement of the device. Similarly, too long intervals may inhibit detecting a drift as early as possible. A compromise then becomes necessary. Yet, the frequency of calibration is also not necessarily constant. The time intervals between inspections may be shortened when the results of previous comparisons do not allow ensuring a permanent accuracy of the measuring instrument. They will be extended if these comparisons indicate that the accuracy does not deteriorate.

Legal metrology. Is the intervention of the government to ensure the quality of measuring instruments or measuring operations affecting the public interest: safety of people, health and environmental protection, and fair transactions. The International Organization of Legal Metrology (OIML), gathering together government agencies analogous to the Sub-Directorate of Metrology, is responsible for establishing international guidelines on measuring instruments.

Attributes of the metrologist. A metrologist should behave and think in good agreement with the required measurements and precision. This presupposes showing faculties such as *curiosity and the aptitude to always doubt, honesty, sense of observation, order, and methodology* as well as good knowledge of basic sciences (mathematics and physics).

Curiosity is evoked here because a metrologist must be knowledgeable about the equipment or instruments that he/she uses under specific conditions, hence

appropriate influence quantities. Is this enough? No. A metrologist must also learn about what goes on in laboratories worldwide and not content himself in isolation...

A *warned* metrologist always calls into *doubt*: the calibration, the reference, and measurement conditions to then opt for a method accordingly. Clearly, the metrologist should criticize processes to further improve. He/she must check periodically that references are properly calibrated by the device. He must also check the conditions of temperature, humidity, and absolute pressure. Why doubt? Doubt leads to repeated measurements via different instruments and preferably with different methods.

Hence, we find again the concept of repeatability and reproducibility of measurements. Doubt, for a metrologist, is not synonymous with suspicion that psychology would better situate it in its proper context. Doubt, for the metrologist means mastery of the measurement uncertainty. For this reason, we would have preferred to call this handbook “Metrology in crafts” and to a lesser degree in the arts. Pablo Picasso said something like: “I do not seek. I find, and then I seek.” This sentence summarizes an aspect of natural doubt that should be cultivated by the metrologist.

The word “honesty” has often been overused. It would be inappropriate or unbecoming to add it here, but for the metrologist, honesty means simple acts such as:

- leave a blank there where the measurement raises doubts;
- transcribe and transmit the results even when the values seem anomalous.

This does not mean to deliberately transmit to “deceive or mislead”, but rather for the fidelity of equipment and instruments being used. The list would be long (unstable apparatus, effects of influence quantities, etc.). Thus, we note that the metrology is not and will not be a fad phenomenon. It is a broad discipline that has its roots in science and various instrumentation techniques.

A keen sense of observation and culture of “things” shown by the metrologist will be useful in the analysis of assemblies. In such cases, an individual who has this sense of observation will quickly notice, for example, that a gauge block is scratched or if a part is reversed on its non-grinded side during the analysis of a surface state, etc.

Ultimately, this handbook, written for the students of schools of art and crafts is also useful for enterprises including departments for dimensional metrology and quality control. That is why we addressed what boils down to study the following chapters:

1. Errors and uncertainties;

2. Geometric Product Specifications: GPS;
3. Linear and angular etalon standards;
4. The CMM Coordinate Measuring Machine;
5. Optical metrology by profile projector and interferometry;
6. Roughness of machined solid materials;
7. Controlling surfaces of revolution;
8. Control of threads, gears and grooves;
9. Hardness testing and mechanical tests;
10. An overall summary.

Further information on regrouping some resolute laboratories, tutorials, and reference tables (benchmarks) will be added as references from a website under construction. The user may become inspired from its laboratories by making its own data.

Each of the chapters dealing with control using instruments or apparatus is supported by case studies and photographic or schematic exhibits. The courtesy of manufacturers Mitutoyo, (Montreal and Toronto, Canada) and Starrett (Athol, MA, USA) as well as the NRC Ottawa (standards) has allowed us to reproduce, with their respective authorizations, some instruments used in various laboratories including ours. Throughout the chapters, the reader will learn the rules to follow to ensure the quality of his/her measurements in both training laboratories and in company laboratories.

The drafting of this book took place during several years of teaching dimensional metrology, design, and GPS in mechanical manufacturing. The content of this book has been put into practice with Cégep students (Quebec). Reviews and comments by students have been taken into account. Writing this book is primarily intended to provide the student with simple and consistent tools to carry out his laboratories and other tutorials.

Teaching experience has taught us that many students are still experiencing difficulty in mastering basic mathematical concepts that are necessary to grasp metrology, similarly for vocabulary issues.

Most chapters are illustrated with photographs, often from either our own laboratories or manufacturer Mitutoyo. Also, we have added at the end of each chapter one (or more) laboratory model(s). The latter remains suggestive because, not all laboratories are equipped with similar equipments as ours. Also any

presented examples taken from a work referenced in the technical literature have been the subject of careful test by our own experimental data.

In our view, practical works should be more emphasized on exercises related to laboratories to enable the student to have more confidence and the ability to more easily translate data in the drafting of his laboratories. We also add to this the importance of geometry and trigonometry, which remain essential in achieving the objectives of training standards.

The texts are deliberately illustrated with charts, diagrams, and photos to facilitate the retention of examples.

Ultimately, the reader will have noticed that we also attempted to “mathematize” some examples of metrology. Indeed, we did it deliberately because unfortunately in some technical departments, this discipline is cunningly concealed. We apologise for this! If, in some places, we have made references to some aspects of construction technology, we did so out of concern for the reader, to spare them from being spread between reading tables and other references. Moreover, we can reaffirm that metrology is synonymous with uncertainties. However to calculate the uncertainty, we briefly emphasized that we should also be familiar with all the aspects involved in the measurement being conducted.

As a user of this book, there undoubtedly will be some remarks to make. We humbly accept and will be happy to receive ideas and criticisms. They will be taken into account in future editions.

Glossary

Lexicon of terms frequently used in metrology

The terms used in the field of metrology applied to arts and crafts may, unfortunately, lead to confusion if one does not appropriately define the context and the related scope. Also, in many cases in North America, the full translation from English may be questionable because of the different meanings given to terms used in metrology.

For example, we use the term control in many cases of both exact sciences and human sciences. Similarly for the words *quality* and *uncertainties*. How much confusion resulted from concepts such as: *accuracy*, *fidelity* and *reliability*, and so forth? In metrology, vocabulary is very important. If a term used is not unanimous, the resulting terms and interpretations are found to be less strong and sometimes divert from their respective objectives.

Clearly, a term used in metrology, even if it has a meaning called “named sense” (French: *sens nommé* or SN), may also have meaning called “usual sense” (French *sens usuel* or SU). Although encyclopedias provide definitions for all these terms, it would be cautious to take good care in providing *the true* meaning to the *true value* in metrology. We will attempt in this section to give a set of terms with definitions provided in metrology and from the sciences applied by the latter. Ignoring the sense would mean wasting words.

We often have the habit of using terms that do not always match what we want to express. A reminder of some commonly used terms in dimensional metrology can be found in the following section.

Warning:

What follows is not a dictionary. This is a brief compilation of some words used in metrology and related disciplines. We suggest them to the reader to avoid some common linguistic threads in “some departments”. These definitions are also chosen based on the frequency of use of vocabulary (unfortunately sometimes overused) in workshops and some places where communication raises doubt or is unreliable, or even devious.

A

Accuracy (*of a measuring instrument*): Ability of a measuring instrument to provide information free of bias. In other words, the closeness of agreement between the measurement result and the conventional true value of the quantity.

Accuracy of measurement: “The closeness of agreement between the result of a measurement and a true value of the measurand”.

Accuracy: Includes the concepts of accuracy, repeatability, hysteresis, and resolution.

Adjustment: Relationship resulting from the difference, before assembly, between the dimensions of two elements (bore and shaft) intended to be assembled. Typology: adjustment clearance adjustment, interference adjustment, uncertain adjustment.

Air knife: Very thin layer of air between two surfaces very close to each other.

Air lamination: Controlled strangulation restricting an air flow which passes through an orifice.

Air wedge: Very thin layer of air, of unequal thickness, which is formed between two non-parallel surfaces close to each other. This is common in interferometry (optical disc in polished glass).

Alignment: Arrangement of a series of pieces in a line. The alignment is done following an axis or a reference surface.

Amplification: Enlargement, magnification. Ratio between the actual size and the observed size for the same entity.

Analyser: An optical device (or electronic) known as a multi-light irradiation selector.

Approach ratio: The ratio of the arc of approach to the arc of action.

Approach: Manner of conducting an argument, a method, an procedure etc.

Arc of Action: Arc of the pitch circle through which a tooth travels, from the beginning to the end of contact with the mating tooth.

Axial plane: In a pair of gears, it is the plane that contains the two. In a single gear, it is the plane containing its axis and a given point.

B

Barb: Residual material on a ridge, left either by machining or by the forming process.

Base Helix Angle: Angle between the tooth and the gear axis, at the base cylinder of an involute gear.

Base plate: A sock is a mostly flat surface used to support a part or an object. Include the case of a rectangular plate with recess leading.

Base/Collar: A base is a reinforced part of a piece used as a support.

Basic Circle: Circle from which an involute gear is generated.

Bias error: Systematic error of indication of a measuring instrument.

Bias: Component of measurement error that, in several measurements of the same measurand, remains constant or varies slowly and predictably.

Bleeding: A deep and thin cut.

Blind hole: A blind hole is a hole that does not penetrate an object and stops completely in the area.

Bore: A bore is a counter inside a piece or an object, having a cylindrical or conical shape which is intended to receive a shaft, bearing, etc.

Boss/Pad: Protrusion of a piece intended to limit the machined surface serving as a support or of contact. It is actually a predicted overthickness (often obtained in smelting) on a piece to reduce the bearing surface and limit machining (used in support of fasteners: screws, etc.).

C

Caliber: Checker of size or shape.

Calibrating: Verifying a measurement by comparison with a standard and certifying its conformity.

Calibration: Setting a measuring device using standard, or set of operations that establish, under specified conditions, the relationship between values of quantities indicated by a meter or measuring system, or values represented by material measure or a reference material, and values of the corresponding quantities achieved by standards.

Center Distance: Distance between axes.

Central Plane: In a worm, it is the plane perpendicular to the gear axis and contains the common perpendicular to the gear and worm axes. In the ordinary case where the axes are at right angles, it also contains the worm axis.

Certify: To authenticate a measure, after repeated verifications providing identical results.

Chamfer: A chamfer is a little oblique surface used to join or connect two other surfaces. A chamfer may be external or internal, and depends on two parameters a and b (or a length plus an angle) and allows particularly the suppression of a sharp edge (CAD).

Chatter (vibration): Alternation of relative slidings and relative rests or of peaks and reductions of sliding velocity, occurring spontaneously during a process of dynamic friction.

Chord: Rope length of the arc which constitutes the thickness (thickness of the tooth measured to the pitch-circle with a compass).

Clamping: Absolute value of the difference between dimensions, before assembly, the bore and shaft, where the difference is negative, that is, when the diameter of the tree is greater than the diameter of the hole.

Clearance: Deviation (in mm or μm) between two nominal dimensions.

Clearance: Difference by recess (chamfer) with respect to the ideal geometric form.

Collar: A collar is a ring or a rim projecting from a shaft or an axis.

Collimator: An optical device beam steering of light rays in a given direction.

Combination: Stack of gauge blocks whose sum constitutes a specified size to one micron nearly.

Concept: An abstract representation of a reality or a set of realities, generally stable and designated with one term. Symbolic statement, consisting of profiles and which allows identifying, classifying an object, a person, or an event.

Conjunction: Variance relief from the ideal geometric form, in the form of chamfer or rounding.

Contact Diameter: The smallest diameter on a gear tooth with which the mating gear tooth makes contact.

Contact end/tip: End of a sensor (often removable) of measuring instrument.

Contact point: Contact reduced to a single point.

Contact Stress: The maximum compressive force within the contact area between mating gear tooth profiles. Also known as “Hertz stress”.

Control Data: Information that can initiate or modify the operating cycle of a system.

Control: Action designed to compare the value of a real quantity to a value of a theoretical quantity.

Coordinates: Quantities defining the position of a point in a plane or in space, according to references. We distinguish between rectangular coordinates and polar coordinates.

Counter-bore: A counter-bore is a housing or a small cylindrical bore, usually machined to a port hole, and serves as a bearing surface (thrust washer) or walnut element (socket head cap screw).

Countersink: Conical-shaped recess around a hole or a bore.

Cusp (point): Correct position of alignment achieved during a slight swing between the followers of the measuring instrument. It is at this point that must be done reading.

Cycle: Series of actions which, once started, is repeated indefinitely until it is stopped.

Cycloid: Curve formed by the path of a point on a circle as it rolls along a straight line. When this circle rolls along the outer side of another circle, the curve is

epicycloid; and when it rolls along the inner side of another circle, the curve is *hypocycloid*.

D

Dedendum: The depth of the interval between two teeth below the pitch circle, or radial distance between the pitch circle and the bottom of the tooth space.

Deviation: Algebraic difference between a given size and the corresponding nominal size (standard deviation σ^* : in statistics, it is the deviation compared with the arithmetic mean.).

Dial: Setting circle before which moves a pointer.

Diametral Pitch: Ratio of the number of teeth to the number of inches of pitch diameter. That is to say the number of gear teeth to each inch of pitch diameter. Normal diametral pitch is the diametral pitch calculated in the normal plane and is equal to the diametral pitch divided by the cosine of the helix angle.

Diascopic: The term “dia” means “through.” The adjective “diascopic” qualifies lighting, from behind an objective whose profile is intended to be examined using an optical device.

Dimension: Number designating, in the selected unit, the numerical value of a length inscribed on a drawing. Nominal dimension: without tolerances. Toleranced dimension: with tolerances.

Dimensional Tolerance: Difference between the maximum size and minimum size.

Dovetail (*hirondo*, in Latin): It means a broader form at one end than the other. Generally trapezoidal shape applies to the concept of groove and tenon.

Driver: Element of caliber (or tool) facilitating proper orientation.

E

Eccentricity/excentering/runout: A deviation of position of a geometric center or an axis (in the latter case the French term is *désaxage*).

Edge: The place or zone of a piece where two or many surfaces meet.

Element: Constituent part of a set or a subset, whatever be the nature or size.

Elongation: Interval of length between two successive positions resulting from angular or sinusoidal displacement.

Engineering drawing: Drawing of piece isolated from the mechanism, whose role is mainly to show the functional surfaces. It represents design of a basic product.

Episcopic: The adjective “episcopic” qualifies for incident light focused on the surface of an opaque object, to examine its magnified image displayed on a screen.

Eyepiece/ocular: Optical magnifier device against which the observer places his eye to examine an object’s image formed by the objective.

F

Face of Tooth: Surface of the tooth that is between the pitch circle and the top of the tooth.

Fidelity/Precision (of a measuring instrument): Ability of a measuring instrument to provide very close indications by replicate measurements on the same measurand, under the same measurement conditions.

Field: Surface observed in an optical device.

Fillet: Continuous helical protrusion and with uniform cross-section on a ruled surface of revolution.

Flange: A flange, generally welded end of a pipe, is an element with standardized dimensions allowing bolting junction of pipes. Note: the number of bolts is often a multiple of four to turn the devices 90° around their axis without orientation problems.

Flank (of tooth): Surface between the pitch circle and the root surface of a gear tooth.

Flatness: Defines the geometrical state of a surface-machined plane.

Full Contact: Full contact between two surfaces (plane or at least three points).

Functional analysis: Approach of listing, ordering, characterizing, prioritizing, and/or enhancing the functions.

Functional chain: Set of components organized to obtain an operative function. A functional chain typically includes a component of treatment, a chain of action, the controlled physical part, and an acquisition chain.

Functional specification of charges (CdF): Document issued by the applicant expresses his needs in terms of service functions and constraints. For each defined assessment criteria and their levels of flexibility matching.

