REGIONAL MODEL LIFE TABLES
AND STABLE POPULATIONS

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PART I. DESCRIPTION AND EXPLANATION

INTRODUCTION

The publication of this volume is intended to make available to demographers and statisticians extended tabulations that will expedite a wide range of both demographic analysis and estimation of population statistics. The tables presented here are of two principal varieties: model life tables and stable populations. Both forms have played an increasingly prominent role in recent scientific work on population. The literature on age distribution, on age patterns of mortality, and on estimation by means of stable (or quasi-stable) age distribution is beginning to assume formidable proportions.

In spite of the increasing attention to model life tables and the calculations based on stable populations, the published material that would facilitate their use is sparse. The Population Branch of the United Nations has issued a set of model life tables that have been widely used, but anyone wishing to employ stable age distributions has had no choice but to embark on calculations that are very laborious—almost prohibitively so if advanced methods of calculation are not accessible. Several organizations (including the Office of Population Research, the Population Branch of the United Nations, and the United Nations Centre for Demographic Studies in Santiago) have calculated sets of stable age distributions for internal use. This volume makes available: (a) a set of model life tables somewhat different and more extensive than the United Nations collection; and (b) a very large number (nearly 5,000) of stable populations, with a variety of associated parameters, including the age distributions of deaths.

There are three chapters of the introductory text. The first is a general description of model life tables and stable populations. The second is a brief account of the methods of calculation used to construct the tables presented in this book. This chapter is rather technical, of interest primarily to specialists. The third chapter describes illustrative uses of the tables, and explains the terms and symbols employed.

The reader interested in a non-technical description of the tables and in a guide to their use should skip Chapter 2.

CHAPTER 1. MODELS OF MORTALITY AND AGE COMPOSITION

A. MODEL LIFE TABLES

> Whereas we have found, that of 100 quick Conceptions about 36 of them die before they be six years old, and that perhaps but one surviveth 76, we, having seven Decads between six and 76, we sought six mean proportional numbers between 64, the remainder, living at six years, and the one, which survives 76, and finde, that the numbers following are practically near enough to the truth; for men do not die in exact Proportions, nor in Fractions: from whence arises this Table following.
>
> JOHN GRAUNT, 1662

Life tables provide a succinct description of what is the most prominent aspect of the state of human mortality: they show the varying chances of dying as a function of age. Basic though such a description might seem, even a superficial check on the existing life tables computed from direct statistical information is sufficient to establish the fact that the available documentation is severely limited. The limitations pertain not only to the quality of the available material, but to its geographic and historical coverage as well.

It is these limitations that constitute both the primary justification and the main weakness of an attempt to construct life tables reflecting generalized experience. Model life tables are necessarily intended for use chiefly in situations where no reliable direct information is at hand; yet the very lack of data for a given population renders the imposition of a model mortality pattern a risky enterprise. It follows from these considerations that the quest for "typical" age patterns of mortality to be incorporated in a family of model life tables should take account of the full range of variation in the documented patterns, but only after that documentation has been subjected to a critical examination as to its reliability.

The limitations of the potential source material for the construction of model life tables can be readily appreciated by recalling a few historical facts. Although death must have always been among the preoccupations of man, it was not until the mid-17th century that the patterns of regularities in the chance of dying were subjected to scientific investigation. It is in John Graunt's celebrated *Observations*[1] on the Bills of Mortality of London that we find the first sketch of what later came to be called a life table. Its contents, with the exception of the ingenious estimate of the magnitude of child mortality, were purely conjectural; however, its form set the precedent for the death—and survivors—columns of all future life tables. It was Graunt's table that prompted the astronomer Halley some forty years later to construct the first modern life table, that of the city of Breslau for the years 1687–1691.[2] "At least 'tis desired"—Halley wrote in the closing lines of his remarkable exposition—"that in imitation hereof the Curious in other Cities would attempt something of the same nature, than which nothing perhaps can be more useful." In fact, it was over forty years before the construction of another table was undertaken: Kresseboom's calculations based on records of life annuities in Holland appeared in 1738, and the famous tables of Deparcieux were published in

[1] *Natural and Political Observations Made Upon the Bills of Mortality*, London 1662. American edition edited by Walter F. Willcox, Johns Hopkins Press, Baltimore 1939.

[2] Edmund Halley, *An Estimate of the Degrees of the Mortality of Mankind* (originally in *Philosophical Transactions of the Royal Society, Vol. 17, 1693*); American edition edited by Lowell J. Reed, Johns Hopkins Press, Baltimore 1942.

1746. But from around 1750 these efforts were indeed followed by ever more frequent attempts to obtain measures of mortality, and the life table gradually came to occupy a central place in population studies as a device of description and as an analytical tool.[3]

Although the value of the early tables, not only as pioneering efforts but also as sources of demographic information, is unquestionable, a common weakness they share is the deficiency of the raw statistical material they incorporate. From the point of view of the type of statistical information used, we can distinguish three broad classes of life tables. The first consists of tables based on data supplied by continuous registration systems, such as births and deaths registered by civil or ecclesiastical authorities, hospital records, records of burials and baptisms, etc. Practically all of the early life tables are based on information of this sort. It is true that such records of births and deaths had frequently preceded the efforts to use them for statistical purposes by many years (for example, such records were kept both in London and in Breslau, the source of Graunt's and Halley's analyses, as early as the last years of the 16th century), and thus in many cases these records had attained a reasonable degree of reliability. Where a classification of deaths by age was also available (as was the case, e.g., in Breslau but not in London) a good estimate of the true mortality conditions could be obtained, provided that the age distribution of the population was close to a stationary one, since in such a case the age distribution of deaths is approximately identical to the d_x column of the life table. The crucial condition, that the population must be near-stationary, was already recognized and clearly stated by Halley. However, a deviation from this condition—particularly if it is due to migration, which was likely to be the case with data relating to urban populations—introduces a difficulty for which ordinarily no satisfactory solution can be found. Furthermore, the omnipresence of age misstatements and the frequent occurrence of age-differential omissions in these bodies of data, and, in many cases, the smallness of totals on which the calculations were based, excludes the possibility of using these life tables for the purposes of generalizing on mortality conditions unless the validity of each table is carefully analyzed: an undertaking that would hav ceeded the limits of the present study. Thus, all life tables o above described type were barred from further consideration basis for constructing model life tables.

A second class of tables is made up by life tables constru from census information alone. If a given cohort that belong a closed population is enumerated at two points of time a survi ship rate can be calculated: such rates if based on censuses t not too far apart in time can be used to calculate a life-ta excepting the youngest ages. A number of non-European coun with no tradition of vital registration or with very inadeq birth and death records has censuses permitting such calculati particularly notable among the resulting life tables are those India and Egypt. An examination of the reported census distributions in these countries indicates, however, that dr adjustments have been necessary before realistic survivorship r could be calculated, and, therefore, little reliance can be placed the precise shape of the life table functions so obtained. The s argument holds if the calculation is based on a single census inflating the reported age distribution by an exponential of the f e^{ra} (a denoting the attained age and r denoting the growth r the latter presumed to be stable). Furthermore, infant and chi hood mortality must be estimated for such tables on the basis separate methods, none of which is likely to give satisfactory resu or which simply repeat experience recorded elsewhere. It w decided, therefore, that no life table of this type would be c sidered for the purpose of constructing model life tables.

The above reasoning thus led us to construct our study considering only life tables of the third type: those calculated the basis of the joint use of data originating (a) from continu registration and (b) from an enumeration of the population a given time. The first table of this kind was derived by Wargen for the years 1755–1763 from the records of Sweden; but in ot parts of Northern and Western Europe appropriate data beca available only during the first part of the 19th century. The of populations for which such life tables exist continued to exp during the past hundred years, and now it includes at least so populations from each continent. Up to the present date, howev the majority of the world's population continues to live in count

[3] For a survey see Harald Westergaard, *Contributions to the History of Statistics*, London 1932.

where either vital statistics or censuses, or both, are lacking, and hence the majority lacks the statistical data that would permit the calculation of life tables of sufficient quality to serve as a basis for model tables.

A large collection of life tables of the third type has been assembled at the Office of Population Research of Princeton University, taken partly from various secondary sources, and partly from original publications of national statistical bureaus. Before accepting this collection as a potential basis for constructing model life tables, some further selection was necessary. First, since uniformity of the data was obviously required, the lack of separate treatment of the two sexes eliminated a table from further consideration; similarly, the availability of life table values (or the possibility to compute life table values) for five-year age groups, with separate groups for age 0 and age 1 to 4, was required for inclusion. Second, while the collection of tables was necessarily unbalanced both according to its geographical coverage and to the year of origin of the tables contained, it was thought desirable to reduce this imbalance by excluding many life tables from the past few decades that were obviously repetitive in character, such as life tables referring to consecutive intercensal years routinely calculated in some countries. Third, tables covering years in which a country was in a major war were eliminated. Fourth, it was decided to use life tables representing the experience of whole countries only, partly because of the belief that the general quality of the life tables for sub-regions within countries is less satisfactory, partly, again, to reduce imbalance and repetitiveness. Thus, the large collections of life tables for the individual states of the United States were excluded. This rule was relaxed in a few instances only when the experience of a sub-region showed distinct characteristics that persisted over time: this was the case in particular with the tables for Southern Italy that were kept in the collection separately. Some of the German states of the 19th century represented by individual tables were also retained. A few life tables refer to a non-geographically defined sub-group of the population, because of the non-availability or the lesser accuracy of the data for the remainder: life tables for the United States and New Zealand are those for the white population only.

After the eliminations outlined above a basic collection was obtained containing 326 male and 326 female life tables. The distribution of these tables according to year of origin and geographic location is shown in Table I. These figures bear out the extreme paucity of adequate mortality information for Africa, Asia, and Latin America, and at the same time indirectly show the extraordinary interest of the existing recorded experience. To develop statistical systems that provide data of good reliability is not only a costly but also a very time-consuming process. If one makes the perhaps not too optimistic assumption that currently observed

TABLE I. Distribution of 326 selected life tables according to year of origin and geographic location

Continent	*Before 1870*	*Midpoint of reference period of life table* *1871–1918*	*1919–1945*	*After 1945*	*Total*
Africa			5	10	15
America, North		3	7	8	18
America, Latin		7	10	16	33
Asia		7	9	16	32
Europe	23	63	62	58	206
Oceania		10	6	6	22
Total	23	90	99	114	326

declining mortality trends the world over will continue in the foreseeable future one realizes the uniqueness of the accurate records—mostly those of European countries before World I—that reflect relatively high levels of mortality. This was then the raw material from which the investigation of age patterns of mortality started.

Generally speaking, two approaches to describe such patterns can be envisaged. The first approach is to find some mathematical expression valid for all ages and thus to formulate what in an earlier parlance was often called a "law of mortality." The first such attempt was made as far back as 1725 by the Huguenot mathematician DeMoivre, who, on studying Halley's table, conjectured that the human survivorship function decreases in an arithmetic progression.[4] DeMoivre's hypothesis was soon disproved, and the first sophisticated step was taken a hundred years later by Gom-

[4] A. DeMoivre, *Annuities on Lives*, London 1725.

pertz, who advanced his well-known formula (later modified by Makeham) that found extensive application in the graduation of the q_x function of recorded life tables showing irregularities from age to age and in other actuarial work.[5] The Gompertz-Makeham formula was applicable, however, only for about age 30 and over, and later attempts to compress the mortality function over the whole range of the human age span in a single expression were successful only at the cost of an increasing formal complexity that tends to obliterate the obvious analytical advantages that would result.[6] The alternative approach to describe a typical pattern of mortality is simply to present a series of tables that retain the numerical form of the conventional life table. The use of "model life tables" in a wide sense was practiced during the past two and a half centuries whenever a life table recorded for a given time and place was applied to populations not directly associated with the experience incorporated in that table. Similarly, it can be said that, e.g., the life table of Finland for the years 1901–1910 is a "model" of the Finnish mortality experience for that decade in which no individual year was likely to have had the same mortality as depicted by that "model." In the same vein, it can be said that any projection of a population that is carried out by some more or less detailed classification by age involves some adoption of model life tables. Assumptions on the future course of mortality were, moreover, frequently formulated in a way that implicitly involved the construction of age patterns of mortality experienced by no actual population up to the time the projection was made. Typical assumptions of this sort were that a country of relatively high mortality will reach the mortality pattern recorded in another country in a certain number of years, the form of transition being specified in some definite manner; or that age-specific mortality trends as observed in the past in a given country and described by some appropriate fitted curve will continue in the future, etc. An interesting instance of another "model life table" of somewhat limited applicability was that constructed from observed past relations between two series of life tables: those for the population in question and those for another population.[7]

As far as we know, the first attempt to construct a model life table summarizing age patterns of mortality in several countries in a form that could be associated with any arbitrarily specified value of a parameter indicating the level of mortality was carried out at the Office of Population Research in the early 1940's by one of the authors of this volume in a cooperative work.[8] The parameter chosen was expectation of life at age 10; q_x values were determined by linear regression on $e_{10}°$.

In 1955 the Population Branch of the United Nations published a series of model life tables[9] based on a collection of 158 tables for each sex, computed by using parabolic regressions between q_x values of adjacent age groups starting from a specified value of infant mortality. These series, subsequently published in a revised form,[10] have been widely used in research work on the demography of underdeveloped countries. A criticism of the regression technique employed in the United Nations models led to experimentation with life tables where each q_x value was obtained by a linear regression on infant mortality, using the same source collection as the United Nations.[11] A modified series of the United Nations model life tables has been in use by the Latin American Demographic Center.[12]

Experimentation with stable population estimates and the resulting desire to take into account the effect of different mortality patterns on such estimates were primarily responsible for the develop-

[5] Benjamin Gompertz, "On the Nature of the Function Expressive of the Law of Human Mortality," *Philosophical Transactions of the Royal Society*, 1825, Part II.

[6] Concerning this topic and for a survey of such attempts, see Maurice Frechet, "Sur les expressions analytiques de la mortalité valables pour la vie entière," *Journal de la Société de Statistique de Paris*, Vol. 88 (1947), Nos. 7–8.

[7] L. I. Dublin, A. J. Lotka, and M. Spiegelman, "The Construction of Life Tables by Correlation," *Metron* (Rome) Vol. 12 (1935), No. 2.

[8] F. W. Notestein *et al., The Future Population of Europe and the Soviet Union*, League of Nations, Geneva 1944. See especially p. 189.

[9] United Nations, *Age and Sex Pattern of Mortality*, Population Studies No. 22, New York 1955.

[10] United Nations, *Methods for Population Projections by Sex and Age*, Population Studies No. 25, New York 1956.

[11] K. R. Gabriel and Ilana Ronen, "Estimates of Mortality from Infant Mortality Rates," *Population Studies* (London), Vol. 12, No. 2 (November 1958).

[12] Centro Latinoamericano de Demografia, "Poblaciones modelos estables, quasi-estables y en transicion demografica," Santiago 1961, mimeo. (It is indicated that these 78 tables constitute an appendix to a study in preparation by L. Tabah.)

ment of the model life tables contained in this volume. They were worked out approximately in their present form during 1960–1962 and were in use in the Office of Population Research since that time, having undergone in the meantime two revisions. The basic approach that was followed was essentially the one used in the 1944 study cited above with the exceptions that two sets of regression equations were calculated to provide a more satisfactory fit; that the underlying source material was much more extensive; and, most importantly, that within the experience incorporated in the extended source material four types of mortality patterns were identified and developed into four sets of model life tables. The resulting collection is best thought of as four families of one-parameter life tables, available separately for each sex. The one parameter is any arbitrarily selected index of the life table[13] to which any value within the range of human mortality experience can be assigned. The four families have distinct age patterns of mortality—patterns of a "regional" nature that we have named "West," "North," "South," and "East."[14]

B. STABLE POPULATIONS

> Cependant on voit bien, que cette détermination ne sauroit être développée en général: mais, pour chaque hypothèse de mortalité, si l'on calcule le rapport M/N [population/births, or 1/birth rate] pour plusieurs valeurs de n [one plus the intrinsic rate of increase], & qu'on en dresse une table, il sera aisé d'assigner réciproquement pour chaque rapport donné $M:N$, qui exprime la fécondité, l'augmentation annuelle de tous les vivans, qui est la même que celle des naissances.
>
> LEONARD EULER, 1760

Thus did Euler 200 years ago suggest, in effect, the construction of tables of stable populations.[15] Euler appears to have been the first to visualize a closed population experiencing the same mortality risks for many years, and births that change in number in a geometric progression (i.e., change at constant proportionate rate) from year to year. Such a population has an unvarying age distribution, increases at a constant rate, and has constant birth and death rates. It can be termed a stable population (or a Malthusian population, to use a term of Lotka's).[16] The age distribution is given by equation (1):

$$c(a) = be^{-ra}p(a) \qquad (1)$$

where $c(a)$ is the proportion of the population at age a, b is the birth rate, r the annual rate of increase, and $p(a)$ the proportion surviving from birth to age a.