G

Gasket or seal: Interchangeable element which may exist during the constitution of a sealing device.

Gauge: Localized contact gauge whose size is usually fixed.

Gauge-block: Standard (parallel or prismatic) of high precision or extra piece used for locating parts in correct position.

Goniometer: Device for measuring angles.

Graduated drum: Cylinder with a graduation.

Graduated drum: Disc with external graduation. It is used for angular measurements.

Graduation: Regular division of a measuring instrument generally features.

Groove: Hack into a piece to receive a tenon. Type: Straight, T-shaped, “dovetail” V-shaped.

H

Hack: Removing a significant portion of an object by machining (milling).

Helical pitch: The distance a helical gear or worm would travel along its axis in one complete turn, if the axial movement was possible.

Helix angle: The angle made by a helical gear tooth with the gear axis at the pitch circle, unless specified otherwise.

Hygrometry: Atmospheric conditions laid down by the relative humidity in a given space.

Hysteresis: Information given when an instrument provides, for the same value of the measured quantity, a different result, depending on whether it is obtained by increasing or decreasing value or decreasing value.

I

Indicator: Sensitive device showing significant deviations of linear dimension compared with measurements.

Interchangeable part: Part whose characteristics allow the substitution of an original part while retaining its core functions.

Internal diameter: The diameter of a circle coinciding with the tip surface of the teeth of an internal gear.

Involute: Curve formed by the path of a point on a straight line called generatrix, as it rolls along a convex base curve. The base curve is usually a circle. This curve is used in drawing the profile of the gear tooth.

K

Kinematic: Model representation mechanisms, from perfect bonding, involving only the kinematic equivalence classes.

Krypton: Chemical element of the periodic table of Mendeleev. The Krypton 86 is a former international standards for assessing the meter. $1\text{ m} = 1\,650\,763.73\lambda$.

L

Lens: Glass disc with two sides typically spherical (sometimes one of them flat), used in optical devices to achieve a magnification.

Lifetime: Duration for which an entity has completed the function it has been assigned. Bearings are generally calculated as such.

Light Box: Table of polished glass, illuminated from below and used to control profiles from/following a gauge.

Line of Action: Line of contact point of an involute gear. It is the straight line passing through the pitch point and is tangent to the two base circles.

Linear Contact: Contact reduced to a straight line (at least two points) or a curve.

Localized Contact: Contact reduced to a surface portion.

Locating: A locating is a centering organ consisting of two perfectly coaxial cylinders. It is used less frequently than the positioning axes of the previous paragraph.

Logic diagram: Chart standard expression of a logic function.

M

Magnification or (enlargement): It is an optical amplification. Relation between the apparent sizes of an object seen in an optical device, and then observed by eyes (individual vision variable).

Material: A term commonly used to designate a solid, plastic, liquid, or fluid object based on its nuance. Example: Steel 1045 (Steel according to SAE = carbon steel or iron alloy with 0.45% C).

Measurand: The measurand is a particular quantity subjected to measurement.

Measure: Value of a quantity assigned to its unit; Result of a measurement.

Measurement process (see also SPC): A process that encompasses all aspects relating to performance and quality measurement. It includes the principle, method, procedure, standards etc.

Measurement: Set of operations aimed at determining the values of various quantities. [It is the action whose outcome is to collect measurement results and NOT the results themselves.]

Meplat: A flat surface present on a piece of circular cross-section or in the tip of a cylindrical or conical piece.

Metrology (s): Metrology is the field of knowledge encompassing all aspects of theoretical, technical, technological, as well as practical aspects of knowledge concerning the operations aimed to determine values of various quantities associated with dimensional characters and geometric characters belonging to physical objects. Some textbooks describe metrology as a sub-discipline of physical sciences dedicated to the calculation of uncertainties. This is partly true before the current importance of quality control and statistical methods relating thereto.

Micrometer (*mechanical*): Vernier or Palmer. Instrument with graduation established on a rule to evaluate a: deviation, size, or position of the object being examined.

Mortise: This is the recessed portion of an object that may receive a pin having the same form (enabling mortise-tenon assembly).

N

Nanometrology: Nanometrology aims to measure the length of very small distances in the fields of microscopy of semiconductors and nanotechnology. A nanometer is millionth of a millimeter ($1/1000,000^{\text{th}}$ mm).

Neutral fiber: It is, in a deformable part, a line not affected by permanent or elastic flexions. It coincides with the longitudinal axis for regular parts with round, square, or regular polygonal cross-section.

Nipple/teat: A nipple is a projection, usually cylindrical and small, designed to fit or fit into the hollow part of another object (groove etc.).

Nominal dimension: Size by reference to which the dimensions are “defined limits” obtained by applying the upper and lower deviations.

Normal plan: A plane normal to the surfaces at a point of contact, and perpendicular to the pitch plane.

Notch: A cut of small size.

NURBS: This stands for Non-Uniform Rational B-Spline. Denotes plane function which is described by plane nodes (as in finite elements). We can, through this function, obtain a 3D modeling of complex elements.

O

Objective: Optical device focusing light rays ensuing from an object being examined by the glass or by projection on optical screen.

Oblong Hole: This is the sum or combination of a light rectangular and two half cylinders. In mechanical manufacturing, a hole of narrow width over its length with two half-cylinders.

Optical lever: Articulated mirror, a small rotation of which deviates a reflected ray thus creating a system with two branches, where both displacements of the mirror and the reflected beam are considered, respectively.

Optical micrometer: Graduation established on a glass slide graticule placed in a glass or microscope to assess: deviation, size, or position of the object being examined.

Optical Splitter: A device that enables measurement of angles with a graduated drum whose precise graduation is observed via a microscope.

Optimeter: (Optometry) Opto-mechanical measuring instrument.

Overall drawing: Drawing whose role is primarily to demonstrate a mechanism and how the manufacturer has constructed and arranged the parts constituting solids.

P

Pairing: Grouping by pair (pairing).

Pebble: A device allowing the mobile element of the device to be maneuvered using a cam or a slide.

Phase: Each of the changes, successive aspects of a phenomenon in evolution. In mechanical manufacturing (machining), a phase imposes a machining without changing the machine tool (turning phase, milling phase).

Pitch: The distance between two similar, equally-spaced tooth surfaces, in a given direction and along a given curve or line. The single word “pitch” is often used without a *qualifying term* to designate (circular) pitch, diametral pitch or the axial pitch; this usage may be confusing and should be avoided.

Pitch base: Corresponding sides of involute teeth are parallel curves, and the base pitch is the constant and fundamental distance between them along a common normal in a plane of rotation. The *normal* base pitch is the base pitch in the *normal* plane, and the *axial* base pitch is the base pitch in the *axial* plane.

Pitch circle: A circle whose radius is equal to the distance between the axis of a gear wheel and its pitch point: specific contact point of the two mating gears.

Pitch diameter: The diameter of pitch circle. In parallel shaft gearing, the pitch diameters can be directly determined from the center distance and the numbers of teeth by proportionality.

Pitch or profile pitch: Distance considered parallel to the axis between two consecutive similar points on the profile of the thread.

Pitch plane: In a pair of gears, it is the plane perpendicular to the axial plane and tangent to the pitch surfaces. In a single gear, it may be any plane tangent to its pitch surface.

Pitch point: Point of tangency of two pitch circles (or a pitch circle and a pitch line), it is located on the line of centers. The pitch point of a tooth profile is located at its intersection with the pitch circle.

Precise, precision: Terms abandoned, replaced by the terms “accurate” and “accuracy”.

Pressure Angle ($\alpha = 20^\circ$): The angle between a tooth profile and a radial line at its pitch point. In involute gear, pressure angle is often defined as the angle between the line of action and the line tangent to the pitch circle.

Pressure Angle: The angle formed at the point of contact, on the pitch circle of a gear (20°) between the tangent to the pitch circle and the normal of involute.

Principle Reference Planes: These are the pitch plane, the axial, and transverse plane, all intersecting at a point and are mutually perpendicular.

Prism: Optical element made of glass, with flat faces which deviate the image once “at simple reflection” or twice “at double reflection”.

Probe: Element of measuring apparatus in contact with the workpiece being measured.

Process: A set of material means necessary to bring to the work material the expected added value e.g., forming, bending, and molding.

Projection: Depth of tooth above the pitch circle, or radial distance between the pitch circle and the tip surface of the tooth (see illustration).

Prototype: Original piece used as a reference or standard.

Protractor: A verification item to control the contact surfaces of a bounded gauge or calibrate a control device to Max and Min dimensions (a dimension protractor).

Q

Quality: The aptitude of a product to comply with any requirements or preset(s).

Quantity value: The value of a quantity is expressed as a number and unity. By a quantity value, it may mean the value deemed to be true, or rather conventionally true.

Quantity/magnitude: It is an attribute (a body) that can be identified and quantified (e.g., length, angle, diameter, and surface). Different units listed in a system of units (e.g., Metric system) correspond to different quantities.

R

Rack: Straight-line gear engaged with a cylindrical gear. It is a gear with teeth spaced along a straight line and is suitable for straight line motion.

Range: Machining range is an optimized sequence that serves as a logical follow reading phase, sub-phases and operations to perform a piece on machine tools.

Ratio of gearing: Ratio of the number of teeth on mating gears. Typically the ratio is obtained by dividing the number of teeth on the larger gear by the number of teeth on the smaller gear or pinion. For example, a ratio of 2 or “2 to 1” means that the pinion makes two revolutions to one revolution for the larger mating gear.

Recess: A recess is a part or a void left in a piece or object to reduce the weight or reduce a support surface (base plate, etc). For example, a non-leading recess practiced by a base plate (foundry) or the example of a leading recess (machining).

Reflection (optics): Strong deviation of a light ray hitting a smooth surface (mirror).

Refractive (optical): Low deflection of a light ray during its passage through various transparent media.

Refractive index: Relation between the speed of light in vacuum and the speed of light through a transparent body (glass, air, water).

Repeatability (of results of measurement): “The closeness of agreement between the results of *successive measurements of the same measurand*, all measurements being made in all of the same measurement conditions.” These conditions are called repeatability conditions.

Reproducibility (of results of measurement): The closeness of agreement between the results of measurements of the same measurand, measurements made by varying the measurement conditions.

Resetting (zeroing): Special calibration of a measuring apparatus with variable dimension, by locating the origin of a graduation in front of an index, when the standard is placed between the probes.

Resolution: The smallest difference indication of a display device that can be meaningfully distinguished.

Result (a measurement): The value of a measurand, obtained by measurement. This result known as:

- *Indication: if the value is provided by the instrument;*
- *Uncorrected result: if this is the value, before correction, of the systematic errors (see def. bias);*
- *Corrected results: if the value is obtained after making corrections to the initial result (uncorrected) and taking into account the stated systematic errors.*

Reticle: Thin parallel-sided blades, mounted on a glass, finely etched lines identifying the position or shape of parts observed (becomes a micrometer when the graduation is gauging).

Ring: Tubular element whose inner surface is the surface of a radial bearing hole in one piece.

Rolling elements: Ball or roller which rolls between tracks.

Root Circle: Circle coincident or tangent to the surface of the bottom of the tooth spaces.

Root Diameter: Diameter of the root circle.

Round: A round is a rounded surface of round conjunction joining two other adjoined surfaces forming an outgoing angle or a sharp edge (usually designed to “break” the sharp edge).

Rounding: This is a connecting surface making the junction between two surfaces forming an inside corner.

S

S.P.C.: Abbreviation for *Statistical Process Control*.

Scratch: Machining operation, terminal, performed manually using a scraper.

Screen: Refers here to projection screen. Illuminated plane table, in polished glass, on which a magnified image of a form is observed.

Screws tooth: Amount by which the hollow of a gear exceeds the projections of the mating gear.

Scuffing: Phenomenon of deformation of surfaces in contact due to adhesion and transfer of material.

Sealing device: A device with or without seal preventing leakage of fluids or the entry of pollutants.

Sensitivity: Characteristic of amplification.

Set: Group of subsets providing one or more technical functions that make it able to perform an operational function.

Sextant: Instrument for reflection, whose graduated drum covers 60° and which allows measuring the heights of a celestial body from a ship or aircraft.

Shaft: A shaft is a piece constituted of cylindrical parts or sections, sometimes conical, with special shapes such as: shoulders, chamfers, flanges, grooves, flats, keyways, and splines.

Shoulder: A special form of a shaft, a shoulder is a support surface obtained by a sudden change of section.

Spacing: Distance between the parallel axes of a spur gear or helical gear, or distance between the intersecting axes of a left helical gear or worm. It is also the distance between the centers of the pitch circle.

Specular: State of a (material) the polished pushed to the extreme (mirror).

Spline: Thin wall connecting two structural elements of a part to stiffen. A rib is also a striking feature or a reinforcement of substantially constant thickness designed to increase strength or stiffness of a part or object.

Standard: Precise piece constituting a unit of measurement or comparison (Object or instrument which embodies one unit of measure and serves as a reference.).

Stop: Bearing built to essentially withstand axial loads, with a nominal angle of contact higher than 45° and less than or equal to 90° . Its key elements are a shaft-ring, a washer housing, and rolling elements with or without cage.

Straightness: Sets the accuracy of a line generated mechanically.

T

Tangent plane: A plane tangent to the tooth surfaces at a point or line of contact.

Telescope: Instrument of astronomical observation whose objective is a concave mirror.

Tenon: Protrusion of an object or a piece which may be housed in a groove or a mortise.

Thread: Volume generated by one or more fillets. **External thread:** Thread performed on the outer surface of revolution.

Through-hole: A through-hole is a hole that traverses from side to side, or completely, a part or object.

Tip relief: An arbitrary modification of a tooth profile by removal of an amount of material near the tip of the gear tooth.

To measure: Action aiming at determining a value of a quantity.

Tolerance: Difference tolerated (accepted) between the maximum and minimum size limits of a rating.

Tongue: A tongue is a form (plane-parallel) allowing performing a sliding connection when paired with a groove.

Total height: The total height of a gear tooth, equal to the sum of the addendum plus dedendum, or equal to effective height (working depth) plus clearance.

Traceability: Property of measurement result or an etalon standard whereby it can be related to stated references, usually national or international etalon standards, through an unbroken chain of comparisons all of which having stated uncertainties. “The unbroken chain of comparisons is called a chain of connection to etalon standard or calibration chain.” It includes: A measuring instrument, the working etalon standard, the reference etalon standard, the transfer etalon standard, national etalon standard, the international etalon standard.

Transverse plane: A plane perpendicular to the axial plane and to the pitch plane. In gears with parallel axes, the transverse and the plane of rotation coincide.

Trochoid: Curve described by a point at the end of the radius of a circle which rolls on a curve or a line.

True Involute Form Diameter: The smallest diameter on a tooth at which the involute exists. Usually this is the point of tangency of the involute tooth profile and the fillet curve. It is referred to as “TIF” diameter.

Turnbuckle: Part of a shaft that supports a radial plain bearing.

Type A-method (uncertainty): Based on the application of statistics (mainly used for quantifying uncertainties of repeatability).

Type B-method (uncertainty): Associated with to the control of the measurement (specifications, calibration certificates, influence factors).

U

Uncertainty of measurement: *The closeness of agreement between the result of a measurement and the true value of the measurand.* A parameter, associated with the result of a measurement that characterizes the dispersion of values, that could reasonably be attributed to the measurand. In a little clearer language, we can say that it is the maximum guaranteed value of the difference between the result of the measure and the conventionally true value of the measured quantity. “We estimate an uncertainty” ΔX or I_x . Used alone, the term uncertainty refers to the expanded uncertainty U_x^* . Warning/ be careful: uncertainty is different from error.

Undercut/groove: Narrow undercut such as a notch, in a piece of circular cross-section. A groove is also an alternate rounded performed on a shaft or in a bore.

Undercutting: It is a particular hollowing realized inside a bore to reduce the scope or the bearing surface (limiting “cylindrical” machining).