Every combination of a life table and a rate of increase implies a determinate age composition, with an associated birth rate and death rate. We have followed Euler's implicit suggestion, and calculated a total of nearly 5,000 stable populations, associated with the 192 model life tables (24 for each sex in each of the four "regional" families). We have exceeded his suggestion to calculate the reciprocal of the birth rate, and have printed many parameters of each stable population, including the proportion in each five-year interval; the cumulated proportion to ages at five-year intervals; and the birth rate, death rate, and rate of increase. Accompanying each stable age distribution is an age distribution of the deaths that would occur in the stable population.

The development of a fully articulated theory of stable populations was the work of Alfred J. Lotka. In 1907, he wrote a brief note[17] in which he stated equation (1) exactly as given here, and little different from an equation employed by Euler 150 years

[13] From the point of view of computation, as is explained below, this parameter [is] the expectation of life at age 10.

[14] A detailed discussion of the construction of these tables appears below, in Chapter 2.

[15] Leonard Euler, "Recherches générales sur la mortalité et la multiplication du genre humain," *Histoire de l'Académie Royale des Sciences et Belles-Lettres*, 1760, pp. 144–64.

[16] Lotka used the term *Malthusian* to describe a population with a constant rate of increase and an unchanging mortality schedule, and the term *stable* for the special case of a Malthusian population when the constant rate of increase results from the prolonged prevalence of an unchanging fertility schedule.

[17] A. J. Lotka, "Relation between Birth Rates and Death Rates," *Science*, N.S. Vol. 26 (1907), No. 653, pp. 21–22.

earlier of which Lotka was apparently unaware. In this article, Lotka showed that the relations inherent in a stable population were closely matched in the population of England and Wales in 1871–1880. Four years later, in a joint paper with F. R. Sharpe, Lotka first outlined the mathematical proof that the continuation of a specified fertility and mortality schedule would lead to an unchanging age distribution, which Sharpe and Lotka labeled "stable."[18] In a stable population, the rate of increase that comes to prevail is the sole real root of equation (2):

$$\int_0^\omega e^{-ra}p(a)m(a)\,da = 1 \qquad (2)$$

where $m(a)$ is the proportion of persons of one sex who annually become parents of a child of the same sex, and ω is the highest age attained. Usually, $m(a)$ expresses the age schedule according to which women bear female babies, although it is equally appropriate for males.

In a series of more than 30 articles and a book, Lotka elaborated the theory of stable populations. He stated a method for calculating the one real root and the principal complex roots of equation (2), and showed how these values of r (real and complex) illuminate the approach to stability and the nature of the ultimate stable population. He discovered and stated a large number of interrelationships among parameters in the stable populations; he illustrated the use of stable population concepts to promote a better understanding of what fertility and mortality schedules imply, specifically noting that the *intrinsic* (stable) rate of increase in the United States of 1920 was well below the then current (crude) rate of natural increase; he used stable population concepts to estimate the average size of family in Colonial America; he showed how the establishment of a stable population from the progeny of a single population element could be derived by considering births by *generations* rather than on an annual basis; and he analyzed the incidence of orphanhood, the extinction of families, and the growth of two species competing for a common food supply. The most systematic exposition of his contributions is in *Théorie analytique des associations biologiques* (Hermann et Cie, Paris 1939). This book also contains a bibliography listing Lotka's principal contributions prior to 1939.

Since Lotka's death in 1949, there has been a revival of interest in stable population analysis, partly because the stable age distribution is often closely approximated under the demographic conditions found in many underdeveloped areas a history of approximately constant fertility and steadily falling mortality. Such age distributions have been designated *quasi-stable*. A recent article summarizes the use of quasi-stability in making estimates of population parameters, and gives a selected list of readings on this subject.[19]

[18] F. R. Sharpe and A. J. Lotka, "A Problem in Age-Distribution," *Philosophical Magazine*, April 1911, pp. 435–38.

[19] A. J. Coale, "Estimates of Various Demographic Measures through the Quasi-Stable Age Distribution," in Milbank Memorial Fund, *Emerging Techniques in Population Research*, Proceedings of the 1962 Annual Conference of the Milbank Memorial Fund, pp. 175–93.

CHAPTER 2. CALCULATION OF MODEL TABLES

A. CALCULATION OF FOUR FAMILIES OF MODEL LIFE TABLES

The four families of life tables presented in this volume are the result of a search for several distinctive patterns in the variation of mortality rates with age. The search originated in the impossibility of finding an acceptable one-parameter representation of all reliably documented mortality experience.

The various conventional functions (l_x, L_x, e_x°, etc.) of a life table can be constructed from a small number of life table death rates—q_x's. For example, knowledge of ($_1q_0$, $_4q_1$, $_5q_5$, $_5q_{10}$, . . . , $_5q_{75}$) permits the calculation of l_x (and L_x) to age 80. The value of T_{80} can be estimated from l_{80} (although not very precisely), and life expectancy at the various ages can then be calculated. Thus, every life table can be considered a point in a Euclidean space of 17 dimensions, where each coordinate axis represents the proportion dying in a particular age interval. A one-parameter set of model life tables is, then, a singly connected sequence of such points—a line" that is as close as possible to the observed life tables. The United Nations model tables form a "line" that is derived by estimating a quadratic relation between the $_nq_x$ values in progressively older age groups; other model tables have been calculated from the data used by the United Nations using a straight line in the space whose coordinates are log q_x's.[1] The factor analytic approach[2] assumes that model tables can best be represented (at least for some purposes) as lying in an ellipsoid having more than one, but fewer than 17 dimensions. In the construction of model life tables by factor analysis, three parameters are considered adequate for a close representation of male and female life tables. The first parameter (closely correlated with e_0°) by itself defines a line passing close to the observed life tables; the other two can be viewed as specifying commonly observed age patterns of deviations from the principal line. To prepare three-parameter model life tables, Bourgeois-Pichat proposes to select values of the second and third parameters that cause maximum expected deviations from the central model tables associated with variations in the first parameter alone.[3]

The model tables presented here are based on a tendency that we noted for the life tables based on accurate data to cluster around four different lines, representing distinct age patterns of mortality in certain geographical regions.

The four families of life tables were isolated as the result of working with a preliminary one-parameter family of model tables designed to represent the entire collection of 326 life tables. The following method was used to construct the preliminary model tables: all of the q_x values were ordered, from highest to lowest, at each age. The values were then plotted as a function of order for each age, and occasional erratic fluctuations removed by hand smoothing. Preliminary model tables were then formed by putting together mortality rates with the same rank. This system of construction has the virtue of simplicity, and of symmetrical treatment of mortality rates at all ages. That is to say, no particular rate or rates are singled out as the basis for estimating others. The usefulness of the preliminary set of tables is questionable, however, because the 326 tables are so widely scattered about the "line" these model tables formed, as we could see by examining two-dimensional scatter diagrams.

K. R. Gabriel and Ilana Ronen, *op.cit.*
Sully Lederman and Jean Breas, "Les dimensions de la mortalité," *Population*, Vol. 14 (October–December 1959), No. 4.
Jean Bourgeois-Pichat, "Application of Factor Analysis to the Study of Mortality," in Milbank Memorial Fund, *op.cit.* pp. 194–299.
United Nations, Population Branch, "Factor Analysis of Sex-Age Specific Death Rates," *Population Bulletin of the United Nations*, No. 6, 1962, pp. 149–[illegible]

[3] In a forthcoming United Nations publication.

The next step was to examine deviations of individual life tables from the age pattern of mortality in the preliminary model tables. The pattern of deviations was measured by the difference between q_x in the given table and q_x in a model life table with the same general level of mortality. The "comparison" model table was selected by first noting the expectation of life at birth in the model life tables with the same $_nq_x$ as the given life table, at $x = 0, 1, 5, \ldots, 75$. The median model table (the table with the median e_0°) was the "comparison" table. The graph showing $_nq_x - ({}_nq_x)_{\text{model}}$ displays, for ready visual comprehension, the age pattern of mortality relative to the age pattern in the model tables, in a way that does not depend on the over-all level of mortality. Graphs of this sort were constructed for all 326 life tables. The following general observations emerged from an examination of the patterns:

1. Some life tables have large variations from the model patterns. Large deviations were especially frequent under age 10 and over age 60.

2. The biggest deviations were found in life tables where the quality of the underlying data is suspect: life tables for Western Europe prior to 1850; certain Eastern European life tables—for Russia in 1897 and 1926, for Bulgaria and Greece; and life tables in underdeveloped countries in Asia, Africa, and Latin America. In most of these tables, age-reporting in the censuses, and doubtless in the mortality register, is inaccurate. The completeness of death registration—especially for infants—is uncertain.

3. Where data are known to be accurate, deviations are usually moderate. Statistics that are comparatively free of age misstatement and omission are found in some European countries since 1870, in 20th-century Canada, the United States, Australia, and New Zealand. Unfortunately, these areas have a rather narrow range of cultural diversity and doubtless represent a narrow sample of age patterns of mortality. It is of special interest to note, then, that the life tables of Taiwan and the more recent Japanese life tables both based on accurate data—conform as well as do European tables to the preliminary model tables.

Two features of the deviations found within life tables based on accurate data led to the construction of four separate families of model tables: (a) The fact that the pattern of deviations is often similar among life tables expressing the mortality of the same population at different times, and (b) the fact that several groups of geographically linked populations exhibited similar patterns of deviations.

Figure 1 shows patterns of deviation in certain Scandinavian life tables; in tables from Germany, Austria, Czechoslovakia, Poland and North Italy; and in tables from South Italy, Spain, and Portugal. Within each group the similarity of deviations is easily visible, although from group to group the patterns differ.

On the basis of an examination of the patterns of deviations in the 326 life tables for each sex, correlation matrices were calculated for various groups of life tables—zero-order correlations among the variables: log $_nq_x$ at various ages ($x = 0, 1, 5, 10, \ldots, 75$), and expectation of life at age 0 and age 10. We experimented with nine principal sets of correlations for each sex: (1) tables before 1870; (2) tables for Russia and certain Balkan areas; (3) tables for selected Central European areas; (4) tables for Scandinavian countries; (5) tables for Spain, Portugal, and Southern Italy; (6) tables for Switzerland; (7) tables for countries with reliable data not included in (3), (4), (5), or (6); (8) tables reflecting mortality when there is an unusually high incidence of tuberculosis and (9) modern tables based on relatively inaccurate data—primarily from Asia, Africa, and Latin America.

The nature of the life tables underlying each of the "regional" model tables is as follows:

1. Tables underlying "East" model tables. The life tables of Austria, Germany (including tables in 1878 and the 1890's for Bavaria and Prussia), Czechoslovakia, North and Central Italy, Hungary, and Poland show deviations from the preliminary model life tables characterized by high mortality rates in infancy, and increasingly high rates over age 50 (Figure 1, central panel). Switzerland's life tables show deviations very similar to this group until 1920, although the early Swiss life tables have a less conspicuous positive deviation in infancy. After 1920, the Swiss life tables show zero or negative deviations in infant mortality. Hungarian life tables exhibit substantial deviations in an age pattern indicating an extraordinary incidence of tuberculosis. Inclusion of the Hungarian life tables lowers the correlation coefficients from age 5 to 35, but has little other effect, and they were omitted from

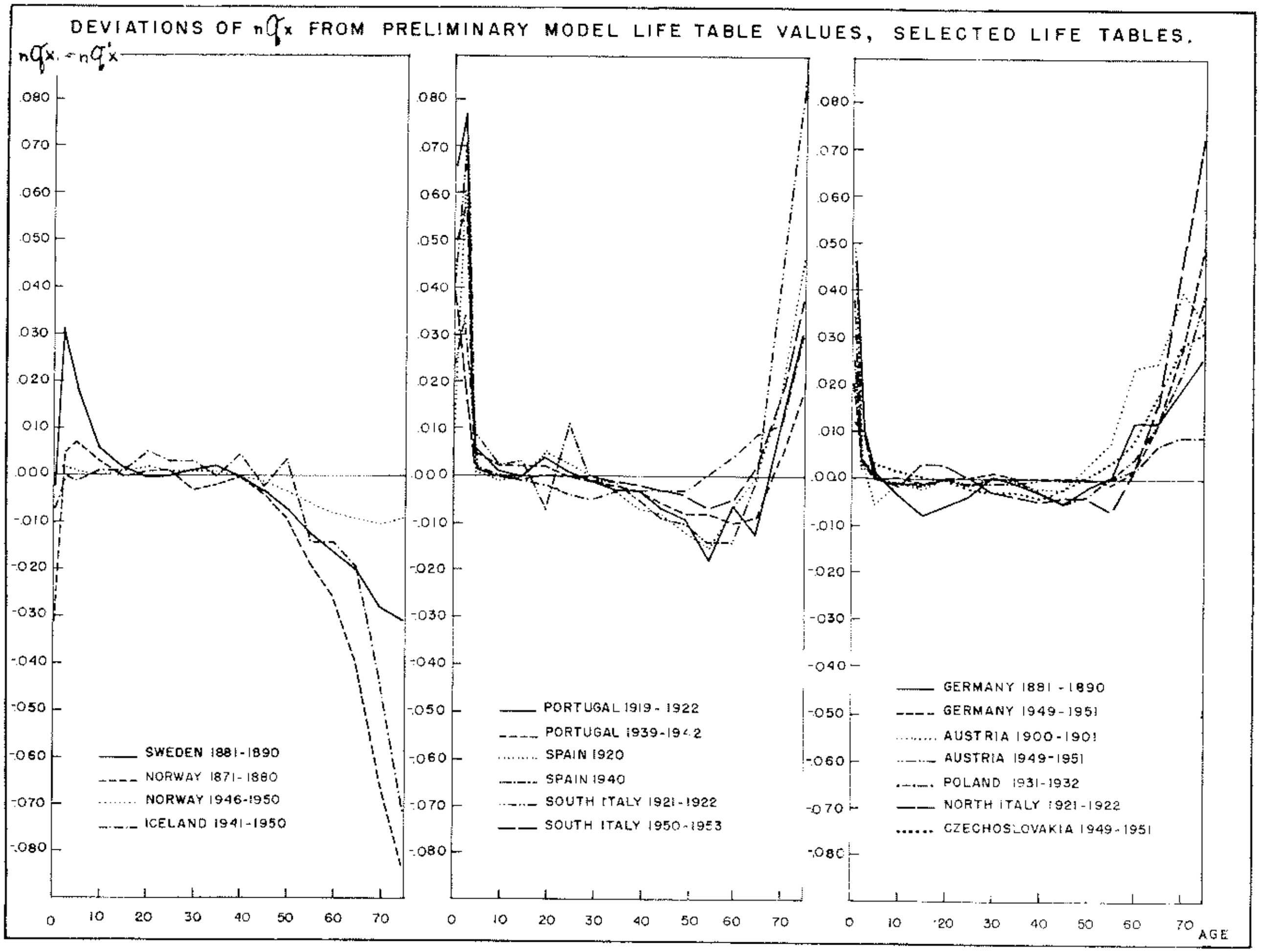

FIGURE 1. Deviations of mortality rates in three regional collections of life tables ($_nq_x$) from mortality rates in a preliminary model life table ($_nq_x'$). Selected tables for females.

the calculation of the "East" model tables, as were the Swiss life tables. The tables in the "East" group include 13 from Germany (of which 3 are from Prussia or Bavaria), 5 from Austria, 3 from Poland, 4 from Czechoslovakia, and 6 from North or Central Italy.

2. *Tables underlying "North" model tables.* The life tables of Norway, Sweden until 1920, and Iceland deviate from the preliminary model tables in having low infant mortality rates, and rates that are lower than the model rates by an increasing margin at ages beyond 45 or 50. Later Swedish life tables do not have this characteristic pattern (Figure 1, left panel). In the life tables of all three "North" countries from 1890 or 1900 to 1940, there are deviations in the mortality rates from age 5 to 35 or 40 indicating the

effect of an unusual incidence of tuberculosis. Model tables incorporating this experience would be suitable only for populations with a high endemicity of tuberculosis. Consequently, the "North" model tables are based on Swedish mortality from 1851 to 1890 (4 tables), Norwegian mortality from 1856 to 1880 and from 1946 to 1955 (4 tables), and the Iceland life table for 1941–1950.

3. *Tables underlying "South" model tables.* The life tables of Spain, Portugal, and Southern Italy have high mortality under age 5, low mortality from about age 40 to 60, and high mortality over age 65, relative to the preliminary model tables (Figure 1, right panel). Early tables—prior to 1912—for all Italy had these same characteristics. The "South" model life tables were based on 5 tables for all Italy (1876 to 1910), 8 tables for Portugal (1919 to 1958), 1 table for Sicily (1951), 3 for South Italy (1921 to 1957), and 5 for Spain (1910 to 1940).

4. *Tables underlying "West" model life tables.* The "West" model life tables were based on mortality experience recorded in populations known to have relatively good vital statistics, and not showing a persistent systematic pattern of deviations from the preliminary model tables. In other words, the tables underlying the "West" models are a residual collection after the "East," "South," and "North" tables have been removed. (The Swiss tables were also omitted, having a recent pattern resembling the "West" up to age 45 or 50, and the "East" above age 50.) The tables underlying the "West" models include 7 from Australia (1881 to 1955), 4 from Belgium (1880 to 1949), 7 from Canada (1926 to 1959), 11 from Denmark (1895 to 1955), 11 from England and Wales (1871 to 1959), 1 from Estonia (1933), 4 from Finland[4] (1881 to 1955), 16 from France (1871 to 1959), 5 from Ireland (1925 to 1952), 3 from Israel (1949 to 1959), 6 from Japan[5] (1949 to 1960), 1 from Latvia (1936), 1 from Luxemburg (1947), 10 from the Netherlands (1870 to 1955), 12 from New Zealand (1906 to 1959), 2 from Northern Ireland (1925 to 1959), 7 from Scotland (1891 to 1959), 5 from Sweden (1931 to 1959), 3 from Taiwan[6] (1921 to 1959), 4 from the Union of South Africa white population (1920 to 1947), and 10 from the United States (1901 to 1958). Tables before 1870 were eliminated because most of those examined (from France, the Netherlands, and England and Wales) had irregular patterns that appeared to arise from faulty data.