V

Value recognized as true = Result(s) of measurement(s) + decision to give reliance

Verification/Inspection: Confirmation by examination and provision of evidence that specified requirements have been fulfilled. A verification’s results can be recorded in a verification statement.

Warping: Elastic distortion found on a piece generally poorly wedged/blocked and biased torsion.

Wavelength (λ): It is the pitch of the sine curve representing graphically a vibration phenomenon.

Wear: Result of the phenomenon of wear (or wear process) expressed in specified units.

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Appendix 1

IT calculation: fundamental tolerances (µm = 0.001 mm) as a step-function (mm)																		
Grade	01	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
≤3	0.3	0.5	0.8	1.2	2	3	4	6	10	14	25	40	60	100	140	250	400	600
>3–6	0.4	0.6	1	1.5	2.5	4	5	8	12	18	30	48	75	120	180	300	480	750
>6–10	0.4	0.6	1	1.5	2.5	4	6	9	15	22	36	58	90	150	220	360	580	900
>10–18	0.5	0.8	1.2	2	3	5	8	11	18	27	43	70	110	180	270	430	700	1,100
>18–30	0.6	1	1.5	2.5	4	6	9	13	21	33	52	84	130	210	330	520	840	1,300
>30–50	0.6	1	1.5	2.5	4	7	11	16	25	39	62	100	160	250	390	620	1,000	1,600
>50–80	0.8	1.2	2	3	5	8	13	19	30	46	74	120	190	300	460	740	1,200	1,900
>80–120	1	1.5	2.5	4	6	10	15	22	35	54	87	140	220	350	540	870	1,400	2,200
>120–180	1.2	2	3.5	5	8	12	18	25	40	63	100	160	250	400	630	1,000	1,600	2,500
>180–250	2	3	4.5	7	10	14	20	29	46	72	115	185	290	460	720	1,150	1,850	2,900
>250–315	2.5	4	6	8	12	16	23	32	52	81	130	210	320	520	810	1,300	2,100	3,200
>315–400	3	5	7	9	13	18	25	36	57	89	140	230	360	570	890	1,400	2,300	3,600
>400–500	4	6	8	10	15	20	27	40	63	97	155	250	400	630	970	1,550	2,500	4,000

Table A1.1. Fundamental tolerances for normalized adjustments

Letter →	a*	b*	c	d	E	f	g	h
Grade ↓	All grades according to 286-2							0
≤3	−270	−140	−60	−20	−14	−6	−2	0
>3–6	−270	−140	−70	−30	−20	−10	−4	0
>6–10	−280	−150	−80	−40	−25	−13	−5	0
>10–14	−290	−150	−95	−50	−32	−16	−6	0
>14–18	−290	−150	−95	−50	−32	−16	−6	0
>18–24	−300	−160	−110	−65	−40	−20	−7	0
>24–30	−300	−160	−110	−65	−40	−20	−7	0
>30–40	−310	−170	−120	−80	−50	−25	−9	0
>40–50	−320	−180	−130	−80	−50	−25	−9	0
>50–65	−340	−190	−140	−100	−60	−30	−10	0
>65–80	−360	−200	−150	−100	−60	−30	−10	0
>80–100	−380	−220	−170	−120	−72	−36	−12	0
>100–120	−410	−240	−180	−120	−72	−36	−12	0
>120–140	−460	−260	−200	−145	−85	−43	−14	0
>140–160	−520	−280	−210	−145	−85	−43	−14	0
>160–180	−580	−310	−230	−145	−85	−43	−14	0
>180–200	−660	−340	−240	−170	−100	−50	−15	0
>200–225	−740	−380	−260	−170	−100	−50	−15	0
>225–250	−820	−420	−280	−170	−100	−50	−15	0
>250–280	−920	−480	−300	−190	−110	−56	−17	0
>280–315	−1,050	−540	−330	−190	−110	−56	−17	0
>315–355	−1,200	−600	−360	−210	−125	−62	−18	0
>355–400	−1,350	−680	−400	−210	−125	−62	−18	0
>400–450	−1,500	−760	−440	−230	−135	−68	−20	0
>450–500	−1,650	−840	−480	−230	−135	−68	−20	0

*Not expected to be less than 1 mm

Table A1.2. Upper deviations for shafts

Letter →	j			k	
Grades ↓	5 and 6	7	8	4–7	≤3 or >7
≤3	–2	–4	–6	0	0
>3–6	–2	–4	–	1	0
>6–10	–2	–5	–	1	0
>10–14	–3	–6	–	1	0
>14–18	–3	–6	–	1	0
>18–24	–4	–8	–	2	0
>24–30	–4	–8	–	2	0
>30–40	–5	–10	–	2	0
>40–50	–5	–10	–	2	0
>50–65	–7	–12	–	2	0
>65–80	–7	–12	–	2	0
>80–100	–9	–15	–	3	0
>100–120	–9	–15	–	3	0
>120–140	–11	–18	–	3	0
>140–160	–11	–18	–	3	0
>160–180	–11	–18	–	3	0
>180–200	–13	–21	–	4	0
>200–225	–13	–21	–	4	0
>225–250	–13	–21	–	4	0
>250–280	–16	–26	–	4	0
>280–315	–16	–26	–	4	0
>315–355	–18	–28	–	4	0
>355–400	–18	–28	–	4	0
>400–450	–20	–32	–	5	0
>450–500	–20	–32	–	5	0

Table A1.3. Upper deviations for shafts

Letter →	m	n	p	r	d	t	u	v	x	y	z
Grade ↓	All grades according to 286-2										
≤3	2	4	6	10	14	–	18	–	20	–	26
3–6	4	8	12	15	19	–	23	–	28	–	35
6–10	6	10	15	19	23	–	28	–	34	–	42
10–14	7	12	18	23	28	–	33	–	40	–	50
14–18	7	12	18	23	28	–	33	39	45	–	60
18–24	8	15	22	28	35	–	41	47	54	63	73
24–30	8	15	22	28	35	41	48	55	64	75	88
30–40	9	17	26	34	43	48	60	68	80	94	112
40–50	9	17	26	34	43	54	70	81	97	114	136
50–65	11	20	32	41	53	66	87	102	122	144	172
65–80	11	20	32	43	59	75	102	120	146	174	210
80–100	13	23	37	51	71	91	124	146	178	214	258
100–120	13	23	37	54	79	104	144	172	210	254	310
120–140	15	27	43	63	92	122	170	202	248	300	365
140–160	15	27	43	65	100	134	190	228	280	340	415
160–180	15	27	43	68	108	146	210	252	310	380	465
180–200	17	31	50	77	122	166	236	284	350	425	520
200–225	17	31	50	80	130	180	258	310	385	470	575
225–250	17	31	50	84	140	196	284	340	425	520	640
250–280	20	34	56	94	158	218	315	385	475	580	710
280–315	20	34	56	98	170	240	350	425	525	650	790
315–355	21	37	62	108	190	268	390	475	590	730	900
355–400	21	37	62	114	208	294	435	530	660	820	1,000
400–450	23	40	68	126	232	330	490	595	740	920	1,100
450–500	23	40	68	132	252	360	540	660	820	1,000	1,250

Table A1.4. Upper deviations for shafts

Letter →	A*	B*	C	D	E	FG	G	H
Grade ↓	All grades according to 286-2							
≤3	270	140	60	20	14	4	2	0
>3–6	270	140	70	30	20	6	4	0
>6–10	280	150	80	40	25	8	5	0
>10–14	290	150	95	50	32	–	6	0
>14–18	290	150	95	50	32	–	6	0
>18–24	300	160	110	65	40	–	7	0
>24–30	300	160	110	65	40	–	7	0
>30–40	310	170	120	80	50	–	9	0
>40–50	320	180	130	80	50	–	9	0
>50–65	340	190	140	100	60	–	10	0
>65–80	360	200	150	100	60	–	10	0
>80–100	380	220	170	120	72	–	12	0
>100–120	410	240	180	120	72	–	12	0
>120–140	460	260	200	145	85	–	14	0
>140–160	520	280	210	145	85	–	14	0
>160–180	580	310	230	145	85	–	14	0
>180–200	660	340	240	170	100	–	15	0
>200–225	740	380	260	170	100	–	15	0
>225–250	820	420	280	170	100	–	15	0
>250–280	920	480	300	190	110	–	17	0
>280–315	1,050	540	330	190	110	–	17	0
>315–355	1,200	600	360	210	125	–	18	0
>355–400	1,350	680	400	210	125	–	18	0
>400–450	1,500	760	440	230	135	–	20	0
>450–500	1,650	840	480	230	135	–	20	0

*Not expected to be less than 1 mm

Table A1.5. Upper deviations for shafts

Letter →	J					K		M	
Grade ↓	6	7	8	≤8	>8	≤8	>8	≤8	>8*
≤3	2	4	6	0	0	-2	-2	-4	-4
>3–6	5	6	10	-1 + Δ	0	-4 + Δ	-4	-8 + Δ	0
>6–10	5	8	12	-1 + Δ	0	-6 + Δ	-6	-10 + Δ	0
>10–14	6	10	15	-1 + Δ	0	-7 + Δ	-7	-12 + Δ	0
>14–18	6	10	15	-1 + Δ	0	-7 + Δ	-7	-12 + Δ	0
>18–24	8	12	20	-2 + Δ	0	-8 + Δ	-8	-15 + Δ	0
>24–30	8	12	20	-2 + Δ	0	-8 + Δ	-8	-15 + Δ	0
>30–40	10	14	24	-2 + Δ	0	-9 + Δ	-9	-17 + Δ	0
>40–50	10	14	214	-2 + Δ	0	-9 + Δ	-9	-17 + Δ	0
>50–65	13	18	28	-2 + Δ	0	-11 + Δ	-11	-20 + Δ	0
>65–80	13	18	28	-2 + Δ	0	-11 + Δ	-11	-20 + Δ	0
>80–100	16	22	34	-3 + Δ	0	-13 + Δ	-13	-23 + Δ	0
>100–120	16	22	34	-3 + Δ	0	-13 + Δ	-13	-23 + Δ	0
>120–140	18	26	41	-3 + Δ	0	-15 + Δ	-15	-27 + Δ	0
>140–160	18	26	41	-3 + Δ	0	-15 + Δ	-15	-27 + Δ	0
>160–180	18	26	41	-3 + Δ	0	-15 + Δ	-15	-27 + Δ	0
>180–200	22	30	47	-4 + Δ	0	-17 + Δ	-17	-31 + Δ	0
>200–225	22	30	47	-4 + Δ	0	-17 + Δ	-17	-31 + Δ	0
>225–250	22	30	47	-4 + Δ	0	-17 + Δ	-17	-31 + Δ	0
>250–280	25	36	55	-4 + Δ	0	†	-20	-34 + Δ	0
>280–315	25	36	55	-4 + Δ	0	†	-20	-34 + Δ	0
>315–355	29	39	60	-4 + Δ	0	-21 + Δ	-21	-37 + Δ	0
>355–400	29	39	60	-4 + Δ	0	-21 + Δ	-21	-37 + Δ	0
>400–450	33	43	66	-5 + Δ	0	-23 + Δ	-23	-40 + Δ	0
450–500	33	43	66	-5 + Δ	0	-23 + Δ	-23	-40 + Δ	0

*N is not expected for upper grades below 1 mm

†exception, for M6 256–315 μm instead of -11 (UD = -9)

Table A1.6. Upper deviations (UD) for shafts

Letter →	P	R	D	T	U	V	X	Y	Z
Grade ↓	>7 All grades according to 286-2								
≤3	−6	−10	−14	−	−18	−	−20	−	−26
>3–6	−12	−15	−19	−	−23	−	−28	−	−35
>6–10	−15	−19	−23	−	−28	−	−34	−	−42
>10–14	−18	−23	−28	−	−33	−	−40	−	−50
>14–18	−18	−23	−28	−	−33	−39	−45	−	−60
>18–24	−22	−28	−35	−	−41	−47	−54	−63	−73
>24–30	−22	−28	−35	−41	−48	−55	−64	−75	−88
>30–40	−26	−34	−43	−48	−80	−68	−80	−94	−112
>40–50	−26	−34	−43	−54	−70	−81	−97	−114	−136
>50–65	−32	−41	−53	−66	−87	−102	−122	−144	−172
>65–80	−32	−43	−59	−75	−102	−120	−146	−174	−210
>80–100	−37	−51	−71	−91	−124	−146	−178	−214	−258
>100–120	−37	−54	−79	−104	−144	−172	−210	−254	−310
>120–140	−43	−63	−92	−122	−170	−202	−248	−300	−365
>140–160	−43	−65	−100	−134	−190	−228	−280	−340	−415
>160–180	−43	−68	−108	−148	−210	−252	−310	−380	−465
>180–200	−50	−77	−122	−166	−236	−284	−350	−425	−520
>200–225	−50	−80	−130	−180	−258	−310	−385	−470	−575
>225–250	−50	−84	−140	−196	−284	−340	−425	−520	−640
>250–280	−56	−94	−158	−218	−315	−385	−475	−580	−710
>280–315	−56	−98	−170	−240	−350	−425	−525	−650	−790
>315–355	−62	−108	−190	−268	−390	−475	−590	−730	−900
>355–400	−62	−114	−208	−294	−435	−530	−660	−820	−1,000
>400–450	−68	−126	−232	−330	−490	−595	−740	−920	−1,100
>450–500	−68	−132	−252	−360	−540	−660	−820	−1,000	−1,250

Table A1.7. Upper deviations for shafts

Grade ↓	3	4	5	6	7	8
≤3	–	–	–0	0	–	–
>3–6	1	1.5	1	3	4	6
>6–10	1	1.5	2	3	6	7
>10–14	1	2	3	3	7	9
>14–18	1	2	3	3	7	9
>18–24	1.5	2	3	4	8	12
>24–30	1.5	2	3	4	8	12
>30–40	1.5	3	4	5	9	14
>40–50	1.5	3	4	5	9	14
>50–65	2	3	5	6	11	16
>65–80	2	3	5	6	11	16
>80–100	2	4	5	7	13	19
>100–120	2	4	5	7	13	19
>120–140	3	4	6	7	15	23
>140–160	3	4	6	7	15	23
>160–180	3	4	6	7	15	23
>180–200	3	4	6	9	17	26
>200–225	3	4	6	9	17	26
>225–250	3	4	6	9	17	26
>250–280	4	4	7	9	20	29
>280–315	4	4	7	9	20	29
>315–355	4	5	7	11	21	32
>355–400	4	5	7	11	21	32
>400–450	5	5	7	13	23	34
>450–500	5	5	7	13	23	34

Table A1.8. *Values of Δ , in micrometer*

Correspondence ANSI-ISO (inch) (ANSI $\approx$ ACNOR) (1 in. = 25.4 mm)									
RC		LC		LT		LN		FN	
Clearance		Clearance		Uncertain		Clamping		Strong interference	
ANSI	ISO	ANSI	ISO	ANSI	ISO	ANSI	ISO	ANSI	ISO
RC1	H5/g4	LC1	H6/h5	LT1	H7/js6	LN1	**	FN1	**
RC2	H6/g5	LC2	H7/h6	LT2	H8/js7	LN2	H7p6	FN2	H7/s6
RC3	H7/f6	LC3	H8/h7	LT3	H7/k6	LN3	H7n6	FN3	H7/t6
RC4	H8/f7	LC4	H10/h9	LT4	H8/k7	LN4	**	FN4	H7/u6
RC5	H8/e7	LC5	H7/g6	LT5	H7/n6	LN5	**	FN5	H8/x7
RC6	H9/e8	LC6	H9/f8	LT6	H8/n7	LN6	**	—	—
RC7	H9/d8	LC7	H10/e9	—	—	—	—	—	—
RC8	H10/c9	LC8	H10/d9	—	—	—	—	—	—
RC9	H11*	LC9	H11/c10	—	—	—	—	—	—
—	—	LC10	H12*	—	—	—	—	—	—

Table A1.9. Application example on shaft/bore with $\varnothing 50$ h6/D7*; h6/F7*; h6/H7; h6/K7**; h6/M7**; h6/N7**; h6/P7**; h6/S7; and h6Z7 (mm)¹

Clearances and interferences	Adjustments with clearances				Adjustments with clearances			Tightened adjustments		
	50h6 with	50D7	50F7	50G7	50H7	50K7	50M7	50N7	50P7	50S7
Clearance max	0.121	0.066	0.050	0.041	0.023	0.016	0.008	—	—	—
Clearance min	0.080	0.025	0.009	0	0	0	0	—	—	—
Interference max	—	—	—	—	0	0	0	0.001	0.018	0.152
Interference min	—	—	—	—	0.018	0.025	0.033	0.042	0.059	0.111

Table A1.10. Examples of adjustments achievable with the three main cases: clearance max, clearance min, and tightened min–max

¹ (*) and (**): it is advisable to avoid the use of these if possible

Tolerance class		Nominal dimension ranges (mm)		
Designation	Description	0.5* up to 3 mm	>3 up to 6	>6
F	fine	0.2	0.5	1
M	medium	0.2	0.5	1
C	coarse	0.4	1.0	2
V	very coarse	0.4	1.0	2

*For values below 0.5 mm, the deviation shall be indicated adjacent to the nominal size

Table A1.11. Permissible deviations for linear dimensions of broken edges (radii and chamfers) (source: ISO 2768-1)

Tolerance class		Ranges of length (mm) (the shorter side of the angle in question)				
Designation	Description	Up to 10 mm	>10 up to 50	>50 up to 120	>120 up to 400	>400
F	fine	1°	0°30'	0°20'	0°10'	0°5'
M	medium	1°	0°30'	0°20'	0°10'	0°5'
C	coarse	1°30'	1°	0°30'	0°15'	0°10'
V	very coarse	3°	2°	1°	0°30'	0°20'

Table A1.12. Permissible deviations for angular dimension, excerpts (source: ISO 2768-1)

NOTE.— The results shown in Figures A1.1a–A1.1c are deliberately presented in imperial units (inches). The nominal diameter is expressed in millimeter. The main reason is to show irrationality sometimes in the conversion millimeters → inches and vice versa (see ISO correspondent tables for adjustments).