CORRELATION MATRICES OF MORTALITY RATES

Intercorrelations among log $_1q_0$, log $_4q_1$, log $_5q_x$ (x ranging from 5 to 75), and $e_0°$ and $e_{10}°$ are shown in Tables II–IX for the data underlying the "West," "North," "East," and "South" model life tables, females and males. Table X compares the correlations between $e_{10}°$ and log $_nq_x$ in "West," "North," "East," and "South" life tables with the correlations in pre-1870 European life tables; in Russia and the Balkans, and in life tables from Asia, Africa, and Latin America.

It is evident in these tables that the grouping of areas having similar age patterns of mortality (as evidenced in similar deviations from the preliminary model tables) produces very high correlations. The highest correlations are in the "North," "East," and "South" tables; the "West" group—a residual collection characterized by non-systematic deviations from the preliminary model tables—is not characterized by a distinctive pattern, and has somewhat lower correlation coefficients among mortality rates by age.

A conspicuous feature of these intercorrelations is that with few exceptions (mostly in the "East" group) the intercorrelations are lower among male than among female mortality rates. The biggest differences are in the "West" correlations, especially in comparisons of male and female correlations between mortality rates for ages over 50 on the one hand, and under 30 on the other. The principal cause of the lower correlation among male mortality rates is that the pattern of male mortality has changed with the passage of time—

[4] Finnish tables from 1890 to 1940 were omitted because of a strongly tubercular age pattern of mortality.

[5] Japanese tables prior to 1949 have a strongly tubercular age pattern of mortality.

[6] Taiwanese life tables before 1921 were omitted because of an especially deficient registration of births and infant deaths in earlier years. (George W. Barclay, *Colonial Development and Population in Taiwan*, Princeton University Press, Princeton, N.J., 1954, pp. 159ff.)

[illegible] (x = 0, 1, 5, . . . , 75) and $-e_0^\circ$, $-e_{10}^\circ$, in female life tables underlying "West"

Variable: log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0^\circ$	$-e_{10}^\circ$
0	—	961	952	947	948	929	938	947	954	950	938	928	921	921	930	910	867	963	953
1		—	975	949	938	930	941	951	960	958	950	943	936	931	931	912	851	966	960
5			—	973	968	958	963	971	972	965	951	939	929	925	925	906	828	958	961
10				—	980	961	960	961	958	950	928	936	905	924	903	889	820	945	958
15					—	989	985	979	971	959	943	936	919	919	916	893	821	929	953
20						—	996	988	970	957	940	932	916	914	911	891	817	910	940
25							—	993	979	966	952	940	929	920	921	898	822	919	948
30								—	990	981	965	955	944	938	935	912	836	939	963
35									—	992	980	966	960	948	946	917	832	955	974
40										—	990	974	968	959	950	916	826	961	978
45											—	975	981	957	956	911	816	948	968
50												—	981	986	959	929	839	952	975
55													—	973	973	924	833	942	963
60														—	973	951	867	953	973
65															—	968	901	947	962
70																—	942	926	937
75																	—	870	869
$-e_0^\circ$																		—	989

TABLE III. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0^\circ$, $-e_{10}^\circ$, in male life tables underlying "West"

Variable: log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0^\circ$	$-e_{10}^\circ$
0	—	960	942	930	932	926	924	923	913	905	864	819	733	723	697	721	735	947	890
1		—	982	941	943	942	946	943	930	914	872	825	746	740	718	749	738	955	909
5			—	954	954	949	947	941	927	906	862	804	723	711	686	714	698	939	895
10				—	967	944	936	935	920	908	846	813	696	711	641	666	671	925	892
15					—	984	969	960	947	929	885	839	743	743	696	708	693	937	917
20						—	988	976	957	938	897	854	764	761	720	738	726	935	926
25							—	994	980	963	929	887	802	789	754	768	753	940	944
30								—	989	981	949	912	830	817	776	790	773	948	958
35									—	988	970	929	860	835	801	810	781	944	962
40										—	985	959	898	879	840	842	809	952	977
45											—	972	945	908	885	870	815	930	970
50												—	948	952	912	901	852	914	966
55													—	964	959	918	836	848	918
60														—	965	944	875	849	919
65															—	968	895	820	887
70																—	944	827	882
75																	—	813	851
$-e_0^\circ$																		—	976

TABLE IV. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0^\circ$, $-e_{10}^\circ$, in female life tables underlying "North"

Variable	log $_nq_x$, for listed values of x																	
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0^\circ$	$-e_{10}^\circ$
0	—	995	995	981	955	925	950	959	978	960	984	958	985	979	977	963	938	997	992
1		—	999	989	964	933	959	968	985	964	984	951	971	961	961	948	908	993	990
5			—	993	969	939	963	973	987	969	987	958	972	963	964	951	908	993	992
10				—	990	970	987	993	998	988	990	970	955	946	948	922	862	978	985
15					—	993	998	998	992	991	979	964	923	917	920	879	810	947	961
20						—	996	990	975	985	958	958	892	890	893	838	764	913	931
25							—	998	990	992	974	966	918	912	914	867	800	941	956
30								—	996	996	984	974	933	927	929	888	819	955	969
35									—	993	994	977	958	950	951	920	861	976	987
40										—	990	990	948	945	948	907	840	959	974
45											—	987	981	976	977	953	901	987	995
50												—	967	969	972	934	879	962	974
55													—	998	996	985	966	992	990
60														—	999	984	968	985	983
65															—	988	964	985	983
70																—	976	975	968
75																	—	945	927
$-e_0^\circ$																		—	997

TABLE V. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0^\circ$, $-e_{10}^\circ$, in male life tables underlying "North"

Variable	log $_nq_x$, for listed values of x																	
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0^\circ$	$-e_{10}^\circ$
0	—	995	993	986	926	904	916	917	944	962	987	958	968	959	959	961	944	993	980
1		—	996	994	944	923	932	930	951	966	980	951	953	946	946	946	917	991	980
5			—	997	952	934	945	949	968	982	987	967	955	954	954	952	922	993	987
10				—	964	946	954	955	971	982	983	962	945	944	943	940	901	989	989
15					—	997	997	988	982	969	929	919	847	869	861	840	783	931	949
20						—	998	989	979	959	907	903	816	845	837	810	751	910	931
25							—	994	986	970	921	918	833	863	855	831	775	922	942
30								—	995	984	934	943	855	890	883	858	804	931	953
35									—	995	962	969	899	929	923	901	855	959	976
40										—	981	984	932	954	951	936	895	977	989
45											—	989	982	985	983	978	955	997	996
50												—	970	989	986	973	944	981	990
55													—	991	993	996	987	980	969
60														—	999	993	977	979	977
65															—	996	980	979	975
70																—	988	976	965
75																		[illegible]	[illegible]

Variable	log $_nq_x$, for listed values of x																		
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0°$	$-e_{10}°$
0	—	979	972	969	958	960	975	979	980	975	975	971	970	965	958	944	925	970	960
1		—	991	992	977	973	985	991	992	992	987	985	979	981	974	972	950	983	985
5			—	992	978	973	984	990	992	992	991	991	986	986	978	974	946	979	985
10				—	991	986	991	992	991	989	986	985	978	978	969	968	935	968	980
15					—	997	995	988	982	978	976	972	962	964	955	957	924	946	966
20						—	996	990	983	978	975	971	961	962	954	955	923	945	964
25							—	998	993	989	987	984	976	975	969	968	943	964	977
30								—	998	996	993	990	984	984	978	974	952	978	986
35									—	998	996	994	989	988	982	978	954	984	990
40										—	998	997	993	993	989	984	961	988	994
45											—	998	995	994	991	985	964	986	993
50												—	998	998	994	986	960	986	993
55													—	998	995	983	958	987	991
60														—	996	988	963	988	994
65															—	992	978	986	991
70																—	983	976	988
75																	—	964	969
$-e_0°$																		—	994

TABLE VII. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0°$, $-e_{10}°$, in male life tables underlying "East"

Variable	log $_nq_x$, for listed values of x																		
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0°$	$-e_{10}°$
0	1	974	970	966	964	956	973	978	981	975	966	947	932	921	939	946	911	973	964
1		—	990	990	985	974	985	989	990	981	968	945	925	925	940	965	950	980	982
5			—	988	982	970	985	992	993	985	970	951	934	935	947	960	934	979	985
10				—	989	979	988	990	986	976	960	941	918	920	933	963	928	962	974
15					—	993	993	988	980	967	946	925	908	916	931	956	918	957	969
20						—	995	983	969	948	925	901	885	894	917	944	903	949	957
25							—	995	986	971	952	930	912	914	934	954	918	969	974
30								—	996	987	973	954	935	933	949	965	931	978	985
35									—	996	985	970	954	950	962	972	935	986	991
40										—	996	985	970	963	968	975	938	986	992
45											—	994	977	965	965	968	938	982	987
50												—	992	981	973	965	912	973	980
55													—	993	984	962	890	967	972
60														—	992	973	895	961	971
65															—	983	914	974	977
70																—	950	969	981
75																	—	940	945
$-e_0°$																		—	994

TABLE VIII. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0°$, $-e_{10}°$, in female life tables underlying "South"

Variable	log $_nq_x$, for listed values of x																		
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0°$	$-e_{10}°$
0	1	955	970	949	934	937	937	953	943	947	926	903	880	884	855	854	828	965	925
1		—	975	968	962	954	959	968	960	971	951	939	924	920	906	902	890	967	951
5			—	984	967	960	966	982	978	983	972	956	940	943	923	911	879	985	970
10				—	990	979	988	994	995	989	987	953	961	953	943	937	896	980	979
15					—	988	994	991	990	978	978	931	945	928	922	925	870	955	958
20						—	974	987	983	978	978	933	942	928	913	908	845	952	952
25							—	988	988	968	968	914	936	915	915	928	872	956	955
30								—	996	991	986	957	958	953	944	944	898	980	980
35									—	989	993	953	970	960	951	948	900	978	983
40										—	989	980	967	972	956	944	914	982	985
45											—	968	987	976	968	952	902	976	987
50												—	964	987	967	941	930	969	980
55													—	983	990	974	927	962	986
60														—	985	962	944	968	989
65															—	986	955	954	984
70																—	952	946	975
75																	—	923	948
$-e_0°$																		—	990

TABLE IX. Correlations (times 1,000) among log $_nq_x$ (x = 0, 1, 5, 10, . . . , 75) and $-e_0°$, $-e_{10}°$, in male life tables underlying "South"

Variable	log $_nq_x$, for listed values of x																		
log $_nq_x$ for listed value of x	0	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	$-e_0°$	$-e_{10}°$
0	1	944	959	942	894	908	909	921	912	893	874	794	814	808	807	830	788	972	913
1		—	968	960	911	961	911	952	935	922	900	813	865	855	861	900	874	970	939
5			—	984	931	952	918	941	923	904	893	813	865	852	846	854	805	985	945
10				—	950	955	922	929	914	879	883	786	862	838	833	842	793	978	941
15					—	930	978	916	948	917	945	874	925	905	916	896	828	944	963
20						—	944	982	956	915	924	794	897	847	858	891	841	948	946
25							—	949	978	949	976	895	944	915	926	916	848	943	972
30								—	978	958	949	849	915	885	890	923	879	948	960
35									—	979	988	901	953	927	937	946	890	943	973
40										—	981	954	947	960	954	954	911	930	970
45											—	942	978	956	962	949	885	923	975
50												—	939	981	965	925	883	860	933
55													—	972	975	952	893	901	972
60														—	982	952	918	892	963
65															—	979	920	887	957
70																—	966	906	951

TABLE X. Correlation coefficients (times 1,000) of $(-e_{10}^{\circ})$ with log $_nq_x$, various collections of life tables, females and males

Age x	European tables pre-1870	Tables from Russia and the Balkans	Tables from Asia, Africa, Latin America	"West" regional tables	"North" regional tables	"East" regional tables	"South" regional tables
				Females			
0	606	699	705	953	992	960	925
1	767	876	746	960	990	985	951
5	408	913	828	961	992	985	970
10	386	941	884	958	985	980	979
15	674	872	928	953	961	966	958
20	297	830	832	940	931	964	952
25	903	857	918	948	956	977	955
30	887	895	928	963	969	986	980
35	808	939	952	974	987	990	983
40	944	974	950	978	974	994	985
45	843	995	943	968	995	993	987
50	908	951	930	975	974	993	980
55	601	407	762	963	990	991	986
60	594	826	891	973	983	994	989
65	629	726	879	962	983	[illegible]	984
70	547	641	880	937	968	988	975
75	528	307	699	869	927	969	948
				Males			
0	566	614	700	890	980	964	913
1	733	873	725	909	980	982	939
5	499	877	778	895	987	985	945
10	132	867	822	892	989	974	941
15	435	827	927	917	949	969	963
20	799	800	938	926	931	957	946
25	889	883	960	944	942	974	973
30	904	899	970	958	953	985	960
35	847	937	975	962	976	991	973
40	858	938	976	977	989	992	970
45	716	939	970	970	996	987	975
50	694	902	948	966	990	980	933
55	707	888	920	918	969	972	972
60	790	805	906	919	977	971	963
65	699	666	864	887	975	977	957
70	626	640	865	882	965	981	951
75	393	296	772	851	930	945	908

notably in a tendency in recent life tables toward higher death rates above age fifty relative to the rates under 30.[7]

CONSTRUCTION OF FOUR SETS OF MODEL LIFE TABLES

The principal steps in the calculation of the four sets of model life tables were as follows:

1. Intercorrelation matrices for ${}_nq_x$ and $\log_{10}({}_nq_x)$ were calculated for the "North," "South," "East," and "West" data. [$e_0°$ and $e_{10}°$ were left untransformed (no logarithms taken) in the second sets of intercorrelations.]

2. Least-square linear regressions of ${}_nq_x$ and of log ${}_nq_x$ on $e_{10}°$ were fitted for both sexes in all four "regions." The regression coefficients are given in Table XI.

3. The values of ${}_nq_x$ estimated from the logarithmic regression are always above those from the regression of untransformed mortality rates at the high and low extremes of observed life expectancies, and the logarithmic regression values are always lower in the middle range. In other words, the two regression lines always intersect twice within the range of observations. In constructing the model life tables, ${}_nq_x$ values were taken from the simple regression at all points to the left (i.e., at points with lower life expectancy) of the first intersection of the regression lines; and to the right of the second intersection, ${}_nq_x$ values were taken from the logarithmic regression. Between the two intersections, the mean of the ${}_nq_x$ values from the two regressions was used.

4. From various values of the independent variable ($e_{10}°$), ${}_nq_x$'s at ages 0, 1, 5, 10, . . . , 75 were calculated. From each such set of ${}_nq_x$'s, l_1, l_5, l_{10}, . . . , l_{80} were computed with l_0 taken as 100,000.

[7] We have calculated multiple correlations with each mortality rate as a dependent variable, and time (date of the life table) as an additional independent variable. The multiple correlation coefficients for males typically exceed the corresponding zero order correlation by a wide margin, and the difference for females is typically trivial. Thus, zero-order and multiple correlation between log ${}_5q_{65}$ and $-e_{10}°$ are about the same (.952). Among "West" males, the corresponding correlations are: multiple $R = .932$ and zero-order $r = .887$. In consequence, the female model life tables are more closely representative of mortality experience in the areas from which the data are drawn then are the male model life tables.

5. ${}_nL_x$ and $e_x°$ were estimated on the use of the foll[owing] formulae:

$$
\begin{aligned}
{}_1L_0 &= k_0l_0 + (1 - k_0)l_1 \\
{}_4L_1 &= k_1l_1 + (4 - k_1)l_5 \\
{}_5L_x &= 2.5(l_x + l_{x+5}),\ x = 5, 10, \ldots, 75 \\
e_{80}° &= 3.725 + 0.0000625l_{80} \\
T_{80} &= e_{80}°l_{80} \\
T_x &= \sum_{x}^{75} L_x + T_{80} \\
e_x° &= \frac{T_x}{l_x}
\end{aligned}
$$

The values of k_0 were as follows, when ${}_1q_0 \geq 0.100$:

	For females	For males
West, North, South "Regions"	0.35	0.33
East "Region"	0.31	0.29

The values of k_0, ${}_1q_0 < 0.100$, were given by the followin[g ex]pressions:

	For females	For males
"West," "North," "South"	$k_0 = 0.050 + 3.00\ {}_1q_0$	$k_0 = 0.0425 +$ [illegible]
"East"	$k_0 = 0.010 + 3.00\ {}_1q_0$	$k_0 = 0.0025 +$ [illegible]

The values of k_1 were as follows, when ${}_1q_0 \geq 0.100$:

	"West"	"North"	"East"	"South"
For females	1.361	1.570	1.324	1.239
For males	1.352	1.558	1.313	1.240

The values of k_1, ${}_1q_0$, < 0.100, were given by the followin[g ex]pressions:

	For females	For males
"West"	$1.524–1.625\ {}_1q_0$	$1.653–3.013\ {}_1q_0$
"North"	$1.733–1.627\ {}_1q_0$	$1.859–3.013\ {}_1q_0$
"East"	$1.402–1.627\ {}_1q_0$	$1.541–3.013\ {}_1q_0$
"South"	$1.487–1.627\ {}_1q_0$	$1.614–3.013\ {}_1q_0$

6. Age-specific mortality rates (${}_nm_x$) were calculated fro[m the] formula ${}_nm_x = {}_nd_x/{}_nL_x$ where ${}_nd_x = l_x - l_{x+n}$

7. Five-year survival rates for projecting five-year age gr[oups] (${}_5P_x$) were calculated by the formula ${}_5P_x = {}_5L_{x+5}/{}_5L_x$, x [=] 5, . . . , 70.

TABLE XI. Regression coefficients relating ${}_nq_x$ and $\log_{10}(10{,}000\ {}_nq_x)$ to e_{10}° for regional model life tables, male and female (coefficients A_x and B_x in ${}_nq_x = A_x + B_x e_{10}^{\circ}$; and A_x' and B_x' in $\log_{10}(10{,}000\ {}_nq_x) = A_x' + B_x' e_{10}^{\circ}$)

	"WEST" LIFE TABLES				"NORTH" LIFE TABLES			
	Female				*Female*			
Age (x)	A_x	B_x	A_x'	B_x'	A_x	B_x	A_x'	B_x'
0	0.53774	−0.008044	5.8992	−0.05406	0.47504	−0.006923	5.7332	−0.05133
1	0.39368	−0.006162	7.4576	−0.08834	0.45025	−0.006805	7.6298	−0.08909
5	0.10927	−0.001686	6.2018	−0.07410	0.19376	−0.002928	7.1271	−0.08647
10	0.08548	−0.001320	5.9627	−0.07181	0.10041	−0.001497	6.1089	−0.07192
15	0.10979	−0.001672	5.9335	−0.06812	0.10126	−0.001480	5.4984	−0.05955
20	0.13580	−0.002051	5.9271	−0.06577	0.11261	−0.001618	5.2649	−0.05372
25	0.15134	−0.002276	5.8145	−0.06262	0.13137	−0.001893	5.2547	−0.05236
30	0.17032	−0.002556	5.6578	−0.05875	0.15448	−0.002239	5.3691	−0.05339
35	0.18464	−0.002745	5.3632	−0.05232	0.17693	−0.002566	5.3186	−0.05136
40	0.19390	−0.002828	4.9600	−0.04380	0.18440	−0.002612	4.9099	−0.04261
45	0.20138	−0.002831	4.5275	−0.03436	0.19440	−0.002712	4.6164	−0.03627
50	0.25350	−0.003487	4.4244	−0.03004	0.22364	−0.003011	4.3673	−0.02961
55	0.31002	−0.004118	4.3131	−0.02554	0.30043	−0.004053	4.4363	−0.02858
60	0.43445	−0.005646	4.3439	−0.02295	0.41033	−0.005394	4.4163	−0.02511
65	0.53481	−0.006460	4.2229	−0.01773	0.56691	−0.007187	4.4030	−0.02152
70	0.69394	−0.007713	4.1838	−0.01376	0.77206	−0.009334	4.3826	−0.01784
75	0.84589	−0.008239	4.1294	−0.00978	0.96175	−0.010681	4.3108	−0.01355
	Male				*Male*			
Age (x)	A_x	B_x	A_x'	B_x'	A_x	B_x	A_x'	B_x'
0	0.63726	−0.009958	5.8061	−0.05338	0.54327	−0.008251	5.6151	−0.05022
1	0.40548	−0.006653	7.1062	−0.08559	0.46169	−0.007290	7.2025	−0.08475
5	0.10393	−0.001662	5.4472	−0.06295	0.18983	−0.002974	6.1947	−0.07195
10	0.07435	−0.001183	5.0654	−0.05817	0.09551	−0.001476	5.3488	−0.06047
15	0.09880	−0.001539	4.8700	−0.05070	0.09666	−0.001422	4.5662	−0.04322
20	0.14009	−0.002183	5.0677	−0.05156	0.13472	−0.001968	4.6970	−0.04277
25	0.15785	−0.002479	5.2660	−0.05471	0.14325	−0.002103	4.7661	−0.04372
30	0.18260	−0.002875	5.3438	−0.05511	0.15280	−0.002244	4.7248	−0.04236
35	0.21175	−0.003312	5.2792	−0.05229	0.17535	−0.002589	4.7568	−0.04197
40	0.25049	−0.003864	5.0415	−0.04573	0.20924	−0.003083	4.7280	−0.03986
45	0.27894	−0.004158	4.6666	−0.03637	0.24673	−0.003605	4.6020	−0.03578
50	0.33729	−0.004856	4.4506	−0.02961	0.28578	−0.004016	4.3499	−0.02857
55	0.38425	−0.005190	4.2202	−0.02256	0.36171	−0.005037	4.3718	−0.02682
60	0.48968	−0.006300	4.1851	−0.01891	0.45849	−0.006124	4.2977	−0.02244
65	0.59565	−0.007101	4.1249	−0.01491	0.59986	−0.007677	4.2858	−0.01913
70	0.73085	−0.007911	4.1051	−0.01161	0.82662	−0.010241	4.3482	−0.01710
75	0.89876	−0.008695	4.1133	−0.00895	1.03681	−0.011906	4.3197	−0.01357

(Continued)

TABLE XI (*continued*). Regression coefficients relating ${}_nq_x$ and $\log_{10}$ (10,000 ${}_nq_x$) to e_{10}° for regional model life tables, male and female coefficients A_x an
${}_nq_x = A_x + B_x\, e_{10}^\circ$; and A_x' and B_x' in $\log_{10}$ (10,000 ${}_nq_x$) $= A_x' + B_x' e_{10}^\circ$)

"EAST" LIFE TABLES

Female

Age (x)	A_x	B_x	A_x'	B_x'
0	0.78219	−0.011679	5.8529	−0.05064
1	0.46584	−0.007284	7.2269	−0.08351
5	0.13739	−0.002136	6.3204	−0.07590
10	0.07600	−0.001166	5.6332	−0.06684
15	0.10067	−0.001529	5.5780	−0.06295
20	0.13039	−0.001973	5.5872	−0.06081
25	0.15401	−0.002335	5.6149	−0.06004
30	0.16941	−0.002559	5.4593	−0.05616
35	0.18184	−0.002718	5.1881	−0.05000
40	0.18555	−0.002718	4.8186	−0.04209
45	0.19407	−0.002746	4.4509	−0.03368
50	0.24415	−0.003376	4.3702	−0.02966
55	0.34490	−0.004723	4.4480	−0.02807
60	0.49585	−0.006651	4.4917	−0.02544
65	0.68867	−0.008874	4.4702	−0.02152
70	0.88452	−0.010551	4.3759	−0.01640
75	1.07727	−0.011513	4.2972	−0.01191

Male

Age (x)	A_x	B_x	A_x'	B_x'
0	1.07554	−0.017228	6.3796	−0.06124
1	0.55179	−0.009201	7.8944	−0.09934
5	0.15292	−0.002523	6.4371	−0.08076
10	0.06856	−0.001096	5.1199	−0.05978
15	0.10060	−0.001578	4.9229	−0.05182
20	0.14725	−0.002312	5.1056	−0.05225
25	0.15127	−0.002381	5.1036	−0.05207
30	0.17022	−0.002686	5.1685	−0.05244
35	0.20786	−0.003277	5.1986	−0.05131
40	0.24876	−0.003868	5.0221	−0.04577
45	0.28685	−0.004320	4.6915	−0.03697
50	0.32623	−0.004654	4.3492	−0.02767
55	0.38906	−0.005243	4.1849	−0.02171
60	0.49337	−0.006341	4.1647	−0.01842
65	0.66168	−0.008182	4.2175	−0.01634
70	0.84188	−0.009644	4.2171	−0.01324
75	1.03876	−0.010780	4.2155	−0.01035

"SOUTH" LIFE TABLES

Female

A_x	B_x	A_x'	
0.52069	−0.007051	4.5097	[illegible]
0.68268	−0.010453	5.9815	[illegible]
0.17066	−0.002657	5.6479	[illegible]
0.09000	−0.001380	5.1045	[illegible]
0.12189	−0.001851	5.2384	[illegible]
0.15083	−0.002279	5.1708	[illegible]
0.16073	−0.002412	5.0949	[illegible]
0.16719	−0.002505	4.9291	[illegible]
0.17408	−0.002583	4.8035	[illegible]
0.17278	−0.002504	4.4917	[illegible]
0.17800	−0.002513	4.2693	[illegible]
0.22639	−0.003140	4.1982	[illegible]
0.30167	−0.004130	4.2724	[illegible]
0.47682	−0.006501	4.4242	[illegible]
0.67440	−0.008891	4.4554	[illegible]
0.92943	−0.011532	4.4348	[illegible]
1.16023	−0.013009	4.3542	[illegible]

Male

A_x	B_x	A_x'	
0.61903	−0.008974	4.7096	[illegible]
0.70613	−0.011375	6.3246	[illegible]
0.16455	−0.002674	5.6400	[illegible]
0.07634	−0.001207	4.6816	[illegible]
0.11449	−0.001810	4.9454	[illegible]
0.17104	−0.002693	5.2748	[illegible]
0.17171	−0.002710	5.1168	[illegible]
0.16483	−0.002535	4.8459	[illegible]
0.17905	−0.002734	4.7660	[illegible]
0.20606	−0.003081	4.5796	[illegible]
0.23208	−0.003370	4.3559	[illegible]
0.28000	−0.003917	4.1918	[illegible]
0.35245	−0.004765	4.1492	[illegible]
0.49465	−0.006569	4.2479	[illegible]
0.66947	−0.008608	4.3069	[illegible]
0.89759	−0.010843	4.3251	[illegible]
1.10111	−0.011806	4.2684	[illegible]

and B_x in

B_x'
-0.02566
-0.05532
-0.06136
-0.05537
-0.05494
-0.05171
-0.04945
-0.04590
-0.04280
-0.03615
-0.03092
-0.02717
-0.02588
-0.02491
-0.02190
-0.01775
-0.01296

B_x'
-0.02980
-0.06433
-0.06389
-0.05008
-0.05170
-0.05458
-0.05152
-0.04547
-0.04292
-0.03738
-0.03116
-0.02547
-0.02193
-0.02063
-0.01863
-0.01552
-0.01123

The first survival rate is the proportion surviving to the end of a five-year time interval of persons born during the interval, estimated as $_5L_0/5l_0$. The last survival rate is of persons over 75 at the beginning of an interval (and over 80 at the end), estimated as T_{80}/T_{75}.

8. Both male and female tables are calculated by regression of $_nq_x$ on e_{10}°. The values of e_{10}° that were used as the independent variable in constructing the female tables were chosen so as to give even 2.5-year intervals of e_0° from 20 to 77.5 years. The values of e_{10}° for males were chosen so as to correspond with the female e_{10}°'s in a way that preserves the typical relation of e_{10}° for males and females at each level of mortality within each family of life tables. The relationship posited was as follows:

$$(e_{10}^\circ)_m - (\overline{e_{10}^\circ})_m = \frac{\sigma_m}{\sigma_f}\left[(e_{10}^\circ)_f - (\overline{e_{10}^\circ})_f\right]$$

where σ_m and σ_f are the standard deviations of expectation of life at age 10 for males and females. This expression is the equation for the straight line with a slope intermediate between the regression of $(e_{10}^\circ)_m$ on $(e_{10}^\circ)_f$, and the inverse of the regression of $(e_{10}^\circ)_f$ on $(e_{10}^\circ)_m$. The correlation between e_{10}° for the two sexes is more than .99 in all instances, so that the two regression lines are almost identical.

9. Single year values of l_x for ages between 1 and 5 were determined after l_x at ages 1, 5, 10, . . . , 80 had been calculated by the methods already described. These values are given in Table [illegible]A at end of Chapter 3. The method of calculation was to determine weights (α_2, α_3, and α_4) so that l_i could be estimated as $\alpha_i l_1 + (1 - \alpha_i) l_5$. Constant values of α_i were used when $_1q_0 \geq 0.100$, as follows:

	Females			Males		
	α_2	α_3	α_4	α_2	α_3	α_4
"West"	.489	.260	.112	.484	.258	.110
"North"	.589	.336	.145	.584	.331	.143
"East"	.473	.249	.102	.466	.244	.103
"South"	.457	.207	.075	.458	.208	.074

For low levels of infant mortality ($_1q_0 < 0.100$) variable weights, were used, of the form $\alpha_i' = \alpha_i + b_i\ (0.100 - {_1q_0})$.

The values of b_i were as follows:

	b_2	b_3	b_4
For females	0.656	0.601	0.370
For males	1.353	1.089	0.571

THE USE OF TWO REGRESSIONS IN CALCULATING $_nq_x$ IN MODEL TABLES

There are two considerations that make questionable the use of regression equations in constructing model life tables: (1) A regression of a particular functional form (linear, quadratic, logarithmic, etc.) often fits the data better over some parts of the range of observations than others, and often represents the data poorly at extremes, and may in particular provide an implausible extension or extrapolation of relationships beyond the range of observations; (2) "least squares" regression equations of any given functional form establish two relations between any pair of variables, as each is considered the independent variable, and the other dependent. In an estimation of one variable from a known value of another, the use of one variable as dependent and the other as independent is clearly appropriate and there is a logical reason for employing one regression rather than the other. But there is no logical basis for selecting one mortality rate or another as independent when trying to estimate a set of interrelationships among mortality rates at different ages. In the original United Nations model life tables, the selection of $_1q_0$ as the sole purely independent variable, and the calculation of a chain of regressions, with each mortality rate first being a dependent and then an independent variable, created a tendency (because of the well-known "regression toward the mean") to incorporate less of the observed range of variability at higher ages than at lower ages. A regression chain starting at the highest ages and ending with $_1q_0$ would have been equally plausible, and would have produced somewhat different model tables.

The use of two linear regressions (logarithmic and non-logarithmic) and the selection of e_{10}° as the index of mortality (the independent variable in calculating mortality rates for every age group) result in a procedure that lacks elegance, but takes account of the considerations stated in the preceding paragraph.

The expectation of life at age 10 is an approximate general index of the level of mortality, an index that is not strongly dependent on the mortality rates for any one age group. (In contrast, e_0° is strongly influenced by the value of ${}_1q_0$.) Correlations with e_{10}° are near to unity at most ages in all "regions." The choice of e_{10}° as the basis for regression estimates is not an arbitrary fixing of any one mortality rate in the resultant model life tables. All mortality rates are separately subject to "regression toward the mean." Hence if an e_{10}° two standard deviations above the mean is chosen as the basis for estimating a model life table, *all* of the mortality rates will be somewhat less than two standard deviations below their mean value; but the sequence of mortality rates is not ever more compressed toward the mean as age advances.

One final oddity: the e_{10}° in a given model life table (calculated from T_{10}/l_{10}) is not the same as the e_{10}° that served as the basis for estimating the various mortality rates in the given table. There are two e_{10}°'s associated with each model life table: the e_{10}° that served as the index by which all ${}_nq_x$'s were estimated, and the e_{10}° implied by the ${}_nq_x$'s themselves. The relationship between the calculated e_{10}° and ${}_nq_x$ (for any x) is curvilinear, even over the range where estimation by linear regression of ${}_nq_x$ on e_{10}° is employed. The line representing the relation between the calculated e_{10}° in the model life table and ${}_nq_x$ for each x fits the observed ${}_nq_x$'s better than the regression line.

The facts that led us to combine estimates from regressions of ${}_nq_x$ and log ${}_nq_x$ on e_{10}° were as follows:

1. The two sets of correlation coefficients were comparable in magnitude. There were 48 instances where the non-logarithmic correlation exceeded the logarithmic by at least .003; 55 where the logarithmic coefficients were larger by this margin; and 33 instances where the difference was less than .003. (Table XII). However, there is a different relative effect on the two kinds of correlation coefficients of "scatter" in mortality rates at high levels of mortality on the one hand, and low levels of mortality on the other. At low levels of e_{10}°, the associated mortality rates usually have large arithmetic deviations one from another, but not especially large logarithmic deviations, since the latter depend on the ratios of mortality rates one to another. Hence, the "scatter" found among high mortality rates depresses the nonlogarithmic correlation coefficient, but not the logarithmic. Conversely, the logarithmic co
tion coefficient is depressed by the large relative differences
may accompany small arithmetic differences among low mor
rates. In short, the relative size of the non-logarithmic and log
mic correlations may be more a reflection the special geo
among high and among low mortality rates than an indicati
whether the underlying relationship is linear or exponentia
fact, the position of the logarithmic regression line is aff
more than that of the non-logarithmic line by the geometry o
cluster of observations at lower mortality levels, and the po
of the non-logarithmic regression line is affected more than
of the logarithmic by the scatter of observations at higher mor
levels.