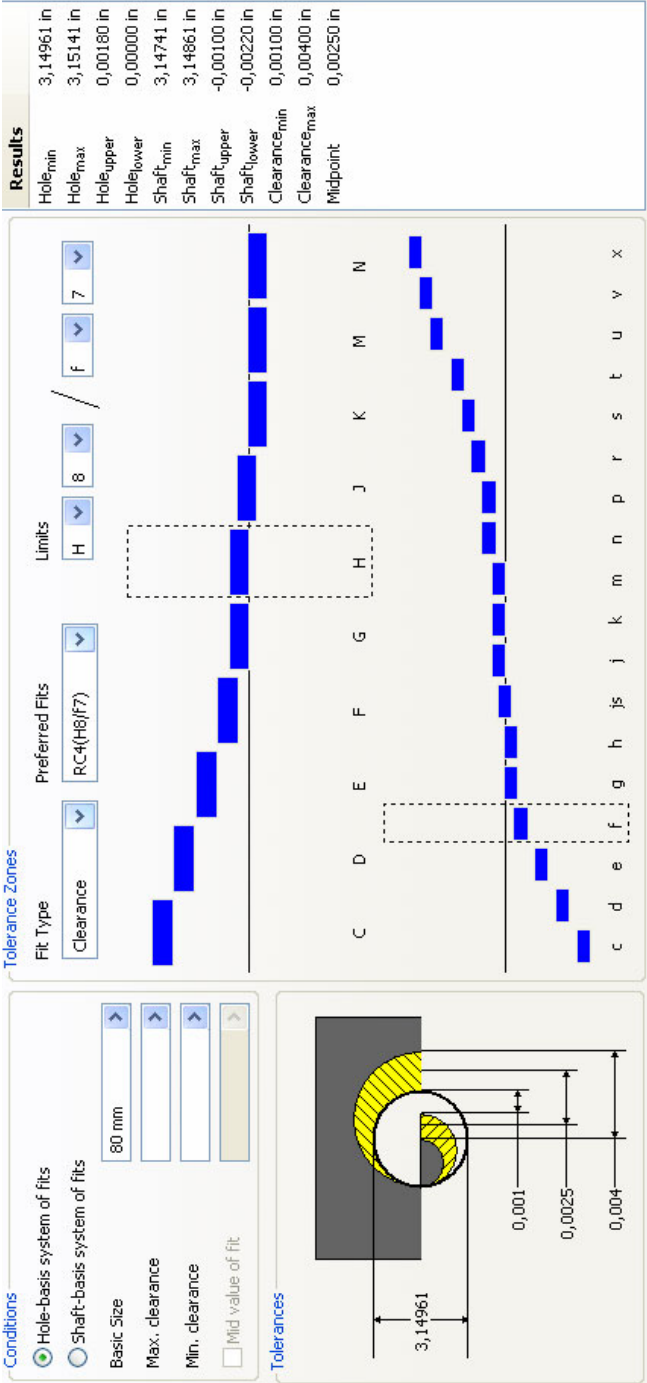
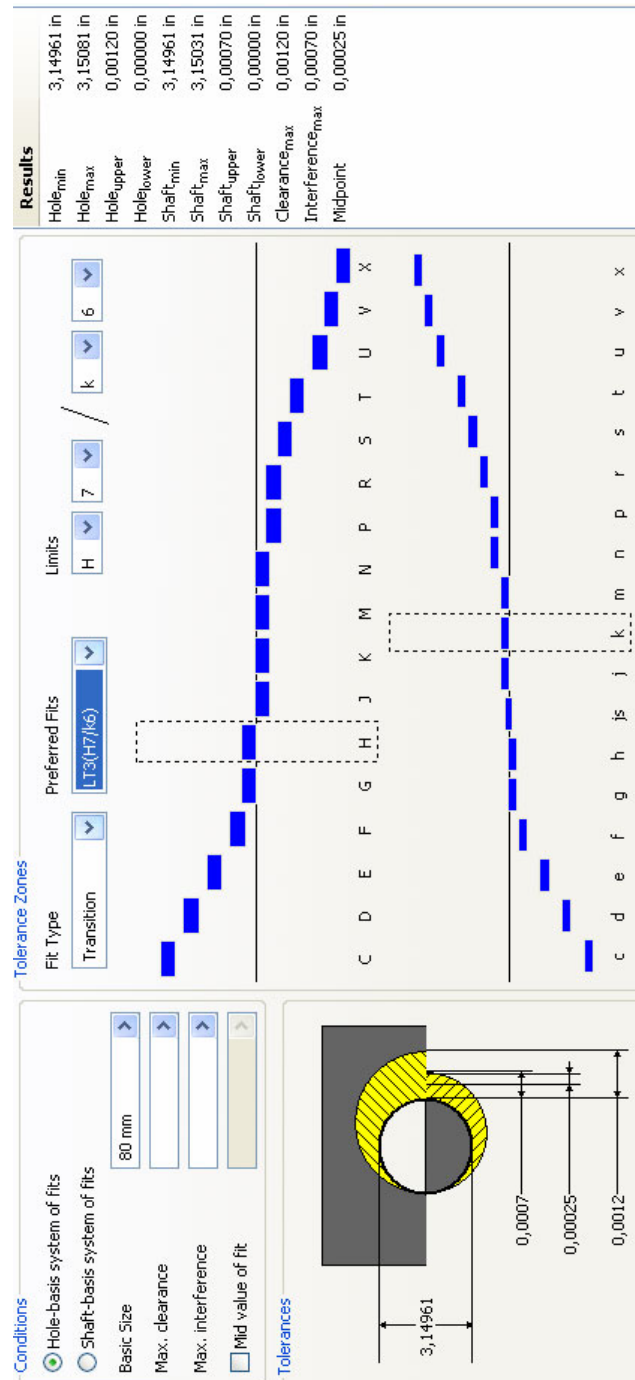


Figure A1.1a. Results of the adjustment calculations (H7/f6, ISO or RC4, ANSI (USA), and CSA (Canada) using the software Autodesk Inventor Pro 09)



Required Load

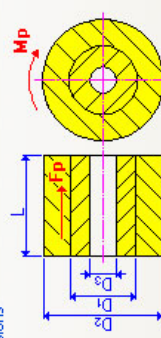
Required Force

Torque

Force

Safety Factor

Dimensions



Outer Diameter

Shaft Diameter

Inner Diameter

Connection Length

Limits and Fits

Fit Symbol

Fundamental Deviation

☐ Specify manually

ES

EI

Advanced

☐ Assembly Clearance

☐ Surface Smoothness

Clamping Factor

Clamping Factor (pressing)

Temperature

Limitation of

Hub Warming up

Base Temperature

Shaft Cooling

Hub Material

☐ User material

Modulus of Elasticity

Allowable Stress

Poisson's Ratio

Thermal Expansion

Surface Texture

Shaft Material

☐ User material

Modulus of Elasticity

Allowable Stress

Poisson's Ratio

Thermal Expansion

Surface Texture

Results

F_{pmax} 3733,424 lbf

F_{pmin} -3749,177 lbf

P_{pmax} 2880,935 psi

P_{pmin} -2893,091 psi

σ_{D1} 0,0004925 in

σ_{D2} -0,0004945 in

σ_{D3} 0,0003201 in

σ_{D4} -0,0003215 in

T_{dMax} -923,0927 lbf/ft

F_{pdMax} -8861,6903 lbf/ft

P_{Ah} 22306,1927 psi

P_{As} 24375,000 psi

δ_{min} -1,2024794 in

δ_{max} 0,002 in

δ'_{min} -0,000 in

δ'_{max} 0,0080927 in

δ'_{min} 0,0011336 in

Figure A1.1c. Simulation on adjustment calculations ($\approx H7/p6$, $H7/js6$) using the software Autodesk Inventor Pro 09

Appendix 2

Measuring range of chart number 511.			Range from 0.7" to 1.4"		
Capacity (inches)	Measuring ranges (inches)	Size of the ring	Capacity (inches)	Measuring ranges (inches)	Size of the ring
0.7	0.70–0.74	0.02	0.9	0.90–0.94	0.02
	0.72–0.76	0.04		0.92–0.96	0.04
	0.74–0.78	0.02 0.04		0.94–0.98	0.02 0.04
	0.76–0.80	0.08		0.96–1.00	0.08
0.8	0.80–0.84	0.02	1.0	1.00–1.04	0.02
	0.82–0.86	0.04		1.02–1.06	0.04
	0.84–0.88	0.02 0.04		1.04–1.08	0.02 0.04
	0.86–0.90	0.08		1.06–1.10	0.08
1.1	1.10–1.14	0.02	1.3	1.30–1.314	0.02
	1.12–1.16	0.04		1.32–1.36	0.04
	1.14–1.18	0.02 0.04		1.314–1.38	0.02 0.04
	1.16–1.20	0.08		1.36–1.40	0.08
1.2	1.20–1.24	0.02	1.4	1.40–1.44	0.02
	1.22–1.26	0.04		1.42–1.46	0.04
	1.24–1.28	0.02 0.04		1.44–1.418	0.02 0.04
	1.26–1.30	0.08		1.46–1.500	0.08

Measuring range of chart number 511.			Range from 1.4"–2.4"		
Capacity (inches)	Measuring ranges (inches)	Size of the ring	Capacity (inches)	Measuring ranges (inches)	Size of the ring
1.4	1.40–1.44	0.02	2.0	2.00–2.04	0.02
	1.42–1.46	0.04		2.02–2.06	0.04
	1.44–1.48	0.02 0.04		2.04–2.08	0.02 0.04
	1.46–1.50	0.08		2.06–2.10	0.08
	1.48–1.52	0.02 0.04		2.08–2.12	0.02 0.04
	1.50–1.54	0.12		2.10–2.14	0.12
	1.52–1.56	0.02 0.12		2.12–2.16	0.02 0.12
	1.54–1.58	0.02 0.12		2.14–2.18	0.02 0.12
	1.56–1.60	0.02 0.04 0.12		2.16–2.20	0.02 0.04 0.12
1.6	1.60–1.64	0.02	2.2	2.20–2.24	0.02
	1.62–1.66	0.04		2.22–2.26	0.04
	1.64–1.68	0.02 0.04		2.24–2.28	0.02 0.04
	1.66–1.70	0.08		2.26–2.30	0.08
	1.68–1.72	0.02 0.8		2.28–2.32	0.02 0.04
	1.70–1.74	0.12		2.30–2.34	0.12
	1.72–1.76	0.02 0.12		2.32–2.36	0.02 0.12
	1.74–1.78	0.04 0.12		2.34–2.38	0.02 0.12
	1.76–1.80	0.02 0.04 0.12		2.36–2.40	0.02 0.04 0.12

Table A2.1. Bore control

1.8	1.80–1.84	0.02	2.4	2.40–2.44	0.02
	1.82–1.86	0.04		2.42–2.46	0.04
	1.84–1.88	0.02 0.04		2.44–2.48	0.02 0.04
	1.86–1.90	0.08		2.46–2.50	0.08
	1.88–1.92	0.02 0.8		2.48–2.52	0.02 0.8
	1.90–1.94	0.12		2.40–2.54	0.12
	1.92–1.96	0.02 0.12		2.52–2.56	0.02 0.12
	1.94–1.98	0.04 0.12		2.54–2.58	0.04 0.12
	1.96–2.00	0.02 0.04 0.12		2.66–2.60	0.02 0.04 0.12

Measuring range of chart number 511.			Range from 2.0" to 4.0" and 4.4" to 6.0"		
Capacity (inches)	Measuring ranges (inches)	Size of the ring	Capacity (inches)	Measuring ranges (inches)	Size of the ring
2.0	2.00–2.04	0.02	2.8	2.80–2.84	0.02
	2.02–2.06	0.04		2.82–2.86	0.04
	2.04–2.08	0.02 0.04		2.84–2.88	0.02 0.04
	2.06–2.10	0.08		2.86–2.90	0.08
	2.08–2.12	0.02 0.8		2.88–2.92	0.02 0.8
	2.10–2.14	0.12		2.90–2.94	0.12
	2.12–2.16	0.02 0.12		2.92–2.96	0.02 0.12
	2.14–2.18	0.04 0.12		2.94–2.98	0.04 0.12
2.2	2.16–2.20	0.02 0.04 0.12	3.0	2.96–3.00	0.02 0.04 0.12
	2.20–2.24	0.02		3.00–3.04	0.02
	2.22–2.26	0.04		3.02–3.06	0.04
	2.24–2.28	0.02 0.04		3.04–3.08	0.02 0.04
	2.26–2.30	0.08		3.06–3.10	0.08
	2.28–2.32	0.02 0.8		3.08–3.12	0.02 0.8
	2.30–2.34	0.12		3.10–3.14	0.12
	2.22–2.36	0.02 0.12		3.12–3.16	0.02 0.12
2.4	2.34–2.38	0.04 0.12	3.2	3.14–3.18	0.04 0.12
	2.36–2.40	0.02 0.04 0.12		3.16–3.20	0.02 0.04 0.12
	2.40–2.44	0.02		3.20–3.24	0.02
	2.42–2.46	0.04		3.22–3.26	0.04
	2.44–2.48	0.02 0.04		3.24–3.28	0.02 0.04
	2.46–2.50	0.08		3.26–3.30	0.08
	2.48–2.52	0.02 0.8		3.28–3.32	0.02 0.8
	2.50–2.54	0.12		3.30–3.34	0.12
	2.52–2.56	0.02 0.12		3.22–3.36	0.02 0.12
	2.54–2.58	0.04 0.12		3.34–3.38	0.04 0.12
	2.56–2.60	0.02 0.04 0.12		3.36–3.40	0.02 0.04 0.12

Table A2.1. (continued) Bore control

2.6	2.60–2.64	0.02	3.4	3.40–3.44	0.02
	2.62–2.66	0.04		3.42–3.46	0.04
	2.64–2.68	0.02 0.04		3.44–3.48	0.02 0.04
	2.66–2.70	0.08		3.46–3.50	0.08
	2.68–2.72	0.02 0.8		3.48–3.52	0.02 0.8
	2.70–2.74	0.12		3.50–3.54	0.12
	2.72–2.76	0.02 0.12		3.52–3.56	0.02 0.12
	2.74–2.78	0.04 0.12		3.54–3.58	0.04 0.12
	2.76–2.80	0.02 0.04 0.12		3.56–3.60	0.02 0.04 0.12
3.6	3.60–3.64	0.02	4.0	4.00–4.04	0.02
	3.62–3.66	0.04		4.02–4.06	0.04
	3.64–3.68	0.02 0.04		4.04–4.08	0.02 0.04
	3.66–3.70	0.08		4.06–4.10	0.08
	3.68–3.72	0.02 0.8		4.08–4.12	0.02 0.8
	3.70–3.74	0.12		4.10–4.14	0.12
	3.72–3.76	0.02 0.12		4.12–4.16	0.02 0.12
	3.74–3.78	0.04 0.12		4.14–4.18	0.04 0.12
	3.76–3.80	0.02 0.04 0.12		4.16–4.20	0.02 0.04 0.12
3.8	3.80–3.84	0.02	<i>Source:</i> <i>Mitutoyo Catalog, Canada</i>		
	3.82–3.86	0.04			
	3.84–3.88	0.02 0.04			
	3.86–3.90	0.08			
	3.88–3.92	0.02 0.8			
	3.90–3.94	0.12			
	3.92–3.96	0.02 0.12			
	3.94–3.98	0.04 0.12			
	3.96–3.40	0.02 0.04 .12			

Table A2.1. (continued) Bore control (in inches)

Parameters for reading gears using vernier calipers

Number of teeth	t"	S"	Number of teeth	t"	S"	Number of teeth	t"	S"
6	1.5529	1.1022	51	1.5706	1.0121	96	1.5707	1.0064
7	1.5568	1.0873	52	1.5706	1.0119	97	1.5707	1.0064
8	1.5607	1.0769	53	1.5706	1.0117	98	1.5707	1.0063
9	1.5628	1.0684	54	1.5706	1.0114	99	1.5707	1.0062
10	1.5643	1.0616	55	1.5706	1.0112	100	1.5707	1.0061
11	1.5654	1.0559	56	1.5706	1.011	101	1.5707	1.0061

Table A2.2. Thickness of the core rope of a gear tooth of 1 diametral pitch

12	1.5663	1.0514	57	1.5706	1.0108	102	1.5707	1.006
13	1.567	1.0474	58	1.5706	1.0106	103	1.5707	1.006
14	1.5675	1.044	59	1.5706	1.0105	104	1.5707	1.0059
15	1.5679	1.0411	60	1.5706	1.0102	105	1.5707	1.0059
16	1.5683	1.0385	61	1.5706	1.0101	106	1.5707	1.0058
17	1.5686	1.0362	62	1.5706	1.01	107	1.5707	1.0058
18	1.5688	1.0342	63	1.5706	1.0098	108	1.5707	1.0057
19	1.569	1.0324	64	1.5706	1.0097	109	1.5707	1.0057
20	1.5692	1.0308	65	1.5706	1.0095	110	1.5707	1.0056
21	1.5694	1.0294	66	1.5706	1.0094	111	1.5707	1.0056
22	1.5695	1.0281	67	1.5706	1.0092	112	1.5707	1.0055
23	1.5696	1.0268	68	1.5707	1.0091	113	1.5707	1.0055
24	1.5697	1.0257	69	1.5707	1.009	114	1.5707	1.0054
25	1.5698	1.0247	70	1.5707	1.0088	115	1.5707	1.0054
26	1.5698	1.0237	71	1.5707	1.0087	116	1.5707	1.0053
27	1.5699	1.0228	72	1.5707	1.0086	117	1.5707	1.0053
28	1.57	1.022	73	1.5707	1.0085	118	1.5707	1.0053
29	1.57	1.0213	74	1.5707	1.0084	119	1.5707	1.0052
30	1.5701	1.0208	75	1.5707	1.0083	120	1.5707	1.0052
31	1.5701	1.0199	76	1.5707	1.0081	121	1.5707	1.0051
32	1.5702	1.0193	77	1.5707	1.008	122	1.5707	1.0051
33	1.5702	1.0187	78	1.5707	1.0079	123	1.5707	1.005
34	1.5702	1.0181	79	1.5707	1.0078	124	1.5707	1.005
35	1.5702	1.0176	80	1.5707	1.0077	125	1.5707	1.0049
36	1.5703	1.0171	81	1.5707	1.0076	126	1.5707	1.0049
37	1.5703	1.0167	82	1.5707	1.0075	127	1.5707	1.0049
38	1.5703	1.0162	83	1.5707	1.0074	128	1.5707	1.0048
39	1.5704	1.0158	84	1.5707	1.0074	129	1.5707	1.0047
40	1.5704	1.0154	85	1.5707	1.0073	130	1.5708	1.0047
41	1.5704	1.015	86	1.5707	1.0072	131	1.5708	1.0047
42	1.5704	1.0147	87	1.5707	1.0071	132	1.5708	1.0047
43	1.5705	1.0143	88	1.5707	1.007	133	1.5708	1.0046
44	1.5705	1.014	89	1.5707	1.0069	134	1.5708	1.0046
45	1.5705	1.0137	90	1.5707	1.0068	135	1.5708	1.0045
46	1.5705	1.0134	91	1.5707	1.0068	150	1.5708	1.0025
47	1.5705	1.0131	92	1.5707	1.0067	250	1.5708	1
48	1.5705	1.0129	93	1.5707	1.0067	arc		
49	1.5705	1.0126	94	1.5707	1.0066			
50	1.5705	1.0123	95	1.5707	1.0065			

Table A2.2. (continued) Thickness of the core rope of a gear tooth of 1 diametral pitch
 (source: AGMA Standards: 112.05, 115.01, and 116.01 (see Machinery's Handbook
 25th ed.))

Appendix 3

ISO standard	Corresponding title
ISO 468	Roughness, parameters, values and general rules for specifying requirements
ISO 1302	Indication of surface texture in technical product documentation (e.g., drawings)
ISO 1878	Classifications of instruments and devices for measurement and evaluation of the geometrical parameters of surface
ISO 1879	Instruments for the measurement of surface roughness by the profile method–Vocabulary
ISO 1880	Instruments for the measurement of surface roughness by the profile method–Contact (stylus) instruments of progressive profile transformation–Profile recording instruments
ISO 2632-1	Roughness comparison specimens–Part 1: Turned, ground, bored, milled, shaped, and planed
ISO 2632-2	Roughness comparison specimens–Part 2: Spark-eroded, shot, blasted and grit, blasted and polished
ISO 3274	Nominal characteristics of contact (stylus) instruments
ISO 4287	Terms, definitions and surface texture parameters
ISO 4288	Rules and procedures for the assessment of surface texture
ISO 5436	Calibration specimens–Stylus instruments–Types, calibration and use of specimens
ISO 8785	Surface imperfections–Terms, definitions and parameters
ISO 11562	Metrological characteristics of phase corrected filters
ISO 12085	Motif parameters
ISO 12179	Calibration of contact roughness instruments
ISO 13565-1	Filtering and measurement conditions
ISO 13565-2	Height characterization using the linear material ratio curve

Table A3.1. *ISO standards governing roughness domains*

Appendix 4

An applet display of an AVI file can be found in appendices. This workshop describes the graph of a function and its tangent passing through $(a, f(a))$, for different values of a .

$$\text{Function: } f(x) = x \cdot \sin(x) \quad f'(x) = \frac{d}{dx} f(x)$$

$$\text{Tangent: } y(a, x) = f'(a) \cdot (x - a) + f(a)$$

Parameter defined as function of the variable frame:

$$a = \pi + \frac{\text{Frame}}{5} \quad a = 3.142 \quad x = 0, 0.1 \dots 4 \cdot \pi \quad a = 1\pi$$

The slope of the tangent: $f'(a) = -1\pi$

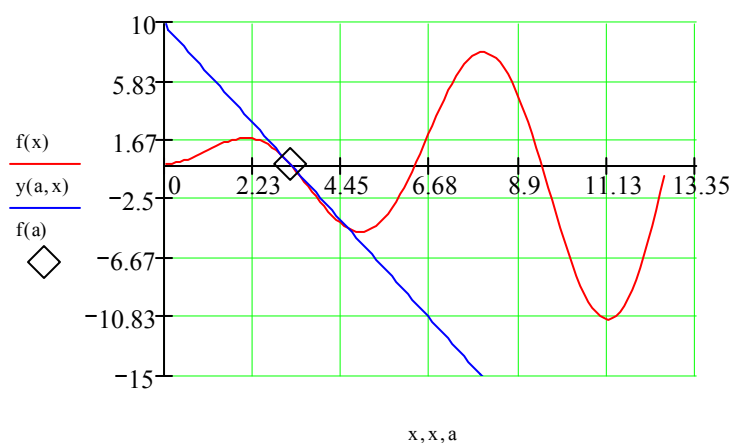


Figure A4.1. Tangent function: spline

Appendix 5

Major Ø	Threads (in.)		Outer Ø tap	Drills for tapping	Decimal equivalent
Tap	NC	NF			
0	—	80	0.060	3/64"	0.4690
1	64	—	0.073	53	0.0595
1	—	72	—	53	0.0595
2	56	—	0.086	51	0.0670
2	—	64	—	50	0.0700
3	48	—	0.099	5/64"	0.0781
3	—	56	—	46	0.0810
4	40	—	0.112	44	0.0860
4	—	48	—	3/32"	0.0937
5	40	—	0.125	39	0.0995
5	—	44	—	37	0.1040
6	32	—	0.138	36	0.1065
6	—	40	—	33	0.1130
8	32	—	0.164	29	0.1360
8	—	36	—	29	0.1360
10	24	—	0.190	26	0.1470
10	—	32	—	5/32"	0.1562
12	24	—	0.216	11/64"	0.1719
12	—	28	—	15	0.1800
14	20	—	0.242	10	0.1935
14	—	24	—	7	0.2010

Table A5.1. Machine screw taps (source: KAR, TAP Drill Reference, http://www.physics.ncsu.edu/pearl/Tap_Drill_Chart.html)

Major Ø	Threads (in.)	Drills for tapping
1/8	27	11/32"
1/4	18	7/16"
3/8	18	37/64"
1/2	14	23/32"
3/4	14	59/64"
1	11 1/2	1 5/32"
1 1/4	11 1/2	1 1/2"
1 1/2	11 1/2	1 47/64"
2	11 1/2	2 7/32"
2 1/2	8	2 5/8"
3	8	3 1/4"
3 1/2	8	3 3/4"
4	8	4 1/4"

Table A5.2. Thread taps, National American Standard, NC and NF (source: KAR, TAP Drill Reference, http://www.physics.ncsu.edu/pearl/Tap_Drill_Chart.html)

Major Ø	Threads (in.)		Outer Ø	Decimal equivalent
Tap	NC	NF	Tap	
1/4	20	–	7	0.2010
1/4	–	28	3	0.2130
5/16	18		F	0.2570
5/16	–	24	I	0.2720
3/8	16	–	5/16	0.3125
3/8	–	24	Q	0.3320
7/16	14	–	U	0.3680
7/16	–	20	W	0.3860
1/2	13	–	27/64	0.4219
1/2	–	20	29/64	0.4531
9/16	12	–	31/64	0.4840
9/16	–	18	1/2	0.5000
5/8	11	–	17/32	0.5312
5/8	–	18	9/16	0.5625
3/4	10	–	41/64	0.6406
3/4	–	16	11/16	0.6875
7/8	9	–	49/64	0.7656
7/8	–	14	51/64	0.7969
1	8	–	7/8	0.8750
1	–	14	59/64	0.9218
1 1/8	7	–	63/64	0.9844
1 1/8	–	12	1 3/64	1.0469
1 1/4	7	–	1 7/64	1.1094
1 1/4	–	12	1 11/64	1.1719
1 3/8	6	–	1 13/64	1.2187
1 3/8	–	12	1 19/64	1.2969
1 1/2	6	–	1 11/32	1.3437
1 1/2	–	12	1 27/64	1.4219

Table A5.3. Thread Taps, National American Standard, NC and NF (source: KAR, TAP Drill Reference, http://www.physics.ncsu.edu/pearl/Tap_Drill_Chart.html)

Major diameter, d = D (mm)	Coarse pitch P (mm)	v on the pitch, d_2 = D_2	v core screw, d_3 (mm)	Resistant section Seq(mm ²)	v inner screw nut D_1 (mm)	Fine pitch recommended
1	0.25	0.838	0.693	0.460	0.729	
(1.1)	0.25	0.938	0.793	0.588	0.829	
1.2	0.25	1.038	0.893	0.732	0.929	
(1.4)	0.30	1.205	1.032	0.983	1.075	0.2
1.6	0.35	1.373	1.171	1.27	1.221	0.2
(1.8)	0.35	1.573	1.371	1.70	1.421	0.2
2	0.4	1.740	1.509	2.07	1.567	0.25
(2.2)	0.45	1.908	1.648	2.48	1.713	0.35
1.6	0.35	1.373	1.171	1.27	1.221	0.2
(1.8)	0.35	1.573	1.371	1.70	1.421	0.2
2	0.4	1.740	1.509	2.07	1.567	0.25
(2.2)	0.45	1.908	1.648	2.48	1.713	0.35
2.5	0.45	2.208	1.948	3.39	2.013	0.35
3	0.5	2.675	2.387	5.03	2.459	0.35
(3.5)	0.6	3.110	2.764	6.78	2.850	0.35
4	0.7	3.545	3.141	8.78	3.242	0.5
(4.5)	0.75	4.013	3.580	11.3	3.688	0.5
5	0.8	4.480	4.019	14.2	4.134	0.5
6	1	5.350	4.773	20.1	4.918	0.75
(7)	1	6.350	5.773	28.9	5.918	0.75
8	1.25	7.188	6.466	36.6	6.647	1-(0.75)
10	1.5	9.026	8.160	58.0	8.376	1.25-(1-0.75)
12	1.75	10.863	9.853	84.3	10.106	1.5-(1.25-1)
(14)	2	12.701	11.546	115	11.835	1.5-(1.25-1)
16	2	14.701	13.546	157	13.835	1.5-(1)
(18)	2.5	16.376	14.933	192	15.294	2-(1.5-1)
20	2.5	18.376	16.933	245	17.294	2-(1.5-1)
(22)	2.5	20.376	18.933	303	19.294	2-(1.5-1)
24	3	22.051	20.319	353	20.752	2-(1.5-1)
(27)	3	25.051	23.319	459	23.752	2-(1.5-1)
30	3.5	27.727	25.706	561	26.211	2-(1.5-1)
(33)	3.5	30.727	28.706	694	29.211	2-(1.5)
36	4	33.402	31.093	817	31.670	3-(2-1.5)
(39)	4	36.402	34.093	976	34.670	3-(2-1.5)
42	4.5	39.077	36.479	1.121	37.129	4-(3-2-1.5)
(45)	4.5	42.077	39.479	1.306	40.129	4-(3-2-1.5)
48	5	44.752	41.866	1.473	42.587	4-(3-2-1.5)
(52)	5	48.752	45.866	1.758	46.587	4-(3-2-1.5)
56	5.5	52.428	49.252	2.030	50.046	4-(3-2-1.5)
(60)	5.5	56.428	53.252	2.362	54.046	4-(3-2-1.5)
64	6	60.103	56.639	2.676	57.505	4-(3-2-1.5)

Values in brackets are to be avoided.