2. The logarithmic regression line at the highest observe
pectations of life is closer to the observations, and represents a
plausible extension of the observations, than the non-logari
regression line. This closer fit is caused partly by the sensitiv
the logarithmic regression to relative rather than absolute devia
and partly by the fact that declining linear exponential functio
necessarily asymptotic to zero, and never give negative value
contrast, the non-logarithmic regression line falls *below* th
served mortality rates for the higher values of e_{10}°, and some
indicates negative mortality estimates near the upper limits o
served values of e_{10}°—a patent absurdity.

3. At the other end—high mortality, low expectation of lif
the range of observed values, the non-logarithmic regression l
usually closer to empirical mortality rates, and, in this range
represents a more plausible extension of observed relation
exponential necessarily becomes steeper at lower expectatio
life, and would yield absurdly high estimated mortality ra
e_{10}° were assumed to have a low value.

Through the middle range—between the two intersections o
lines—sometimes one line is closer, and sometimes the other t
observed mortality rates. To have a uniform rule, and mi
discontinuity in the sequence of model tables, we estimated
tality rates from the non-logarithmic regression up to th
intersection, used the mean of the two regression estimates be
the two intersections, and used the estimates from the logar

TABLE XII. Correlation coefficients (times 1,000) between $(-e_{10}^{\circ})$ and $_nq_x$ and between $(-e_{10}^{\circ})$ and log $_nq_x$ in life tables underlying model life tables "West," "North," "East," and "South"

	"WEST"				"NORTH"				"EAST"				"SOUTH"			
	Females		*Males*		*Females*		*Males*		*Females*		*Males*		*Females*		*Males*	
Age x	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$	$_nq_x$	log $_nq_x$
0	947	953	907	890	978	992	977	980	950	960	963	964	912	925	880	913
1	904	960	906	909	984	990	979	980	960	985	970	982	972	951	900	939
5	917	961	884	895	984	992	980	987	954	985	963	985	930	970	856	945
10	866	958	842	892	990	985	986	989	978	980	980	974	981	979	897	941
15	948	953	935	917	982	961	947	949	974	966	984	969	996	958	957	963
20	977	940	949	926	975	931	930	931	982	964	968	957	982	952	940	946
25	984	948	971	944	979	956	959	942	993	977	984	974	981	955	965	973
30	983	963	976	958	987	969	984	953	991	986	991	985	992	980	965	960
35	980	974	977	962	995	987	994	976	990	990	991	991	996	983	986	973
40	978	978	976	977	993	974	989	989	991	994	988	992	992	985	984	970
45	964	968	962	970	993	995	974	996	993	993	985	987	990	987	976	975
50	970	975	957	966	970	974	960	990	993	993	980	980	959	980	931	933
55	956	963	911	918	960	990	942	969	988	991	969	972	974	986	953	972
60	969	973	924	919	950	983	945	977	991	994	971	971	968	989	955	963
65	967	962	891	887	957	983	950	975	990	991	978	977	974	984	953	957
70	945	937	887	882	949	968	946	965	989	988	985	981	969	975	952	951
75	874	869	857	851	908	927	914	930	967	969	949	945	943	948	905	908

gression at expectations of life at age 10 above—to the right of the second intersection.

ESTIMATION OF LIFE TABLE VALUES OTHER THAN $_nq_x$

The weights (k_0 and k_1, the "separation factors") that relate $_1L_0$ to l_0 and l_1, and $_4L_1$ to l_1 and l_5, can be shown to equal the age at death of those members of the life table population who die under age 1 (for k_0), and the age at death minus 1 of those who die between ages 1 to 4 (for k_1). The values of the factor k_0 were therefore determined by examining the average age at death under 1 year in the records of the populations whose mortality experience was the basis for the four regional model life tables. No consistent variation in average age of infant deaths was found at infant mortality levels above 0.100, nor any consistent differences among the "regions," except that k_0 for "East" infants was consistently less than in the other "regions," and k_0 for males was generally slightly less than for females. When $_1q_0$ is very low—in the range from 0.015 to 0.100—there is a clearly apparent tendency for k_0 to vary, because at very low levels of infant mortality infant deaths are much more concentrated immediately after birth. This tendency was represented by allowing k_0 to rise linearly from a typical level at the lowest observed infant mortalities until at $_1q_0 = 0.100$ it reaches the plateau typical of higher infant mortality.

The value of k_1 in the expression $_4L_1 = k_1 l_1 + (4 - k_1) l_5$ was determined from the estimates of l_2, l_3, and l_4 given in Table XV at the end of Chapter 3. In fact $_4L_1$ was calculated on the assumption that l_x can be considered linear in each single-year age interval from 1 to 5; thus $k_1 = 0.5 + \alpha_2 + \alpha_3 + \alpha_4$, when $l_2 = \alpha_2 l_1 + (1 - \alpha_2) l_5$, etc. Each α_i was based on the relation of l_i to l_1 and l_5 observed in the life tables upon which each family of model tables

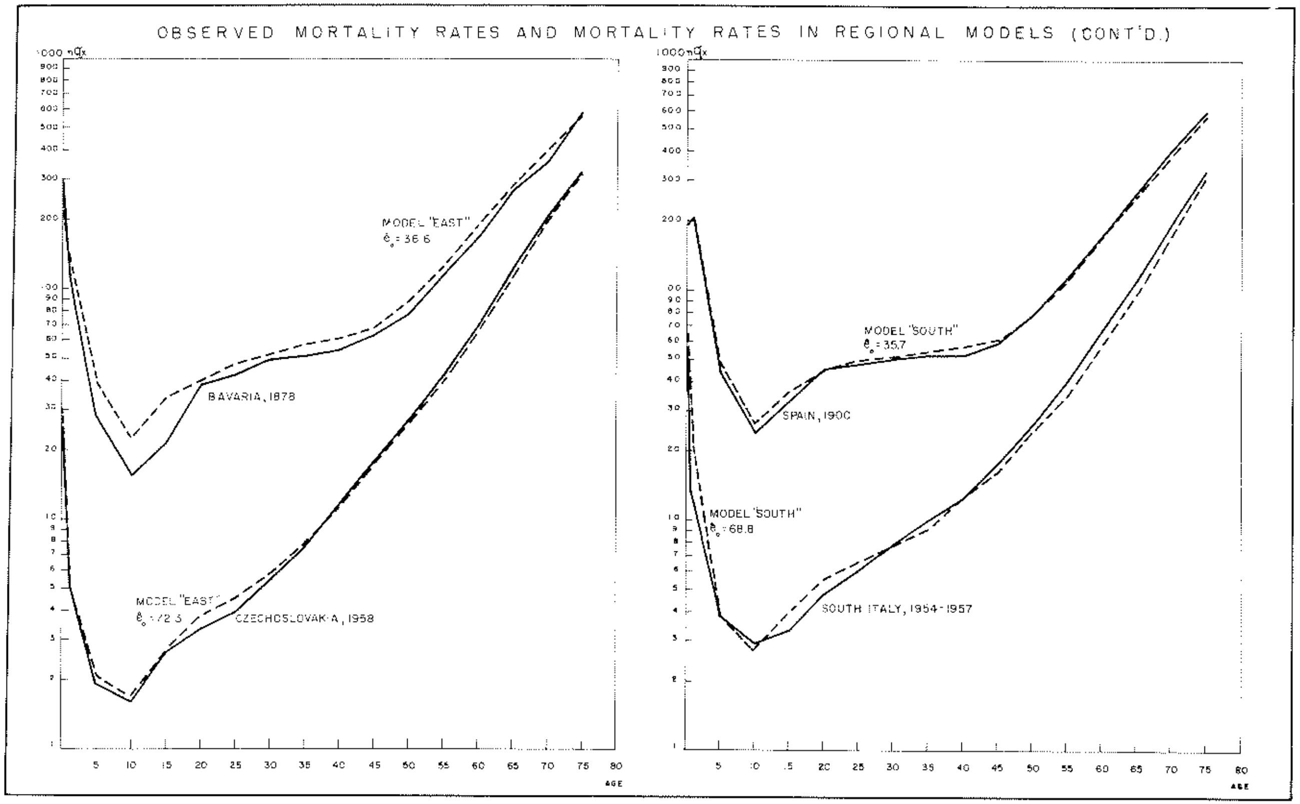

FIGURE 2. Values of $_nq_x$ in female life tables with highest and lowest $e_0°$ among life tables in the collections underlying the four families of model life tables compared with values of $_nq_x$ in life tables with the same $e_0°$ of the corresponding model.

was based. As with k_0, it was noted that when $_1q_0 > 0.100$, the values of each α_i showed no tendency to vary as a function of the level of mortality, although at levels of mortality below $_1q_0 = 0.100$ there is a tendency for the deaths from age 1 to age 5 to be more evenly distributed, and for l_x in this range to be more nearly linear. Hence at low levels of mortality, each α_i was allowed to fall as $_1q_0$ increases in a manner analogous to the rise in k_0. The values of α_2, α_3, and α_4 are somewhat different in each region. It is reassuring to note that when $_1q_0 > 0.100$, the α_i's observed in the life tables underlying each regional set of model tables clustered about an average typical of the set, and differing from the av value of α_i for the other "regions." In other words, each r has a distinctive pattern of mortality from 1 to 5, as well a other ages.

The formula for expectation of life at age 80 is based on a fitted to observations in 70 life tables (35 for males and 35 females) from the Netherlands, Norway, Sweden, Switzerl and Japan. These tables were selected because age reporting in census and mortality records in these countries for persons 80 appeared especially trustworthy.

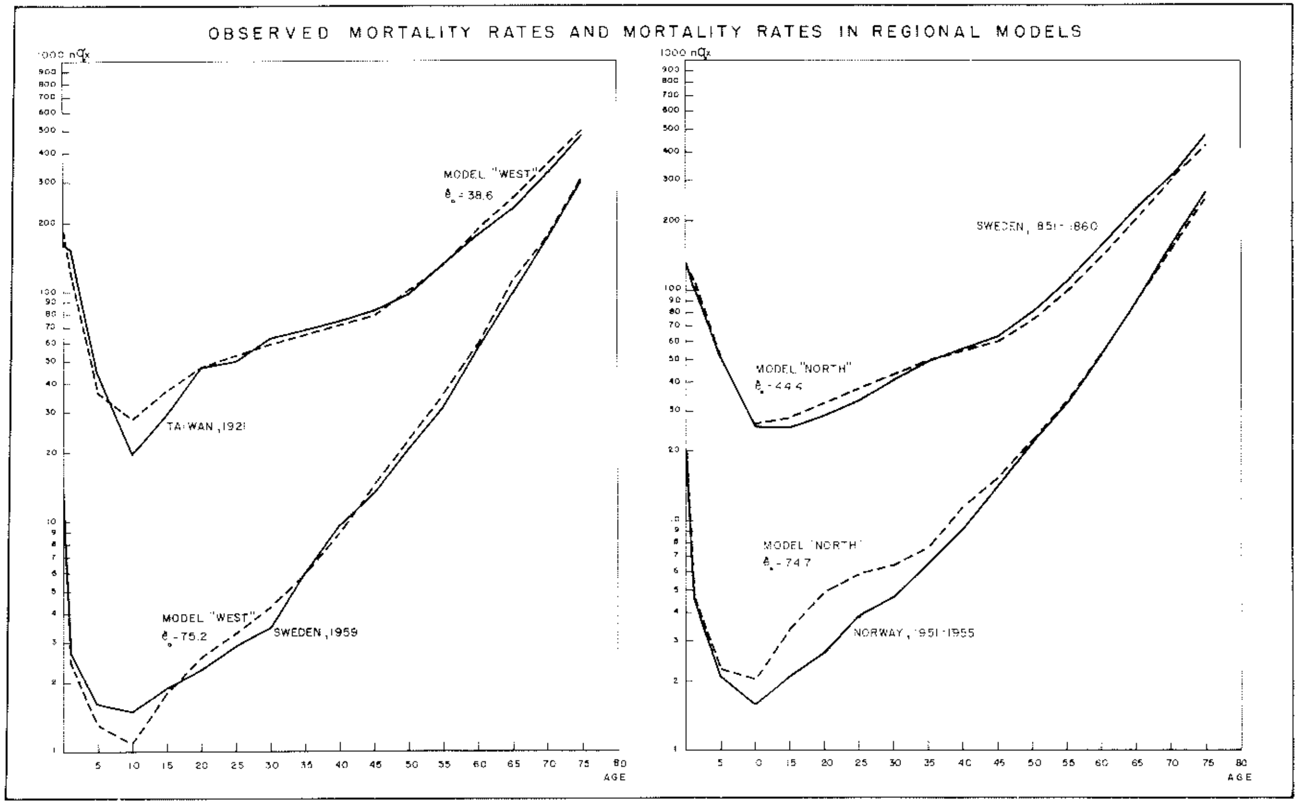

FIGURE 2 (*continued*)

THE VALUE OF FOUR FAMILIES OF MODEL LIFE TABLES

The separate families of model life tables provide estimates that in our judgment are quite reliable when utilized judiciously for populations within the areas upon which each family of model tables is based. This point is illustrated by Figure 2 in which the mortality rates in the highest and lowest mortality tables (for females) in each group are compared with the mortality rates in the model table with the same expectation of life at birth. Figure 3 shows how the four female model tables differ at various levels of mortality.

The closeness of fit encourages us to believe that the "East" family of age patterns of mortality provides a better basis for estimating life tables in, for example, Poland or North Italy than would one of the other families, or a single collection of life tables such as the United Nations' models. We also feel (with less conviction) that the extrapolated model life tables approximate the different patterns of mortality that likely prevailed or will prevail at life expectancies below or above the range of observed experience within the regions covered.

A part of our motivation in constructing these families of model life tables was to provide a research tool that promises to be useful in our own program of studying population change in the provinces of Europe during the period of the so-called demographic transi-

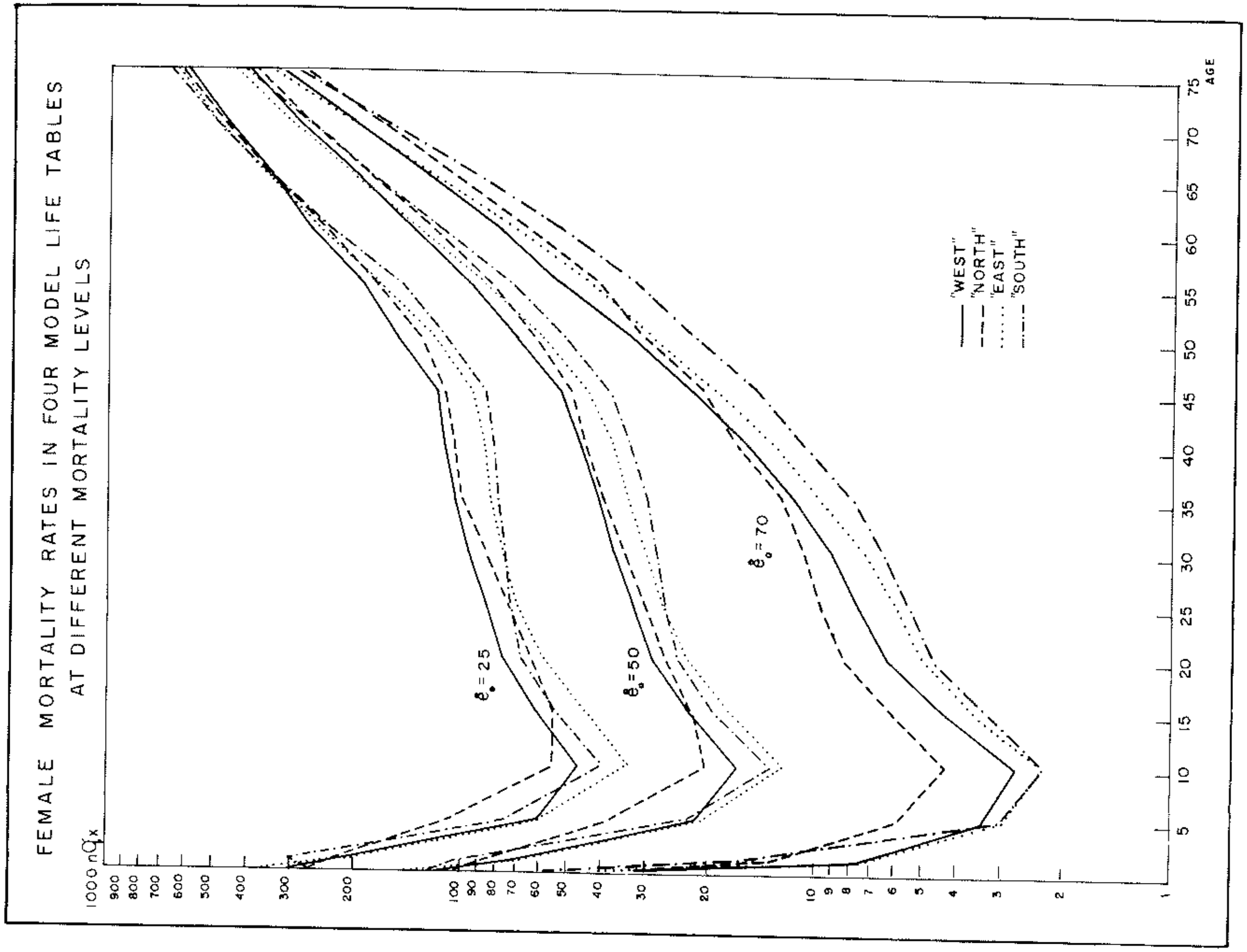

FIGURE 3. Values of $_nq_x$ in the four families of model life tables ("West," "North," "East," and "South") when e_0° = 25, 50, and 70 years, for females.

tion. We expect to estimate vital rates from age distributions to supplement registered data on births and deaths, and feel that the four families of life tables will be superior for this purpose to any single set of one-parameter model tables.

The four families of model tables will also provide a basis for estimating life tables in populations outside of the ones that underlie the model tables themselves. We would suggest utilizing the "West" family in the usual circumstances of underdeveloped countries where there is no reliable guide to the age pattern of mortality that prevails. The other families can be moderately useful in these circumstances to illustrate the variability in estimates that arises from assuming well-established age patterns of mortality other than the pattern underlying the "West" tables. We must concede that this use of the four families is far from satisfactory, because there is no strong reason for supposing that the age patterns of mortality exhibited in these four families covers anything like the full range of variability in age patterns in populations under different circumstances. At the same time, we feel that the age patterns of mortality exhibited in life tables from underdeveloped areas are usually quite untrustworthy because of defects in the underlying data. The question of what is the pattern of mortality in a population of an underdeveloped area is essentially unresolvable, because there exists no way to determine exactly the age of an illiterate person who does not know it himself, and *a fortiori* of a deceased person who did not know his own age at the time of death. By the time a population has reached the stage where age-specific mortality rates can be measured with confidence, the level and age pattern of mortality may have changed, so that the pattern of mortality during the underdeveloped period may never be known.

B. CALCULATION OF FOUR FAMILIES OF STABLE POPULATIONS

Each of the nearly 5,000 stable age distributions is a combination of a model life table and a rate of increase, as in equation (1), Chapter 1. The proportions of persons in 18 age intervals were calculated, for males and females, the intervals being: under 1, 1–5, 5–10, 10–15, . . . , 75–80, 80+. The exact expression for the proportions in each interval is:

$$\int_{a_1}^{a_2} c(a)\, da = b \int_{a_1}^{a_2} e^{-ra} p(a)\, da \tag{3}$$

and

$$b = \frac{1}{\int_0^{\omega} e^{-ra} p(a)\, da} \tag{4}$$

In our calculations, the following approximation was employed:

$$\int_{a_1}^{a_2} e^{-ra} p(a)\, da \doteq e^{-r\bar{a}} \int_{a_1}^{a_2} p(a)\, da \tag{5}$$

This approximate formula is exactly accurate if the appropriate interior value of $\bar{a}$ is used. For every interval except the last, $\bar{a}$ was taken as $(a_1 + a_2)/2$. For the interval 80+, $\bar{a}$ was taken as 80 plus a linear expression with e_{80}° as the independent variable.[8] $\int_{a_1}^{a_2} p(a)\, da$ is taken from the L_x column in the given model life table up to age 80; T_{80} is used for $\int_{80}^{\omega} p(a)\, da$.

An important characteristic of the stable population is the underlying *fertility schedule*. Although a fertility schedule and a mortality schedule fully determine the intrinsic rate of increase, there is a *family* of fertility schedules that, in conjunction with a given mortality table, would produce the same intrinsic rate of increase, and hence the same stable population. The relationship between fertility and the intrinsic rate of increase is given in equation (2), Chapter 1.

Lotka calculates r from given fertility and mortality schedules by deriving a power series with coefficients that are the cumulants of the net fertility function, $\phi(a) = p(a)\, m(a)$.

$$\mu_1 r - \mu_2 \frac{r^2}{2!} + \mu_3 \frac{r^3}{3!} \cdots - \log R_0 = 0 \tag{6}$$

where $R_0 = \int_0^{\omega} p(a) m(a)\, da$ (the net reproduction rate), μ_1 is the mean of $\phi(a)$, and μ_i is the ith cumulant of $\phi(a)$. In Lotka's calculation of r, terms above $(\mu_2/2)r^2$ are neglected, implying that moments of $\phi(a)$, or

[8] Specifically, $\bar{a}_{80+} = 80 + 0.6 e_{80}^{\circ} + 0.92$. This relation was derived from the same life tables (for Japan, the Netherlands, Norway, Sweden, and Switzerland) by which the formula relating e_{80}° to l_{80} was determined.

of $m(a)$, higher than the second have little effect on the relation between fertility and the intrinsic rate of increase. This is an empirical property of the age schedules of human female fertility. The form of fertility schedules has only limited variability. In all populations where reliable records have been kept, fertility is zero until about age 15, rises smoothly to a single peak, and falls smoothly to zero by age 45 to 50. The mean age of the fertility schedule is usually between about 26 and 33 years.

Variations in the form of the age schedule of fertility occur because of different customs governing age of cohabitation, the use of contraception or abortion, etc. But in all schedules the higher moments about the mean are not large enough to affect r, and there is such a nearly constant relationship of the variance to the mean that knowledge of the gross reproduction rate (the area under the maternity schedule) and the mean age of the schedule is sufficient to estimate r with a very small margin of error.

Although variations in the higher moments of the fertility schedule are not important in determining r, the mean age of the fertility schedule has a strong influence on the rate of increase, especially when fertility is high.[9] For example, with the same mortality schedule ("West" females, $e_0^\circ = 50$ years), a GRR (the gross reproduction rate) of 3.500 connotes an r of 0.0363 when $\bar{m} = 27$ years, and of only 0.0281 when $\bar{m} = 33$ years. Thus each stable population is consistent with a different GRR for every different mean age of the fertility schedule, and there is included with each stable population four GRR's, appropriate to mean ages of 27, 29, 31, and 33 years.

The fertility schedules with mean ages of 27, 29, 31, and 33 years are scalar multiples of basic schedules having these mean ages, and a GRR of 1.000. The four basic schedules were obtained in two steps: (1) Schedules from the large collection of national age-specific fertility rates in the United Nations *Demographic Yearbook* of 1959 were selected in groups having mean ages of about 27, 29, 31, and 33 years. All schedules were divided by total fertility so as to yield "maternity" schedules with a GRR of 1.000. (2) Synthetic ("basic") schedules with mean ages of exactly 27, 29, 31, and 33 years were constructed by taking weighted averages of the selected

[9] A. J. Coale and C. Y. Tye, "The Significance of Age-Patterns of Fertility in High Fertility Populations," *The Milbank Memorial Fund Quarterly*, Vol. 39, No. 4 (October 1961).

schedules with mean ages near these values. The four basic fe[…] schedules are shown in Table XIII.

TABLE XIII. Fertility schedules (annual female births per woman in [...] age group) with mean age equal 27, 29, 31, and 33 yea[rs]

Age group	Annual female births per woman		
	$\bar{m} = 27$	$\bar{m} = 29$	$\bar{m} = 31$
15–19	.029	.018	.008
20–24	.055	.042	.032
25–29	.054	.056	.054
30–34	.037	.044	.050
35–39	.020	.028	.034
40–44	.004	.010	.018
45–49	.001	.002	.004
GRR	1.000	1.000	1.000

In the "growth rate" set of tables, the values of r wer[e] (-0.010, -0.005, . . . , $+0.050$), and the calculation needed to find GRR for $\bar{m} = 27$, 29, 31, and 33. This calculation was [...] by evaluating the integral in equation (7):

$$\text{GRR}(\bar{m}) = \frac{1}{\int_0^\omega e^{-ra} m'(a) p(a)\, da}$$

where $m'(a)$ is the basic maternity schedule, with the give[n] [...] age, and GRR = 1. Equation (7) follows from the fact that:

$$K \int_0^\omega e^{-ra} p(a) m'(a)\, da = 1$$

where K is the scalar multiple relating the fertility schedule [...] the GRR in question to $m'(a)$, which by definition has a GR[R] [of] 1.000.

In the "GRR" tables, the "given" quantity is the gross repro[duc]tion rate, where $\bar{m} = 29$ years. It was necessary to calculate [...] implied by this given fertility schedule and the relevant mor[tality] schedule. r was calculated by a recursive method. An estim[ated] value (r') was substituted in the integral in equation (2), a[...]

second (more accurate) estimate (r'') derived from the divergence of the integral from unity.[10] Iteration was employed until the integral in (2) differed from unity by less than one part in a billion. The value of the gross reproduction rate for $\bar{m}$ other than 29 was then calculated by equation (7).

The death rate was taken as the difference between b (equation (1)) and r. It was also calculated by summing the age-specific death rate (${}_nm_x$) for each interval multiplied by the proportion of the population in each interval; and after an inspection of rates thus computed over a wide range of mortality and growth rates, it appeared that the two methods of calculation would rarely, if ever, produce different results within the decimal places printed. These products (proportion of population times age-specific death rates) are the basis of the age distribution of deaths that is printed with each stable age distribution.

The average age at death was calculated by summing the proportion of deaths in each age interval multiplied by the mean age at death in the stationary population of those dying in the interval. This mean age is equal to k_0 for deaths under age 1, and to $1 + k_1$ for deaths to persons 1 to 5. The mean age was assumed to be the midpoint of each five-year interval from age 5 to 80, and is $80 + e_{80}^{\circ}$ for persons dying over age 80.

A. J. Coale, "A New Method of Estimating Lotka's r," *Population Studies* (London), Vol. 2, No. 1 (July 1957), pp. 92–94.

The first approximation was obtained by the formula $r' = (\log R_0)/(0.7 \log R_0)$ which is based on these considerations: (a) $r = (\log R_0)/T$; (b) $T = \mu_1 - \frac{1}{2}\mu_2 r + \ldots$, and (c) $\mu_2/2\mu_1 = 0.7$ for human fertility schedules. This value of r' is so close to r that the integral in equation (2) usually equals 1.000 ± 0.00001.

CHAPTER 3. USES OF THE TABLES

A. MODEL STABLE POPULATIONS

TABULATION OF STABLE POPULATIONS

The large number of parameters tabulated for each stable population, and the large number of age distributions included in this volume are designed to make the tables as widely useful as possible, given the limitations inherent in the data upon which they are based.

The scheme of tabulation is designed with two considerations in mind: (a) to provide stable age distributions for the full range of mortality and fertility levels likely to be relevant either in fitting the tables to observed data, or in analyzing the hypothetical consequences of fertility and mortality schedules; and (b) to tabulate values at small enough intervals so that linear interpolation is sufficiently exact.

There are two overlapping sets of stable population tables giving information that is to a large extent duplicated. Each set of tables includes a page of age distributions (and also a page of age distributions of deaths) for every model life table. In the first set of tables (the "growth rate," or "r," tables) the basic index of variation for a given mortality schedule is the rate of increase r; and in the second set (the "GRR" tables), the index of variation is the gross reproduction rate. In the first set, stable age distributions with intrinsic rates of natural increase from —0.010 to +0.050 are provided, at intervals of 0.005 in r. The variation in GRR ($\bar{m}$ = 29 years) in these tables is from 2.39 to 12.16 when e_0° = 20 years, and from 0.76 to 4.09 when e_0° = 77.5 years ("West" mortality). Combinations of very rapid growth and high mortality are clearly impossible, because of the impossible level of fertility implied. Age distributions with GRR's above 4.0 or 5.0 will not be used very often, except for illustration.

The second set of tables provides age distributions for every life table with gross reproduction rates (when $\bar{m}$ = 29 years) th from 0.800 to 6.000 in increments that become larger at levels of fertility. The growth rates implied vary from —0.0 0.0233 when e_0° = 20 years, and from —0.0082 to 0.064 e_0° = 77.5 years ("West" mortality). The distributions repre combinations of very low fertility and high mortality, and o high fertility and low mortality will doubtless be used very ra

The two sets of tables cover slightly different universes of fe mortality combinations. The user should note that at high le mortality (short expectation of life) the "growth rate" set in higher fertility levels, and the "GRR" set covers lower f levels. At the other end of the mortality scale (low mortalit life expectation) the two sets start at about the same low f boundary, but the "GRR" set extends to higher fertility level purpose of providing two sets of tables is to make analys interpolation more convenient. The user will find the "growth set of tables more convenient in exploring the implication example) of different recorded intercensal rates of increase the "GRR" set more convenient in analyzing the effects of diff levels of fertility.

MULTIPLE INTERPOLATION IN THE STABLE AGE DISTRIBUTION TA

In using these tables, it is essential to recall that each stabl distribution is the consequence of a mortality schedule and a gr rate, that there are a variety of parameters (birth rate, death mean age) that are implied by each distribution, and that all is implied about fertility in any age distribution is that the sch must be one of many possibilities having an appropriate com tion of GRR and $\bar{m}$.

In the universe of stable age distributions associated with one of the four families of model life tables, it is possible to l a unique stable population from knowledge of two non-redu parameters, such as the birth rate and $e_0°$, the rate of increase and $e_0°$, the proportion under 20 and the rate of increase, or the average age and the death rate. However, it must be borne in mind that measures of total fertility (such as the GRR) do not constitute in themselves an adequate parameter in identifying a unique stable population, but must be supplemented by the mean age of the fertility schedule, and by some other variable as well.

An appropriate stable age distribution can be located by a process of interpolation (more than one is usually required), which is illustrated in the following examples:

Example (1). What is the proportion under 30 in the male stable age distribution with "West" mortality, $e_0° = 21.60$ years, $r = 0.0112$?

First find by linear interpolation in the "growth rate" set of "West" model age distributions that at mortality level 2($e_0° = 20.443$ for males)[1] the proportion under 30 is $(0.24)(0.7354) + (0.76)(0.7038) = 0.7114$ when $r = 0.0112$; and at level 3($e_0° = 22.851$ for males) the proportion under 30 is $(0.24)(0.7220) + (0.76)(0.6891) = 0.6970$ when $r = 0.0112$. Then, interpolating between these two values, we find that the proportion under 30 is $(0.5195)(0.7114) + (0.4805)(0.6970) = 0.7045$ when $e_0°$ is 21.60. This process has located a stable age distribution that lies (horizontally) between columns headed "10.0" and "15.0" and (vertically) between pages headed "mortality level 2" and "mortality level 3." By other interpolations of this sort, other parameters of this stable population can be found. Thus, the birth rate of the stable age distribution in question is 60.75 per thousand, the death rate 49.55, the mean age 24.21 years, etc.

Example (2). What is the female intrinsic birth rate, death rate, and rate of increase in a population with a GRR of 1.695, $\bar{m} = 26.432$, and $e_0° = 73.4$ years (U.S. females, 1962)?

By extrapolation in the column headed "GRR = 1.750" ($\bar{m} = 29$ years) in the second ("GRR") set of model "West" female tables, on the page headed "mortality level 22" ($e_0° = 72.5$ years), it is found that GRR = 1.667 when $\bar{m} = 26.432$. Similarly, in the column headed "GRR = 2.000," it is found that GRR = 1.883 when $\bar{m} = 26.432$. By interpolating between the values of b and r for these two stable populations, it is found that when GRR = 1.695 ($\bar{m} = 26.432$), $b = 25.93$ (per thousand), and $r = 18.69$ (when $e_0° = 72.5$). A parallel calculation on the page headed "mortality level 23" ($e_0° = 75$ years) gives $b = 25.75$ and $r = 19.25$. By a final interpolation (for $e_0° = 73.4$ years) the desired b and r are found to be 25.87 and 18.89 per thousand. The intrinsic death rate is, by subtraction, 6.98 per thousand. The published values (calculated from the actual U.S. fertility and mortality schedules for 1962 rather than from these "model schedules") are 25.8, 18.8, and 7.0 respectively.[2]

ANALYTICAL USES OF THE STABLE AGE DISTRIBUTIONS

These extensive tables of stable populations can be used to illustrate relationships among demographic variables, since the stable population shows the ultimate implications of a fertility and a mortality schedule. For example, a comparison of the intrinsic rates of births, deaths, and natural increase for U.S. females in 1962 (25.87, 6.98, 18.89 per thousand as calculated above) with the corresponding *observed* values for the female population (21.54, 8.07, and 13.47 per thousand) shows that the age structure of the female American population causes a lower birth rate and higher death rate than would the age distribution implied by the fertility and mortality schedules themselves. This comparison (of crude and intrinsic rates) was much emphasized in the 1920's and 1930's when the crude rate of increase in Western countries was well *above* the intrinsic rate.

Table XIV shows that the interplay of mortality, fertility, and age composition can be illustrated by numbers extracted from the model stable populations. Note, for example: the small effect of mortality and the large effect of fertility on the mean age of the population; and the existence of a minimum death rate with each life table a minimum that occurs at a GRR between 2.00 and 2.50

[1] The expectation of life at birth is indicated directly on the first set of age distributions as Pop. size, B(0) = 1, for $r = 0$, because of the well-known fact that in a stationary population, expectation of life at birth is the reciprocal of the birth rate.

[2] U.S. Department of Health, Education, and Welfare, Public Health Service, National Vital Statistics Division, *The Vital Statistics of the United States, 1962*, Vol. 1, *Natality*, pp. 1–6.