Table A5.4. ISO threads at 60° (metric coarse pitch) (source: ISO)

Major diameter	Pitch	Drill hole diameter	Height	Effective diameter	v inner screw nut (mm)
$D = d$	P		H	$D_2 = d_2$	D_1
1	0.25	0.75	0.217	0.838	0.729
1.1	0.25	0.85	0.217	0.938	0.829
1.2	0.25	0.95	0.217	1.038	0.929
1.4	0.30	1.10	0.260	1.205	1.075
1.6	0.35	1.20	0.303	1.373	1.221
1.8	0.35	1.40	0.303	1.573	1.421
2	0.40	1.60	0.346	1.740	1.567
2.2	0.45	1.70	0.390	1.908	1.713
2.5	0.45	2	0.390	2.208	2.013
3	0.50	2.50	0.433	2.675	2.459
3.5	0.60	2.90	0.520	3.110	2.851
4	0.70	3.30	0.606	3.545	3.242
4.5	0.75	3.70	0.650	4.013	3.688
5	0.80	4.20	0.693	4.480	4.134
6	1.00	5	0.866	5.351	4.918
7	1.00	6	0.866	6.351	5.918
8	1.25	6.70	1.083	7.188	6.647
8	1.00	7	0.866	7.351	6.918
10	1.50	8.50	1.299	9.026	8.376
10	1.25	8.70	1.083	9.188	8.647
12	1.75	10.20	1.516	10.863	10.106
12	1.25	10.70	1.083	11.188	10.647
14	2.00	12	1.732	12.701	11.835
14	1.50	12.50	1.299	13.026	12.376
16	2.00	14	1.732	14.701	13.835
16	1.50	14.50	1.299	15.026	14.376
18	2.50	15.50	2.165	16.376	15.294
18	1.50	16.50	1.299	17.026	16.376
20	2.50	17.50	2.165	18.376	17.294
20	1.50	18.50	1.299	19.026	18.376
22	2.50	19.50	2.165	20.376	19.294
22	1.50	20.50	1.299	21.026	20.376
24	3.00	21	2.598	22.052	20.753
24	2.00	22	1.732	22.701	21.835

Table A5.5. ISO threads at 60°

27	3.00	24	2.598	25.052	23.753
27	2.00	25	1.732	25.701	24.835
30	3.50	26.50	3.031	27.727	26.211
30	2.00	28	1.732	28.701	27.835
33	3.50	29.50	3.031	30.727	29.211
33	2.00	31	1.732	31.701	30.835
36	4.00	32	3.464	33.402	31.670
36	3.00	33	2.598	34.052	32.753
39	4.00	35	3.464	36.402	34.670
39	3.00	36	2.598	37.052	35.753
42	4.50	37.50	3.897	39.077	37.129

To create a threaded hole of 10 (pitch 1.50 mm), drill it to 8.5 mm.

Table A5.5. (continued) ISO threads at 60° (metric) with the hole diameter being included (source: ISO)

Major v , $d =$ D (mm)	Fine pitch, P (mm)	v pitch diameter, $d_2 = D_2$	v core screw, d_3 (mm)	Resistance section, Seq (mm ²)	v inner screw nut, D_1 (mm)
8	1	7.350	6.773	39.2	6.917
10	1	9.350	8.773	64.5	8.917
10	1.25	9.188	8.466	61.2	8.647
12	1.25	11.188	10.466	92.1	10.647
12	1.5	11.026	10.160	88.1	10.376
(14)	1.5	13.026	12.376	125	12.376
16	1.5	15.026	14.160	167	14.376
(18)	1.5	17.026	16.160	216	16.376
20	1.5	19.026	18.160	272	18.376
20	2	18.701	17.546	258	17.835
(22)	1.5	21.026	20.160	333	20.376
24	1.5	23.026	22.160	401	22.376
24	2	22.701	21.546	384	21.835
(27)	2	25.701	24.546	496	24.835
30	2	28.701	27.546	621	27.835
(33)	2	31.701	30.546	761	30.835
36	3	34.051	32.319	865	32.752
(39)	3	37.051	35.319	1,028	35.752

Values in brackets are to be avoided.

ISO metric threads, fine pitch triangular thread

Table A5.6. ISO threads at 60° (fine pitch metric) (source: [ISO])

Common designation		Outer diameter, D (mm)	Pitch P (no. of threads/inch)	Pitch P (mm)	Pitch diameter, δ (mm)	Inner diameter, D (mm)	Drill * hole diameter
Inches	Millimeters						
1/8	5–10	9.728	28	0.9071	9.147	8.565	8.60
1/4	8–13	13.157	19	1.3368	12.301	11.445	11.50
3/8	12–17	16.662	19	1.3368	15.806	14.950	15
1/2	15–21	20.955	14	1.8143	19.794	18.633	18.70
5/8	16–23	22.911	14	1.8143	21.750	20.589	20.70
3/4	–	26.441	14	1.8143	25.281	24.120	24.20
7/8	24–31	30.201	14	1.8143	29.040	27.879	28
1	26–34	33.249	11	2.3091	31.770	30.292	30.40
1 1/8	–	37.897	11	2.3091	36.418	34.939	39
1 1/4	33–42	41.910	11	2.3091	40.432	38.953	45
1 1/2	40–49	47.803	11	2.3091	46.325	44.846	51
1 3/4	45–55	53.746	11	2.3091	52.268	50.790	56.80
2	50–60	59.614	11	2.3091	58.136	56.657	63
2 1/4	60–66	65.710	11	2.3091	64.232	62.753	72.50
2 1/2	66–76	75.184	11	2.3091	73.706	72.227	78.80
2 3/4	72–82	81.534	11	2.3091	80.056	78.577	85.30
3	80–90	87.884	11	2.3091	86.406	84.927	*...
3 1/2	90–102	100.330	11	2.3091	98.852	97.373	–
4	102–114	113.030	11	2.3091	111.552	110.073	–
4 1/2	115–127	125.730	11	2.3091	124.252	122.773	–
5	127–140	138.430	11	2.3091	136.952	135.473	–
5 1/2	140–152	151.130	11	2.3091	149.652	148.173	–
6	152–165	163.830	11	2.3091	162.352	160.873	–

*It is recommended to refer to manuals specializing in mechanical manufacturing for choosing the appropriate drill bit for a pilot-hole.

Note: These drill* hole diameters are not the same in North America (USA and Canada) and Europe. First, please refer specialized books to choose the right diameter of drill. When it comes to imperial driller, we try to not convert dimension to millimeter. It never works correctly.

Table A5.7. BSP straight thread *

Common designation (in.)	Pitch P thread		Pitch diameter (mm)		Major (outer) diameter (mm)			Core diameter (mm)		Effective thread length	Hand tight engagement
	Inches	Millimeters	At the end of the bolt δ	In the gauge plane δ_1	At the end of the bolt D	In the gauge plane D_1	Of the bolt C	At the end of the bolt d	In the gauge plane d_1	E (mm)	F (mm)
1/8	28	0.9071	8.899	9.147	9.480	9.728	10.319	8.317	8.565	7.371	3.970
1/4	19	1.3368	11.925	12.301	12.781	13.157	13.494	11.069	11.445	11.024	6.012
3/8	19	1.3368	15.409	15.806	16.265	16.662	17.462	14.553	14.950	11.361	6.350
1/2	14	1.8143	19.284	19.794	20.445	20.955	21.431	18.123	18.633	14.966	8.164
5/8	14	1.8143	21.352	21.749	22.514	22.911	23.813	—	—	15.875	6.350
3/4	14	1.8143	24.686	25.281	25.846	26.441	26.987	23.525	24.120	16.327	9.525
7/8	14	1.8143	28.444	29.039	29.606	30.201	30.956	—	—	19.050	9.525
1	11	2.3091	31.121	31.770	32.600	33.249	34.131	29.643	30.292	19.050	10.391
1 1/4	11	2.3091	39.638	40.432	41.116	41.910	42.862	38.159	38.953	21.359	12.700
1 1/2	11	2.3091	45.531	46.325	47.009	47.803	48.419	44.052	44.846	21.359	12.700
1 3/4	11	2.3091	51.275	52.267	52.754	53.746	54.769	—	—	28.575	15.875
2	11	2.3091	57.144	58.136	58.622	59.614	60.325	55.665	56.657	25.687	15.875
2 1/4	11	2.3091	63.140	64.231	64.619	65.710	66.675	—	—	31.750	17.462
2 1/2	11	2.3091	72.615	73.706	74.093	75.184	76.200	71.136	72.227	30.163	17.463
2 3/4	11	2.3091	78.765	80.055	80.244	81.534	82.550	—	—	34.925	20.637
3	11	2.3091	85.116	86.406	86.594	87.884	88.900	83.637	84.927	33.338	20.638
3 1/4	11	2.3091	91.112	92.501	92.591	93.980	95.250	—	—	38.100	22.225
3 1/2	11	2.3091	97.463	98.852	98.941	100.330	101.600	95.984	97.373	34.925	22.225
3 3/4	11	2.3091	103.812	105.201	105.291	106.680	107.950	—	—	38.100	22.225
4	11	2.3091	109.964	111.552	111.442	113.030	114.300	108.485	110.073	39.256	25.400
4 1/2	11	2.3091	122.664	124.251	124.143	125.730	127.000	—	—	41.275	25.400
5	11	2.3091	135.166	136.952	136.644	138.430	139.700	133.687	135.473	43.584	28.575
6	11	2.3091	160.566	162.352	162.044	163.830	—	159.087	160.873	43.584	28.575

Table A5.8. *BSP thread tapered*

Common designation (in.)	Pitch P		Pitch diameter (mm)		Outer (major) diameter (mm)			Inner diameter (mm)		Effective thread length	Hand tight engagement
	No. of threads/inch	Millimeter	ϕ at the end of the bolt	ϕ_i in the gauge plane	At the end of the bolt D	In the gauge plane D_1	Outer (major) diameter (mm) of pipe C	At the end of the bolt d	In the gauge plane D_1	E (mm)	F (mm)
1/8	28	0.9071	8.899	9.147	9.480	9.728	10.319	8.317	8.565	7.371	3.970
1/4	19	1.3368	11.925	12.301	12.781	13.157	13.494	11.069	11.445	11.024	6.012
3/8	19	1.3368	15.409	15.806	16.265	16.662	17.462	14.553	14.950	11.361	6.350
1/2	14	1.8143	19.284	19.794	20.445	20.955	21.431	18.123	18.633	14.966	8.164
5/8	14	1.8143	21.352	21.749	22.514	22.911	23.813	—	—	15.875	6.350
3/4	14	1.8143	24.686	25.281	25.846	26.441	26.987	23.525	24.120	16.327	9.525
7/8	14	1.8143	28.444	29.039	29.606	30.201	30.956	—	—	19.050	9.525
1	11	2.3091	31.121	31.770	32.600	33.249	34.131	29.643	30.292	19.050	10.391

Table A5.9. Characteristics of a Whitworth thread

Major diameter, D		Pitch, P		Pith	Core	Drill hole
Inches	Millimeter	Thread/inch	Millimeter	diameter, δ	diameter, d	diameter, D_p
3/16	4.763	32	0.7938	4.255	3.747	3.80
7/32	5.558	28	0.9071	4.976	4.394	4.50
1/4	6.350	26	0.9769	5.725	5.100	6.30
9/32	7.142	26	0.9769	6.518	5.893	6.10
5/16	7.938	22	1.1545	7.198	6.459	6.70
3/8	9.525	20	1.2700	8.712	7.899	8.20
7/16	11.113	18	1.4111	10.208	9.304	9.50
1/2	12.700	16	1.5875	11.684	10.668	11
9/16	14.288	16	1.5875	13.272	12.256	12.50
5/8	15.875	14	1.8143	14.714	13.553	13.80
11/16	17.463	14	1.8143	16.302	15.141	15.40
3/4	19.050	12	2.1167	17.694	16.337	16.70
13/16	20.638	12	2.1167	19.281	17.925	18.30
7/8	22.225	11	2.3091	20.747	19.268	19.80
1"	25.400	10	2.5400	23.774	22.149	22.60
1 1/8	28.575	9	2.8222	26.769	24.963	25.50
1 1/4	31.750	9	2.8222	29.944	28.138	28.60
1 3/8	34.925	8	3.1750	32.893	30.861	31.60
1 1/2	38.100	8	3.1750	36.068	34.036	34.50
1 5/8	41.275	8	3.1750	39.243	37.211	37.80
1 3/4	44.450	7	3.6286	42.126	39.801	40.70
2"	50.800	7	3.6286	48.476	46.152	47
2 1/4	57.150	6	4.2333	54.440	51.730	52.80
2 1/2	63.500	6	4.2333	60.760	58.080	59.20
2 3/4	69.850	6	4.2333	67.140	64.430	65.50
3"	76.200	5	5.0800	72.946	69.693	71
3 1/4	82.550	5	5.0800	79.286	76.043	
3 1/2	88.900	4.5	5.6444	85.286	81.671	
3 3/4	95.250	4.5	5.6444	91.636	88.021	
4"	101.600	4.5	5.6444	97.986	94.371	

Table A5.10. Characteristics of a Whitworth thread (series B.S.F., fine pitch) (source: BSW)

Common designation	Pitch P		Pitch diameter		Outer diameter			Effective thread length E	Hand tight engagement F
	Inches	Millimeter	End of the bolt A	At the notch of the gauge B	End of the bolt L	At the notch of the gauge M	Of the bolt C		
1/16	27	0.9407	6.888	7.142	7.640	7.895	7.938	6.632	4.064
1/8	27	0.9407	9.233	9.489	9.985	10.242	10.287	6.703	4.102
1/4	18	1.4111	12.126	12.487	13.254	13.616	13.716	10.206	5.786
3/8	14	1.4111	15.545	15.926	16.673	17.055	17.145	10.358	6.096
1/2	14	1.8143	19.264	19.772	20.715	21.223	21.336	13.556	8.128
3/4	14	1.8143	24.579	25.117	26.030	26.568	26.670	13.861	8.611
1"	11 1/2	2.2087	30.826	31.461	32.592	33.227	33.401	17.343	10.160
1 1/4"	11 1/2	2.2087	39.551	40.218	41.317	41.984	42.164	17.953	10.668
1 1/2"	11 1/2	2.2087	45.621	46.287	47.387	48.053	48.260	18.377	10.668
2"	11 1/2	2.2087	57.633	58.325	59.399	60.091	60.325	19.215	11.074
2 1/2"	8	3.1750	69.076	70.159	71.616	72.699	73.025	28.892	17.323
3"	8	3.1750	84.852	86.068	87.392	88.608	88.900	30.480	19.456
3 1/2"	8	3.1750	97.473	98.776	100.013	101.316	101.600	31.750	20.853
4"	8	3.1750	110.093	111.433	112.633	113.973	114.300	33.020	21.438
5"	8	3.1750	136.925	138.412	139.465	140.952	141.300	35.720	23.800
6"	8	3.1750	163.731	165.252	166.271	167.792	168.275	38.418	24.333

Table A5.11. Characteristics of a BRIGGS tapered thread (ASA-B2 -1 to 1960). Cone 6.25%

No. of diameter (in.)	Pitch P		Outer diameter, D	Pitch diameter, E	Core diameter, K	Drill hole diameter
	No. of threads/inch	Millimeters				
N°1	64	0.3969	1.854	1.598	1.425	1.50
N°2	56	0.4536	2.184	1.890	1.694	1.70
N°3	48	0.5292	2.515	2.172	1.941	2
N°4	40	0.6350	2.845	2.433	2.156	2.20
N°5	40	0.6350	3.175	2.764	2.487	2.60
N°6	32	0.7938	3.505	2.990	2.647	2.70
N°8	32	0.7938	4.166	3.650	3.307	3.40
N°10	24	1.0583	4.826	4.138	3.680	3.80
N°12	24	1.0583	5.486	4.798	4.341	4.50
1/4	20	1.2700	6.350	5.525	4.976	5.10
5/16	18	1.4111	7.938	7.021	6.411	6.60
3/8	16	1.5875	9.525	8.494	7.805	8
7/16	14	1.8143	11.113	9.934	9.149	9.40
1/2	13	1.9538	12.700	11.430	10.584	10.60
9/16	12	2.1167	14.288	12.913	11.996	12.10
5/8	11	2.3091	15.875	14.376	13.376	13.50
3/4	10	2.5400	19.050	17.399	16.299	16.50
7/8	9	2.8222	22.225	20.391	19.169	19.50
1"	8	3.1750	25.400	23.338	21.963	22.30