TABLE XIV. Selected parameters of "West" female stable populations for different expectations of life at birth, and gross reproduction rates (mean age of fertility schedule 29 years).

e_0°	Gross reproduction rate ($\bar{m}$ = 29)					
	0.80	1.00	1.50	2.00	3.00	4.00
			Proportion under age 15			
20	.098	.128	.197	.256	.349	.419
30	.117	.151	.229	.293	.392	.462
40	.129	.167	.250	.318	.419	.490
50	.138	.178	.266	.336	.438	.509
60	.146	.188	.279	.350	.454	.524
70	.149	.192	.286	.359	.464	.534
			Proportion at ages 65 and over			
20	.167	.135	.085	.059	.032	.020
30	.178	.142	.087	.058	.031	.019
40	.187	.148	.089	.058	.030	.018
50	.195	.153	.091	.059	.030	.017
60	.200	.155	.091	.058	.029	.017
70	.212	.164	.095	.061	.030	.017
			Mean age (years)			
20	44.6	41.6	35.8	31.7	26.2	22.7
30	43.8	40.5	34.2	29.9	24.4	21.1
40	43.4	39.8	33.2	28.8	23.3	20.0
50	43.2	39.4	32.6	28.1	22.6	19.4
60	43.0	39.0	32.0	27.5	22.0	18.8
70	43.3	39.2	31.9	27.2	21.7	18.5
			Birth rate (per thousand)			
20	9.6	13.3	22.6	31.5	47.6	61.3
30	9.8	13.4	22.5	31.0	45.8	58.1
40	9.8	13.4	22.3	30.5	44.5	55.8
50	9.7	13.3	22.1	30.0	43.4	54.2
60	9.7	13.3	21.9	29.7	42.6	52.9
70	9.6	13.1	21.7	29.3	41.9	51.8
			Death rate (per thousand)			
20	56.9	53.1	48.7	47.7	49.5	52.9
30	43.7	39.8	35.0	33.5	33.9	35.7
40	34.9	30.9	25.8	23.9	23.4	24.2
50	28.4	24.4	19.1	16.9	15.7	15.9
60	23.4	19.3	13.9	11.4	9.7	9.3
70	19.4	15.3	9.7	7.1	5.0	4.4

when $e_0^\circ = 20$, between 2.500 and 3.00 when $e_0^\circ = 30$, between 3.00 and 4.00 when $e_0^\circ = 50$, and above 4.00 when $e_0^\circ = 70$ years.[3]

USE OF THE STABLE AGE DISTRIBUTIONS TO ESTIMATE DEMOGRAPHIC PARAMETERS IN POPULATIONS WITH HISTORIES OF CONSTANT FERTILITY AND CONSTANT OR GRADUALLY CHANGING MORTALITY

The immediate purpose of calculating the preliminary version of these tables several years ago in the Office of Population Research was to estimate birth rates, death rates, and approximate age distributions in research on populations for which there were only incomplete or inaccurate data. Euler proposed estimation by methods such as these in the article cited earlier. In 1907, Lotka noticed how closely the stable age distribution implied by the life table and rate of natural increase of England and Wales fitted the actual age distribution; and since World War II there has been a revival of interest in the stable population because of evidence that age distributions in the so-called underdeveloped areas are not very different from stable age distributions. Various techniques have been employed to select or calculate a stable age distribution best fitting the data available. Once a stable age distribution has been selected, its characteristics can be assumed to approximate those of the population in question.

Three examples illustrate this use of the tables.

Example (1). Given the female age distribution for England and Wales in 1881, and the fact that e_0° was 44.62 in 1871–1880, estimate the birth rate for the years preceding the census, on the assumption that the age distribution approximates the stable.

The cumulative proportions of females in the 1881 census to age 5, 10, 15, 20, 25 and 30 are 0.1332, 0.2516, 0.3565, 0.4524, 0.5436, and 0.6236. These proportions are utilized with the given e_0° to locate, by interpolation, a sequence of female "West" stable populations, having the following sequence of birth rates: 33.6, [illegible] 34.0, 34.0, 34.5, and 34.7 per 1,000.

For comparison, the female registered female birth rate in 1851–1860 was 32.65; in 1861–1870, 33.5; and in 1871–1880, 33.7. The rates adjusted for under-registration by Glass[4] for these periods are 34.0, 34.2, and 34.0.

Example (2). Estimate the birth rate, death rate, e_0°, and age distribution for India, 1911, from the reported age distribution, and the intercensal growth rate.

Age distribution in the Indian censuses are characterized by extreme age-heaping and other indications of faulty data. In addition to the misreporting of ages, it is also likely that persons in some age intervals are subject to exceptional under-enumeration. Hence, the birth rate and the expectation of life at birth estimated from the rate of increase and the proportion cumulated to age 5, 10, . . . , 35 are not nearly as consistent as in 19th-century England and Wales.

Estimates of the female birth rates in India based on (a) the proportions 0-5, 0-10, . . . , 0-35, (b) the intercensal rate of increase 1901–1911, and (c) "West" model age distributions are 44.8, 54.8, 49.7, 43.9, 45.0, 48.2, and 52.6 per thousand. The values of e_0° in these stable populations are 26.3, 21.3, 23.5, 26.8, 26.1, 24.3, and 22.1 years. These varying sequences of estimates reflect the conspicuous relative under-counts of ages 0-4, 10-14, and 15-19 that are seen in a comparison of the "West" stable age distribution that has the same proportion under 35 as in the census, and the intercensal rate of increase (Figure 4). This example illustrates how wide is the range of uncertainty in making estimates based on censuses of illiterate populations. Moreover, there is an additional uncertainty as to whether the "West" pattern of mortality is appropriate. Estimates based on the proportion to age 35 and the intercensal rate of increase in "North," "East," and "South" model age distributions are 55.4, 67.2, and 64.4 per thousand for the birth rate (instead of 52.6 in the "West" distribution), and 21.0, 16.5, and 17.5 years for e_0° (instead of 22.1). There is some reason for believing that the "West" table is the most likely choice at such a high level of mortality. In spite of the obvious uncertainties, we

[3] These stable age distributions are used to illustrate various other interrelations in A. J. Coale, "Birth Rates, Death Rates, and Rates of Growth in Human Populations," Symposium: Research Issues in Public Health and Population [illegible], Graduate School of Public Health, University of Pittsburgh (to be published), and in a forthcoming book by Coale on how age distributions are determined.

[4] D. V. Glass, "A Note on the Under-Registration of Births in Britain in the 19th Century," *Population Studies* (London), Vol. 5, No. 1 (July 1951), pp. 70–88.

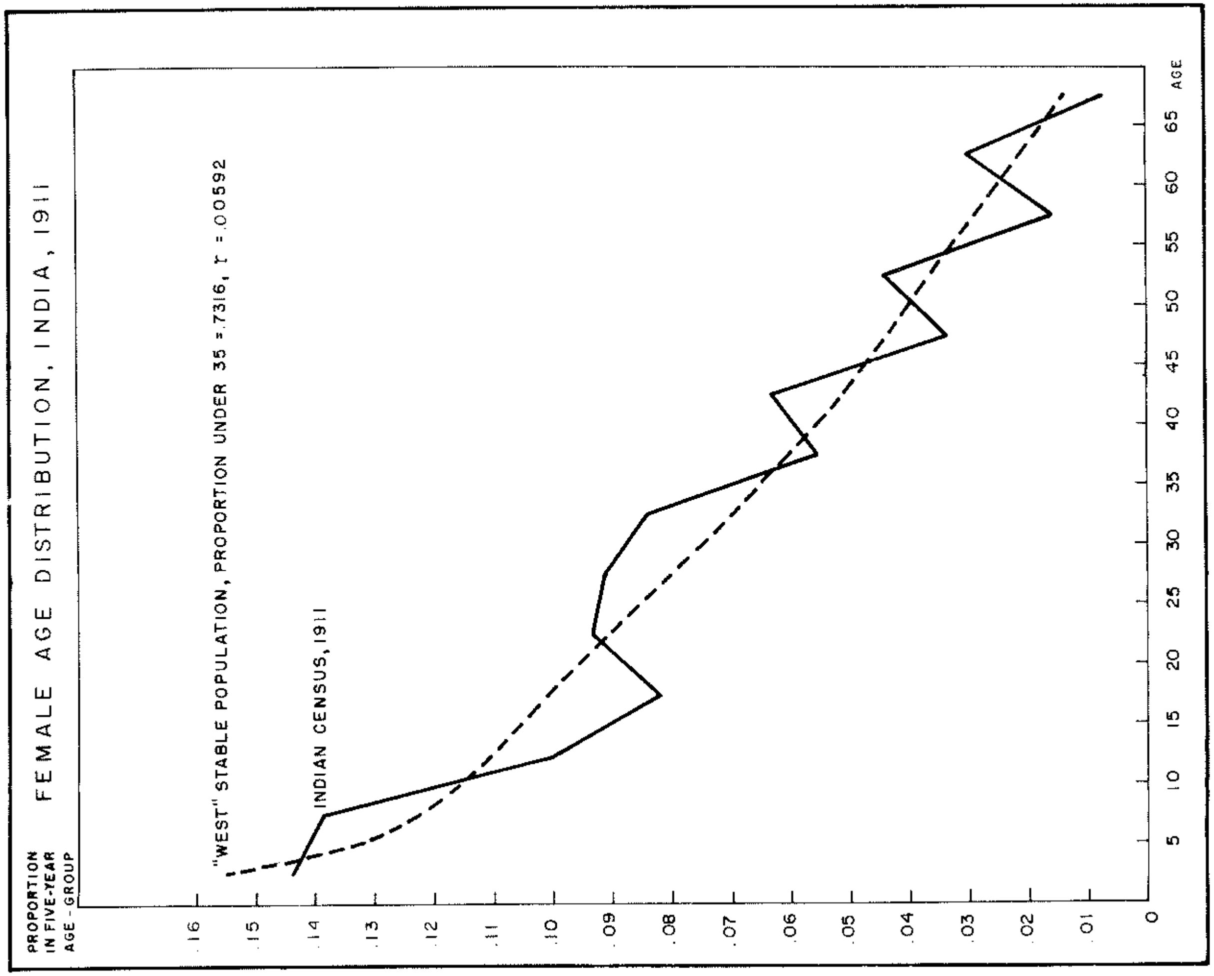

FIGURE 4. Female age distribution in India, 1911, as recorded in census, and approximated from "West" stable population with same proportion under age 35, and a growth rate of 0.00592.

feel that a minimum estimate of 49-51 for the birth rate, and a maximum estimate of 22-24 for e_0° is warranted.

The 1911 age distribution of India has been taken as an example rather than data from a more recent census because the 1921 age distribution was affected by the extraordinary influenza epidemic that began in 1918, and because post-World War I age distributions have been affected by declining mortality. Declining mortality changes the age distribution in a way that lowers the estimates of birth rates. If the decline is gradual, the effects may be ignored, and the resultant estimates can be useful, even if slightly biased. The effect of declining mortality has been discussed briefly in two recent articles[5] and will be analyzed more fully in a manual that the Office of Population Research is preparing for the United Nations.

Example (3). Estimate the birth rate for England and Wales for 1871–1880 from the age distribution of deaths and the rate of natural increase.

In a pioneering article on estimating vital rates by stable population analysis, Bourgeois-Pichat suggested using the age distribution of deaths[6] as a means of identifying a stable age distribution closely fitting the population in question. Even if the registration of deaths is incomplete, deaths as registered may be reasonably representative in their distribution by age of actual deaths, at least at ages above infancy. The age distribution of registered mortality provides a means independent of population censuses of selecting a stable age distribution. The proportion of the deaths over age 5 among females in England and Wales that occurred over age 30, 50, and 70 was 0.766, 0.552, and 0.357. Given the female rate of natural increase in 1871–1880 (0.0140, adjusted for under-registration of births), and interpolating in the tables showing the age distributions of deaths, the following estimates of the birth rate are obtained: 33.3, 33.8, and 33.3. Note how closely these rates compare with those derived in an earlier example.

B. USE OF MODEL LIFE TABLES, BY THEMSELVES, OR WITH MODEL STABLE POPULATIONS

The four families of model life tables are closely representative of age schedules of mortality in the "regions" upon which they are based, with these provisos: (a) Any extraordinary incidence of a cause of death that is highly age-and-sex specific produces a mortality schedule that naturally does not conform to the model tables. Examples are Norwegian mortality at the turn of the century, when tuberculosis was an extraordinary cause of death, populations affected by the influenza pandemic of 1918–1920, or populations experiencing heavy military casualties. (b) The most recent life tables in many countries have male mortality rates over age 45 or 50 higher than in the corresponding model tables. (c) Within the large residual "region"—the "West"—there are populations with more deviant mortality patterns than in populations in the other "regions." (d) In all of the model life tables, the person-years lived beyond age 80 is a very rough estimate, so that the expectation of life above 65 or 70, and also the survival rates for projecting the oldest age group are less firmly based than corresponding figures at earlier ages.

Subject to these reservations, the model life tables may be used as substitutes for tables calculated by the usual actuarial methods in applications involving populations within the "regions." This substitution is especially useful for standard demographic calculations, such as population projections.

Recently, for example, a request came to the Office of Population Research to estimate the growth of world population until the end of the 20th century, on the assumption that the female expectation of life at birth rises continuously to about 66 years by 1990 and that fertility falls continuously to a level that in 1990 would give a net reproduction rate of 1.000—exact replacement of the maternal generation. To obtain an approximate answer quickly, we took

[5] A. J. Coale, "Estimates of Various Demographic Measures Through the Quasi-Stable Age Distributions," *Milbank Memorial Fund, op. cit.*; Paul Demeny, "Estimation of Vital Rates for Populations in the Process of Destabilization," paper presented at the June 1964 Meetings of the Population Association of America, to be published in *Demography* (Chicago 1965) Vol. 2.

[6] Bourgeois-Pichat, "Utilization de la notion de population stable pour mesurer la mortalité et la fécondité des populations des pays sous-développés," *Bulletin de l'Institut International de Statistique* (Stockholm 1958), Vol. 36, Part 2, pp. 94-121.

note of recent estimates that the rate of increase of the world population in 1960 was about 20 per thousand, and that the world birth rate was about 40 per thousand; assumed that the age distribution of the world population in 1960 conformed approximately to the "West" stable distributions; located the model distribution with this birth rate and rate of increase; and assumed that the corresponding model life table described world mortality in 1960. From the model age distribution, it was also possible to read the ratio of births/women 15-44. The next step was to estimate $e_0°$ for the years 1960–1965, 1965–1970, . . . , 1995–2000 by linear interpolation (arriving at 66 in 1990 and thereafter constant) to read the ratio of births/women 15-44 for the stationary population with $e_0° = 66$, and to interpolate values of this ratio for 1965, 1970, . . . , 1985. Thus, with little labor, we obtained the ingredients for a standard population projection by five-year survival rates, and a general fertility rate instead of age-specific fertility rates. Although the result is not very precise, it cannot differ much from what would be obtained by alternative approaches to a question that calls for only an approximate answer. (The projection showed the world population growing from 3 billion in 1960 to about 5.5 billion in 2000 under the stated assumptions.)

The model life tables can be used in selecting an appropriate model stable population. Sometimes mortality estimates for only a restricted age range are available. William Brass has devised a way of estimating survival rates for children to age 1, 2, 3, and 5 from the proportion of children-ever-born reported as surviving by women 15-19, 20-24, etc.[7] The availability of these estimates makes it possible to select a model life table (in any of the four families, or in all four, to illustrate the resulting variations) on the basis of the value of l_1, l_2, l_3, l_4, and l_5 given in Table XV at end of this chapter; and if conditions warrant the assumption that the population has approximately stable characteristics, knowledge of only one additional parameter—say the proportion of the population under 20—would serve to fix the choice of a model age distribution.

[7] William Brass, "The Construction of Life Tables from Childhood Survivorship Ratios," Union internationale pour l'étude scientifique de la population, *International Population Conference; Congrès International de la Population, New York 1961*, London 1963, Vol. 1, pp. 294–301.

C. GLOSSARY OF SYMBOLS USED IN TABLES

GENERAL REMARKS

The main body of this volume consists of photographic rep[...] tions of computer tables. Limitations on printout formats [...] some peculiarities of the notation, in particular the exclus[...] of capital letters. Readability and mnemonic convenience [...] than formal elegance, were sought in designating the variou[...] printed in the tables. A description of these designations [...] below. First the symbol is shown as it is used in the tabl[...] lowed by the conventional notation, if any, in square bracke[...] a brief explanation of the meaning of the symbol.

NOTATION IN MODEL LIFE TABLES

Q(X)	$[{}_nq_x]$	Probability at age x of dying before reaching age [...]
D(X)	$[{}_nd_x]$	Number of deaths between age x and $x + n$ out of [...] cohort of 100,000.
M(X)	$[{}_nm_x]$	Death rate in the life table population (number [...] per person-years lived) between age x and $x + n$.
l(X)	$[l_x]$	Number of survivors at age x out of an original [...] 100,000.
L(X)	$[{}_nL_x]$	Number of person-years lived between age x and [...] an original cohort of 100,000. This column give[...] distribution in a stationary population with 100,0[...] births and deaths.
P(X)	$\left[\frac{{}_5L_{x-5}}{{}_5L_x}\right]$	The proportion of persons in a given five year age [...] the stationary population alive five years lat[...] however that the entry after age zero in this [...] P(BIRTH), $[{}_5L_0/5l_0]$; and that the entry after [...] P(0–4), $[{}_5L_5/{}_5L_0]$; and that the entry after age 75 [...] T(75), $[T_{80}/T_{75}]$.
T(X)	$[T_x]$	Number of person-years lived at age x and ov[...] original cohort of 100,000.
E(X)	$[e_x°]$	Average number of years remaining to be lived (exp[...] of life) at age x.