Table A5.12. Sellers thread standard series NC

1"1/8	7	3.6286	28.575	26.218	24.648	25
1"1/4	7	3.6286	31.750	29.393	27.823	28
1"3/8	6	4.2333	34.925	32.174	30.343	30.80
1"1/2	6	4.2333	38.100	35.349	33.518	34
1"3/4	5	5.0800	44.450	41.151	38.951	39.50
2"	4.5	5.6444	50.800	47.135	44.689	45
2"1/4	4.5	5.6444	57.150	53.485	51.039	51.50
2"1/2	4	6.3500	63.500	59.375	56.627	57
2"3/4	4	6.3500	69.850	65.725	62.977	63.50
3"	4	6.3500	76.200	72.075	69.327	69.50
3"1/4	4	6.3500	82.550	78.425	75.677	
3"1/2	4	6.3500	88.900	84.775	82.027	
3"3/4	4	6.3500	95.250	91.125	88.377	
4"	4	6.3500	101.600	97.475	94.727	

Table A5.12. (continued) Sellers thread standard series NC
(National Coarse = coarse pitch), standard ASA-B1.1949 (USA)

No. or diameter (in.)	Pitch <i>P</i>		Outer diameter, <i>D</i>	Pitch diameter, <i>E</i>	Core diameter, <i>K</i>	Drill hole diameter
	No. of threads/pitch	Millimeter				
N°0	80	0.317	1.524	1.318	1.181	1.20
N°1	72	0.353	1.854	1.626	1.473	1.50
N°2	64	0.397	2.184	1.928	1.755	1.80
N°3	56	0.454	2.515	2.220	2.024	2.10
N°4	48	0.529	2.845	2.502	2.271	2.30
N°5	44	0.577	3.175	2.799	2.550	2.60
N°6	40	0.635	3.505	3.094	2.817	2.90
N°8	36	0.706	4.166	3.708	3.401	3.50
N°10	32	0.794	4.826	4.310	3.967	4.10
N°12	28	0.907	5.486	4.897	4.530	4.60
1/4	28	0.907	6.350	5.761	5.367	5.50
5/16	24	1.058	7.938	7.249	6.792	6.90
3/8	24	1.058	9.525	8.837	8.379	8.50
7/16	20	1.270	11.113	10.287	9.738	9.80
1/2	20	1.270	12.700	11.875	11.326	11.50
9/16	18	1.411	14.288	13.371	12.761	13
5/8	18	1.411	15.875	14.958	14.348	14.50
3/4	16	1.588	19.050	18.019	17.330	17.50
7/8	14	1.814	22.225	21.046	20.262	20.50
1"	14	1.814	25.400	24.221	23.437	23.50
1"1/8	12	2.117	28.575	27.201	26.284	26.50
1"1/4	12	2.117	31.750	30.376	29.459	29.60
1"3/8	12	2.117	34.925	33.551	32.634	32.80
1"1/2	12	2.117	38.100	36.726	35.809	36

Table A5.13. Sellers fine-pitch thread standard, NF fine-pitch series

Major diameter		Coarse pitch Coarse thread, UNC			Fine pitch Fine thread series, UNF			Extra-fine pitch Extra-fine thread, UNEF		
<i>D</i> (in.) and sizes	(mm)	Threads/inch	Pitch <i>P</i> (in.)	Ø <i>d</i> ₂ core screw	Threads/ inch	Pitch <i>P</i> (in.)	Ø <i>d</i> ₂ core screw	Threads/ inch	Pitch <i>P</i> (in.)	Ø <i>d</i> ₂ core screw
0.006 (0)	1.52	—	—	—	80	0.0519	0.0447	—	—	—
0.073 (1)	1.85	64	0.0629	0.0538	72	0.0640	0.0560	—	—	—
0.086 (2)	2.18	56	0.0744	0.0641	64	0.0759	0.0668	—	—	—
0.099 (3)	2.51	48	0.0855	0.0734	56	0.0874	0.0771	—	—	—
0.112 (4)	2.84	40	0.0858	0.0813	48	0.0985	0.0864	—	—	—
0.125 (5)	3.17	40	0.1088	0.0943	44	0.1102	0.0971	—	—	—
0.138 (6)	3.50	32	0.1177	0.0997	40	0.1218	0.1073	—	—	—
0.164 (8)	4.16	32	0.1437	0.1257	36	0.1460	0.1299	—	—	—
0.190 (10)	4.83	24	0.1629	0.1389	32	0.1697	0.1517	—	—	—
0.216 (12)	5.49	24	0.1889	0.1649	28	0.1928	0.1722	32	0.1957	0.1777
0.250 (1/4)	6.35	20	0.2175	0.1887	28	0.2268	0.2062	32	0.2297	0.2117
0.3125 (5/16)	7.94	18	0.2764	0.2443	24	0.2854	0.2614	32	0.2922	0.2742
0.375 (3/8)	9.52	16	0.3344	0.2983	24	0.3479	0.3239	32	0.3547	0.3367
0.4375 (7/16)	11.11	14	0.3911	0.3499	20	0.4050	0.3762	28	0.4143	0.3937
0.500 (1/2)	12.70	13	0.4500	0.4056	20	0.4675	0.4387	28	0.4768	0.4562
0.5625 (9/16)	14.29	12	0.5084	0.4603	18	0.5264	0.4943	24	0.5354	0.5114
0.625 (5/8)	15.87	11	0.5660	0.5135	18	0.5889	0.5568	24	0.5979	0.5739
0.6875 (11/16)	17.46	—	—	—	—	—	—	24	0.6604	0.6364
0.750 (3/4)	19.05	10	0.6850	0.6273	16	0.7094	0.6733	20	0.7175	0.6887
0.8125 (13/16)	20.64	—	—	—	—	—	—	20	0.7800	0.7512

Table A5.14. Complementary series supplementing the earlier series (Unified Profile (USA) Unified Standard ANSI/ASME B1.1 Main series (1 in. = 25.4 mm))

0.875 (7/8)	22.22	9	0.8028	0.7387	14	0.8286	0.7874	20	0.8425	0.8137
0.9375	23.81	—	—	—	—	—	—	20	0.9050	0.8762
(15/16)	25.40	8	0.9188	0.8466	12	0.9459	0.8978	20	0.9675	0.9387
1.000 (1)	26.99	—	—	—	—	—	—	18	1.0264	0.9943
1.0625 (1 1/16)	28.57	7	1.0322	0.9497	12	1.0709	1.0228	18	1.0889	1.0568
1.125 (1 1/8)	30.16	—	—	—	—	—	—	18	1.1514	1.1193
1.1875 (1 3/16)	31.57	7	1.1572	1.0747	12	1.1959	1.1478	18	1.2139	1.1818
1.250 (1 1/4)	33.34	—	—	—	—	—	—	18	1.2764	1.2443
1.3125 (1 5/16)										
1.375 (1 3/8)	34.92	6	1.2667	1.1705	12	1.3209	1.2728	18	1.3389	1.3068
1.4375 (1 7/16)	36.51	—	—	—	—	—	—	18	1.4014	1.3693
1.500 (1 1/2)	38.10	6	1.3917	1.2955	12	1.4459	1.3978	18	1.4639	1.4318
1.5625 (1 9/16)	36.69	—	—	—	—	—	—	18	1.5264	1.4943
1.625 (1 5/8)	41.27	—	—	—	—	—	—	18	1.5889	1.5568
1.6875 (1 11/16)	42.86	—	—	—	—	—	—	18	1.6514	1.6193
1.750 (1 3/4)	44.45	5	1.6201	1.5046	—	—	—	16		
2.000 (2)	50.80	4.5	1.8557	1.7274	—	—	—	16		
2.250 (2 1/4)	57.15	4.5	2.1057	1.9774	—	—	—			
2.500 (2 1/2)	63.50	4	2.3376	2.1933	—	—	—			
2.750 (2 3/4)	69.85	4	2.5876	2.4433	—	—	—			
3.000 (3)	76.20	4	2.8376	2.6933	—	—	—			
3.250 (3 1/4)	82.55	4	3.0876	2.9433	—	—	—			
3.500 (3 1/2)	88.90	4	3.3376	3.1933	—	—	—			
3.750 (3 3/4)	95.25	4	3.5876	3.4433	—	—	—			
4.000 (4)	101.60	4	3.8376	3.6933	—	—	—			

Source: ANSI/ASME B1.1.7M-1984
(R 1992)

Table A5.14. continued

Major diameter $\varnothing d$		No. of threads/inch			Major diameter $\varnothing d$		No. of threads/inch		
Inches	$\varnothing$ (mm)	8N series	12N series	16N series	Inches	$\varnothing d$ (mm)	8N series	12N series	16N series
0.500 (1/2)	12.70	—	12	—	1.750 (1 3/4)	44.45	8	12	16
0.5625 (9/16)	14.29	—	12	—	1.8125 (1 7/8)	46.04	—	—	16
0.625 (5/8)	15.87	—	12	—	1.875 (1 13/16)	47.62	8	12	16
0.6875 (11/16)	17.46	—	12	—	1.9375 (1 15/16)	49.21	—	—	16
0.750 (3/4)	19.05	—	12	16	2.000 (2)	50.80	8	12	16
0.8125 (13/16)	20.64	—	12	16	2.0625 (2 1/16)	52.39	—	—	16
0.875 (7/8)	22.22	—	12	16	2.125 (2 1/8)	53.98	8	12	16
0.9375 (15/16)	23.81	—	12	16	2.1875 (2 3/16)	55.56	—	—	16
1.000 (1)	25.40	8	12	16	2.250 (2 1/4)	57.15	8	12	16
1.0625 (1 1/16)	26.99	—	12	16	2.3125 (2 5/16)	58.74	—	—	16
1.125 (1 1/8)	28.57	8	12	16	2.375 (2 3/8)	60.32	—	12	16
1.1875 (1 3/16)	30.16	—	12	16	2.4375 (2 7/16)	61.91	—	—	16
1.250 (1 1/4)	31.75	8	12	16	2.500 (2 1/2)	63.50	8	12	16
1.3125 (1 5/16)	33.34	—	12	16	2.625 (2 5/8)	66.68	—	12	16
1.375 (1 3/8)	34.92	8	12	16	2.750 (2 3/4)	69.85	8	12	16
1.4375 (1 7/16)	36.51	—	12	16	2.875 (2 7/8)	73.02	—	12	16
1.500 (1 1/2)	38.10	8	12	16	3.000 (3)	76.20	8	12	16
1.5625 (1 9/16)	39.69	—	—	16	3.125 (3 1/8)	79.38	—	12	16
1.625 (1 5/8)	41.27	8	12	16	3.250 (3 1/4)	82.55	8	12	16
1.6875 (1 11/16)	42.86	—	—	16	3.375 (3 3/8)	85.72	—	12	16

Table A5.15. Unified profiles, Unified standard ANSI/ASME B1.1 (complementary series 8N, 12N, and 16N (1 in. = 25.4 mm))

			ACME THREADS				Stub ACME Threads	
Major size (in.)	Major size (mm)	No. of threads/inch	Basic height H_1 (in.)	2G-3G-4G 2C-3C-4C $\varnothing D$ (in.)	2G-3G-4G 2C-3C-4C helix (°)	5C and 6C $\varnothing D$ (in.)	Basic height H_1 (in.)	Angle° helix
1/4	0.2500	16	0.03125	0.2500	5°12'	—	0.01875	4°54'
5/16	0.3125	14	0.03571	0.3125	4°42'	—	0.02143	4°28'
3/8	0.3750	12	0.04167	0.3750	4°33'	—	0.02500	4°20'
7/16	0.4375	12	0.04167	0.4375	3°50'	—	0.02500	3°41'
1/2	0.5000	10	0.05000	0.5000	4°3'	0.4823	0.03000	3°52'
5/8	0.6250	8	0.06250	0.6250	4°3'	0.6052	0.03750	3°52'
3/4	0.7500	6	0.08333	0.7500	4°33'	0.7284	0.05000	4°20'
7/8	0.8750	6	0.08333	0.8750	3°50'	0.8516	0.05000	3°41'
1	1.0000	5	0.10000	1.0000	4°3'	0.9750	0.06000	3°52'
1 1/8	1.1250	5	0.10000	1.1250	3°33'	1.0985	0.06000	3°25'
1 1/4	1.2500	5	0.10000	1.2500	3°10'	1.2220	0.06000	3°4'
1 3/8	1.3750	4	0.12500	1.3750	3°39'	1.3457	0.07500	3°30'
1 1/2	1.5000	4	0.12500	1.5000	3°19'	1.4694	0.07500	3°12'
1 3/4	1.7500	4	0.12500	1.7500	2°48'	1.7169	0.07500	2°43'
2	2.0000	4	0.12500	2.0000	2°26'	1.9646	0.07500	2°22'
2 1/4	2.2500	3	0.16667	2.2500	2°55'	2.2125	0.10000	2°50'
2 1/2	2.5000	3	0.16667	2.5000	2°36'	2.4605	0.10000	2°32'
2 3/4	2.7500	3	0.16667	2.7500	2°21'	2.7085	0.10000	2°18'
3	3.0000	2	0.25000	3.0000	3°19'	2.9567	0.15000	3°12'
3 1/2	3.5000	2	0.25000	3.5000	2°48'	3.4532	0.15000	2°43'
4	4.0000	2	0.25000	4.0000	2°26'	3.9500	0.15000	2°22'
4 1/2	4.5000	2	0.25000	4.5000	2°8'	4.4470	0.15000	2°6'
5	5.0000	2	0.25000	5.0000	1°55'	4.9441	0.15000	1°53'

Table A5.16a. ACME and Stub ACME trapezoidal threading (U.S. standards)

Main trapezoidal threads ISO 2902					
Major $\varnothing$, $d = D$ (mm)	Pitch P (mm)	Major $\varnothing$ $d = D$ (mm)	Pitch P (mm)	Major $\varnothing$ $d = D$ (mm)	P (mm)
8	1.5	44	7	160	16
(9)	2	(46)	8	(170)	16
10	2	48	8	180	18
(11)	2	(50)	8	(190)	18
12	3	52	8	200	18
(14)	3	(55)	9	(210)	20
16	4	60	9	220	20
(18)	44	(65)	10	(230)	20
20	4	70	10	240	22
(22)	5	(75)	10	(250)	22
24	5	80	10	260	22
(26)	5	(85)	12	(270)	24
28	5	90	12	280	24
(30)	6	(95)	12	(290)	24
32	6	100	12	300	24
(34)	6	(110)	12		
36	6	120	14		
(38)	7	(130)	14		
40	7	140	14		
(42)	7	(150)	16		

Values in brackets are to be avoided

Table A5.16b. Main trapezoidal threads (source [ISO 2902])