In the first and last columns of the life tables the exact ages [...] 1, 5, . . . , 80 are shown. Note that the width of age in[...] denoted by the subscript n in the conventional notation is [...]

in the sequence of these exact ages. Thus the width of the first interval is one year (between exact age 0 and 1); the width of the second interval is four years (between exact age 1 and 5); and from then on the width of interval is five years up to age 80. The last line of the tables refers to the "open" age interval of exact age 80 and over, except for columns 1, P, T, and E, where the concept of interval is not applicable.

NOTATION IN MODEL STABLE POPULATIONS

Symbols in Column Headings

R	Growth rate, i.e., intrinsic rate of increase [*r*] for 1,000 persons of given sex.
GRR	Female gross reproduction rate with a mean of 29 years for the underlying maternity schedule.

Symbols in Stubs

AGE	For the conventional age distributions (upper section of table) the relevant age intervals are indicated in terms of age at last birthday. For the cumulated age distributions the upper *exact* age limit is indicated for each interval. The lower limit is age 0 in each instance.
BIRTH RATE, DEATH RATE	Births and deaths of given sex per 1,000 persons of given sex; i.e., intrinsic birth and death rates per 1,000.
GROWTH RATE	Same as symbol R used in column headings and defined above.
GRR (M)	Gross reproduction rate for given sex when the mean age of the underlying maternity (or paternity) schedule is M years.
NRR (M)	Net reproduction rate for given sex when the mean age of the underlying maternity schedule is M years.
PROP. 15-44	Number of people aged 15-44 in the stable population per 100 persons.
BIRTHS/P. 15–44	Births of given sex per person of given sex aged 15–44. With reference to the female population, sometimes called the birth-woman ratio.
POP. −4/15–44	Persons of given sex aged 0–4 per person of 15–44 of given sex. With reference to the female population, sometimes called the child-woman ratio.
POP. 5–14/5+OVR	Persons of given sex aged 5–14 per person 5 and over of given sex. An index of the age distribution proposed by M. Bourgeois-Pichat.
DEPNDCY RATIO	Persons aged 0–14 and over 60 of given sex per person 15–59 of given sex. Sometimes called the dependency ratio.
POP. SIZE, B (0) = 1	The reciprocal of the birth rate, i.e., the size of the stable population for a unit-size current-year (year zero) birth cohort.
EXP. OF LIFE	Expectation of life at birth in the life table identified in the table heading by pattern, level, and sex. This index is printed in the "GRR" set of tables only, in the last line under the first column.
DR OVR AGE 1	Death rate of persons above age 1, i.e., deaths to persons over age 1 of given sex per 1,000 persons over age 1.
AVG. A AT DEATH	Average age at death in stable populations of given sex.
AVG. A AT DEATH OVR 5	Average age at death in stable population of given sex of persons dying at age 5 and over.
DEATHS (30+/5+)	Deaths in stable populations to persons of given sex over 30 per death to persons of given sex over 5.

D. HINTS ABOUT USING THE TABLES

USE OF CUMULATIVE AGE DISTRIBUTION

In the stable population tables the second array of figures are cumulative age distributions either for the population (on the left-hand pages of the open book) or for deaths (on the right-hand pages). Because of limitations of space it runs only to age 65 in the R ("growth rate") set, to age 40 in the female "GRR" set and to age 75 in the male "GRR" set. It is printed to age 80 in the death distributions where the proportion of deaths at advanced ages is possibly more important. The cumulative age distributions make it possible to find the proportion of persons (or deaths) for various age intervals (e.g., 15–64) by a single subtraction.

IDENTIFICATION OF MORTALITY LEVELS UNDERLYING MODEL STABLE POPULATIONS

Underlying each of the 13 stable populations printed on two consecutive pages facing each other is a single life table identified on each page by the regional mortality pattern, sex, and mortality level in the upper left and upper right corners. Any desired characteristics of the corresponding life table (e.g., the value of infant mortality) is easily determined by consulting the table at the given level in the model life table section. Note, however, that a convenient cardinal measure of the underlying life table—the expectation of life at birth—can always be determined without reference to the life table section. In the "growth rate" set of stable age distributions this is performed by reference to the stationary age distribution—the age distribution when R = 0. The last line on the left-hand pages is the reciprocal of the birth rate, which *for the stationary population* is also the expectation of life at birth of the corresponding life table. In the GRR set of tables the expectation of life at birth in the corresponding model life table is printed in the bottom line on the left hand side pages under the first column.

USE OF THE RECIPROCAL OF THE BIRTH RATE

The reciprocal of the birth rate printed in the tables for stable populations is, by definition, the size of the stable population expressed in terms of a unit-sized current birth cohort, hence the notation: POP. SIZE, B(0) = 1. Notice that this figure facilitates the calculations of a combined male and female stable age distribution implicit in the female fertility schedules, the male and female mortality schedules, and a sex ratio at birth.

Example: What is the sex ratio in the age group 10 years and over in a stable population with a female gross reproduction rate of 3.00 ($\bar{m} = 29$ years) if $e_0^\circ = 40.0$ years for females, if it is assumed that the age pattern and sex differentials of mortality are those characteristic of model "North," and the sex ratio at
1.05?

The relevant mortality level for both sexes is Level 9. Th
the total female population over age 10 for one unit of "cur
male births at GRR (29) = 3.00 is 22.710 × (1 − 0.3006)
Similar calculation gives the male population as 2
(1 − 0.3082) = 14.779. Since for each female birth there
male births the answer to the question is given by the
(14.799 × 1.05)/15.883 = 0.977. Note that the figures for th
population are taken from a column identified by the level
female GRR.

THE GRR FIGURES IN THE TABLES FOR MALE STABLE POPULATIONS

In the "growth rate" sets of tables for stable populations (p
tions with specified evenly spaced growth rates) the tab
males indicate the level of GRR calculated on the b
the same four "maternity" distributions that are used f
females, i.e., with mean ages of 27, 29, 31, and 33 years. It
be kept in mind, however, that the mean age of the male f
schedule is often several years greater than in the female sc
of the same population. Thus there may be instances in wh
GRR for males could be estimated only by a major extrap
beyond age 33. Also the shape of male schedules can be
different from female schedules, possibly disturbing the r
among the gross reproduction rate, the mean of the fertility s
ule, and the rate of increase. But even with these reservatio
felt there will be occasions on which the male GRR's can
useful.

In the GRR sets of tables the information on male gross rep
tion rates was suppressed to avoid confusion. The tables fo
in these sections contain only one GRR value for each pop
The GRR value that appears in the heading of these tab
as the title of the tables indicate, a *female* GRR (with a m
29 years for the underlying maternity function). The asso
of this female GRR with the given male stable population
it would produce, jointly with the *female* mortality of th
level as the given male life table, a rate of increase equal t
shown for the male population.

ROUNDING OF FIGURES, INTERPOLATION

The last digit of all figures printed in the tables is rounded. Thus a death rate of 19.218 . . . is printed as 19.22 and so on.

The tabulation of both the life tables and stable population characteristics is sufficiently "dense" so that as a rule simple linear interpolation can be applied to obtain values intermediate between tabulated figures.

DISCREPANCIES ARISING FROM THE USE OF AGE INTERVALS OF FINITE WIDTH

The use of age intervals (0–1, 1–4, 5–9, etc.) is for most practical purposes a fine enough division to support calculations as accurate as is warranted in the light of available data, but we wish nonetheless to call attention to an instance where grouped data introduce biases. Population projection by use of the factor ${}_5L_{x+5}/{}_5L_x$ is based on an assumption that the age distribution within each five-year interval of the stationary population and the population being projected is the same. If the population projected is growing rapidly, there is a detectable difference between the actual survival rate for children born during the five-year projection interval, and the factor ${}_5L_0/5l_0$. (The latter is in this instance too small.) The next survival ratio—${}_5L_5/{}_5L_0$—is slightly too large for a growing population. The use of the last survival factor—T_{80}/T_{75}—assumes that the population over 75 has the stationary distribution, and is too small for a growing population.

These effects imply, among other small-scale effects, that stable populations obtained by projection with five-year survival factors differ slightly from those printed here, which make allowance for subdivision within the first five years.

See next two pages for Table XV.

TABLE XV. Values of l_x by single years of age from 1 to 5 for regional model life tables (l_0 = 100,000) at mortality levels 1 to 24

MODEL WEST

LEVEL	Females l_1	Females l_2	Females l_3	Females l_4	Females l_5	Males l_1	Males l_2	Males l_3	Males l_4	Males l_5
1	63483	55000	51199	48742	46883	58093	50308	46898	44665	43005
2	66538	58557	54936	52595	50824	61657	54152	50865	48712	47112
3	69481	61829	58399	56183	54506	64868	57690	54546	52488	50957
4	72064	64856	61625	59538	57958	67785	60967	57980	56024	54571
5	74427	67671	64643	62686	61205	70454	64015	61195	59348	57976
6	76602	70300	67476	65651	64270	72911	66865	64217	62482	61194
7	78614	72765	70145	68451	67169	75183	69537	67064	65445	64242
8	80482	75084	72665	71101	69918	77294	72052	69756	68253	67135
9	82226	77271	75051	73616	72530	79263	74425	72307	70919	69888
10	83857	79340	77315	76007	75017	81105	76671	74728	73457	72511
11	85388	81300	79468	78285	77389	82835	78800	77032	75875	75015
12	86829	83163	81519	80457	79654	84463	80822	79228	78184	77408
13	88169	84939	83492	82556	81848	86058	82912	81534	80632	79961
14	89452	86720	85496	84705	84106	87547	84833	83644	82866	82287
15	90661	88364	87324	86646	86127	88864	86523	85498	84826	84327
16	91823	89936	89066	88490	88041	90143	88164	87292	86720	86293
17	92934	91419	90709	90232	89854	91379	89790	89056	88561	88184
18	93996	92820	92260	91878	91571	92570	91334	90736	90321	90001
19	95006	94143	93724	93436	93201	93713	92796	92332	92002	91744
20	95966	95392	95109	94912	94749	94807	94179	93847	93606	93415
21	96907	96559	96385	96263	96160	95909	95508	95285	95121	94989
22	97738	97530	97425	97350	97286	96925	96675	96531	96422	96334
23	98484	98377	98321	98282	98248	97856	97719	97636	97573	97521
24	99106	99061	99037	99020	99006	98668	98605	98566	98535	98510

MODEL NORTH

LEVEL	Females l_1	Females l_2	Females l_3	Females l_4	Females l_5	Males l_1	Males l_2	Males l_3	Males l_4	Males l_5
1	68027	59707	54585	50719	47783	62883	54784	49858	46197	43413
2	70798	62931	58089	54433	51658	66077	58341	53637	50141	47482
3	73285	65879	61320	57878	55265	68944	61599	57133	53813	51289
4	75539	68592	64316	61087	58637	71541	64603	60383	57247	54862
5	77595	71104	67108	64091	61801	73911	67387	63419	60470	58227
6	79483	73440	69720	66912	64780	76087	69979	66265	63504	61405
7	81226	75622	72173	69569	67592	78096	72404	68942	66369	64412
8	82842	77668	74483	72078	70253	79959	74678	71467	69080	67265
9	84347	79591	76664	74454	72777	81694	76819	73854	71652	69976
10	85753	81405	78729	76708	75175	83314	78839	76117	74095	72557
11	87070	83119	80687	78851	77457	84833	80749	78266	76421	75017
12	88305	84739	82544	80886	79628	86256	82556	80306	78633	77361
13	89451	86319	84391	82936	81831	87589	84328	82344	80870	79749
14	90498	87810	86140	84874	83906	88772	85920	84186	82897	81916
15	91512	89247	87813	86714	85864	89926	87456	85954	84838	83990
16	92492	90605	89388	88447	87710	91045	89005	87707	86725	85969
17	93435	91887	90871	90078	89450	92126	90473	89368	88518	87855
18	94337	93096	92267	91615	91093	93167	91855	90937	90218	89652
19	95199	94236	93583	93064	92645	94164	93156	92416	91829	91362
20	96018	95311	94822	94432	94114	95115	94378	93811	93355	92989
21	96795	96323	95993	95727	95508	96019	95524	95126	94801	94538
22	97535	97287	97111	96968	96850	96870	96595	96365	96175	96021
23	98177	98036	97935	97852	97783	97647	97484	97341	97222	97124
24	98782	98713	98662	98620	98585	98394	98310	98234	98170	98117

TABLE XV (*Continued*). Values of l_x by single years of age from 1 to 5 for regional model life tables (l_0 = 100,000) at mortality levels 1 to 24

MODEL EAST

LEVEL	Females l_1	Females l_2	Females l_3	Females l_4	Females l_5	Males l_1	Males l_2	Males l_3	Males l_4	Males l_5
1	57215	49832	46694	44635	43206	49494	42963	40248	38524	37264
2	60670	53531	50496	48504	47123	53551	47104	44424	42721	41478
3	63820	56971	54059	52149	50823	57249	50961	48346	46686	45473
4	66712	60186	57412	55592	54329	60644	54570	52044	50441	49269
5	69382	63204	60578	58854	57659	63778	57960	55541	54005	52883
6	71859	66045	63574	61953	60828	66686	61156	58857	57397	56330
7	74167	68730	66418	64902	63849	69396	64178	62008	60630	59624
8	76326	71272	69124	67714	66736	71931	67042	65009	63718	62775
9	78352	73685	71702	70400	69497	74309	69762	67872	66671	65794
10	80258	75981	74163	72970	72142	76547	72352	70608	69500	68691
11	82040	78213	76586	75518	74777	78641	74870	73302	72307	71579
12	83700	80317	78879	77935	77280	80562	77196	75797	74909	74259
13	85297	82332	81071	80244	79670	82416	79439	78201	77415	76841
14	86831	84259	83166	82449	81952	84204	81599	80516	79828	79326
15	88302	86103	85168	84555	84129	85923	83676	82742	82148	81715
16	89710	87865	87080	86565	86208	87574	85671	84879	84377	84009
17	91058	89569	88926	88498	88194	89158	87586	86933	86518	86214
18	92343	91191	90684	90341	90092	90672	89442	88918	88580	88329
19	93567	92730	92354	92096	91905	92117	91227	90827	90559	90357
20	94732	94189	93942	93769	93638	93494	92913	92637	92447	92300
21	95907	95550	95384	95266	95176	94857	94466	94271	94132	94023
22	96937	96716	96611	96536	96478	96108	95865	95738	95645	95571
23	97856	97734	97675	97633	97599	97240	97104	97029	96973	96928
24	98640	98582	98554	98534	98517	98218	98153	98115	98087	98064

MODEL SOUTH

LEVEL	Females l_1	Females l_2	Females l_3	Females l_4	Females l_5	Males l_1	Males l_2	Males l_3	Males l_4	Males l_5
1	69300	55531	49192	45845	43943	66445	53955	48194	45107	43401
2	71551	58633	52685	49544	47760	68878	57082	51641	48725	47115
3	73585	61504	55941	53005	51336	71075	59976	54857	52113	50598
4	75437	64176	58992	56254	54699	73076	62670	57869	55296	53876
5	77135	66676	61860	59317	57872	74911	65187	60702	58298	56971
6	78702	69022	64565	62212	60875	76604	67551	63375	61137	59901
7	80153	71231	67124	64955	63722	78172	69776	65903	63828	62681
8	81504	73318	69550	67560	66429	79632	71878	68301	66384	65326
9	82766	75295	71855	70039	69007	80996	73868	70580	68818	67845
10	83935	77177	74065	72422	71489	82248	75682	72654	71030	70134
11	84956	78923	76145	74679	73845	83358	77468	74751	73294	72490
12	85953	80603	78140	76839	76100	84441	79188	76765	75466	74749
13	86925	82219	80052	78909	78259	85498	80845	78700	77549	76914
14	87870	83774	81888	80892	80326	86525	82442	80558	79548	78991
15	88786	85268	83649	82794	82308	87522	83978	82343	81467	80983
16	89673	86705	85339	84617	84207	88487	85455	84056	83307	82893
17	90529	88101	86975	86375	86029	89419	86875	85701	85073	84725
18	91361	89454	88559	88077	87791	90324	88267	87307	86788	86498
19	92285	90801	90094	89708	89474	91382	89888	89165	88762	88528
20	93199	92070	91525	91224	91036	92390	91265	90700	90377	90183
21	94110	93283	92878	92651	92507	93395	92580	92155	91906	91752
22	95013	94436	94151	93989	93883	94395	93833	93528	93345	93230
23	95904	95528	95339	95230	95158	95383	95020	94816	94690	94609
24	96774	96550	96436	96369	96324	96352	96137	96012	95933	95882