List of major diameters recommended (in.)				
1/2	1 1/8	2 1/2	5 1/2	12
9/16	1 1/4	2 3/4	6	14
5/8	1 3/8	3	7	16
11/16	1 1/2	3 1/2	8	18
3/4	1 3/4	4	9	20
7/8	2	4 1/2	10	22
1	2 1/4	5	11	24
Range of major $\varnothing$ (in.) Over up to (included)		Recommended no. of threads/inch		
1/2 (12.7 mm)	11/16	20-16-12		
11/16 (17.46 mm)	1	16-12-10		
1 (25.4 mm)	1 1/2	16-12-10-8-6		
1 1/2 (38.1 mm)	2 1/2	16-12-10-8-6-5-4		
2 1/2 (63.5 mm)	4	16-12-10-8-6-5-4		
4 (101.6 mm)	6	12-10-8-6-5-4-3		
6 (152.4 mm)	10	10-8-6-5-4-3-2.5-2		
10 (254 mm)	16	10-8-6-5-4-3-2.5-2-1.5-1.25		
16 (406.4 mm)	24 (609 mm)	8-6-5-4-3-2.5-2-1.5-1.25-1		

Table A5.17. Asymmetrical trapezoidal profile: Buttress thread (7° and 45°)

Major diameter (in.)	Major Ø(mm)	No. of threads/inch	Inner Ø (in.)	Modified profile (10°) Thickness at the bottom of thread (0.5436 P)
1/4	6.35	10	0.163	0.0544
3/8	9.52	8	0.266	0.0680
1/2	12.70	6.5	0.366	0.0837
3/4	19.05	5	0.575	0.1087
1	25.4	4	0.781	0.1357
1 1/2	38.10	3	1.208	0.1812
2	50.80	2.25	1.612	0.2416
2 1/2	63.50	2	2.063	0.2718
3	76.20	1.75	2.500	0.3160
4	101.60	1.5	3.418	0.3624

Table A5.18. Square profile and modified square thread (U.S. standards)

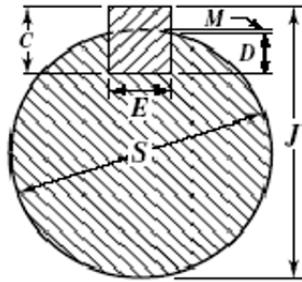
 $J = S - (M + D) + C$ <i>This is an example. All values of M are in inches. To access the complete diagram, the reader should refer to [OBE 96]</i>									
Ø Shift (in.)	1/16	3/32	1/8	5/32	3/16	7/32	1/4	5/16	3/8
0.3125	0.0032	—	—	—	—	—	—	—	—
0.3437	0.0029	0.0065	—	—	—	—	—	—	—
0.3750	0.0026	0.0060	0.0107	—	—	—	—	—	—
0.4060	0.0024	0.0055	0.0099	—	—	—	—	—	—
0.4375	0.0022	0.0051	0.0091	—	—	—	—	—	—
0.4687	0.0021	0.0047	0.0085	0.0134	—	—	—	—	—
0.5000	0.0020	0.0040	0.0079	0.0125	—	—	—	—	—
0.5625	—	0.0039	0.0070	0.0111	0.0161	—	—	—	—
etc	—	—	—	—	—	—	—	—	—
2.2500	—	—	—	—	—	—	0.0070	0.0109	0.0157

Table A5.19. Housing depth and distance from the top of the wedge to the outer face opposite of the shaft (source: [OBE 96])

For more tables, see [OBE 96].

Appendix 6

ISO 204:1997	Metallic materials – Uninterrupted uniaxial creep testing in tension – Method of test
ISO 376:1987	Metallic materials – Calibration of force-proving instruments used for the verification of uniaxial testing machines
ISO/DIS 376	Metallic materials – Calibration of force-proving instruments used for the verification of uniaxial testing machines
ISO 783:1989	Metallic materials – Tensile testing at elevated temperature
ISO/DIS 783	Metallic materials – Tensile testing at elevated temperature
ISO 6892:1998	Metallic materials – Tensile testing at ambient temperature
ISO 7500-1:1986	Metallic materials – Verification of static uniaxial testing machines – Part 1: Tensile testing machines
ISO/DIS 7500-1	Metallic materials – Verification of static uniaxial testing machines – Part 1: Tensile testing machines
ISO 7500-2:1996	Metallic materials – Verification of static uniaxial testing machines – Part 2: Tension creep testing machines – Verification of the applied load
ISO 9513:1989	Metallic materials – Verification of extensometers used in uniaxial testing
ISO / DIS 9513	Metallic materials – Calibration of extensometers used in uniaxial testing
ISO/DIS 15579	Metallic materials – Tensile testing at low temperature
ISO 156:1982	Metallic materials – Hardness test – Verification of Brinell hardness testing machines
ISO 410:1982	Metallic materials – Hardness test – Tables of Brinell hardness values for use in tests made on flat surfaces
ISO 674:1988	Metallic materials – Hardness test – Calibration of standardized blocks to be used for Rockwell hardness testing machines (scales A, B, C, D, E, F, G, H, K)
ISO 716:1986	Metallic materials – Hardness test – Verification of Rockwell hardness testing machines (scales A, B, C, D, E, F, G, H, K)
ISO 726:1982	Metallic materials – Hardness test – Calibration of standardized blocks to be used for Brinell hardness testing machines
ISO 1024:1989	Metallic materials – Hardness test – Rockwell superficial test (scales 15N, 30N, 45N, 15T, 30T and 45T)
ISO 1079:1989	Metallic materials – Hardness test – Verification of Rockwell superficial hardness testing machines (scales 15N, 30N, 45N, 15T, 30T and 45T)
ISO 1355:1989	Metallic materials – Hardness test – Calibration of standardized blocks to be used for Rockwell superficial hardness testing machines (scales 15N, 30N, 45N, 15T, 30T and 45T)
ISO 4545:1993	Metallic materials – Hardness test – Knoop test
ISO 4546:1993	Metallic materials – Hardness test – Verification of Knoop hardness testing machines

Table A6.1. *Titles of standards published by ISO/TC 164 on the mechanical testing of metals*

ISO 4547:1993	Metallic materials – Hardness test – Calibration of standardized blocks to be used for Knoop hardness testing machines
ISO 6506:1981	Metallic materials – Hardness test – Brinell test
ISO/DIS 6506-1	Metallic materials – Brinell hardness test – Part 1: Test method
ISO/DIS 6506-2	Metallic materials – Brinell hardness test – Part 2: Verification and calibration of testing machines
ISO/DIS 6506-3	Matériaux métalliques – Essais de dureté Brinell – 3eme partie : calibration des blocs de reprise
ISO 6507-1:1997	Metallic materials – Vickers hardness test – Part 1: Test method
ISO 6507-2:1997	Metallic materials – Vickers hardness test – Part 2: Verification of testing machines
ISO 6507-3:1997	Metallic materials – Vickers hardness test – Part 3: Calibration of reference blocks
ISO 6508:1986	Metallic materials – Hardness test – Rockwell test (scales A, B, C, D, E, F, G, H, K)
ISO/DIS 6508-1	Metallic materials – Rockwell hardness test – Part 1: Test method (scales A, B, C, D, E, F, G, H, K, N, T)
ISO/DIS 6508-2	Metallic materials – Rockwell hardness test – Part 2: Verification and calibration of testing machines (scales A, B, C, D, E, F, G, H, K, N, T)
ISO/DIS 6508-3	Metallic materials – Rockwell hardness test – Part 3: Calibration of reference blocks (scales A, B, C, D, E, F, G, H, K, N, T)
ISO 10250:1994	Metallic materials – Hardness testing – Tables of Knoop hardness values for use in tests made on flat surfaces
ISO/TR 14577:1995	Metallic materials – Hardness test – Universal test
ISO 83:1976	Steel – Charpy impact test (U-notch)
ISO 148:1983	Steel – Charpy impact test (V-notch)
ISO 148-2:1998	Metallic materials – Charpy pendulum impact test – Part 2: Verification of test machines
ISO 148-3:1998	Metallic materials – Charpy pendulum impact test – Part 3: Preparation and characterization of Charpy V reference test pieces for verification of test machines
ISO/R 442:1965	Verification of pendulum impact testing machines for testing steels
ISO 3785:1976	Steel – Designation of test piece axes
ISO/DIS 3785	Metallic materials – Designation of test specimens (revision of ISO 3785:1976)
ISO/DIS 12135	Metallic materials – Standardized and unified method for determination of quasi-static fracture toughness
ISO 12737:1996	Metallic materials – Determination of plane-strain fracture toughness
ISO/DIS 14556	Steel – Charpy V-Notch Impact Test – Instrumented Test Method

Table A6.1. (continued) Titles of standards published by ISO/TC 164 on the mechanical testing of metals

Type	Application	Material thickness
SHORE A	Soft rubber, elastomers, natural rubber, neoprene, moldable resin, polyester, PVC soft, leather, etc.	4 mm, DIN EN ISO 868 6 mm, DIN 53505 ISO 7619, ASTM D 2240
SHORE A0 L+L/c	Mosses, soft elastic materials, foam lining in car, flying	6 mm
Barcol SHORE D	Plastics reinforced with fiberglass, hard rubber, hard plastic materials, acrylic, thermoplastic, resopal, rollers, sheets of vinyl acetate, cellulosic acetate, etc.	1.5 mm 4 mm DIN EN ISO 868 6 mm DIN 53505 ISO 7619, ASTM D 2240
SHORE B	Moderately hard rubber materials, typewriters rollers, etc.	6 mm
SHORE C	Plastic and rubber materials moderately hard	6 mm
SHORE D0	Plastic and rubber materials: moderately hard to hard	6 mm
SHORE 0	Soft elastic material, print rollers, medium fast textiles, nylon, fabric, orlon, perlon, rayon	6 mm
SHORE 00/000/000 SIRHD/DIDC Super Soft	Rubber: foam and cellular, silicone gel-like material	6 mm 2 mm
IRHD/DIDC M	Thin O-rings, modelable pieces, normalized plates	0.6 mm–5 mm
IRHD/DIDC N	Soft rubber, highly elastic materials, plastically deformable substance	6 mm–10 mm
IRHD/DIDC L	Materials such as IRHD N	10 mm–12 mm
IRHD/DIDC H	Hard materials such as SHORE D	6 mm–10 mm

Table A6.2. *Comparative scales for materials, according to Bareiss testing devices – Barcol tester*

ISO 75-1:1993	Plastics – Determination of temperature of deflection under load – Part 1: General test method
ISO 178:2001	Plastics – Determination of flexural properties
ISO 179-1:2000	Plastics – Determination of Charpy impact properties – Part 1: Non-instrumented impact test
ISO 179-2:1997	Plastics – Determination of Charpy impact properties – Part 2 Instrumented impact test
ISO 180:2000	Plastics – Determination of Izod impact strength
ISO 306:1994	Plastics – Thermoplastic materials – Determination of Vicat softening temperature (VST)
ISO 458-1:1985	Plastics – Determination of stiffness in torsion of flexible materials – Part 1: General method

Table A6.3. *ISO standards for plastics (source ISO)*

ISO/AWI 4545-1	Metallic materials – Knoop hardness test – Part 1: Test method
ISO/AWI 4545-2	Metallic materials – Knoop hardness test – Part 2: Verification and calibration of testing machines
ISO/AWI 4545-3	Metallic materials – Knoop hardness test – Part 3: Calibration of reference blocks
ISO/AWI 4545-4	Metallic materials – Knoop hardness test – Part 4: Table of hardness values
ISO/AWI 6506-1	Metallic materials – Rockwell hardness test – Part 1: Test method
ISO/AWI 6506-2	Metallic materials – Rockwell hardness test – Part 2: Verification and calibration of testing machines
ISO/AWI 6506-3	Metallic materials – Rockwell hardness test – Part 3: Calibration of reference blocks
ISO/AWI 6506-4	Metallic materials – Rockwell hardness test – Part 4: Table of hardness values
ISO/AWI 6507-1	Metallic materials – Rockwell hardness test – Part 1: Test method
ISO/AWI 6507-2	Metallic materials – Rockwell hardness test – Part 2: Verification and calibration of testing machines
ISO/AWI 6507-3	Metallic materials – Rockwell hardness test – Part 3: Calibration of reference blocks
ISO/AWI 6507-4	Metallic materials – Rockwell hardness test – Part 4: Tables of hardness values
ISO/AWI 6508-1	Metallic materials – Rockwell hardness test – Part 1: Test method (scales A, B, C, D, E, F, G, H, K, N, T)
ISO/AWI 6508-2	Metallic materials – Rockwell hardness test – Part 2: Verification and calibration of testing machines (scales A, B, C, D, E, F, G, H, K, N, T)
ISO/AWI 6508-3	Metallic materials – Rockwell hardness test – Part 3: calibration of reference blocks (scales A, B, C, D, E, F, G, H, K, N, T)
ISO/AWI 14577-4	Metallic materials – instrument indentation test for hardness and materials parameters – Part 4: Test method for coating
ISO/FDIS 18265	Metallic materials – Conversion of hardness values

Table A6.4. ISO standards for hardness testing (source: ISO)

Scale HR	Scale HR	Scale HR	Scale HR	Scale HR	Scale HR
HR 15 N	HR 15 N	HR 15 N	HR 15 N	HR 15 N	HR 15 N
HR 30 N	HR 30 N	HR 30 N	HR 30 N	HR 30 N	HR 30 N
HR 45 N	HR 45 N	HR 45 N	HR 45 N	HR 45 N	HR 45 N
HR 15 T	HR 15 T	HR 15 T	HR 15 T	HR 15 T	HR 15 T
HR 30 T	HR 30 T	HR 30 T	HR 30 T	HR 30 T	HR 30 T
HR 45 T	HR 45 T	HR 45 T	HR 45 T	HR 45 T	HR 45 T
HR 15 W	HR 15 W	HR 15 W	HR 15 W	HR 15 W	HR 15 W
HR 30 W	HR 30 W	HR 30 W	HR 30 W	HR 30 W	HR 30 W
HR 45 W	HR 45 W	HR 45 W	HR 45 W	HR 45 W	HR 45 W
HR 15 X	HR 15 X	HR 15 X	HR 15 X	HR 15 X	HR 15 X
HR 30 X	HR 30 X	HR 30 X	HR 30 X	HR 30 X	HR 30 X
HR 45 X	HR 45 X	HR 45 X	HR 45 X	HR 45 X	HR 45 X
HR 15 Y	HR 15 Y	HR 15 Y	HR 15 Y	HR 15 Y	HR 15 Y
HR 30 Y	HR 30 Y	HR 30 Y	HR 30 Y	HR 30 Y	HR 30 Y
HR 45 Y	HR 45 Y	HR 45 Y	HR 45 Y	HR 45 Y	HR 45 Y

Table A6.5. Superficial Rockwell hardness scales

Designations	Ferrous materials				Major non-ferrous materials					
	Iron	Carbon steels	Stainless steels 18/8	Ductile iron GS EN-GJS	Aluminum	Copper	Zinc	Magnesium	Nickel	Titanium
Quantities at 20°C										
Density	7.87	7.85	7.9	7.2	2.7	8.96	7.13	1.74	8.8	4.54
Initial melting °C	1538	1450	1425	–	660	1083	420	650	1453	1668
Expansion coefficient K^{-1}	$12 \cdot 10^{-6}$	$12 \cdot 10^{-6}$	$17 \cdot 10^{-6}$	$12.5 \cdot 10^{-6}$	$23 \cdot 10^{-6}$	$17 \cdot 10^{-6}$	$40 \cdot 10^{-6}$	$27 \cdot 10^{-6}$	$13 \cdot 10^{-6}$	$10 \cdot 10^{-6}$
Specific heat $J \cdot kg^{-1} \cdot K^{-1}$	444	430	513	515	900	385	385	1025	444	523
Thermal conductivity $W \cdot m^{-1} \cdot K^{-1}$	80	100	30	35	238	402	117	155	62	22
Electrical resistivity $\mu\Omega \cdot cm$	10	11	70	50	2.65	1.73	5.9	4.46	9.2	50
Elasticity modulus E (Gpa)	190	200	193	170	67	112	84	45	210	110
Shear modulus G (Gpa)	75	75	75		27	38		17	80	40
Poisson's ratio ν	0.27	0.32	0.27	0.27	0.34	0.36		0.35	0.31	0.33

Table A6.6. Comparative properties of the major metals (source: MathCAD software)

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